rbinvariantslib:

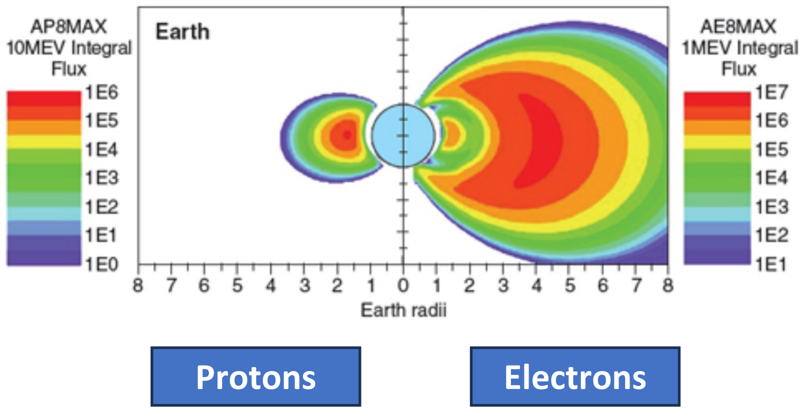
Radiation Belt Invariants Library

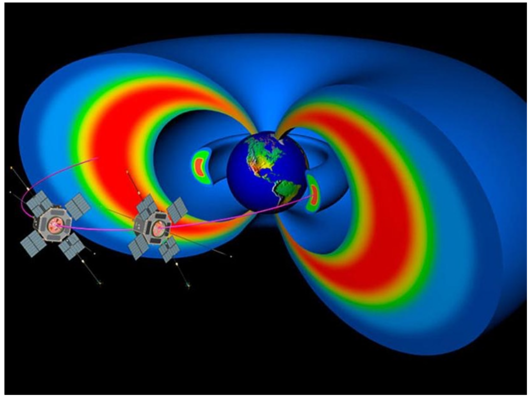
Daniel da Silva, Scot Elkington

GEM Workshop 2025

Introduction and Overview

- Provides tools for radiation belt research to calculate the adiabatic invariants K and L* from gridded models of Earth's magnetic field
  - If you don't know what this means, I will tell you
- Provides supported for empirical and MHD models of Earth's magnetic field (TS05, T96, SWMF, LFM, Arbitrary Structured Grids)
- Method peer reviewed and published in JGR: Space Physics





Adiabatic Invariants, What are they?

Three periodic motions of trapped particles in the radiation belts

- Gyromotion (shortest time scale)
- Bounce Motion
- Drift motion (longest time scale)

Gyromotion

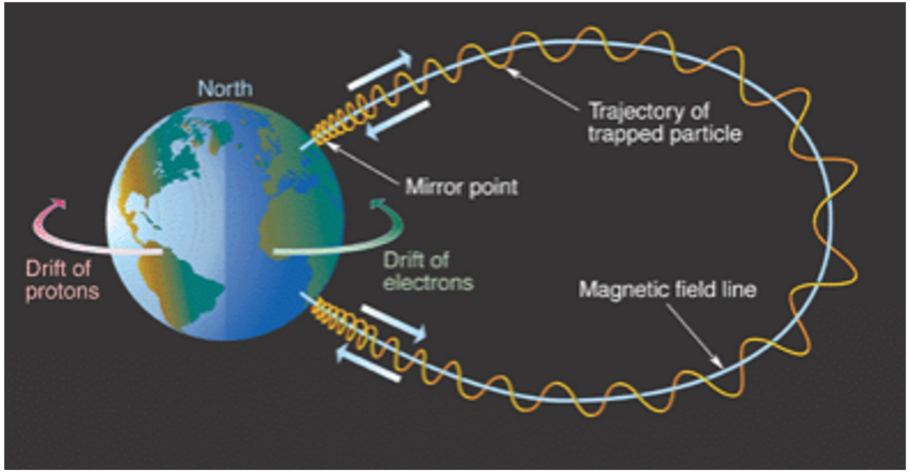
- Happens in any magnetic field

Bounce motion

- Caused by magnetic mirroring
- Mirroring occurs both above and below the magnetic equator

Drift motion

- Particles drift azimuthally around Earth
- Electrons and protons drift in different directions (generates the ring current)



Gyro, Bounce, and Drift Motion
(Source: Hudson 2008)

Adiabatic Invariants: What are they?

- Constants of motion, conserved as magnetic field of earth changes in response to solar wind

- There is one adiabatic invariant for each periodic motion,
  - ***M*: Gyration**

$$M = \frac{p_\perp^2}{2m_0\bar{B}} = \frac{p^2\sin^2(\alpha)}{2m_0\bar{B}}$$

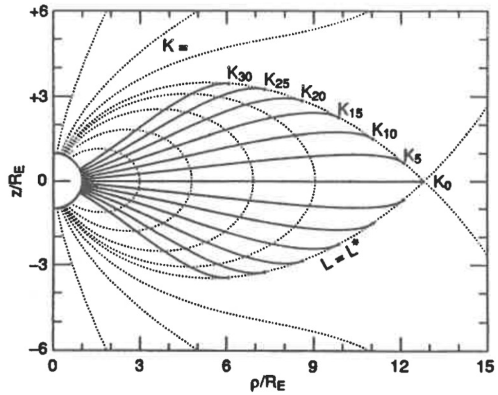
  - ***K*: Bounce Motion**

$$K = \int_{s_1}^{s_2} \sqrt{B_m - B(s)}\,ds$$

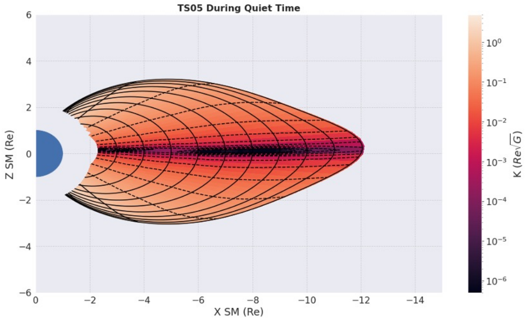
  - ***L**: Azimuthal Drift**

$$L^* = \frac{2\pi B_E R_E^2}{\Phi}$$

$$\Phi = \int_{\substack{drift\\shell}} \boldsymbol{B}\cdot d\boldsymbol{S} \approx B_E R_E^2 \int_0^{2\pi} \sin^2(\theta(\phi))\,d\phi$$



Contours of K for Varying Bounce Lengths (Schulz 1996)



Contours of K for Varying Bounce Lengths (D. da Silva)

How the Code Works

```python
from rbinvariantslib import models, invariants

model = models.get_model(
    "SWMF_CDF",
    "3d__var_1_e20151221-001700-014.out.cdf"
)

# Calculate L*. This takes 20-25 sec.
result = invariants.calculate_LStar(
    model,
    starting_point=(-6.6, 0, 0),
    starting_pitch_angle=60
)

print(f"L* = {result.LStar}")
```

Invariant Calculation Algorithms

Gyration Invariant M

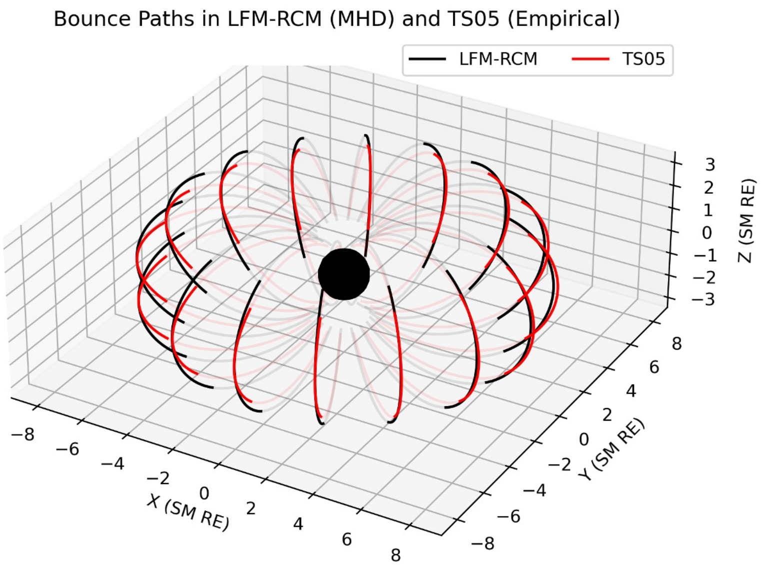
- Easy, use direct analytical equation

Bounce Invariant K

- Trace bounce path
- Numerical integration

Drift Invariant L*

- Use the classic “Roederer Method” (Roederer 1970)
- Optimization problem to determine particle’s drift shell
- Leverages fact that drift shell spans one degree of freedom at each local time (due to K being conserved)
- Avoids doing an expensive particle trace



Magnetic Field Model Support

Empirical Models

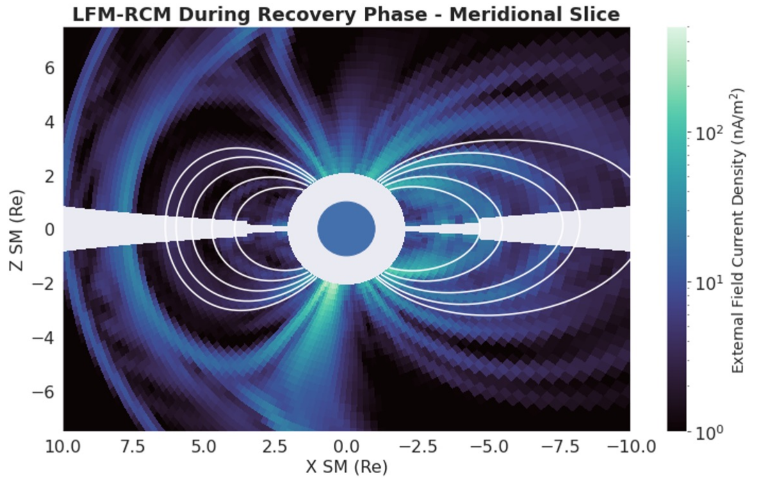
- TS05 (Tsyganenko/Sitnov 2005)
- T96 (Tsyganenko 1996)

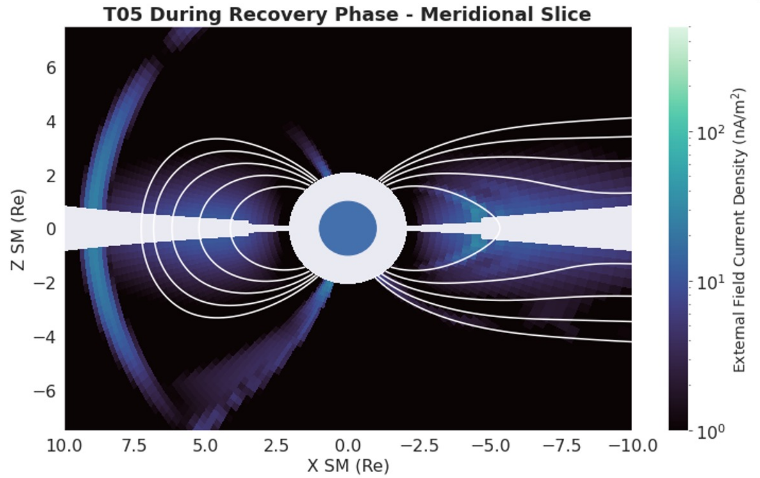
(Code automatically pulls OMNI data from CDAWeb for Empirical Models)

MHD Models

- SWMF (CCMC CDF Output)
- LFM (CCMC HDF4 Output)
- Arbitrary structured grids

(Code automatically regrids SWMF output to Rectilinear grid)





Conclusion

- Provides tools in Python for calculating the adiabatic invariants from empirical and MHD models of Earth’s magnetic field
- Open-source with API documentation online
- Next steps:
    - Adding tools for organizing phase space density measurements (e.g., RBSP) as a function of invariants
- Pull requests and support inquiries welcome

Website:
https://rbinvariantslib.readthedocs.io

JGR Space Physics

Method | Open Access

Numerical Calculations of Adiabatic Invariants From MHD-Driven Magnetic Fields

D. E. da Silva, S. R. Elkington, X. Li, J. Murphy, M. K. Hudson, M. J. Wiltberger, A. A. Chan

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