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Space Environment and Satellite Systems

Improving Radar Observation of Meteors using Compressed Sensing

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Compressed Sensing

- **Theory to guarantee solution of under-determined set of equations**
- **Knowing that the solution is sparse or compressible provides uniqueness**
- **Requirements**
  - Solution known to be sparse
  - Measurements are "incoherent" (global)
  - $m \geq O(s \log n)$

Number of measurements (m)

Sparsity = number of non-zero variables in solution (s)

Total number of variables (n)

For more information: Candès and Wakin, "An Introduction to Compressive Sampling", IEEE Signal Processing Magazine, March 2008.

Compressed Sensing Solution

- **y = Ax** ← Unknown signal vector
  - y: Measurement vector
  - A: Linear system describing measurements
- **Want to find sparsest solution that agrees with measurements**
- **Solve convex optimization problem:**

$$x^* = \arg\min_{\hat{x}} \|x\|_1 \quad s.t. \quad \|y - A\hat{x}\|_2 \leq \sigma$$

- **We use TFOCS software package for MATLAB**

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Radar Model

$$y_q = \sum_{k=1}^{rm} s_{rq-k+1} \left(\sum_{p=0}^{n-1} e^{\frac{2\pi i p q}{n}} h_{p,k} \right)$$

- y_q for q = 1,...,m is the sampled received signal (measurements)
- m is the number of samples
- s_k for k = 1,...,b is a discrete phase shift modulation
- b is the number of bauds in the modulation
- r is the ratio of baud length to sampling period
- n is the number of frequencies
- $h_{p,k}$ are the reflectivity coefficients

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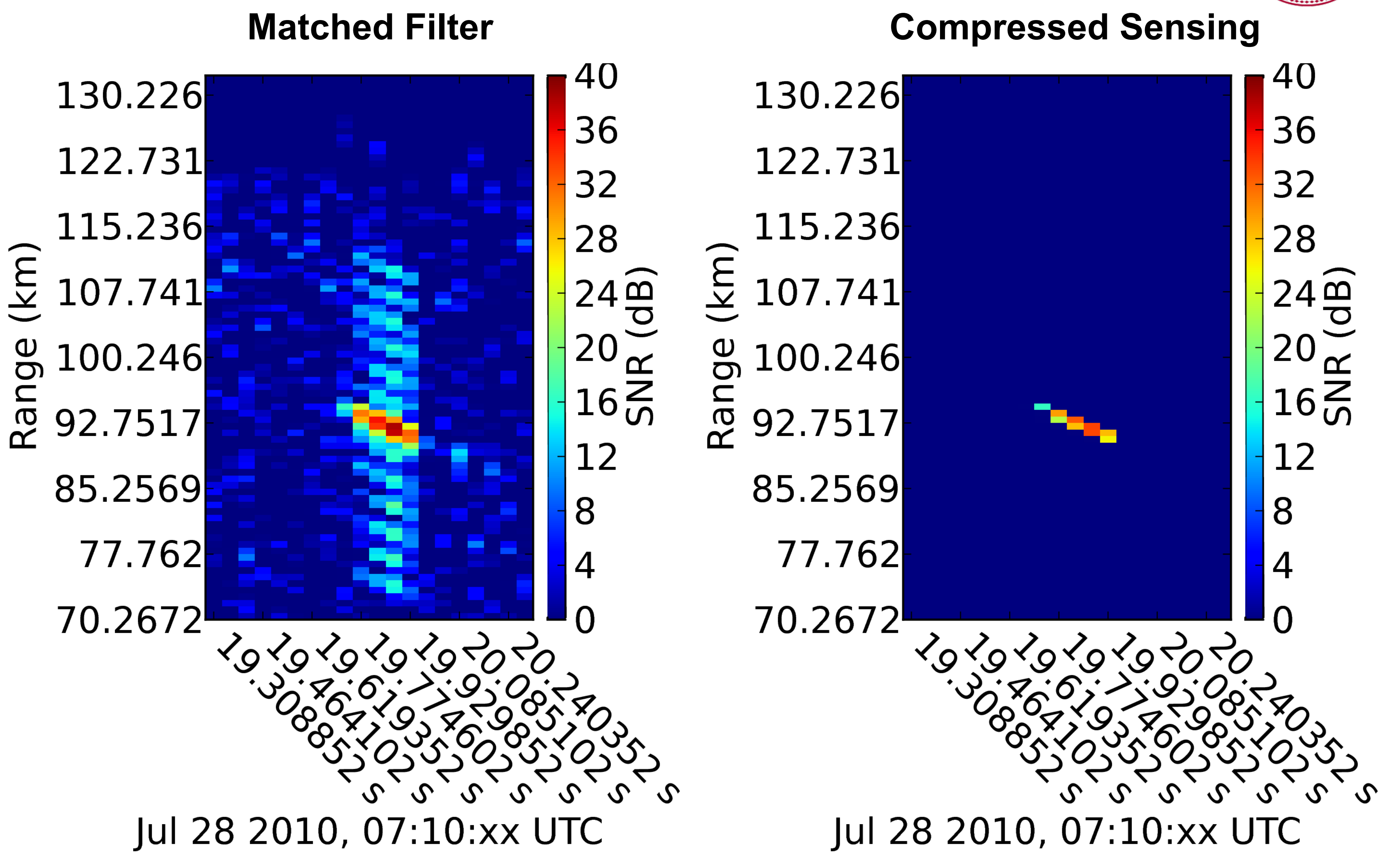
Results



Poker Flat, Barker-13 code, 10 us bauds, 5 us sampling period

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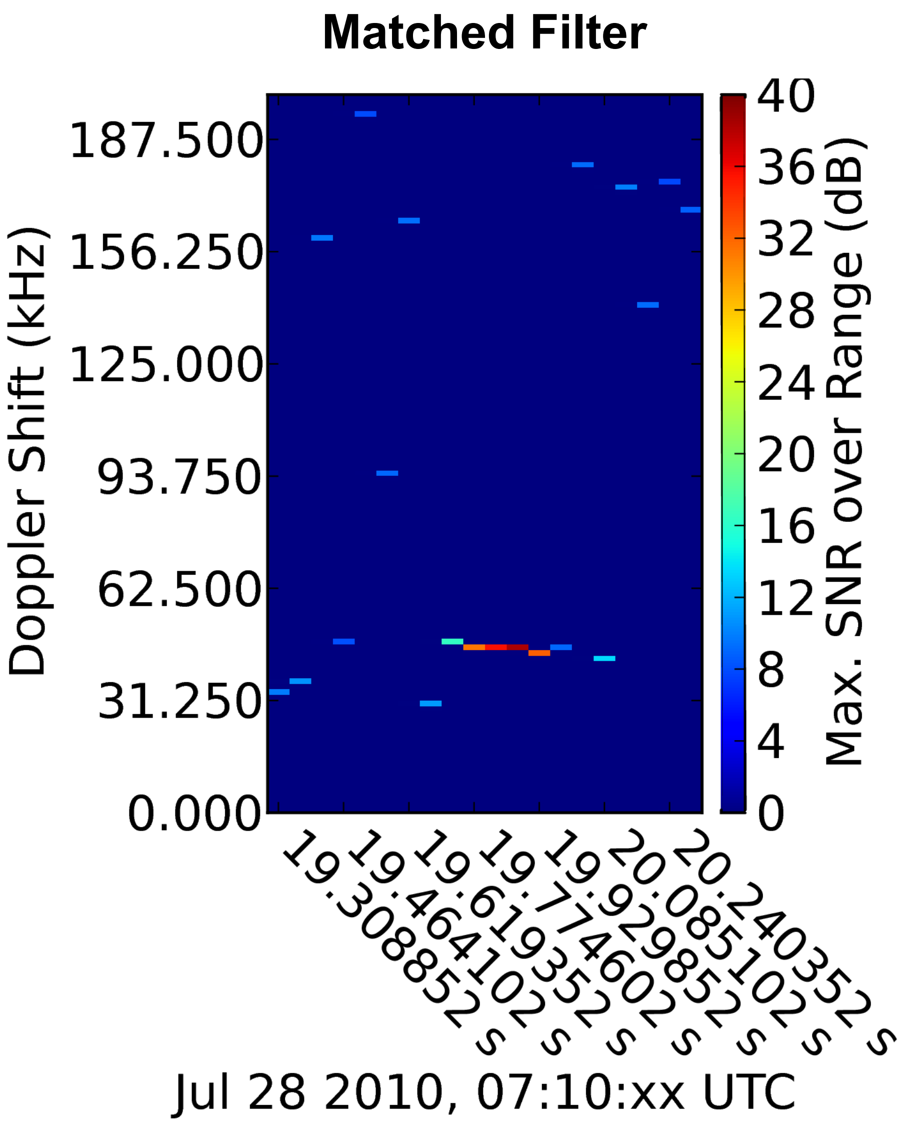
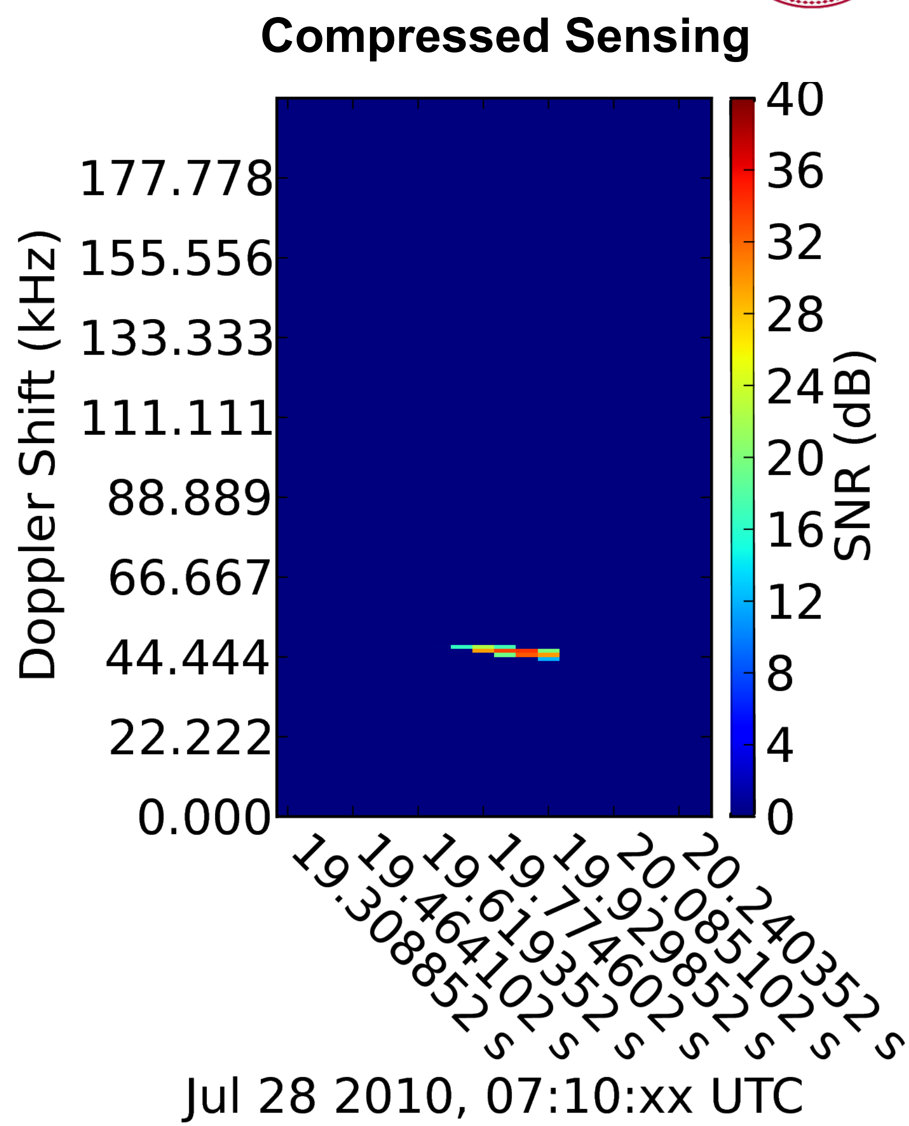
Results





Poker Flat, Barker-13 code, 10 us bauds, 5 us sampling period

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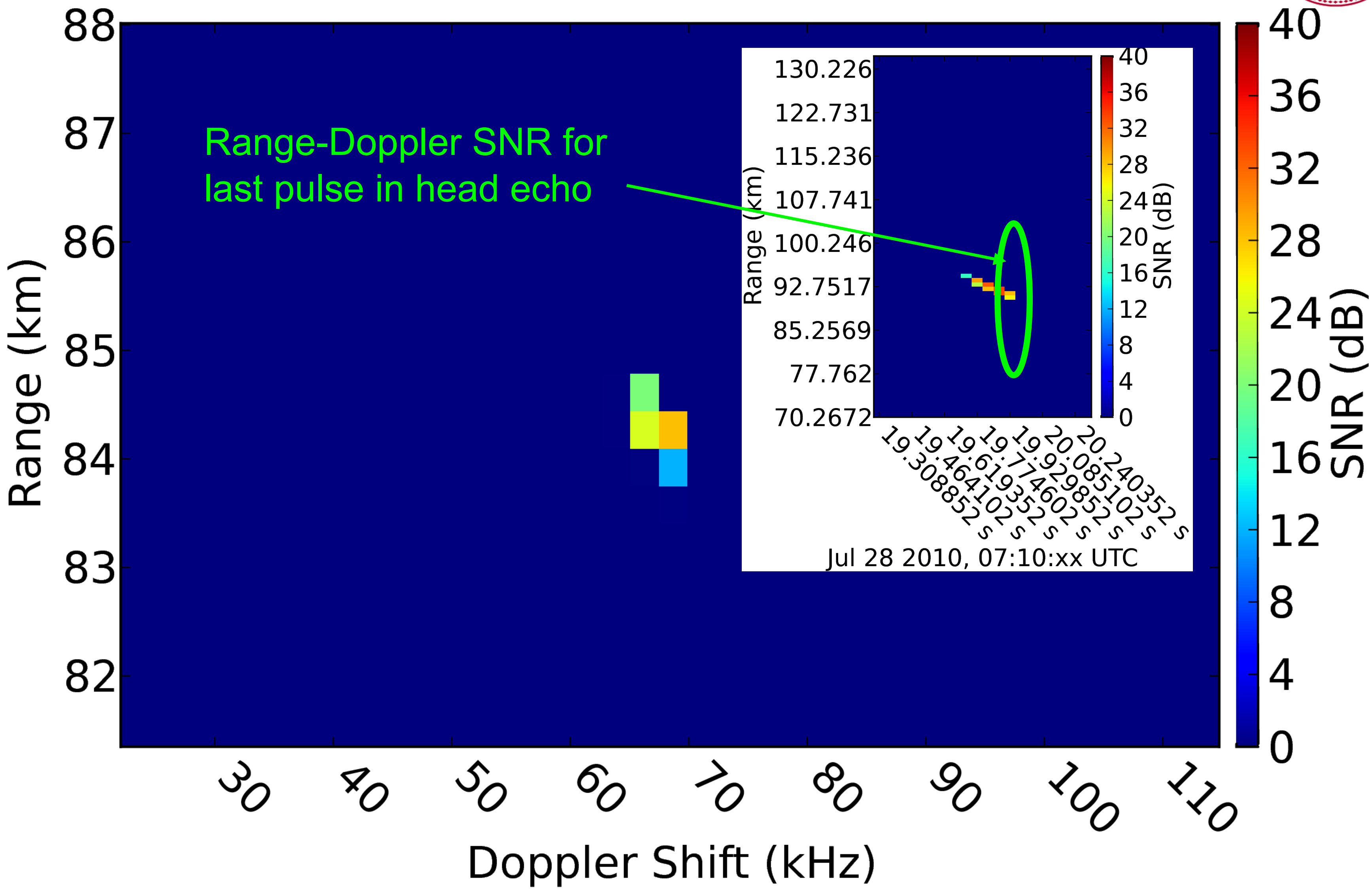
Results



Poker Flat, Barker-13 code, 10 us bauds, 5 us sampling period

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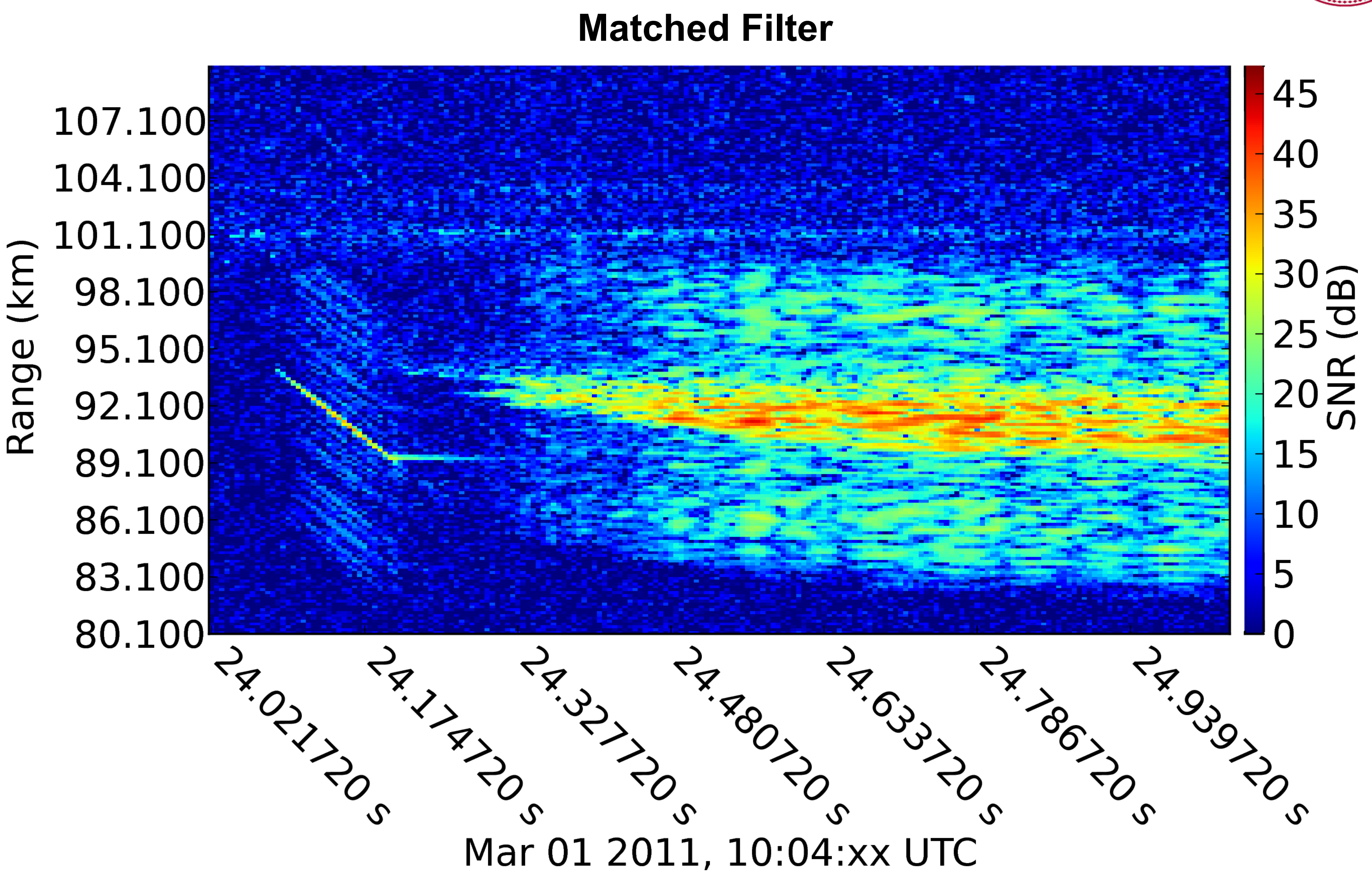
Results



Jicamarca, 51 baud minimum peak sidelobe code, 1 us bauds, 1 us sampling period

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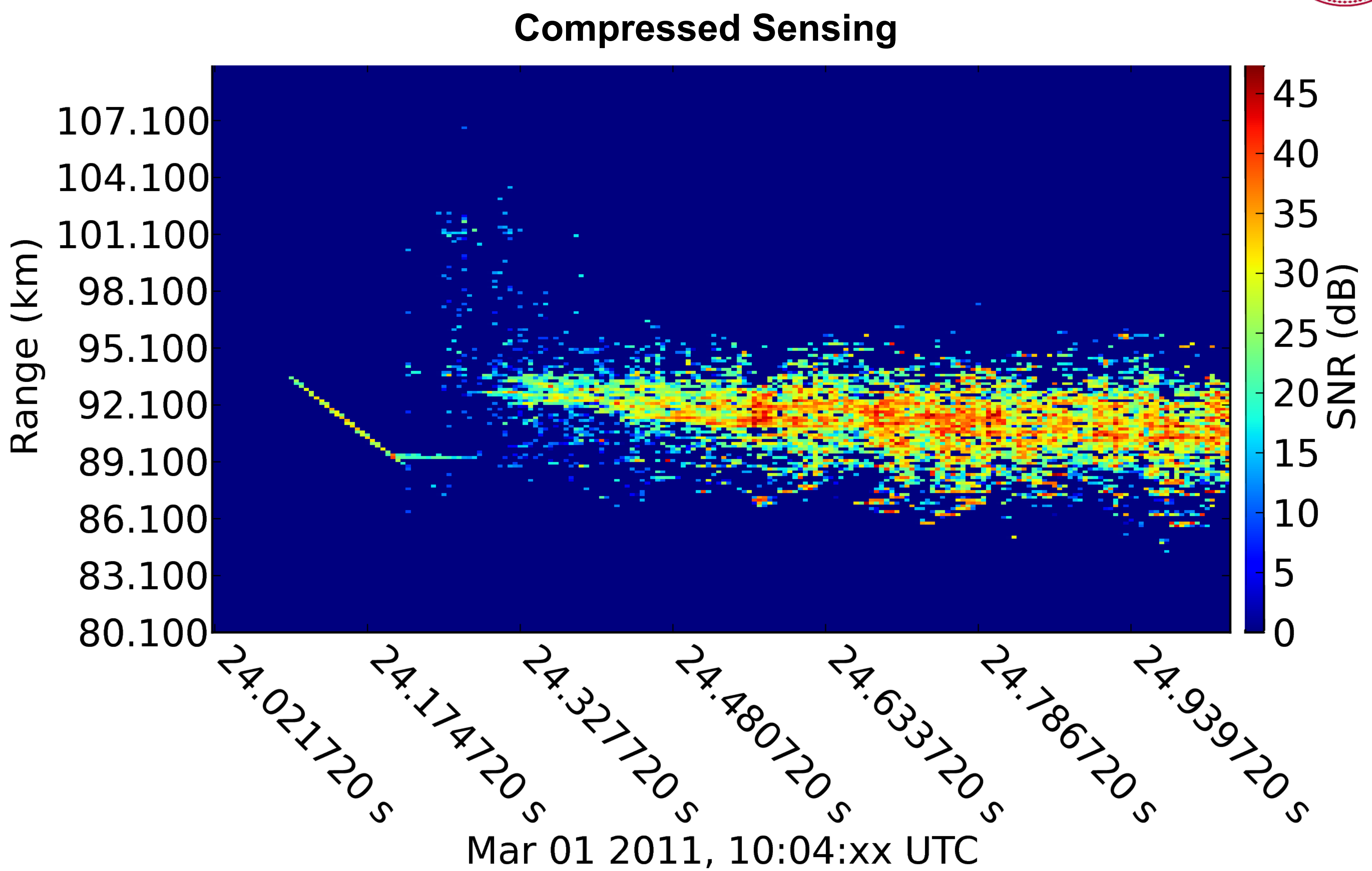
Results



Jicamarca, 51 baud minimum peak sidelobe code, 1 us bauds, 1 us sampling period

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Conclusion

- **Advantages**
  - Complete noise filtering and easy signal identification
  - Imaging with respect to Doppler frequency
  - Easier to increase Doppler and range accuracy
- **Disadvantages**
  - Processing time
  - Needs more development
- **Future Work**
  - Improve radar model
    - Different basis functions (wavelets?)
    - Incorporate simple physics
  - Incorporate correlation between pulses
  - Experiment with new waveforms
  - Achieve the promises of increased accuracy

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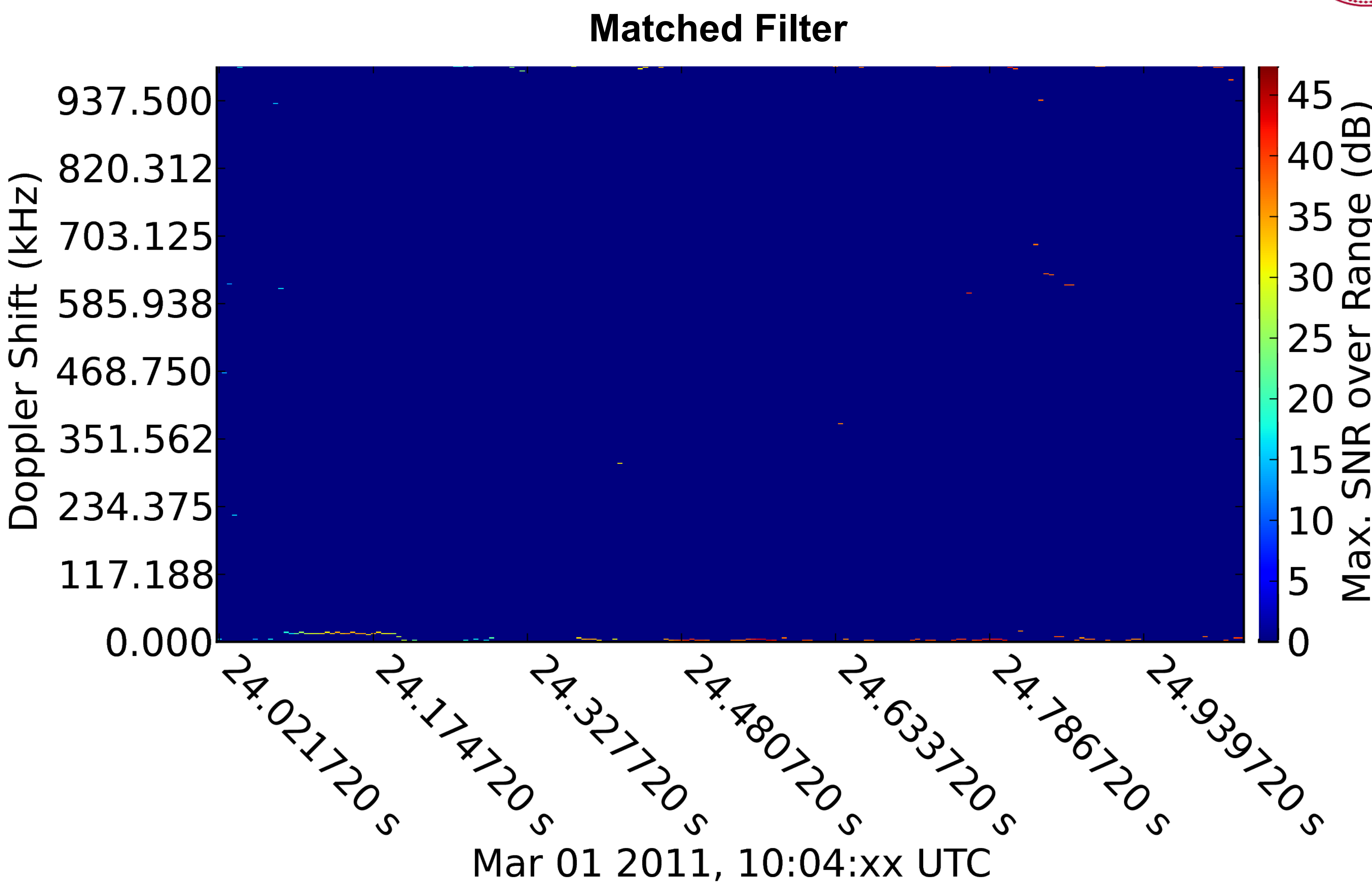
Results



Jicamarca, 51 baud minimum peak sidelobe code, 1 us bauds, 1 us sampling period

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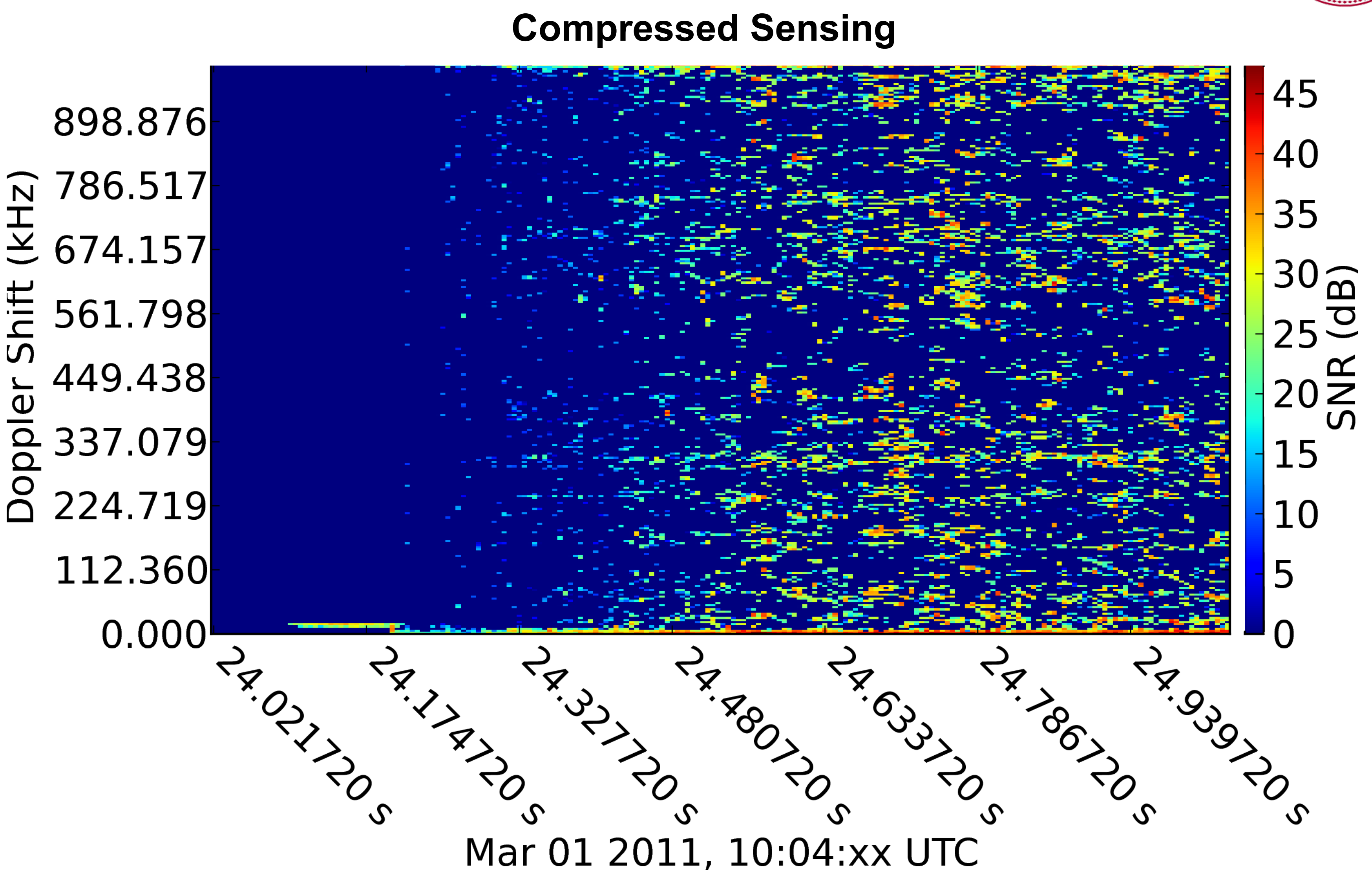
Results



Jicamarca, 51 baud minimum peak sidelobe code, 1 us bauds, 1 us sampling period

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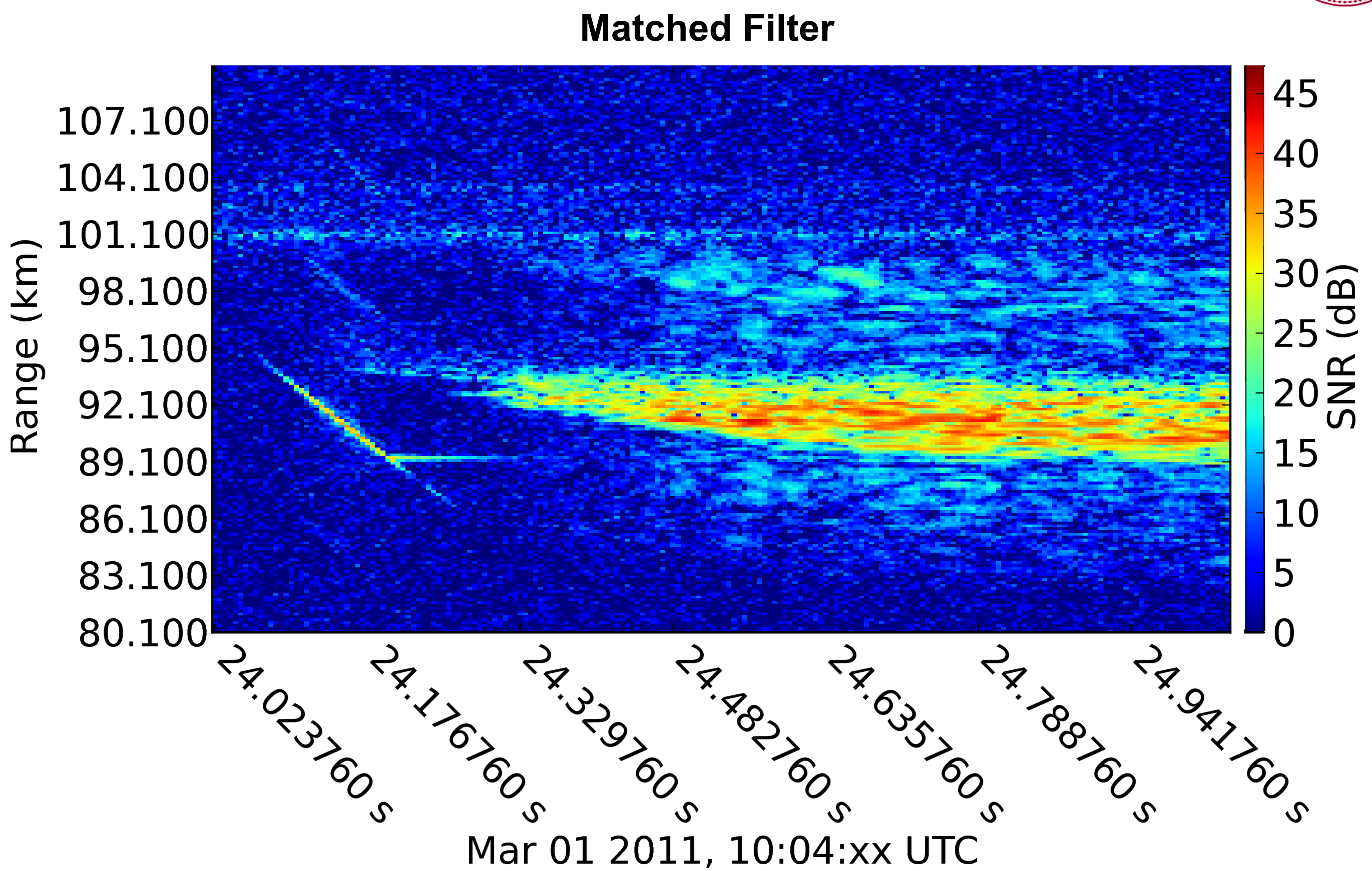
Results



Jicamarca, 51 us LFM chirp, 1 MHz bandwidth, 1 us sampling period

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Radar Model Derivation

$$y(t) = \int_0^T \int_{-B/2}^{B/2} s(t - t_d) e^{2\pi i f_d (t - t_d)} e^{-2\pi i f_0 t_d} h(t_d, f_d) dt_d df_d$$

$$y(q\tau_s) = \sum_{k=1}^{rm} \int_{(k-1)\tau_b}^{k\tau_b} \sum_{p=-\frac{n}{2}+1}^{\frac{n}{2}} \int_{(p-1)\Delta f}^{p\Delta f} s(q\tau_s - t_d) e^{2\pi i f_d (q\tau_s - t_d)} e^{-2\pi i f_0 t_d} h(t_d, f_d) df_d dt_d$$

$$= \sum_{k,p} s_{rq-k+1} \int_{(k-1)\tau_b}^{k\tau_b} \int_{(p-1)\Delta f}^{p\Delta f} e^{2\pi i f_d q\tau_s} e^{-2\pi i (f_0 + f_d) t_d} h(t_d, f_d) df_d dt_d$$

$$= \sum_{k,p} s_{rq-k+1} e^{2\pi i p q \Delta f \tau_s} \int_{(k-1)\tau_b}^{k\tau_b} \int_{(p-1)\Delta f}^{p\Delta f} e^{2\pi i q\tau_s (f_d - p\Delta f)} e^{-2\pi i (f_0 + f_d) t_d} h(t_d, f_d) df_d dt_d$$

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Radar Model Derivation

- **Want to write**

$$y_q = \sum_{k,p} s_{rq-k+1} e^{2\pi i pq\Delta f \tau_s} \int_{(k-1)\tau_b}^{k\tau_b} \int_{(p-1)\Delta f}^{p\Delta f} e^{2\pi i q\tau_s (f_d - p\Delta f)} e^{-2\pi i (f_0+f_d)t_d} h(t_d, f_d) df_d dt_d$$

as

$$y_q = \sum_{k=1}^{rm} s_{rq-k+1} \left(\sum_{p=0}^{n-1} e^{\frac{2\pi i pq}{n}} h_{p,k} \right)$$

- **Approximate reflectivity**

$$h_{p,k} \approx \int_{(k-1)\tau_b}^{k\tau_b} \int_{(p-1)\Delta f}^{p\Delta f} e^{2\pi i q\tau_s (f_d - p\Delta f)} e^{-2\pi i (f_0+f_d)t_d} h(t_d, f_d) df_d dt_d$$

  - Good for very small Δf or for point targets