Polarization Relations of Tidal Waves

Chiao-Yao (Joe) She
Colorado State University
Collaborators: Yuan, Oberheide, Krueger
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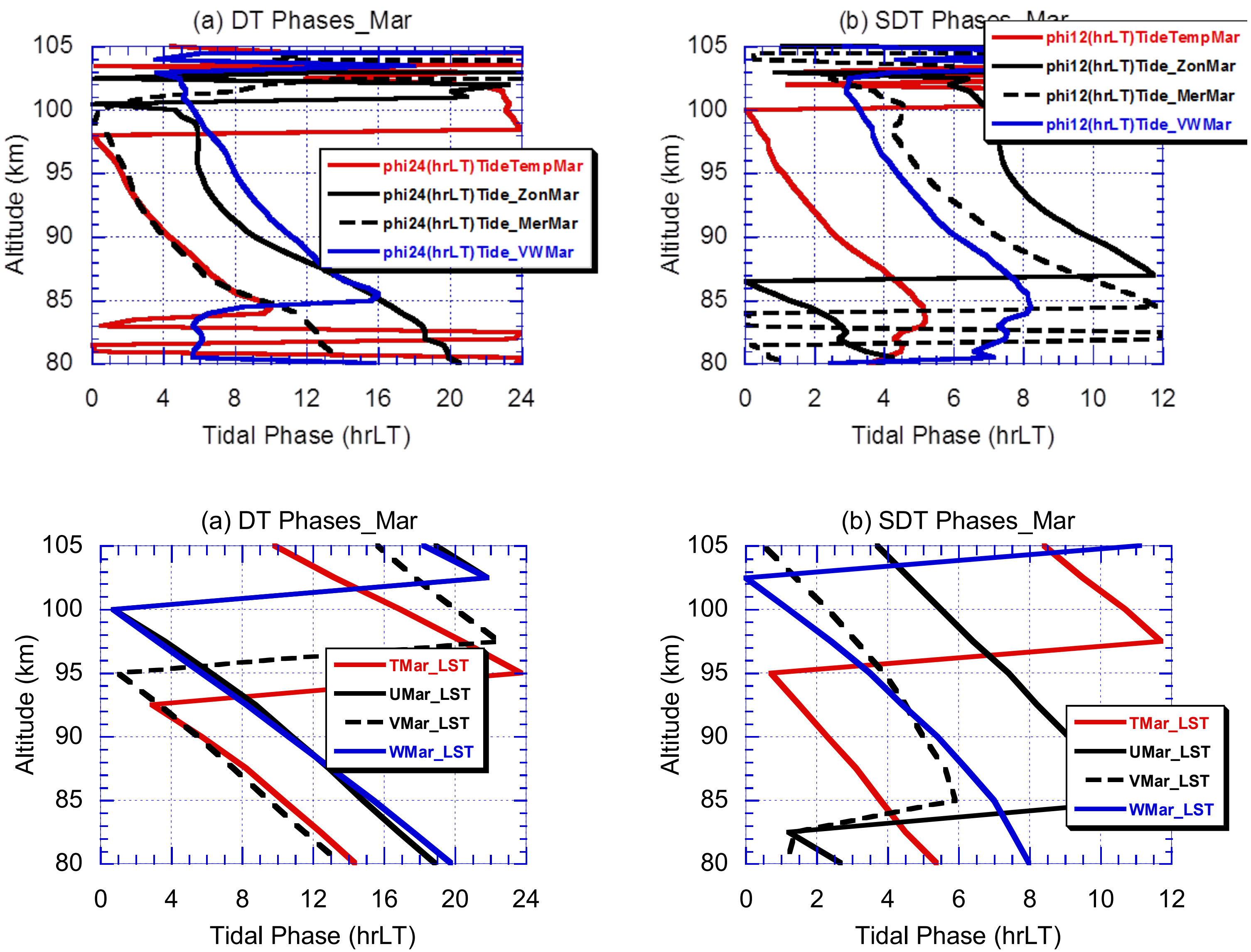
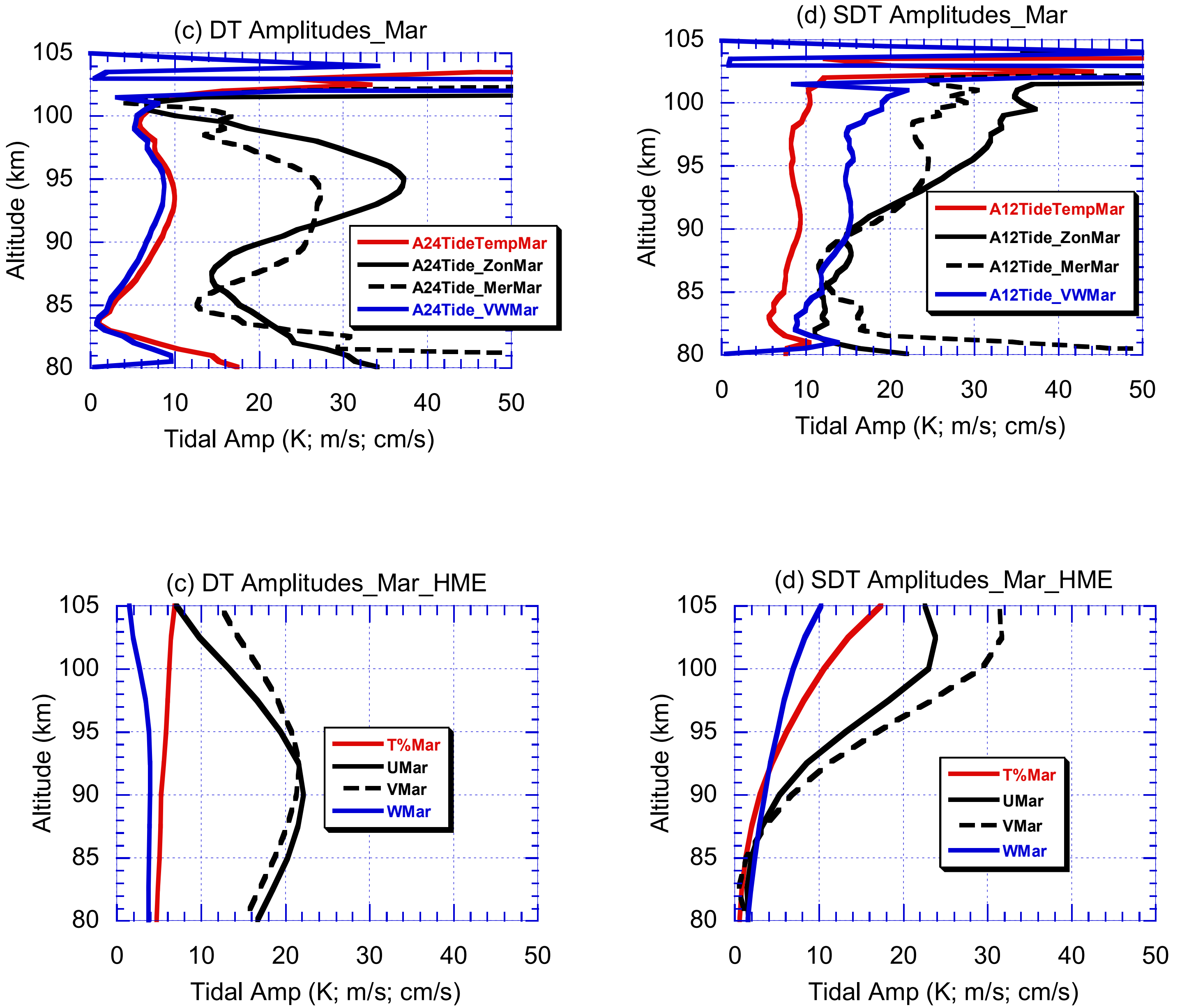
Motivations

- Polarization relations for GWs are well known and frequently utilized to benefit science investigation. The corresponding relations for tidal waves must exist, but are rarely mentioned in the literature.
- Objective for this study: To derive the polarization relations of the global tidal waves (s is an eigen-index) and of the locally observed tidal-period oscillations (only σ can be specified), to apply them to lidar observed tidal-period oscillations based on a long dataset, and to investigate its implications.
- Personal objective: This study provides a chance to understand the difference between classical and modern theory, such as GSWM and HME, and to gain confidence when comparing them to CSU Na lidar observations.

CSU Lidar Data, Full-Diurnal-Cycle Observations (2002 – 2008) determined T, U, V Tidal-Period Perturbations




(c) DT Amplitudes_Mar
A24TideTempMar
A24Tide_ZonMar
A24Tide_MerMar
A24Tide_VWMar
Altitude (km)
Tidal Amp (K; m/s; cm/s)
(d) SDT Amplitudes_Mar
A12TideTempMar
A12Tide_ZonMar
A12Tide_MerMar
A12Tide_VWMar
Altitude (km)
Tidal Amp (K; m/s; cm/s)
(c) DT Amplitudes_Mar_HME
T%Mar
UMar
VMar
WMar
Altitude (km)
Tidal Amp (K; m/s; cm/s)
(d) SDT Amplitudes_Mar_HME
T%Mar
UMar
VMar
WMar
Altitude (km)
Tidal Amp (K; m/s; cm/s)

Starting from the primitive equations (hydrostatic equilibrium) in spherical, log-pressure coordinate.

Wave perturbations to a resting Spherical Atmosphere in log-pressure coordinate are governed by [AHL, p. 115]:

$$u_t' - fv' + (a\cos\phi)^{-1}\Phi_\lambda' = X' \qquad (1a)$$

$$\mathrm{v}_t' + fu' + a\Phi_\phi' = Y' \qquad (1b)$$

$$(\mathrm{a}\cos\phi)^{-1}[u_\lambda' + (v'\cos\phi)_\phi] + \rho_0^{-1}(\rho_0 w')_z = 0 \qquad (1c)$$

$$\Phi'_{zt} + N^2 w' = Q' = \frac{\kappa J'}{H_r}; \quad \mathrm{N}^2 = \frac{R}{H_r}\left[\frac{\partial T}{\partial z} + \frac{\kappa T}{H_r}\right] \qquad (1d)$$

$$\Phi'_z = \frac{\partial \Phi'}{\partial z} = \frac{RT'}{H_r} \qquad (1e)$$

Solution to Eqs. (1a)-(1b) and (1c)-(1e), without excitation sources (homogeneous equations) yield polarization relations (for tides, etc.)

For global waves, the solution may be casted in the following form with perturbation frequency, σ, and zonal wave number, s, specified as eigen-indices.

$$(u',v',w',\Phi',T') = \mathrm{Re}\{[\hat{u}^{s}(\phi,z),\hat{v}^{s}(\phi,z),\hat{w}^{s}(\phi,z),\hat{\Phi}^{s}(\phi,z),\hat{T}^{s}(\phi,z)]e^{i(s\lambda-\sigma t)}\} \qquad (2a)$$

These height-latitude functions may be determined by models (GSWM or HME) or satellite observations (SABER, TIDI).

For situations, in which contributions from waves with more than one zonal wave numbers are important, or contributions from different global modes are not separable, such as tidal-period oscillations at a local site, the solution (or eigen-function) should be casted in the following form; only σ can be specified.

$$(u',v',w',\Phi',T') = \mathrm{Re}\{[\tilde{u}(\phi,\lambda,z),\tilde{v}(\phi,\lambda,z),\tilde{w}(\phi,\lambda,z),\tilde{\Phi}(\phi,\lambda,z),\tilde{T}(\phi,\lambda,z)]e^{-i\sigma t}\} \qquad (2b)$$

Substituting (2b) into (1a) and (1b), and into (1d) and (1e), respectively, we obtain polarization relations (3a) and (3b):

$$\tilde{v} = -i\frac{f}{\sigma}\frac{\alpha e^{i\varphi_\alpha} + \frac{i\sigma}{f}\cos\phi}{\alpha e^{i\varphi_\alpha} + \frac{if}{\sigma}\cos\phi}\tilde{u}, \text{ with } \alpha e^{i\varphi_\alpha} = \frac{\tilde{\Phi}_\lambda}{\tilde{\Phi}_\phi} \quad (3a)$$

Depending on (ϕ, λ)

$$\tilde{T} = -i\frac{H_r N^2}{R\sigma}\tilde{w}, \quad \text{with } N^2 = \frac{R}{H_r}\left[\frac{\partial T_0}{\partial z} + \frac{\kappa T_0}{H_r}\right] \quad (3b)$$

True for all situations

From (1a) and (1b), $\tilde{u}$ may be expressed in terms of $\tilde{\Phi}_\lambda$ and $\tilde{\Phi}_\phi$.

Then, we can now use (1e) to relate $\tilde{T}$ to $\tilde{u}$ approximately as:

$$\tilde{T} \approx \frac{H_s}{R}\left[i\tilde{m}(z) + \frac{1}{2H}\right]\frac{\left(\sigma^2 - f^2\right)\beta e^{i\varphi_\beta}}{\frac{f}{a} - \frac{i\sigma}{a\cos\phi}\alpha e^{i\varphi_\alpha}}\tilde{u}, \text{ where } \beta e^{i\varphi_\beta} = \frac{\tilde{\Phi}}{\tilde{\Phi}_\phi} \quad (4)$$

Using CSU full diurnal observations (from 2002 to 2008), we have determined $\tilde{u}$, $\tilde{v}$, $\tilde{T}$. With (3b), we determine $\tilde{w}$. These were plotted for Mar, as shown previously. With (3a) and (4), one objective is to determine $\alpha e^{i\varphi_\alpha}$ and $\beta e^{i\varphi_\beta}$, and investigate their implications.

Consider 3 special cases:

(a) a tidal component with s, $\tilde{\Phi}_\lambda = is\tilde{\Phi}$
(b) a migrating tide, $s_0 + \frac{\sigma}{\Omega} = 0 \rightarrow \tilde{\Phi}_\lambda = 0, i.e., \tilde{\alpha} \rightarrow 0$, and
(c) a low-frequency GW $\alpha e^{i\varphi_\alpha} = \frac{\tilde{\Phi}_\lambda}{\tilde{\Phi}_\phi} = \frac{is\tilde{\Phi}}{\tilde{\Phi}_\phi} is\beta e^{i\varphi_\beta}$

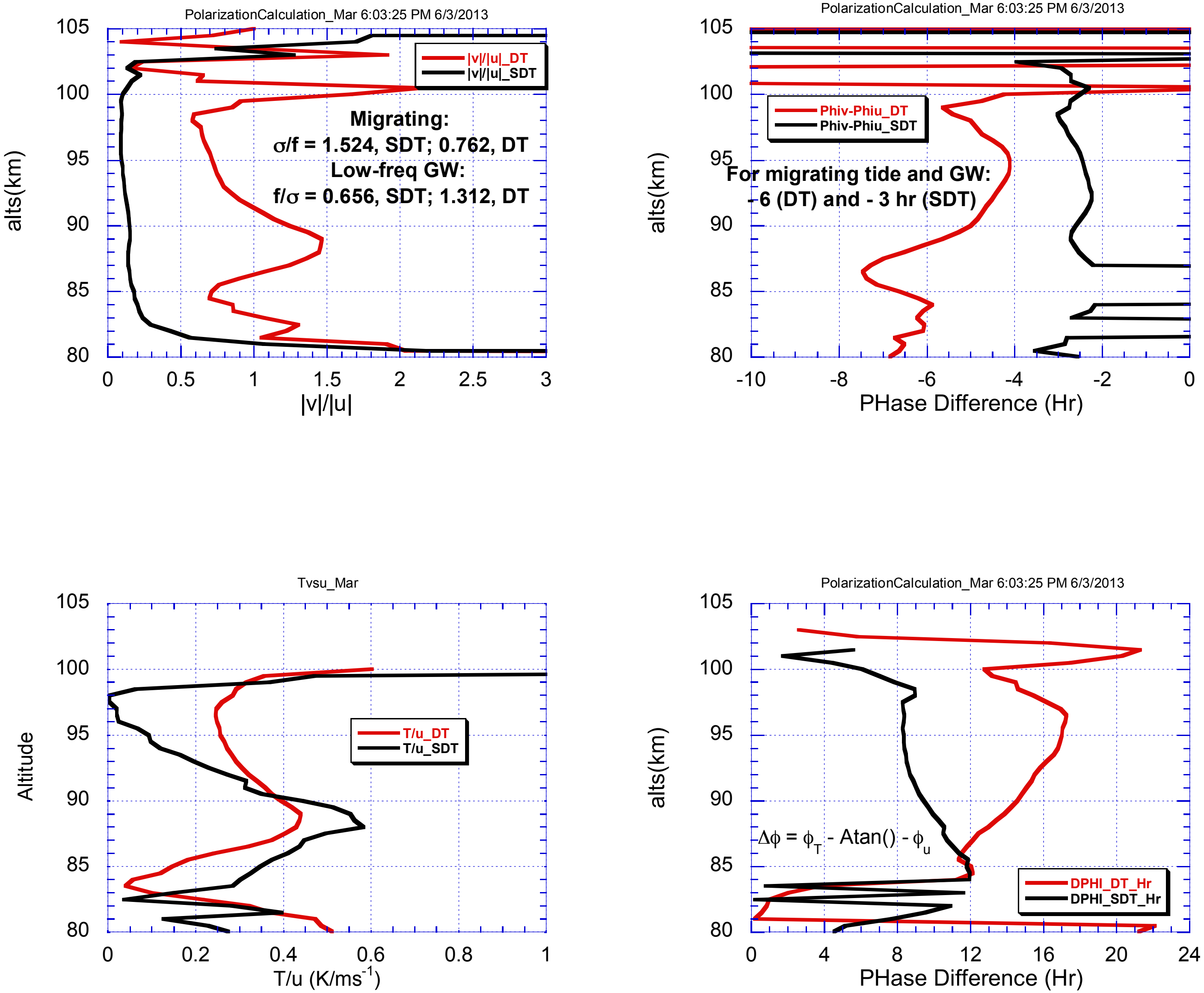
$$\tilde{v} = -i\frac{f}{\sigma}\frac{\alpha e^{i\varphi_\alpha} + \frac{i\sigma}{f}\cos\phi}{\alpha e^{i\varphi_\alpha} + \frac{if}{\sigma}\cos\phi}\tilde{u};\ \tilde{T} \approx \frac{H_s}{sR}\left[i\hat{m}(z) + \frac{1}{2H}\right]\frac{\left(\sigma^2 - f^2\right)\alpha e^{i(\varphi_\alpha - \frac{\pi}{2})}}{\frac{f}{a} - \frac{i\sigma}{a\cos\phi}\alpha e^{i\varphi}}\tilde{u} \quad \text{(8a)}$$

$$\tilde{v} = -i\frac{\sigma}{f}\tilde{u};\ \tilde{T} \approx \frac{H_s}{R}\sqrt{\hat{m}^2(z) + (1/H)^2}\, e^{iA\tan(2\hat{m}H)}\frac{a(\sigma^2 - f^2)\beta e^{i\varphi_\beta}}{f}\tilde{u} \quad \text{(8b)}$$

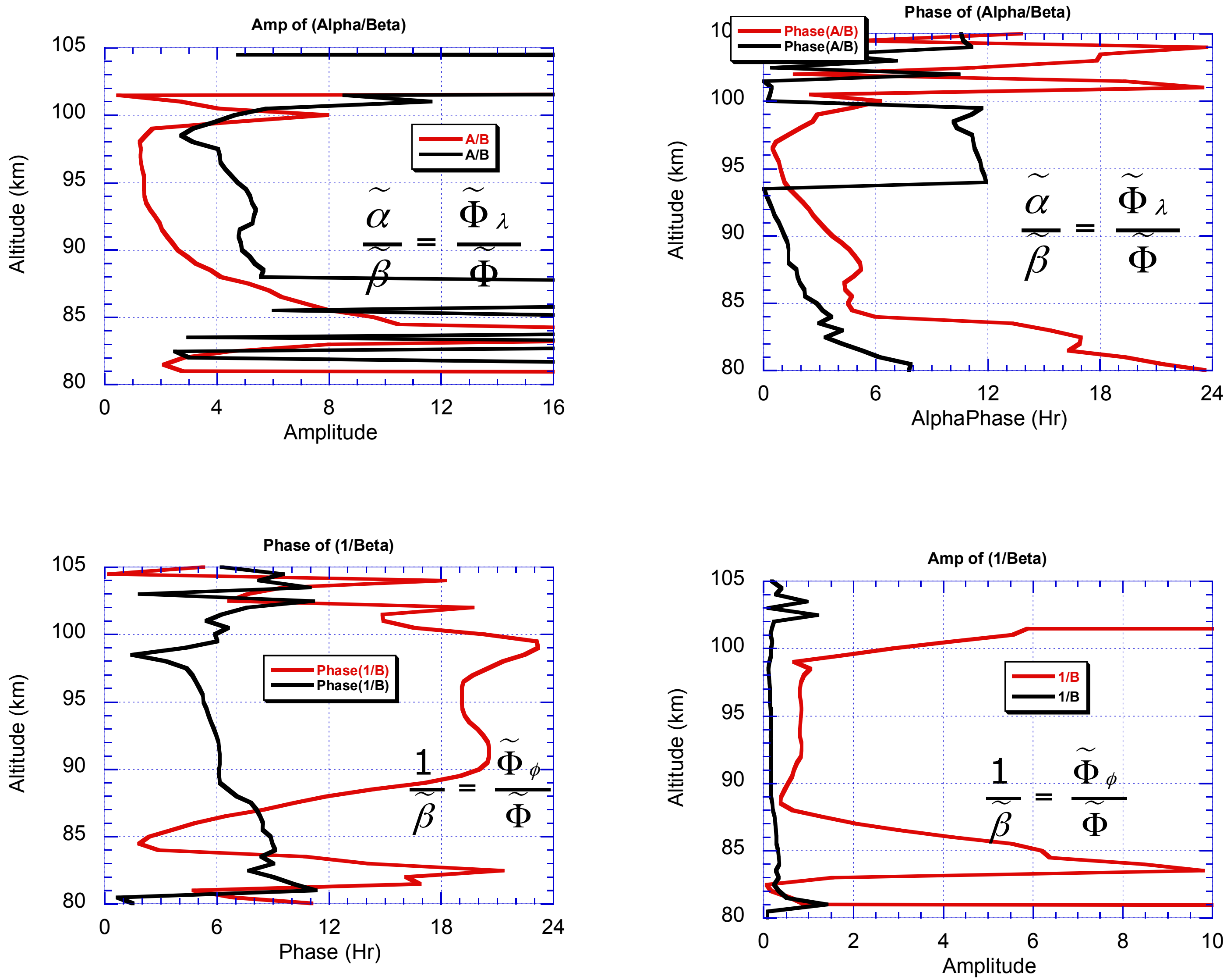
$$\tilde{v} = -i\frac{f}{\sigma}\tilde{u};\ \tilde{T} = \frac{H_s}{R}\sqrt{\hat{m}^2(z) + (1/H)^2}\, e^{iA\tan(2\hat{m}H)}\frac{a\cos\phi\left(\sigma^2 - f^2\right)}{\sigma s}\tilde{u}, \quad \text{(8c)}$$

Let $\lambda_x = \frac{2\pi a\cos\phi}{s}$ in agreement with low-freq GW polarization.

For both cases (b) and (c), v leads u by 90^o;
The phase shift, $\Delta\varphi = \varphi_T - A\tan(2\hat{m}H) - \varphi_u$, may be plotted.



PolarizationCalculation_Mar 6:03:25 PM 6/3/2013
|v|/|u|_DT
|v|/|u|_SDT
Migrating:
σ/f = 1.524, SDT; 0.762, DT
Low-freq GW:
f/σ = 0.656, SDT; 1.312, DT
alts(km)
|v|/|u|
PolarizationCalculation_Mar 6:03:25 PM 6/3/2013
Phiv-Phiu_DT
Phiv-Phiu_SDT
For migrating tide and GW:
-6 (DT) and - 3 hr (SDT)
alts(km)
PHase Difference (Hr)
Tvsu_Mar
T/u_DT
T/u_SDT
Altitude
T/u (K/ms-1)
PolarizationCalculation_Mar 6:03:25 PM 6/3/2013
Δφ = φT - Atan() - φu
DPHI_DT_Hr
DPHI_SDT_Hr
alts(km)
PHase Difference (Hr)


Amp of (Alpha/Beta)
A/B
A/B
$\frac{\tilde{\alpha}}{\tilde{\beta}} = \frac{\tilde{\Phi}_\lambda}{\tilde{\Phi}}$
Altitude (km)
Amplitude
Phase of (Alpha/Beta)
Phase(A/B)
Phase(A/B)
$\frac{\tilde{\alpha}}{\tilde{\beta}} = \frac{\tilde{\Phi}_\lambda}{\tilde{\Phi}}$
Altitude (km)
AlphaPhase (Hr)
Phase of (1/Beta)
Phase(1/B)
Phase(1/B)
$\frac{1}{\tilde{\beta}} = \frac{\tilde{\Phi}_\phi}{\tilde{\Phi}}$
Altitude (km)
Phase (Hr)
Amp of (1/Beta)
1/B
1/B
$\frac{1}{\tilde{\beta}} = \frac{\tilde{\Phi}_\phi}{\tilde{\Phi}}$
Altitude (km)
Amplitude

Conclusion and Future Work

- Polarization relationships relating observed tidal perturbations are derived.
- The relation between w’ and T’ is independent of tidal structure, and may be used as a method to determine w’ from T’.
- The relation between v’ and u’ and between T’ and u’, however, depends on tidal structure via parameters, $\frac{\Phi_{\lambda}}{\Phi}$ and $\frac{\Phi_{\phi}}{\Phi}$, that may be determined from observation.
- Since these parameters depend on tidal structures, the understanding of their meaning and impact require the knowledge of tidal structure, which may be obtained form tidal models, such as HME.