A Novel Data Assimilation Model for the Plasmasphere

CEDAR Workshop 2017

AGS-PRF: No AGS-1231207

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Objective and Motivation

- **Objective:**
  - **Apply the idea of data assimilation to the plasmasphere**
  - **To develop a plasmaspheric data assimilation technique to produce time-evolving maps of plasmasphere density 3D structures.**

- **Motivation**
  - **Data assimilation is a mathematical framework for the statistical union between observations and empirical or physics-based models.**
  - **It allows for ingestion of various types of geospace measurements.**

Methodology – Data Assimilation

- **Inspired by Ionosphere Data Assimilation 4D (IDA4D) [Bust et al. 2004]**
  - **Model:**
    - Global Core Plasma (GCP) Model [Gallagher et al., 2000]:
    - Empirical, Kp driven
  - **Data:**
    - COSMIC Satellites: Total electron content (TEC) from precise orbit determination
      - Utilize upward looking GPS TEC signals
  - **Forward model:**
    - Relating observations to state variables (electron density)
      - Line-of-sight integration

$$TEC = \int N_e(z)dz + \varepsilon_k \qquad \text{or} \qquad y_k = H_k x_k + \varepsilon_k$$

  - **Inverse model:**
      - Define an objective function: least squares solution

$$x_k^a = \arg\min \frac{1}{2}\left(\|y_k - H_k x_k\|^2_{R_k^{-1}} \right)$$

- H is ill-conditioned, in general. Cannot solve directly via $x_k = H^{-1} y_k$

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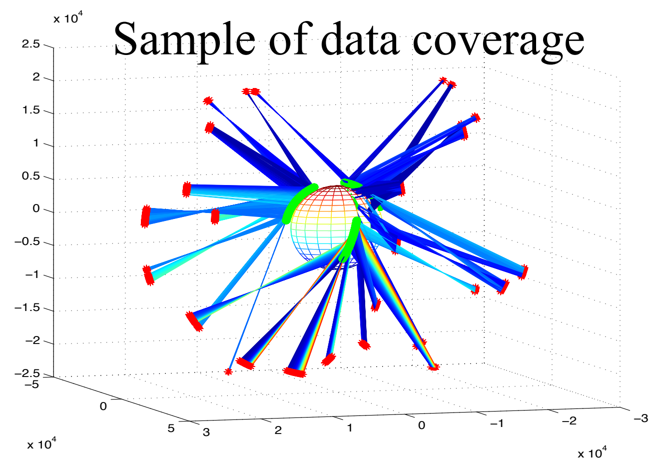

Space APL

Methodology – Data Assimilation

- *A Priori* constraint: add the background model – 3D-VAR
  - Objective function combining the deviation from the data and the model
  - 3D-VAR: estimate 3D structure of the unknown state

$$x_k^a = \arg\min_{x_k} \frac{1}{2}\left(\|y_k - H_k x_k\|^2_{R_k^{-1}} + \|x_k - x_k^b\|^2_{P_b^{-1}}\right)$$

  - Where R_k^{-1} and P_b^{-1} are the data error covariance and model error covariance, respectively.

  - Analytical expressions exist:

$$x_k^a = x_k^b + P_k^b H_k^T \left[R_k + H_k P_k^b H_k^T\right]^{-1} \left(y_k - H_k x_k^b\right)$$
$$P_k^a = P_k^b - P_k^a H_k^T \left[R_k + H_k P_k^b H_k^T\right]^{-1} H_k P_k^b$$

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  - Where R_k^{-1} and P_b^{-1} are the data error covariance and model error covariance, respectively.

➢ **We find that with the ingestion of upward looking data alone, the problem remains ill-posed.**

➢ **Hence, we further constrain the solution to be vertically smooth.**

$$x_k^{aMAP} = \underset{x_k}{\arg\min}\, \frac{1}{2}\left(\|y_k - H_k x_k\|_{R_k^{-1}}^2 + \|x_k - x_k^b\|_{P_b^{-1}}^2 + \lambda^2 \|D(x_k - x_k^b)\|^2 \right)$$

Where D and λ are the regularization functional and regularization parameter, respectively.

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- **The analysis step is carried out in two stages in PDA.**

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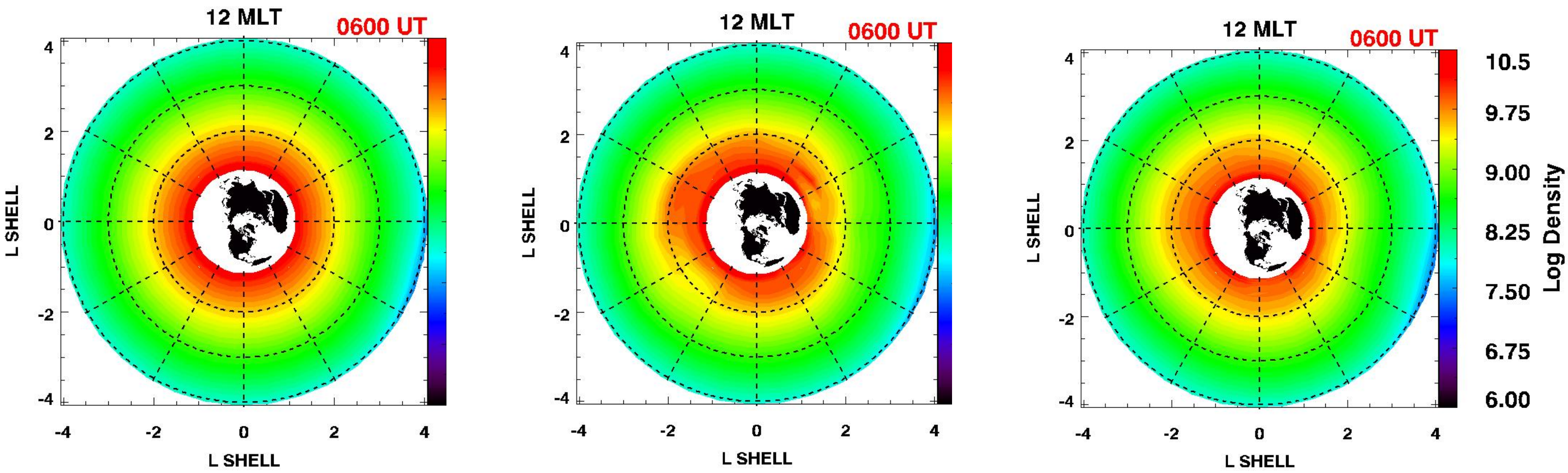
> **The analysis step is carried out in two stages in PDA.**

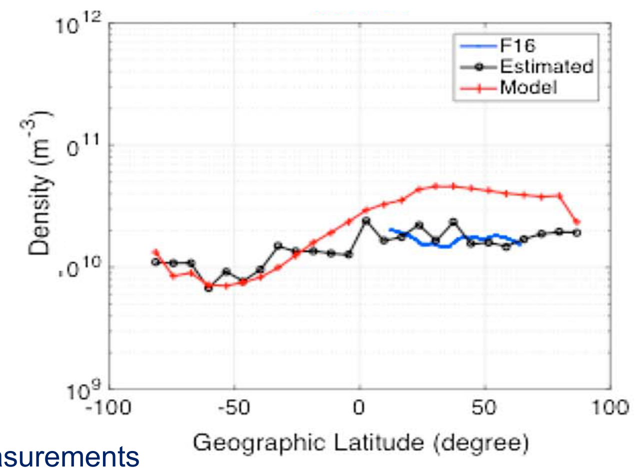
> **Move the state forward in time via Gauss-Markov Kalman.**

An example of PDA Estimation results

Model | 3DVAR | PDA



- Equatorial cross sections of the plasmasphere electron density (m^{-3}).
- The cross sections extend to L=4 (20,000 km).
- Sun is at the top of the image, dawn to the right.
- PDA eliminates unrealistic altitude gradients.
- A good agreement is demonstrated between DMSP-F16 in-situ densities and PDA estimated densities.



The authors are grateful to Dr. Hairston from UT Dallas for providing DMSP density measurements

Summary and Future Directions

- Imposing a smoothness constraint is necessary to avoid non-physical gradients in the vertical direction
- An assimilative model that provides global 3D maps of electron density in the plasmasphere.
- A coupled ionosphere-plasmasphere data assimilation model**
- The extension of the PDA grid to lower ionospheric heights
  - Ground-based GPS TEC
  - ISR measurements
  - Ionosonde
  - FUV 135.6 nm radiance measurements
- The coupled ionosphere-plasmasphere model will allow more accurate specification, nowcast, and forecast of the upper atmosphere.
  - Evolution of the plasmasphere during a geomagnetic storm (plumes)

* NASA-HSR grant #NNX16AG65G, R. Bishop, A. Coster, G. Bust, R. Nikoukar, D. Turner, and C. Lemon, Storm-time Dynamics of the Plasmapause and the Ionosphere/Magnetosphere System, 2014-2016.

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