Numerical Modeling of Lightning-Ionosphere Interactions

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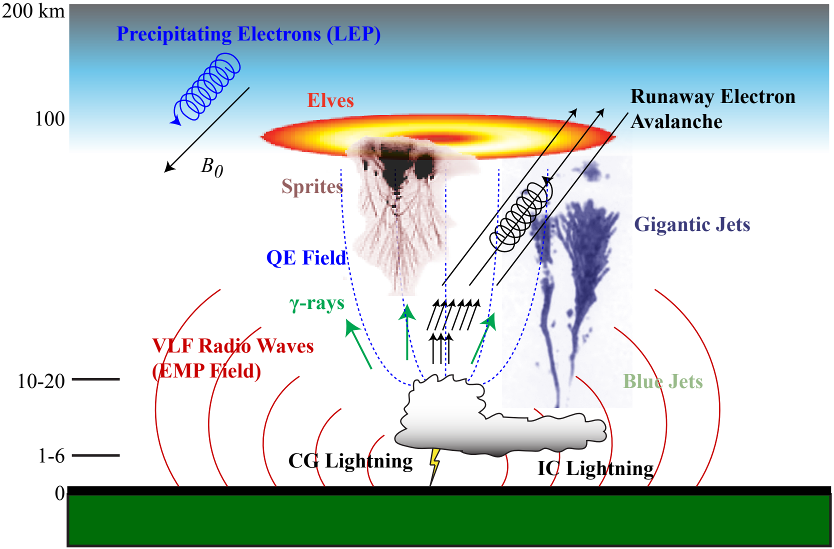
Outline

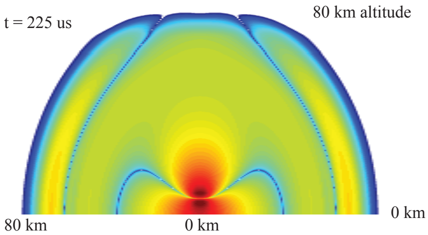
1. Lightning-Ionosphere Interactions
2. Previous Modeling Efforts
3. Modeling in-cloud lightning EMP
4. Towards a full 3D QE/EMP model

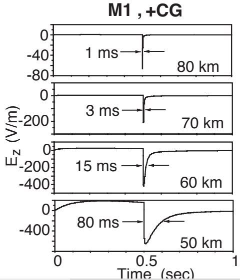
Lightning-generated fields



Lightning-generated fields

- We treat return stroke as a current, and ignore microphysics (leaders, streamers, etc.)
- QE field is electrostatic component of lightning "antenna" field; decay with time makes it "quasi"-electrostatic
- EMP is merely radiation field
- somewhere in between is the induction field
- What happens when fields reach ionosphere?





from *Pasko* [1996] (thesis)

Products of these fields

The QE field:

- Produces sprites ~when $|E| > E_k$, the breakdown field
- Produces halos under similar conditions
- May accelerate a relativistic electron beam

The EMP field:

- Produces elves ~when $|E| > E_k$
- Modifies the electron density, both increases and decreases
- Injects whistler-mode waves into magnetosphere
- Whistlers in turn precipitate electrons from the radiation belts (LEP events)

Both fields "heat" the ionosphere,
by modifying the conductivity,
and thus deposit energy

Questions to Address through Modeling

1. What is the relationship between the lightning parameters (I_k, $I(t)$, h, etc) and the QE field (i.e., altitude and time at which $|E| > E_k$)?
2. What is the relationship between lightning parameters and TLE production: sprites, halos, and elves?
3. How much new ionization is produced?
4. How bright do we expect the optical emissions to be (and how do they compare to observations)?
5. What is the intensity of whistler waves leaking through the ionosphere?
6. How much energy is deposited in the ionosphere (through heating and ionization)?
7. What is the cumulative effect of a storm on the ionosphere?
8. How do questions 1–7 depend on ionosphere conditions and B_0 (magnitude and direction)?

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Inan / Taranenko work

Inan et al. [1991]
Taranenko et al., [1992, 1993a,b]

- 1D time-domain model of EMP only, from 70–100 km altitude
- fully kinetic solutions of Boltzmann equation
- Results: a few to tens of % change in N_e from E_{100} = 10–20 V/m pulses
- Find that electron distribution at 80–90 km altitude becomes stationary in ~10 μs

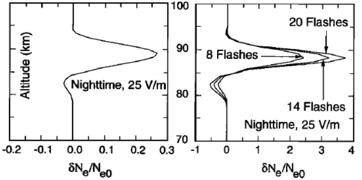


Fig. 5. (Left) The resulting density changes for a single EM pulse with E_{70} = 25 V/m initial amplitude. Fig. 6. (Right) The resulting density changes for 8, 14, and 20 successive EM pulses with 25 V/m initial amplitude. Both for the nighttime conditions.

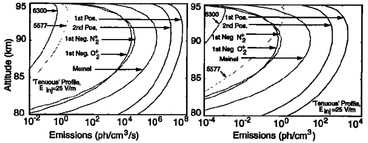
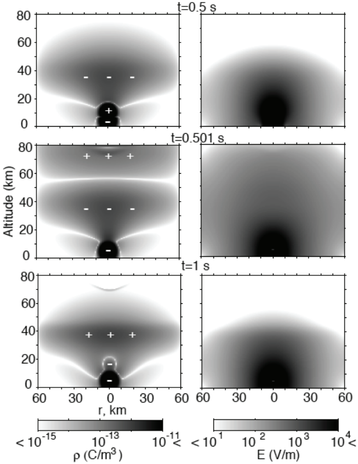


Fig. 3 and 4. (Left) The altitude distributions of maximum intensities stimulated by a E_{inj} = 25 V/m EM pulse for the case of profile (a) of Fig. 1. (Right) The altitude distributions of the emission intensities integrated over the duration of the emissions (~100 μs for the short lasting band emissions, ~0.5 ms at 90 km for the 6300 Å line of O, and ~0.1 s at 90 km for the 5577 Å line of O) stimulated by a E_{inj} = 25 V/m EM pulse for profile (a) of Fig. 1.

Pasko et al. work

Pasko et al. [1995, 1996a,b, 1997, 1998, 1999]

- Cylindrical 2D time-domain model of QE field, from ground to 80 km
- Solutions of Gauss' Law and Continuity equation
- Used analytical descriptions of field-dependent excitation rates for ionization, attachment, mobility, and optical emissions
- Modeled relaxation of electric field due to conductivity changes



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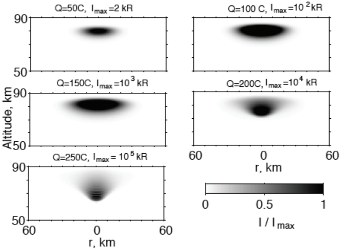

Fig. 3.8. The distribution of time-averaged optical emission intensity. The distribution of the time-averaged intensity (in units of Rayleighs) of emissions of the 1st positive band of N_2 is shown on linear scale and for different values of removed thundercloud charge. It was assumed in all cases that the charge was removed by a lightning discharge from 10 km altitude in 1 ms.

Veronis et al. [1999]

Also used by *Barrington-Leigh et al.*, [2001], *Moore et al.*, [2003]

- Cylindrical 2D time-domain model of EMP field, from ground to 90 km
- FDTD Solutions of Ampere’s and Faraday’s Laws
- Same calculations as Pasko of excitations
- 2D predictions of elves, including "camera" view
- Fully explicit time-domain 2D FDTD model; no B_0

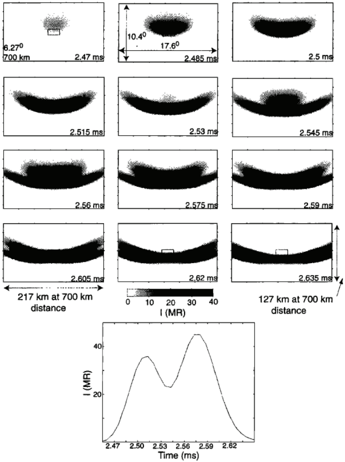

Figure 6. View of the sky with a camera for the case of waveform 3 of Figure 2. The photometric response of pixel P_{20} is also illustrated for comparison.

Cummer (Duke) Research Group

Cummer, [1997], *Hu and Cummer*, [2006], *Hu et al.*, [2007], *Li et al.*, [2008]

- 2D cylindrical FDTD model of Maxwell's equations
- time-dependent processes calculated with same methods as Pasko and Veronis
- Earth curvature correction, PML boundary
- Prediction of breakdown time and altitude due to QE field
- includes B_0 with cylindrical symmetry

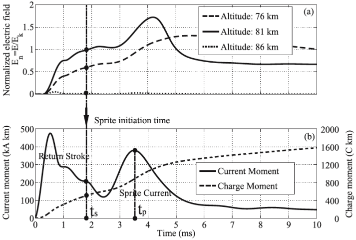

Figure 4. The CB sprite initiation theory testing on a short-delayed sprite with sprite current. (a) The simulated electric field waveforms above the thunderclouds; (b) the extracted current moment waveform of the parent lightning discharge and the corresponding charge moment change.

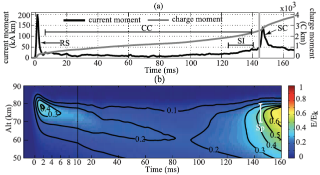

Figure 6. Finite difference time domain (FDTD) simulation results for the typical long-delayed event. (a) Estimated current moment and total charge moment change. (b) Simulated electric fields above the lightning discharge.

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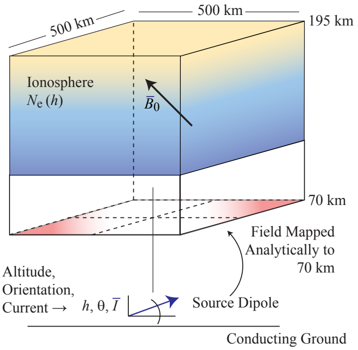
The 3D EMP model

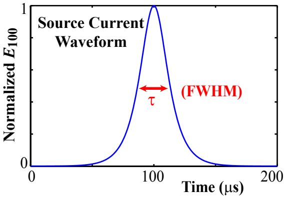


* E_{100}: Electric field in V/m that would be measured at 100 km range on the ground

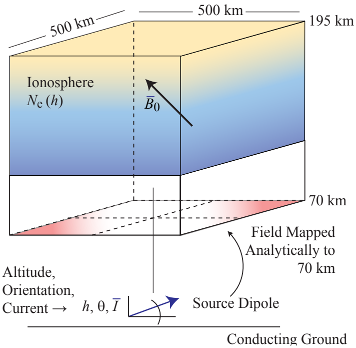
($E_{100} = 30$ V/m $\longrightarrow I_k \simeq 100$ kA)

Source Fields:

- Time-domain solution of Hertz dipole equations
- Mapped to ~60 km lower boundary
- Include near-field and far-field terms
- Dipole at any altitude / orientation



The 3D EMP model



Model solves Maxwell's Equations and the Langevin Equation:

$$\nabla \times \overline{H} = \epsilon_0 \frac{\partial \overline{E}}{\partial t} + \overline{J}_{tot}$$

$$\nabla \times \overline{E} = -\mu_0 \frac{\partial \overline{H}}{\partial t}$$

$$\frac{\partial \overline{J}_a}{\partial t} + \nu_{an}\overline{J}_a = \epsilon_0 \omega_{pa}^2 \overline{E} - \overline{\omega}_{ca} \times \overline{J}_a$$

$\omega_{pa}^2 = \dfrac{N_a q_a^2}{m_a \epsilon_0}$ plasma freq. of species a

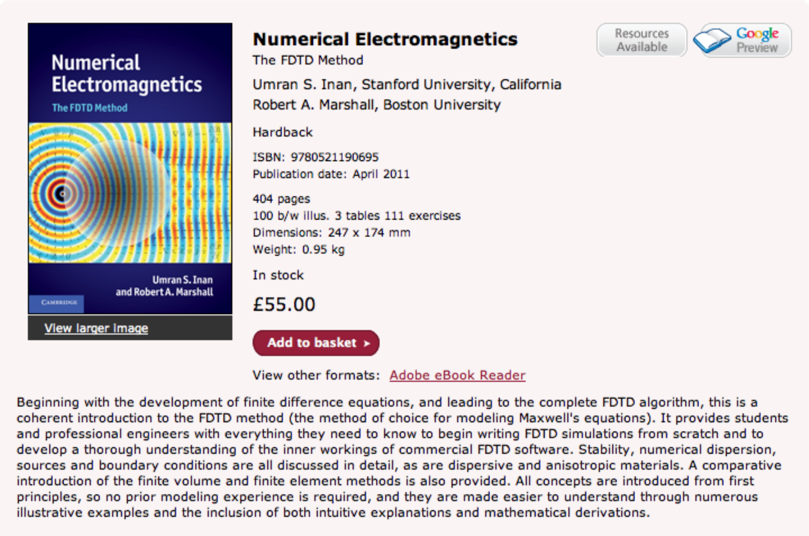
$\overline{\omega}_{ca} = \dfrac{q_a \overline{B}_0}{m_a}$ gyrofreq. of species a

$F = q_a\overline{E} + q_a\overline{v}_a \times \overline{B}_0 - \nu_{an} q_a \overline{v}_a$

$\overline{J}_a = q_a N_a \overline{v}_a$

$\overline{v}_a$ = velocity of species a

Shameless Advertisement



Numerical Electromagnetics

The FDTD Method

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Beginning with the development of finite difference equations, and leading to the complete FDTD algorithm, this is a coherent introduction to the FDTD method (the method of choice for modeling Maxwell's equations). It provides students and professional engineers with everything they need to know to begin writing FDTD simulations from scratch and to develop a thorough understanding of the inner workings of commercial FDTD software. Stability, numerical dispersion, sources and boundary conditions are all discussed in detail, as are dispersive and anisotropic materials. A comparative introduction of the finite volume and finite element methods is also provided. All concepts are introduced from first principles, so no prior modeling experience is required, and they are made easier to understand through numerous illustrative examples and the inclusion of both intuitive explanations and mathematical derivations.

Nonlinear Interactions

Collision Frequency:

$$\frac{\partial \bar{J}}{\partial t} + \boxed{\nu \bar{J}} = \epsilon_0 \omega_p^2 \bar{E} - \bar{\omega}_c \times \bar{J}$$

- ν increases with N (neutrals) → decreases with altitude
- ν increases with $|E|$

Electron Density:

Ionization: $M_2 + e^- \rightarrow M_2^+ + 2\,e^-$

- Yields an increase in electron density

Attachment: $O_2 + e^- \rightarrow O + O^-$

- Yields a decrease in electron density

$$\frac{\partial N_e}{\partial t} = (\nu_i - \nu_a) N_e$$

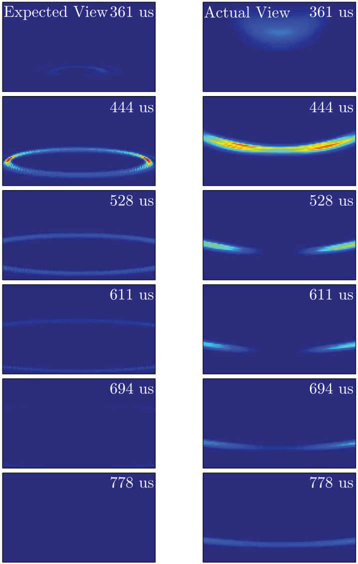
Optical Emissions:

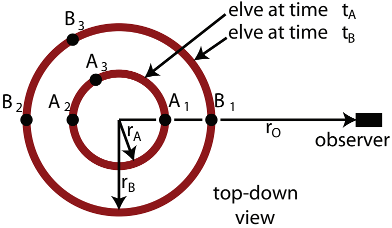
- N_2 First Positive
- N_2 Second Positive
- N_2^+ First Negative
- O_2^+ First Negative
- N_2^+ Meinel

Update number density n_k of particles in excited state k:

$$\frac{\partial n_k}{\partial t} = \nu_k N_e - \frac{n_k}{\tau_k} + \sum_m n_m A_m$$

"Camera View" of Elves

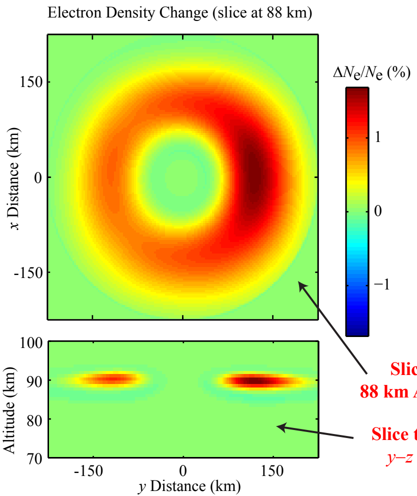


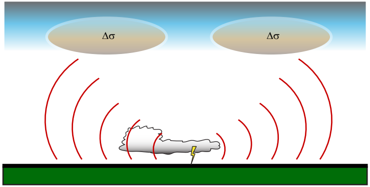


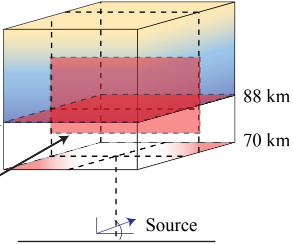
From Newsome [2010] Ph.D thesis

Results: Vertical Discharge (CG)

E_{100} = 20 V/m (75 kA), τ = 20 μs





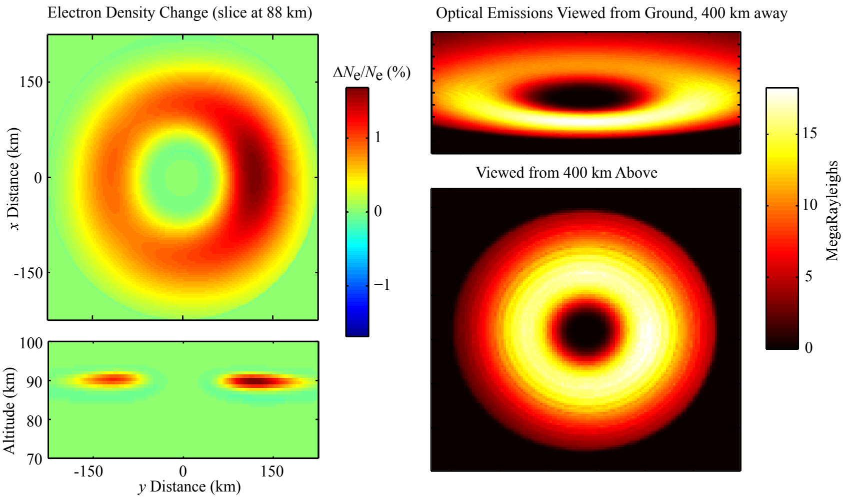


Slice at 88 km Altitude

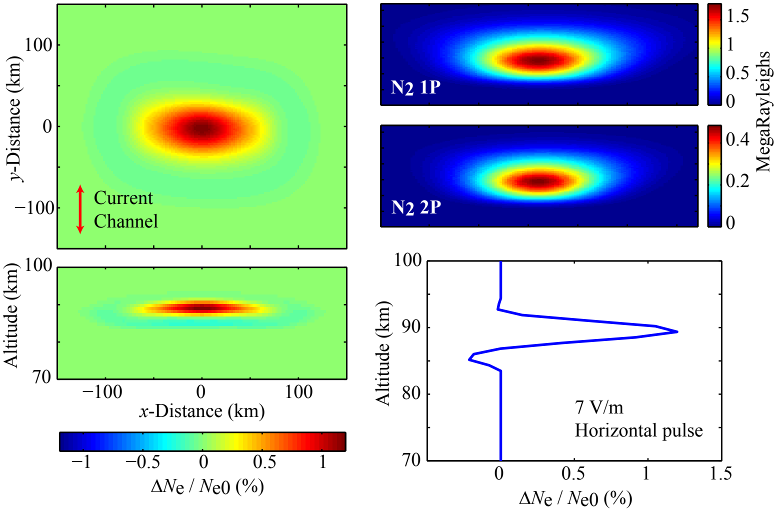
Slice through *y–z* plane

Results: Vertical Discharge (CG)

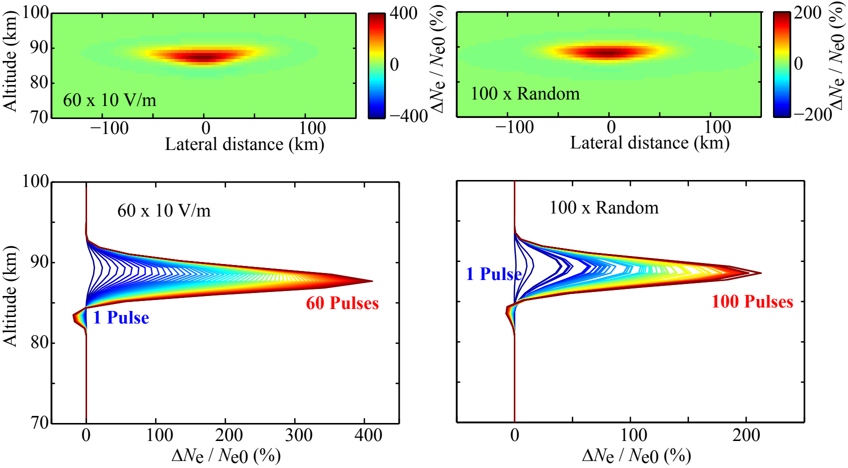
E_{100} = 20 V/m (75 kA), τ = 20 μs



Results: Single In-Cloud Pulse



Results: Multiple IC Pulses



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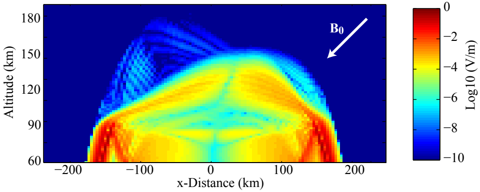
Shortcomings of the old model

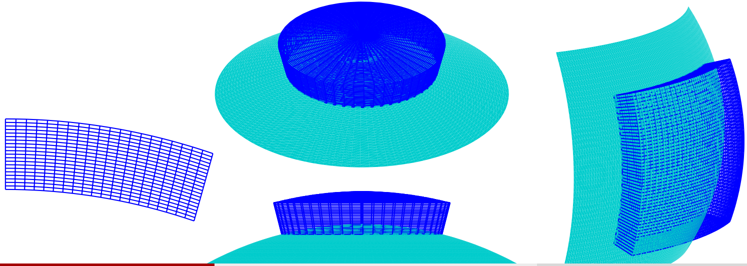
- Analytical description of lightning current not conducive to realistic pulses
- Lower boundary artificially reflects - limited duration
- Poor altitude resolution (∼800 m)
- Outdated method for solving Langevin equation
- Lower boundary does not deal with QE field properly
- Lower boundary limits pulse durations to $\leqslant$ 20 μs
- Written in Fortran77... ugh.



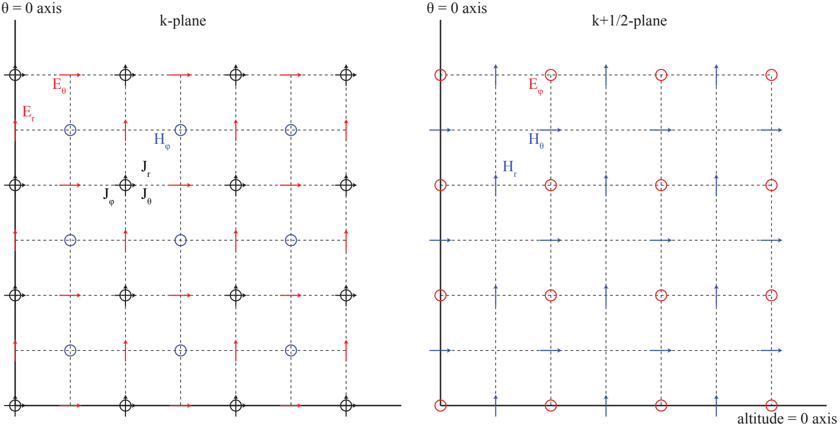
New 3D Model

Features:

- Spherical coordinates (eliminates Earth-curvature problems)
- Nonuniform orthogonal grid (variable Δr, down to 200 m in ionosphere)
- Arbitrary B_0 direction – requires 3D
- PML boundary conditions
- Time-dependent ionosphere parameters (N_e, ν_e, etc.)
- Inclusion of ion species
- Easy parallelization via OpenMP



Lee and Kalluri [1999] Method



Lee and Kalluri [1999] Method

Usual 2nd-order update equations for E and H. J updated according to:

$$\begin{bmatrix} J_r\big|_{i,j,k}^{n+1/2} \\ J_\theta\big|_{i,j,k}^{n+1/2} \\ J_\phi\big|_{i,j,k}^{n+1/2} \end{bmatrix} = \mathbf{A}(\Delta t)\begin{bmatrix} J_r\big|_{i,j,k}^{n-1/2} \\ J_\theta\big|_{i,j,k}^{n-1/2} \\ J_\phi\big|_{i,j,k}^{n-1/2} \end{bmatrix} + \frac{\epsilon_0\omega_p^2}{2}\mathbf{K}(\Delta t)\begin{bmatrix} E_r\big|_{i+1/2,j,k}^{n} + E_r\big|_{i-1/2,j,k}^{n} \\ E_\theta\big|_{i,j+1/2,k}^{n} + E_\theta\big|_{i,j-1/2,k}^{n} \\ E_\phi\big|_{i,j,k+1/2}^{n} + E_\phi\big|_{i,j,k-1/2}^{n} \end{bmatrix}$$

With the matrices:

$$\mathbf{A}(t) = e^{\mathbf{\Omega}t} = e^{-\nu t}\begin{bmatrix} C_1\omega_{br}^2 + \cos(\omega_b t) & C_1\omega_{br}\omega_{b\theta} - S_1\omega_{b\phi} & C_1\omega_{br}\omega_{b\phi} + S_1\omega_{b\theta} \\ C_1\omega_{b\theta}\omega_{br} + S_1\omega_{b\phi} & C_1\omega_{b\theta}^2 + \cos(\omega_b t) & C_1\omega_{b\theta}\omega_{b\phi} - S_1\omega_{br} \\ C_1\omega_{b\phi}\omega_{br} - S_1\omega_{b\theta} & C_1\omega_{b\phi}\omega_{b\theta} + S_1\omega_{br} & C_1\omega_{b\phi}^2 + \cos(\omega_b t) \end{bmatrix}$$

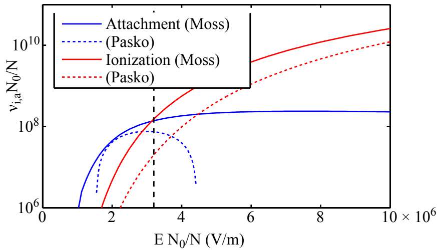
$$\mathbf{K}(t) = \mathbf{\Omega}^{-1}(e^{\mathbf{\Omega}t} - \mathbf{I}) = \frac{1}{\omega_b^2 + \nu^2}\begin{bmatrix} C_2\omega_{br}^2 + C_3 & C_2\omega_{br}\omega_{b\theta} - C_4\omega_{b\phi} & C_2\omega_{br}\omega_{b\phi} + C_4\omega_{b\theta} \\ C_2\omega_{b\theta}\omega_{br} + C_4\omega_{b\phi} & C_2\omega_{b\theta}^2 + C_3 & C_2\omega_{b\theta}\omega_{b\phi} - C_4\omega_{br} \\ C_2\omega_{b\phi}\omega_{br} - C_4\omega_{b\theta} & C_2\omega_{b\phi}\omega_{b\theta} + C_4\omega_{br} & C_2\omega_{b\phi}^2 + C_3 \end{bmatrix}$$

Where:

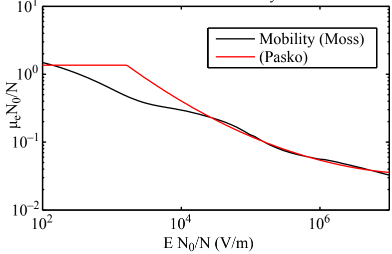
$$\begin{aligned} S_1 &= \sin(\omega_b t)/\omega_b \\ C_1 &= (1 - \cos\omega_b t)/\omega_b^2 \\ C_2 &= (1 - e^{-\nu t})/\nu - \nu e^{-\nu t}C_1 - e^{-\nu t}S_1 \\ C_3 &= \nu(1 - e^{-\nu t}\cos\omega_b t) + e^{-\nu t}\omega_b\sin\omega_b t \\ C_4 &= 1 - e^{-\nu t}\cos\omega_b t - \nu e^{-\nu t}S_1 \end{aligned}$$

Time dependent quantities: Excitation Rates

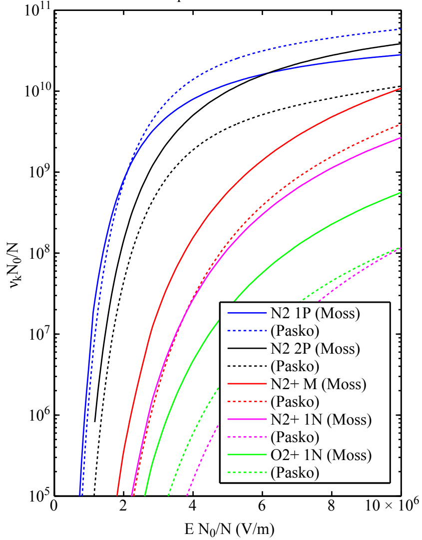
Ionization and Attachment rates



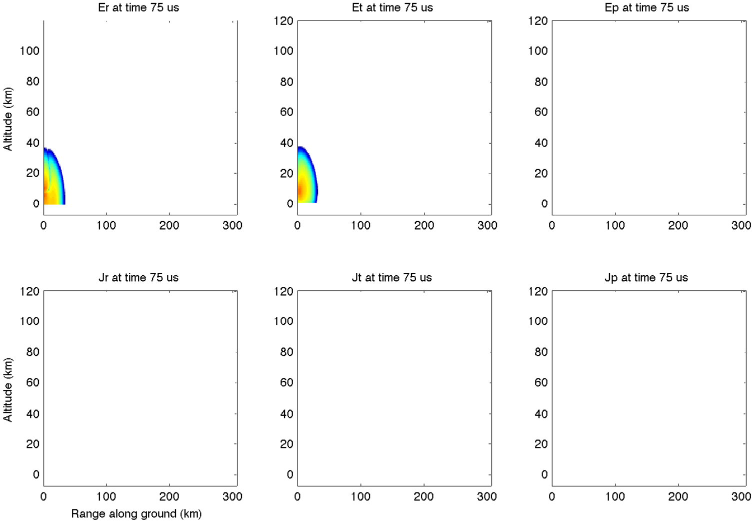
Electron Mobility



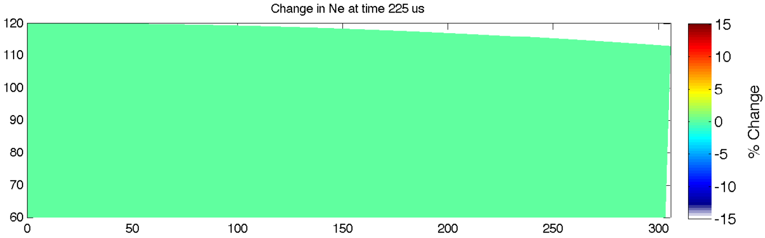
Optical Excitation rates

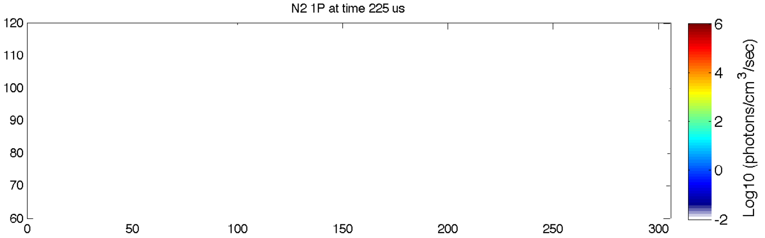


E and J fields in Spherical 2D model

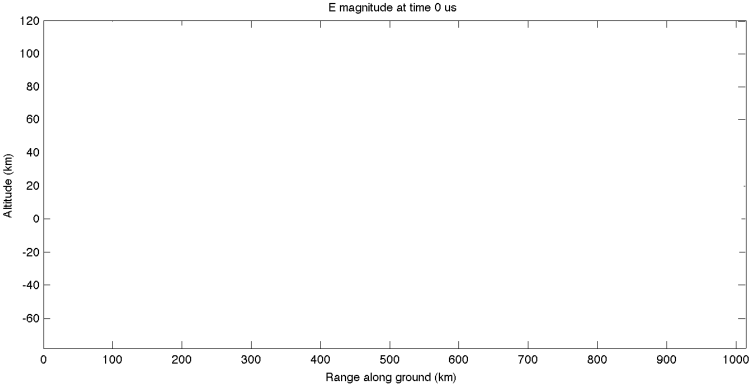


ΔN_e and Optics





Long distance Propagation



To do list:

2D model:

- Include ion species, including $\Delta N_i(t)$
- Run model over variety of ionospheres and lightning parameters
- Calculate energy deposition
- Create a model of energy deposition versus lightning parameters
- Calculate stormwide, global, and seasonal cumulative effects

3D model:

- Incorporate PML and ionosphere equations (Jr[i][j] → Jr[i][j][k])
- Vary magnetic field direction and analyze effect