



Ionospheric Imaging and Data Assimilation

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Overview

- Why Ionospheric Imaging and Data Assimilation
- Description of Inverse Theory and Tomography
- Mathematical Derivation of Ionospheric Tomography
- Current Status of Ionospheric Tomographic Imaging and Data Assimilation
- Examples of Scientific Investigations
- Future Directions: Inverse Theory Beyond Ionospheric Tomography
- Summary



Why Ionospheric Imaging and Data Assimilation

- Ionosphere-Thermosphere is a complicated, complex, coupled system
- External forcing (solar)
- Coupled at boundaries to other systems (plasmasphere, magnetosphere, mesosphere, stratosphere)
- Multiple species momentum, continuity, energy equations
- Ionized plasma and neutrals
- Electric and magnetic fields – Maxwells equations
- *So It is a difficult system to accurately model, predict, understand and interpret*



Why Ionospheric Imaging and Data Assimilation

- Imaging and Data Assimilation methods can accurately specify observed structures and phenomena that modeling cannot
- Can make estimates of important state variables that are difficult to measure
 - Neutral winds
 - Electric fields
 - Electron, Ion, Neutral temperatures
 - Composition
- Which then helps in our understanding and interpretation and helps improve modeling



Examples of Ionosphere Physics Data Assimilation can Contribute to

- Tongues of ionization (TOI), polar patches, boundary blobs
- Storm enhanced densities, sharp gradients
- Equatorial anomalies, equatorial plumes
- Large scale gradients, conductances input to equatorial spread F
- Traveling ionospheric disturbances (TIDS)
- Ionospheric response to stratospheric warming
- Contributing mean state or baseline to other scientific studies of sporadic or anomalous behavior
- Contribute to understanding coupling between different geophysical regions as well as different spatial and temporal scales



Description of Inverse Theory and Tomography

- Forward Predictive Theory
 - Begin with a forward predictive theory/model ($F=ma$ for example)
 - All coefficient / parameters in theory/model are known (gravitational constant for example)
 - Known initial conditions
 - Then predict the time evolution of the system
 - Compare with observations (position versus time, orbit of planet etc)
- Inverse Theory
 - Start with the observations of the system
 - Apply known model or theory that relates the observations to the (known unknown) parameters of the model and or the unknown initial conditions
 - Apply some kind of mathematical/estimation technique to estimate the unknown physical parameters of the system
- Issues in Inverse Theory
 - Incomplete data, errors on the data
 - Incomplete / unknown model or inaccurate model
 - Non-unique solutions, large errors on solutions
 - Flat out wrong solutions (bad models)



Examples Beyond Physics and Math

- Inverse theory is an overall concept and idea that has applications beyond the world of math and physics
- In fact, we often use inverse theory in our daily lives without thinking about it.
 - We make observations of the world around us
 - And using our knowledge of the the world (model) we determine (estimate) what must have been the causes of the observations
 - Sometime (often!!) we are wrong either because
 - » The observations are incomplete and allow more than one interpretation
 - » Our knowledge (model) is incomplete and therefore leads to incorrect interpretation



Sherlock Holmes Example

- Conan Doyle wrote the Sherlock Holmes mysteries in 1880's
- Holmes was a “consulting detective” who reasoned “analytically”
- Doyle simply applied inverse theory
- Quote from “The Study in Scarlet”

“Most people, if you describe a train of events to them, will tell you what the result would be. They can put those events together in their minds, and argue from them that something will come to pass. There are few people, however, who, if you told them a result, would be able to evolve from their own inner consciousness what the **steps were that led up to that result.**”

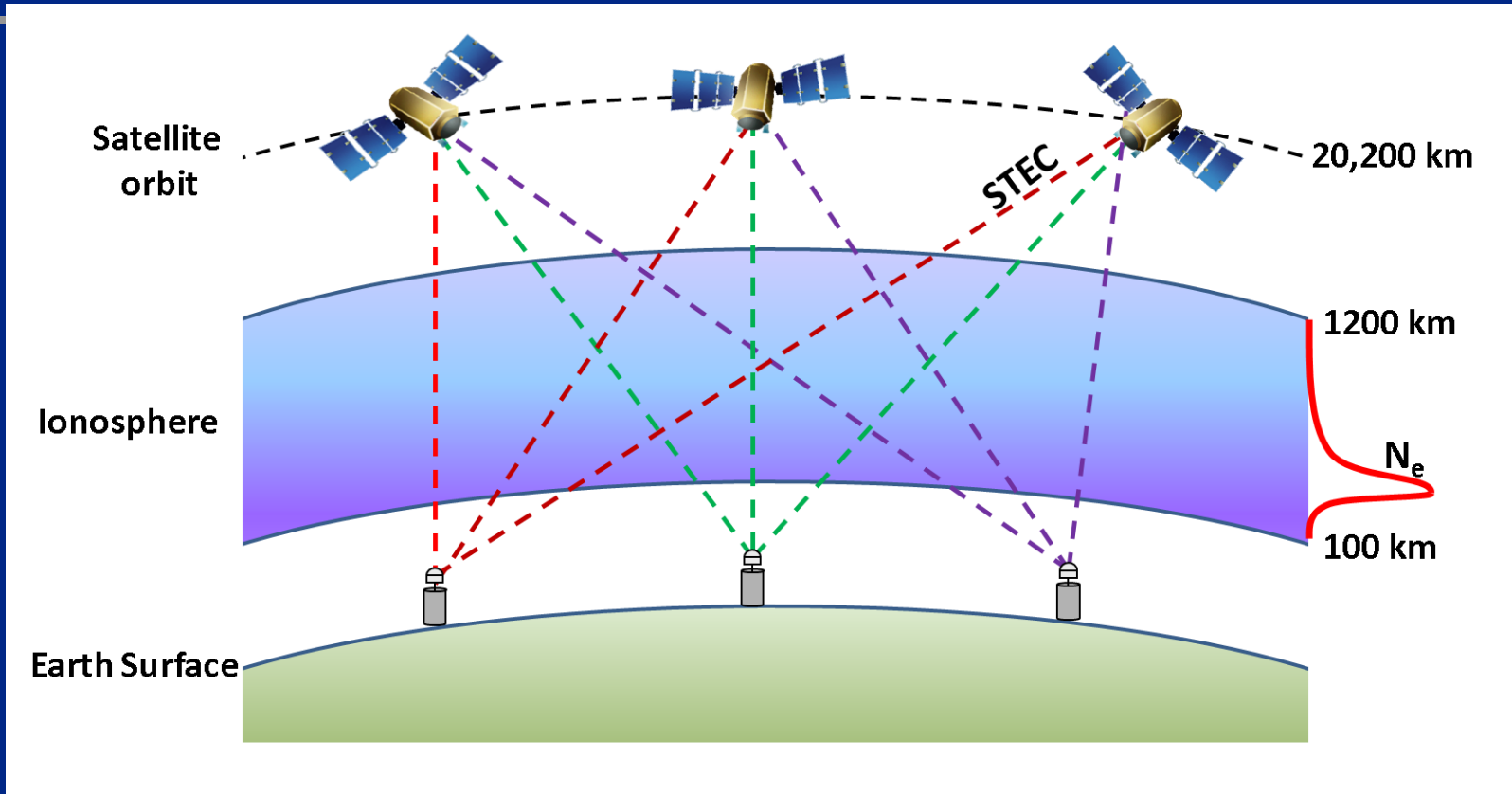
This power is what I mean when I talk of **reasoning backward**, or analytically.”



Example: Ionospheric Tomography

- Forward Modeling:
 - Given 3D distribution of electron density
 - And Geometry between ground GPS receiver and GPS transmitter
 - We can **predict what the Total Electron Content (TEC)** would be.
- Inverse (Tomographic) Modeling (Ideal mathematical sense):
 - Ideal means lines of TEC from every angle that can cut through the volume of interest, with the receiver and transmitter positions completely outside the volume
 - Ideal, also means continuous coverage in angles
 - TEC data known exactly (no measurement errors)
 - We can **exactly and uniquely** recover the electron density distribution
- Inverse Tomography (realistic practical sense):
 - Data is **never complete**
 - limited angles (very difficult to get entirely horizontal rays through the ionosphere)
 - Neither ground GPS receivers or satellite transmitters are continuously distributed around the volume.
 - Data has errors
 - Solution is **not unique and can have large errors**
 - Need for extra information to make solution unique (prior information, regularization)

Lines of sight total electron content (TEC) from low earth orbiting (LEO) satellites



A rapidly moving satellite provides measurements of TEC between the satellite position and an array of ground receivers.

This is a 2D set of measurements (ground position of receiver, and angle to satellite)

Therefore, we should be able to use this 2D set of data to determine the 2D distribution of electron density.



Discrete Linear Estimation Methods

- Start with the integral equation relating observed TEC (T) to electron density N :

$$T = \int_R^S N(r, \theta, \phi) ds$$

- “Pixelize” the density onto a discrete 2D grid
- Then for every measurement “ i ”, we have

$$y_i = \tilde{H}_{ij} x_j$$

- Where now the TEC measurement for a specific ground position and angle is labeled by y_i , the density on 2D grid point is x_j , and the matrix element H_{ij} is the length of the ray through the pixel.



Solutions

- The discrete representation can be turned into a matrix equation:

$$\mathbf{y} = \tilde{\mathbf{H}}\mathbf{x}$$

- The problem is stated:
 - Given measurements \mathbf{y} , with errors, and known \mathbf{H} , Determine \mathbf{x}
- But there is generally
 - ***much less data than unknowns***
 - And generally limited receivers and angles
- Many techniques have been developed to deal with these issues:
 - Algebraic Reconstruction Technique (ART)
 - Multiplicative Algebraic Reconstruction Technique (MART)
 - Orthogonal basis functions
 - Regularization methods with least squares



Tomography Algorithms

- Early Algorithms
 - MART, ART, SIRT
 - Estimate constant of integration
 - Use empirical ionospheres (IRI, Chapman profiles) as prior
- Current Methods
 - Combination of weighted Chapman profiles and MART
 - Empirical orthogonal basis expansion
 - Tikhonov Regularization
 - Statistical minimization
- ART Example:

$$x_j^{k+1} = x_j^k + \lambda_k \frac{y_i - \sum_{m=1}^n \Delta_{im} x_m^k}{\sum_{m=1}^n \Delta_{im} \Delta_{im}} \Delta_{ij}$$

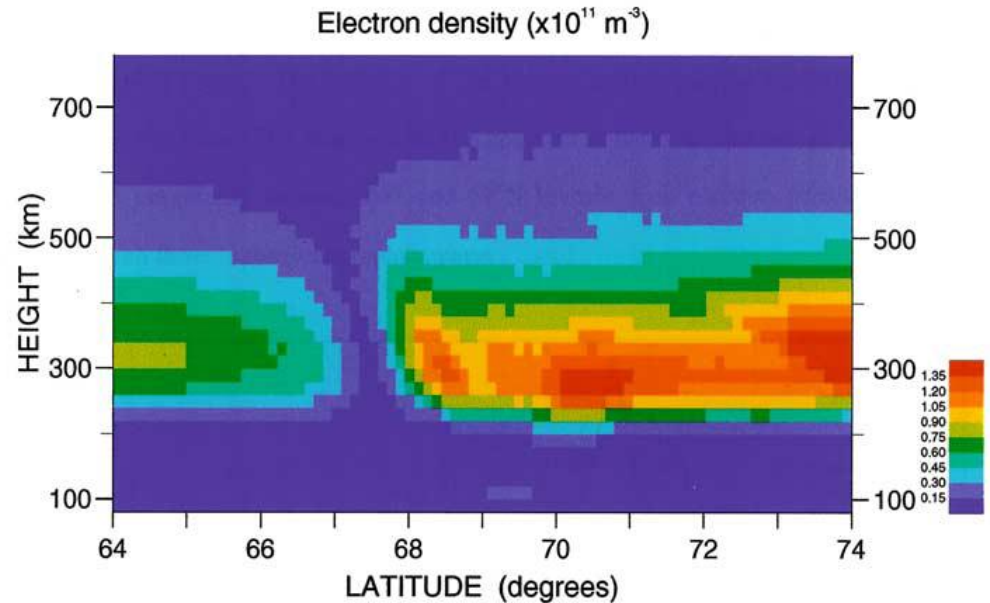


Tomographic image of high-latitude trough

Verification with EISCAT incoherent scatter radar

Reprinted from Mitchell et al. [1995]

Tomographic Image: 15/10/93 21:36 UT



EISCAT BLOB

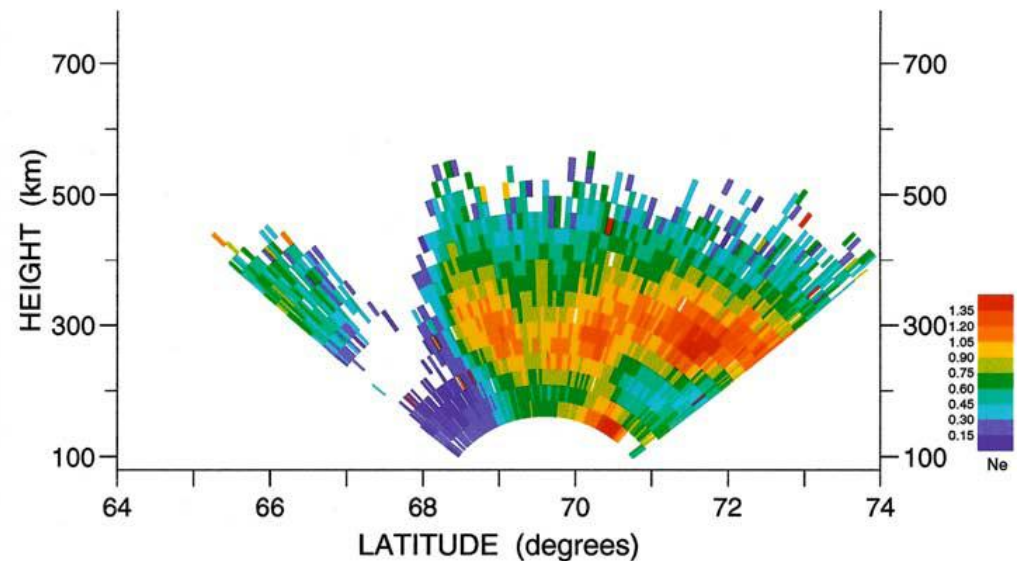
15/10/93

Electron Density ($\times 10^{11} \text{ m}^{-3}$)

21:02 - 21:28 UT

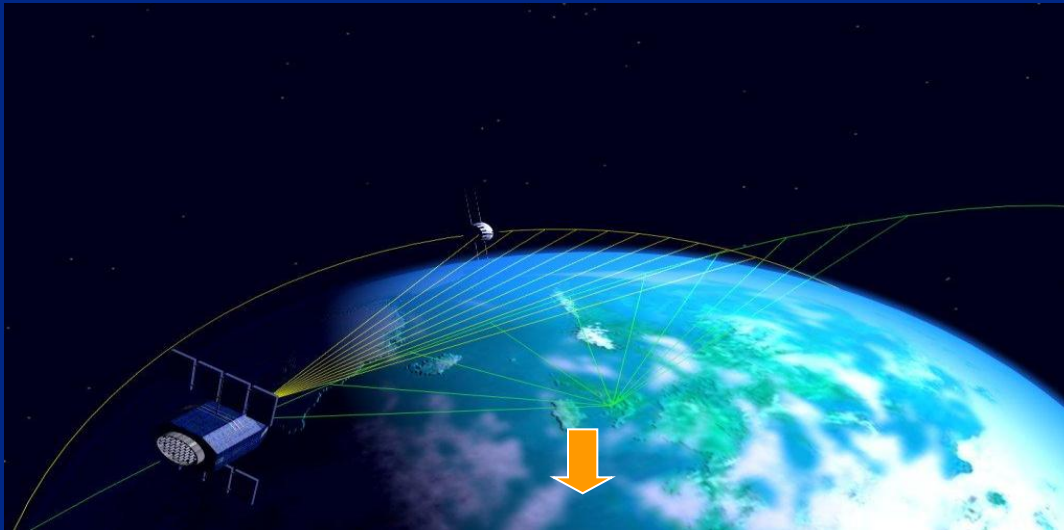
Tromso

Scan N to S

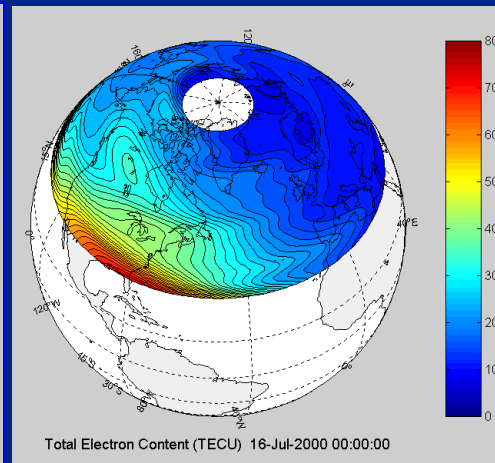
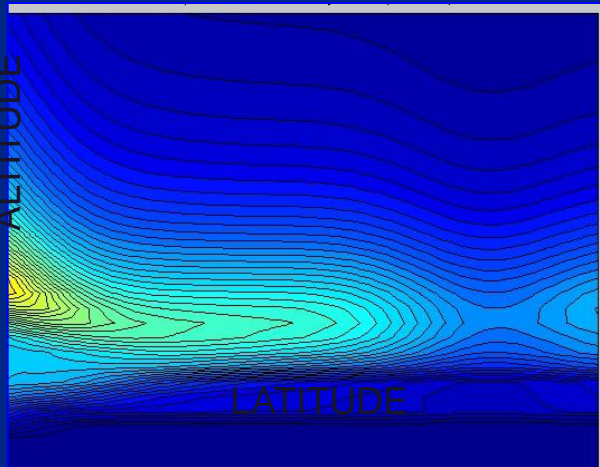




3D Ionospheric imaging



*Measured –
relative values of total
electron content*



*Find –
3D time-evolving
electron density*

(Slide courtesy of C.N. Mitchell University of Bath, UK)



Ionospheric Data Assimilation Four Dimensional (IDA4D)

- Global 3D time-evolving imaging of the ionosphere electron density
 - Gauss Markov Kalman Filter predicts forward in time
- *Solves for log of electron density*
 - *Guarantees positivity*
 - *Errors are more log normal distribution*
- *Completely irregular horizontal grid, vector of vertical grid points*
 - User selectable
 - High resolution where desired
 - Can be dynamically chosen based on data
- Configuration files
 - *User configurable error covariance*
 - model of background ionosphere
 - Amount of and type of data
 - Regional/global
 - Time steps
 - Convergence criteria
 - Data sampling rates, averaging windows, sampling windows, data representation errors



IDA4D Data Sources

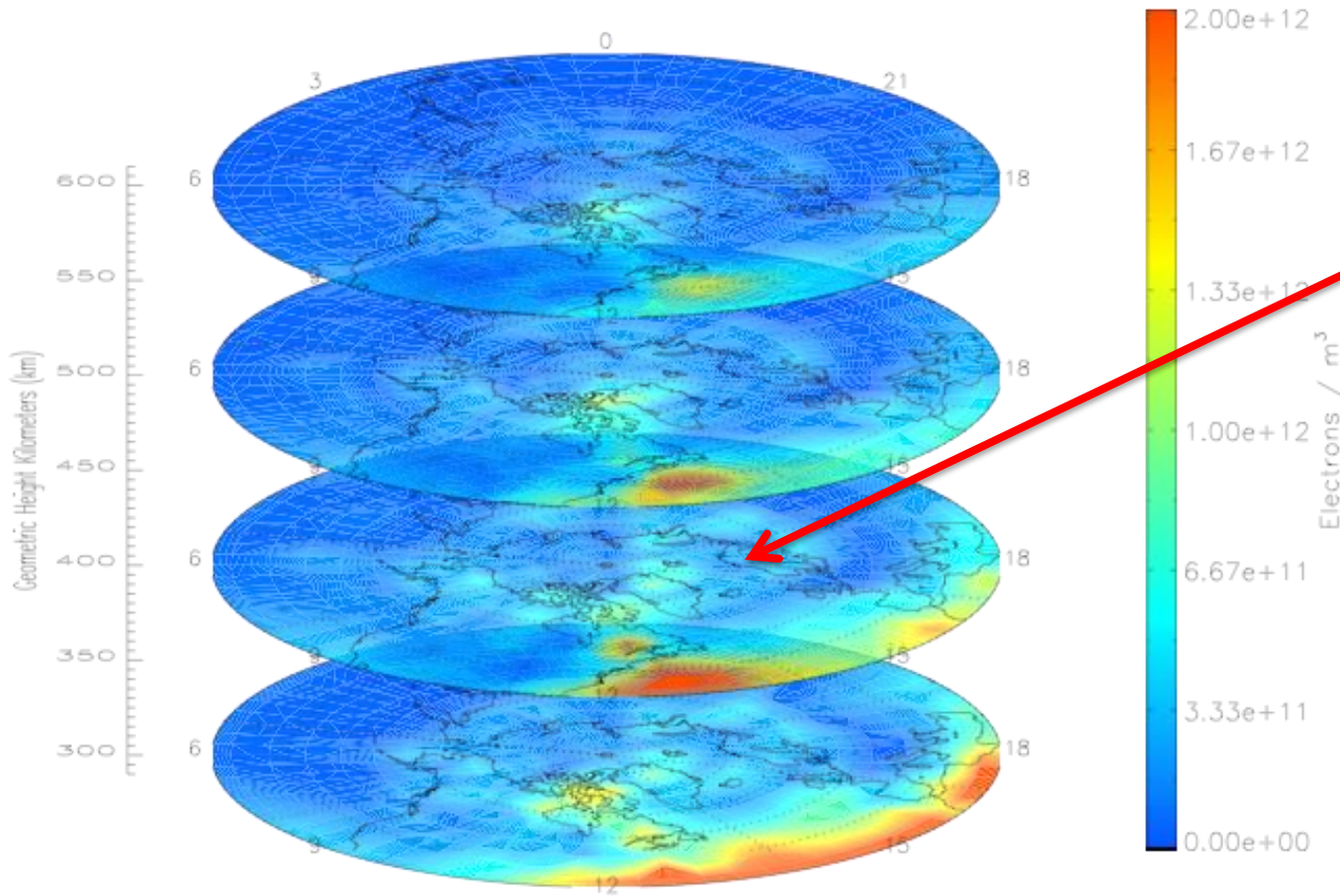
- Linear data sources
 - Ground GPS TEC
 - Ground/Space DORIS TEC
 - Space RF occultations
 - Space Over satellite electron content
 - Space in-situ measurements of electron density
- Non-linear data sources
 - Digisonde virtual height vs. frequency
 - Bi-static HF time-delay vs. frequency
 - HF/VHF ground-space time-delay vs. frequency (Forte)
 - UV 1356 radiance data (SSULI, SSUSI)



Altitude Distribution of Nov 20, 2003 SED Bulge

IDA Map NE
UT = 17.00 DOY = 324

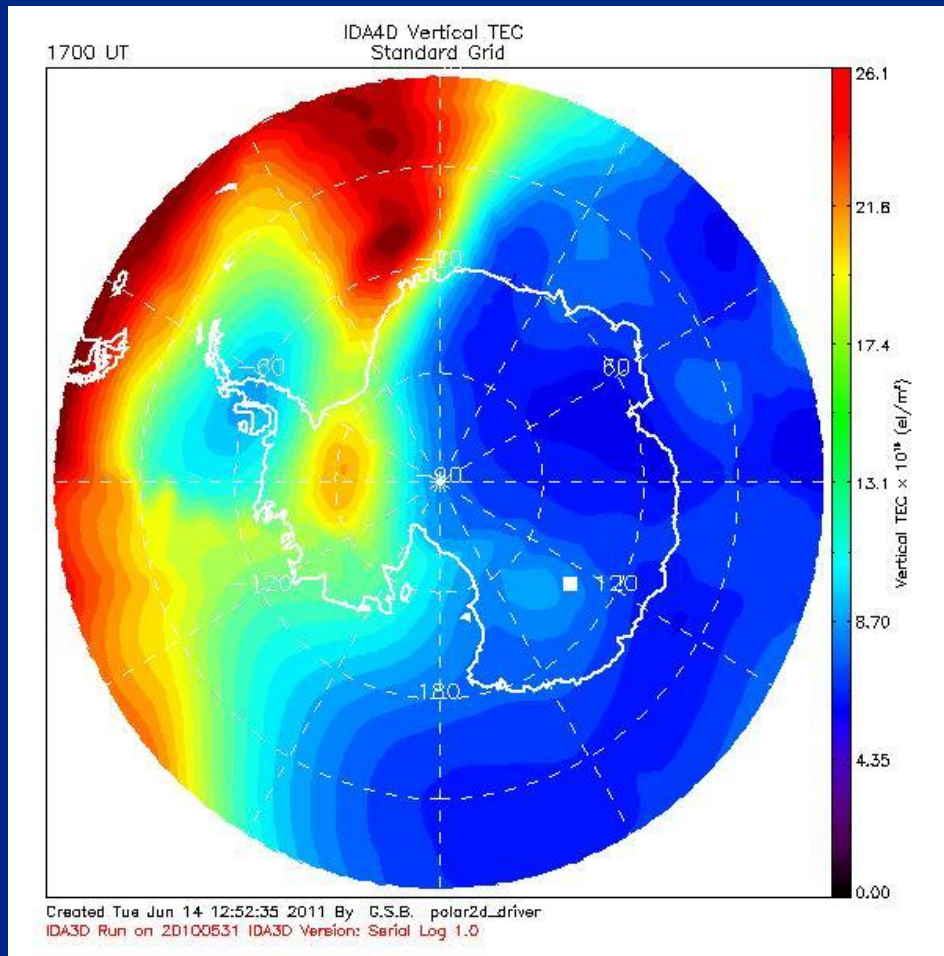
Perim Lat: 30.0



The peak altitude of the bulge seems to be 400-500 kilometers



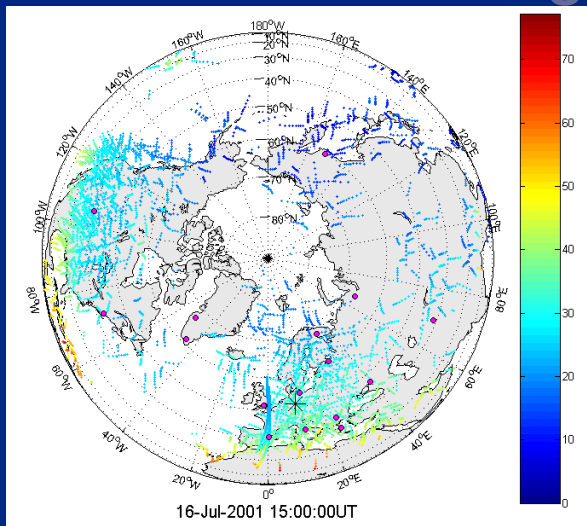
Antarctica April 5, 2010 Storm: 1700 UT





Estimate the **Physical Drivers** that produce the imaged density (Reverse Engineering)

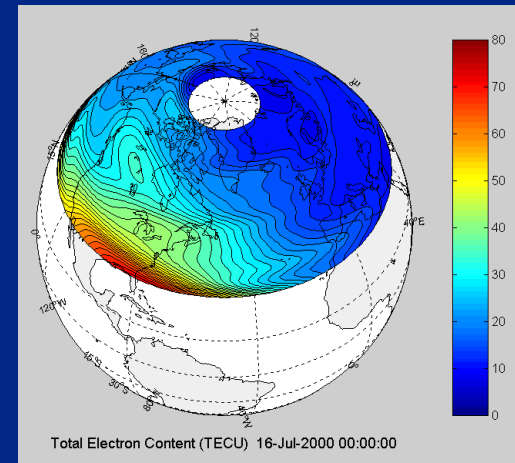
Imaging Issues – next steps



Raw data

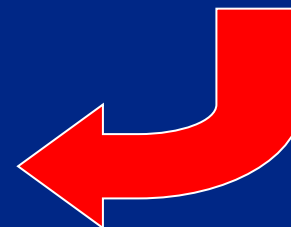


better statistical-imaging methods



Images

Physical drivers producing/moving the ionospheric structures



new inversion/assimilation methods

(Slide courtesy of C.N. Mitchell University of Bath, UK)



Estimating Model Parameters from Ionospheric Reverse Engineering (EMPIRE)

- takes advantage of GNSS-derived images to estimate the ion/electron drifts.

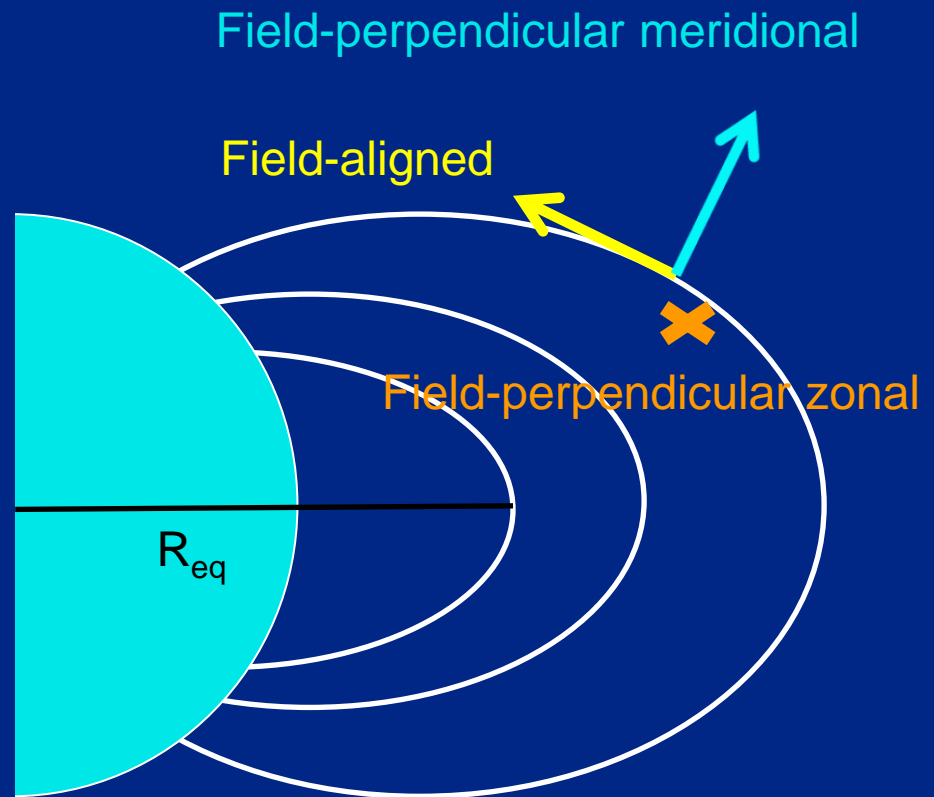
EMPIRE is a tool that can tell us more about the ionospheric drivers, particularly during stormy periods.

- Obtain densities N from IDA4D or other specification technique.
- Formulate the electron density continuity equation as:

$$\frac{dN}{dt} = P - L - \nabla \cdot (Nv) \implies y = Mx + a + \varepsilon, \quad \text{where } y = \frac{dN}{dt}$$

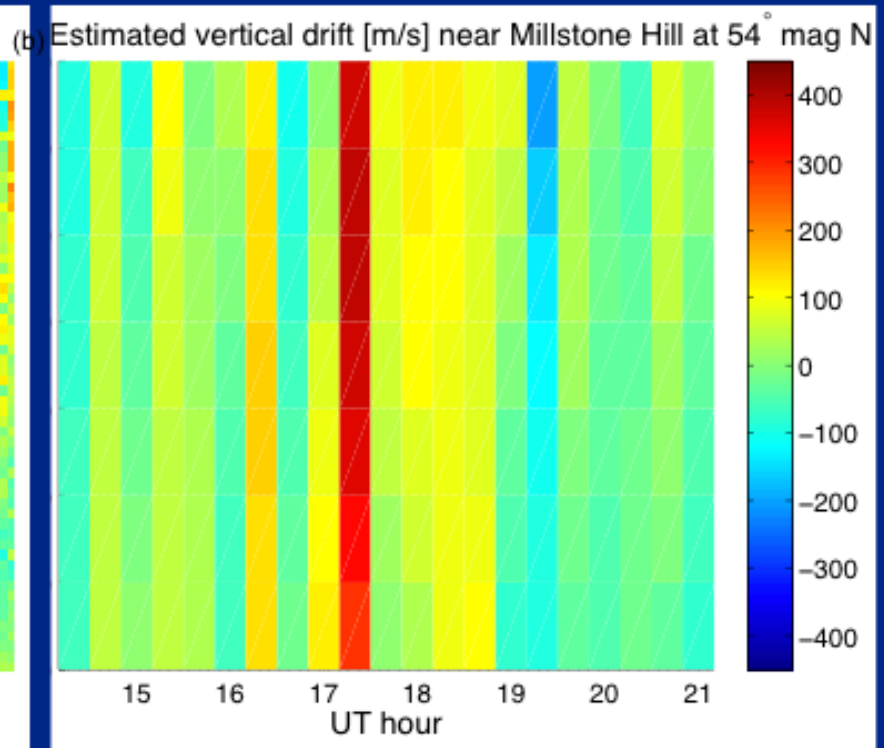
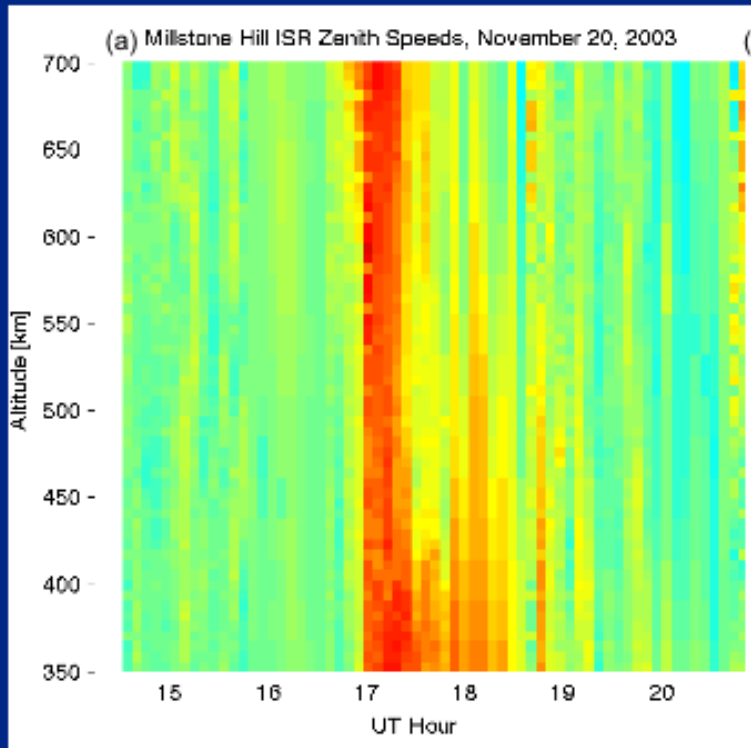
EMPIRE Approach

- Assume a functional form of quantities to be estimated, with coefficients x .
 - Field-aligned drift v_{\parallel} .
 - Electric potential that gives rise to ExB drift v_{exb} .
- Predict drivers via weighted least squares.
- Focus on F2 region.



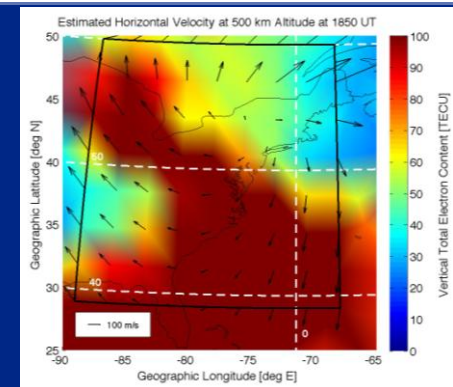
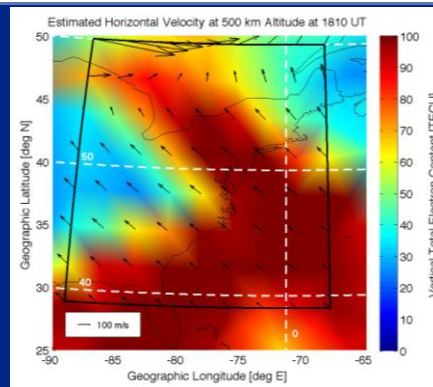
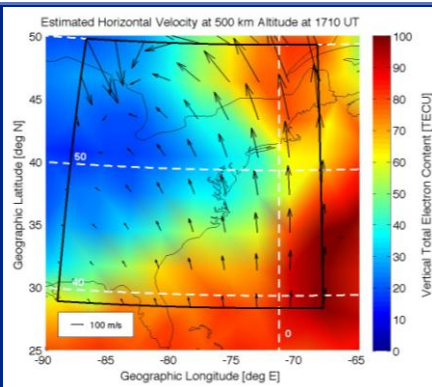
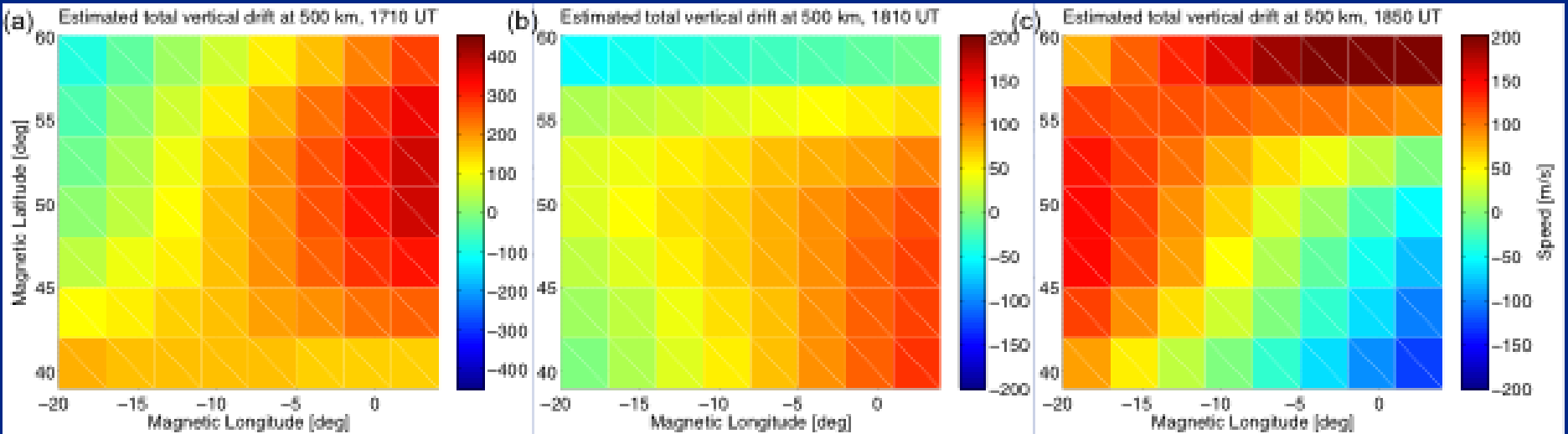


Comparison of EMPIRE Vertical Drifts to ISR (Nov 20, 2003)



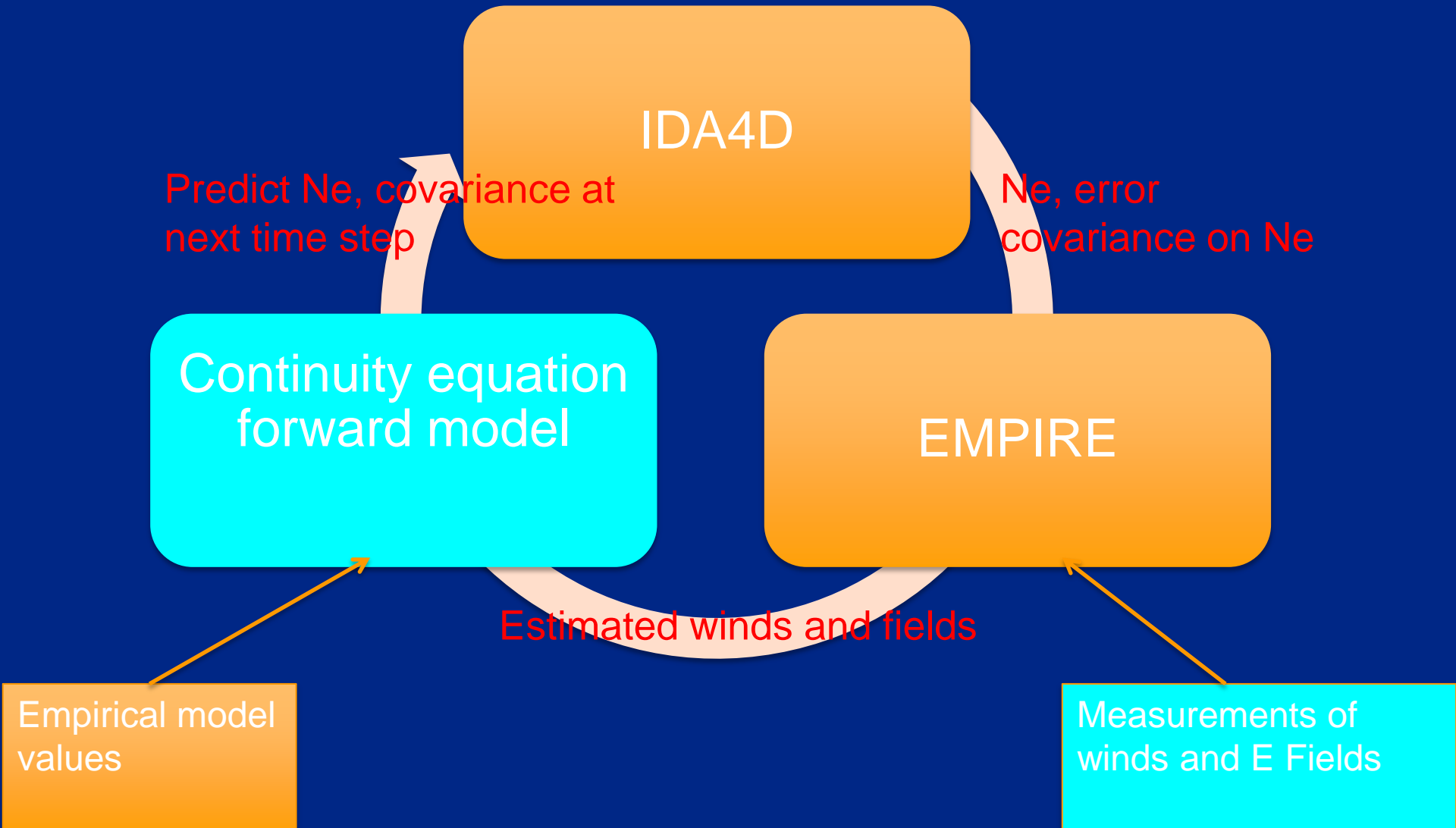


Vertical Drifts vs. Magnetic Latitude and Longitude





Driver adjusted Forward Model Data Assimilation





Summary

- Inverse Theory is simply a way of thinking or reasoning analytically that allows *inferring causes from observations*
- Ionospheric 2D tomography is an implementation and example of inverse theory applied to space weather observations
- 3D time evolving imaging is the generalized global extension of 2D tomography
- Ionospheric Imaging has been used for many scientific as well as applied investigations
- The next level of inverse theory applied to space physics is to *infer the underlying physical drivers from the observations or imaging results*
- Such methods require
 - Observations
 - Knowledge of the physical drivers that are of interest
 - A model to connect the drivers to the observations
 - Some kind of estimation theory to develop a practical algorithm