Optical Tomography in CEDAR Science

Gonzalo Cucho-Padin, Lara Waldrop
Department of Electrical and Computer Engineering – UIUC
Remote Sensing and Space Sciences Group.
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***Everyone is a genius**. But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid.*

- Albert Einstein

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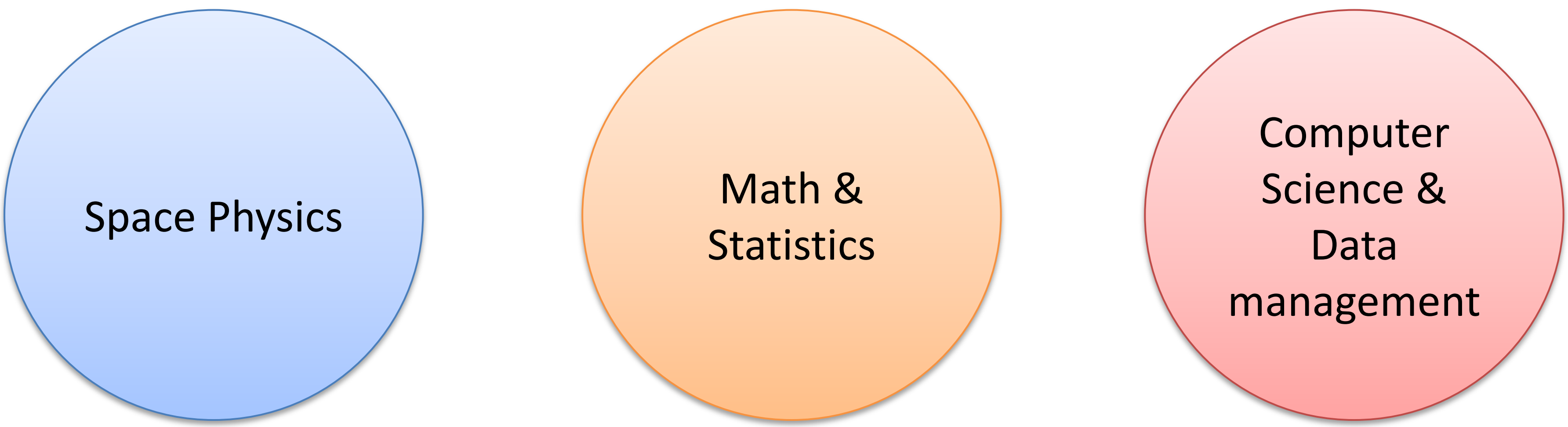
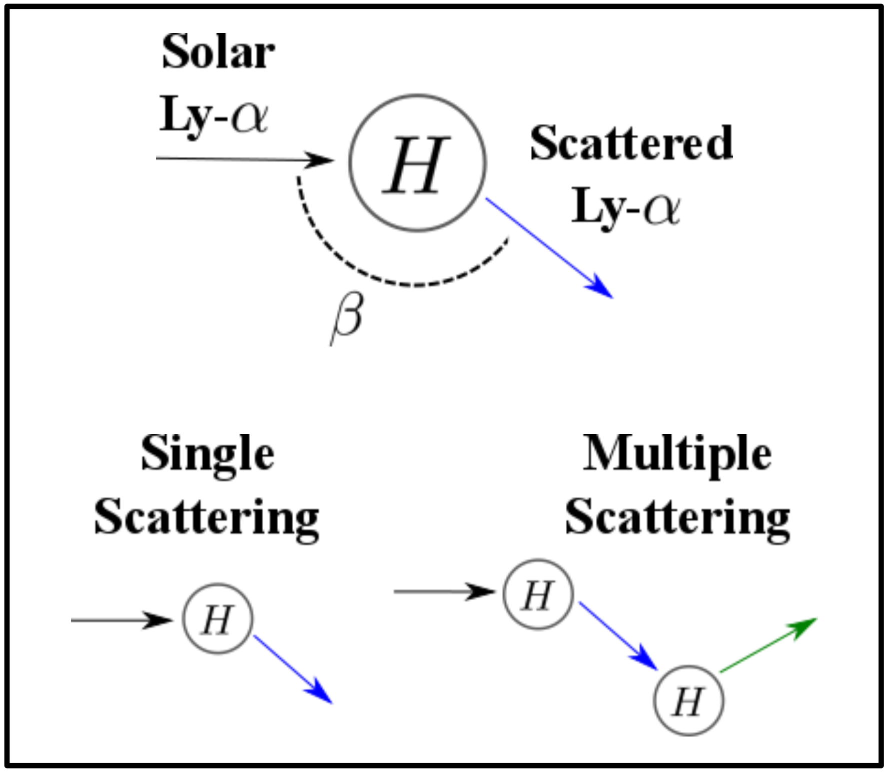
Data science: multi-disciplinary field that aims to extract knowledge from data.

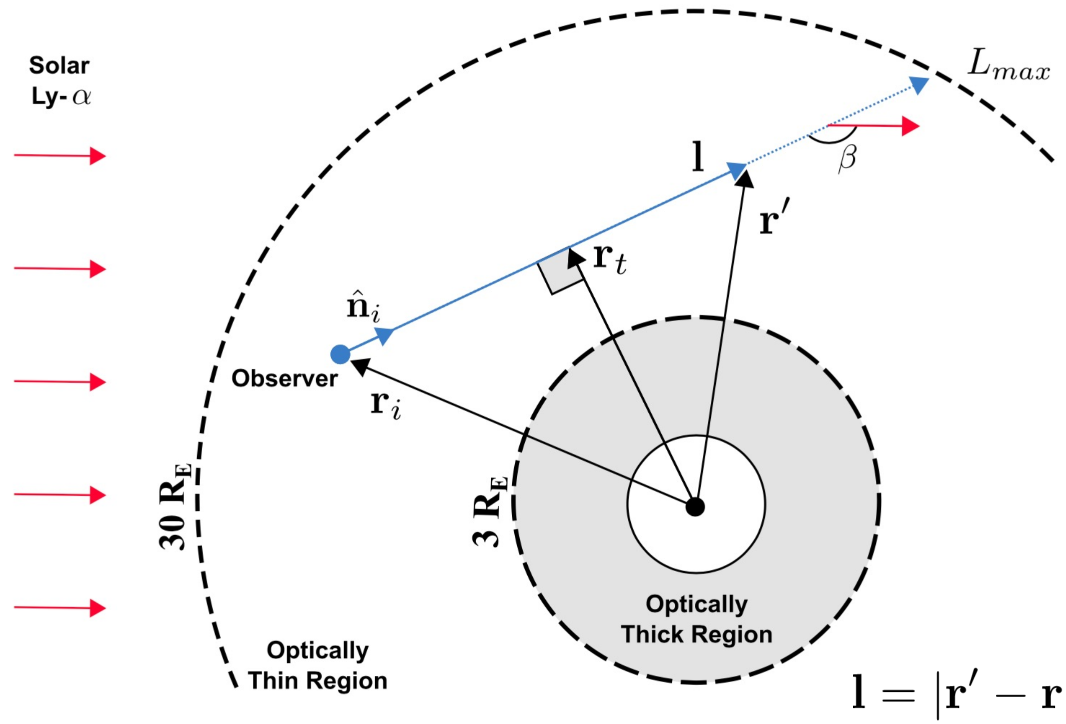
- **Optical tomography** is presented as an example of what we know as data science, multi-disciplinary field that involves, in our case, space physics, math & statistics , computer science (computational resources) and



Space Physics	Math & Statistics	Computer Science & Data management
Exospheric Hydrogen	Signal Processing	Computational resources
Solar-Terrestrial interactions	Tomography	Scientific Computing
Radiative Emission models	Statistical Estimation	Methods to reduce time/resources
Physical Model	**Discrete Model**	**Real Implementation**

Hydrogen density estimation leverages the linearity of the optically thin emission model (>3R$_E$)





$$I(\mathbf{r}, \hat{\mathbf{n}}, t) = \frac{g^*(t)}{10^6} \int_0^{Lmax} n_H(l)\Psi(\beta)dl + I_{IP}(\hat{\mathbf{n}}, t)$$

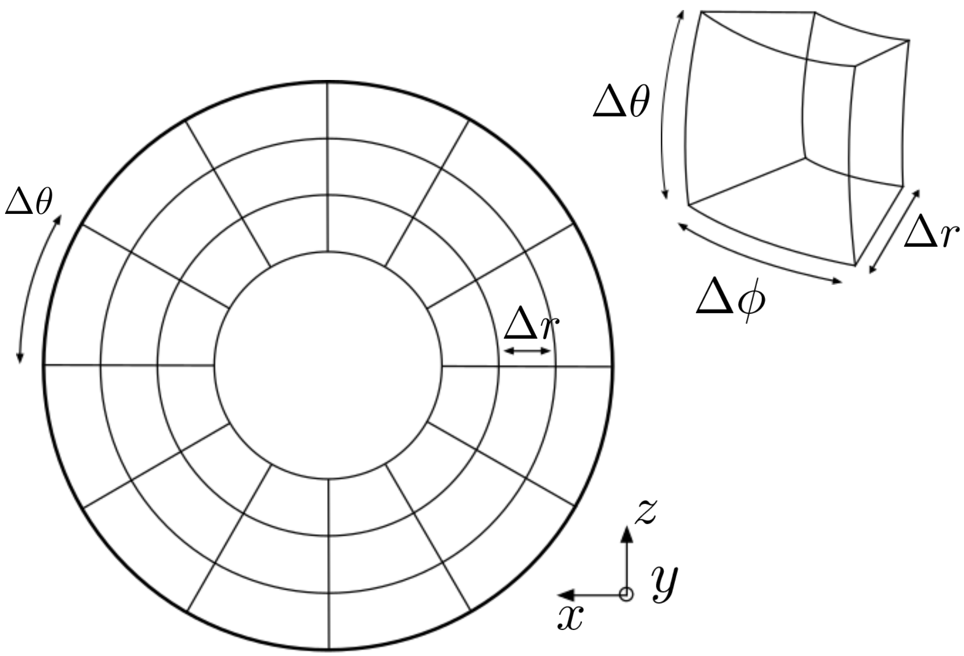
- emission intensity (measured) [R] → $I(\mathbf{r}, \hat{\mathbf{n}}, t)$ — LAD/TWINS
- Ly-alpha resonant scattering rate (measured) [photons.s^{-1}] → $g^*(t)$ — SEE/TIMED
- emitter H density (unknown) [cm^{-3}] → $n_H(l)$
- scattering phase function (known) → $\Psi(\beta)$
- interplanetary Background (measured) [R] → $I_{IP}(\hat{\mathbf{n}}, t)$ — SOHO/SWAN

Discretization of the exospheric volume of interest yields an algebraic linear system.

Math & Statistics

$$I(\mathbf{r}_i, \hat{\mathbf{n}}_i) = \frac{g^*(\mathbf{r}_i)}{10^6} \int_0^{Lmax} n_H(l)\Psi(\hat{\mathbf{n}}_i)dl + I_{IP}(\hat{\mathbf{n}}_i)$$

- **Step 1:** Discretize region into J spherical voxels.



- **Step 2:** Project unknown density function onto J orthonormal basis functions.

$$n_H(r') = \sum_{j=1}^{J} x_j \delta_{H_j}(r'),$$

$$\delta_{Hj}(r') = \begin{cases} 1 & \text{if } r' \in V_j \\ 0 & \text{else} \end{cases}$$

- **Step 3:** Rewrite i^{th} measurement of intensity as a linear equation.

$$y(\mathbf{r}_i, \hat{\mathbf{n}}_i) = \sum_{j=1}^{J} [\frac{g^*(\mathbf{r}_i)}{10^6}\Psi(\hat{\mathbf{n}}_i) \int_0^{Lmax} \delta_{H_j}(l)dl]x_j$$

$$\mathbf{y} = L\mathbf{x}$$

$$\mathbf{y} \in \mathbb{R}^M$$
$$\mathbf{x} \in \mathbb{R}^J$$
$$L \in \mathbb{R}^{M \times J}$$

Solving the estimation problem requires the use of more complex techniques such as regularization

- Observation matrix $L \in \mathbb{R}^{M \times J}$, $M \gg J$ and **is not full column rank** (Voxels with out LOS through them**).**

$$\hat{\mathbf{x}} = \underset{x \geq 0}{\operatorname{argmin}}\, \Phi(\mathbf{x})$$

- Regularization techniques are necessary to obtain a solution.

$$\Phi(\mathbf{x}) = ||L\mathbf{x} - \mathbf{y}||_2^2 + \lambda RRPE(\mathbf{x})$$

Cost Func. — Data misfit term — Regularization term

- The selected regularization method is **Regularized Robust Positive Estimation**.

$$\lambda RRPE(\mathbf{x}) = \lambda_r ||\mathbf{x}||_{D_r} + \lambda_\phi ||\mathbf{x}||_{D_\phi} + \lambda_\theta ||\mathbf{x}||_{D_\theta}$$

Radial dim. — Azimuthal dim. — Polar dim.

- Includes prior knowledge of **physical structure of the Hydrogen density distributions** for each dimension.

$$||\mathbf{x}||_{D_r} = \mathbf{x}^T D_r^T D_r \mathbf{x}$$

Discrete matrix form of 1st and 2nd derivatives

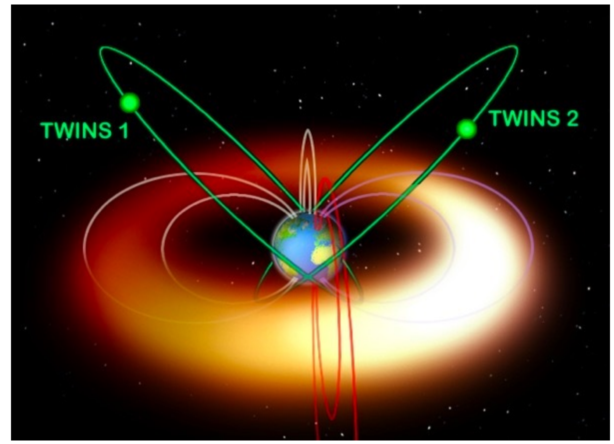
$$D_r \approx \partial^2/\partial r^2$$
$$D_\phi \approx \partial/\partial \phi$$
$$D_\theta \approx \partial/\partial \theta$$

Example of technique feasibility using the NASA's TWINS mission data.

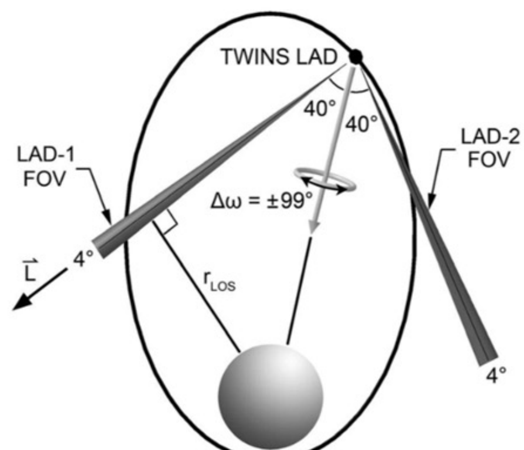
- NASA's Two Wide-angle Imaging Neutral-atom Spectrometers (TWINS) mission provides the capability for **stereoscopically imaging the magnetosphere.**

- Each TWINS1/2 has two **Lyman-alpha detectors (LAD)**, optical sensors.

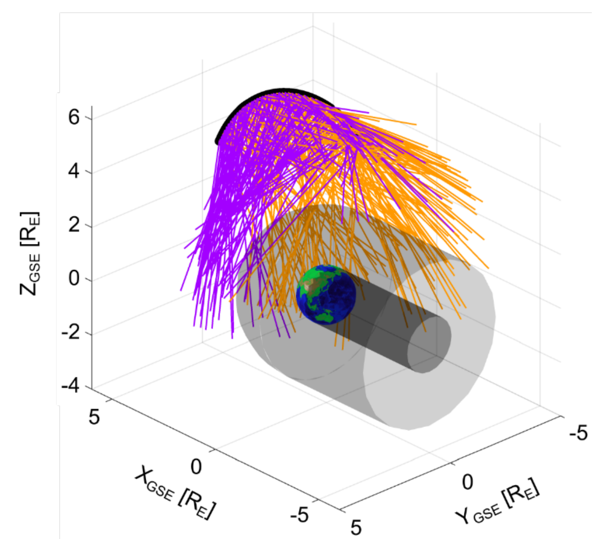
- The selected data in this study is from **11 June 2008**, in order to compare results with those reported by Bailey et al., [2011]



Source: TWINS SWRI website



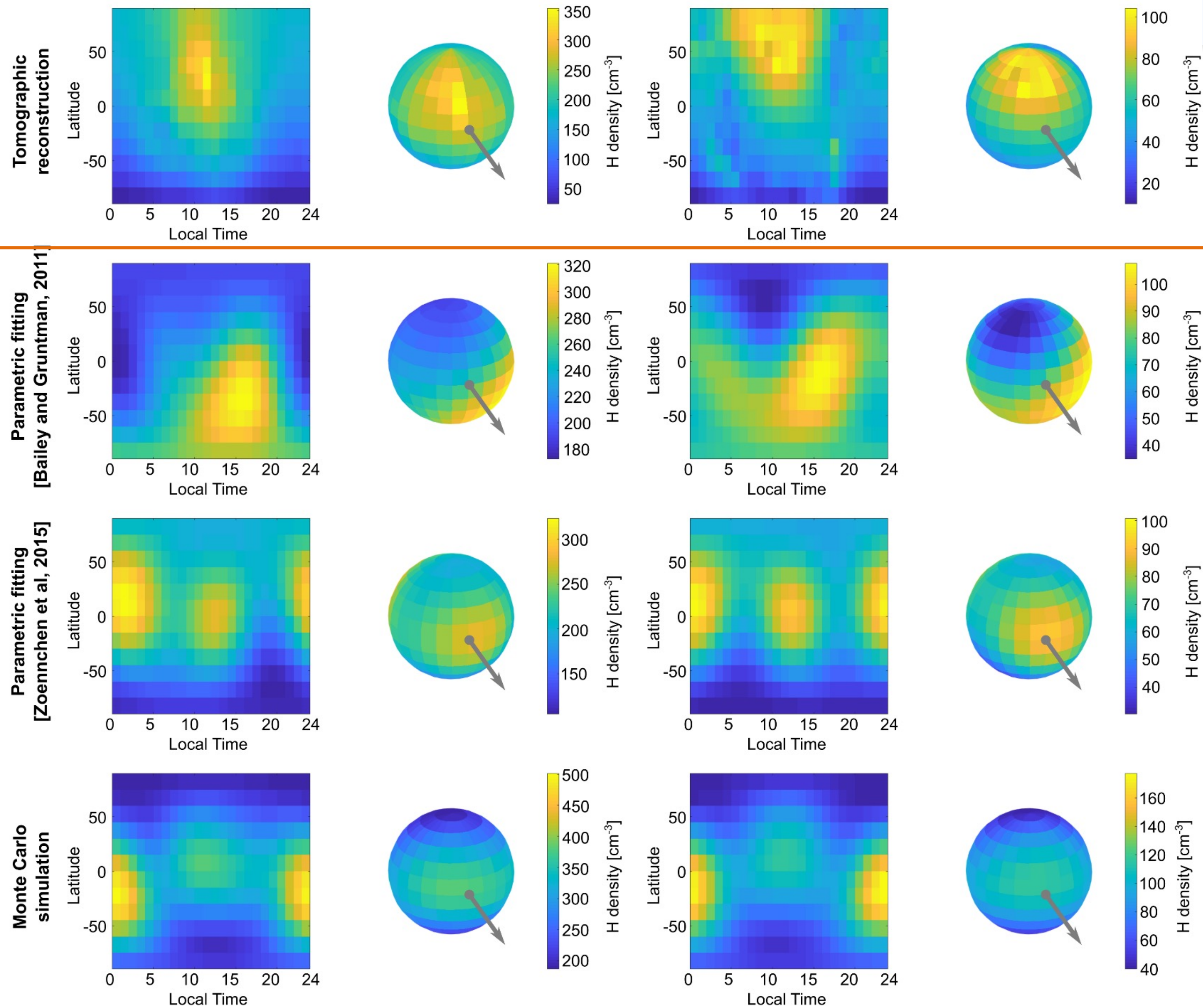
Source: [Bailey et al., 2011]



Space Physics

Radial shell
R = 4.125 RE

Radial shell
R = 6.375 RE



Cucho-Padin and Waldrop, (2018)
Tomographic Estimation of Exospheric Hydrogen Density Distributions, JGR Space Physics

Space-state framework approach for dynamic tomography and Kalman Filter as a solver

As exospheric H densities are prone to be dynamic during storm-time, we use the state-space model as a means for time-varying estimation:

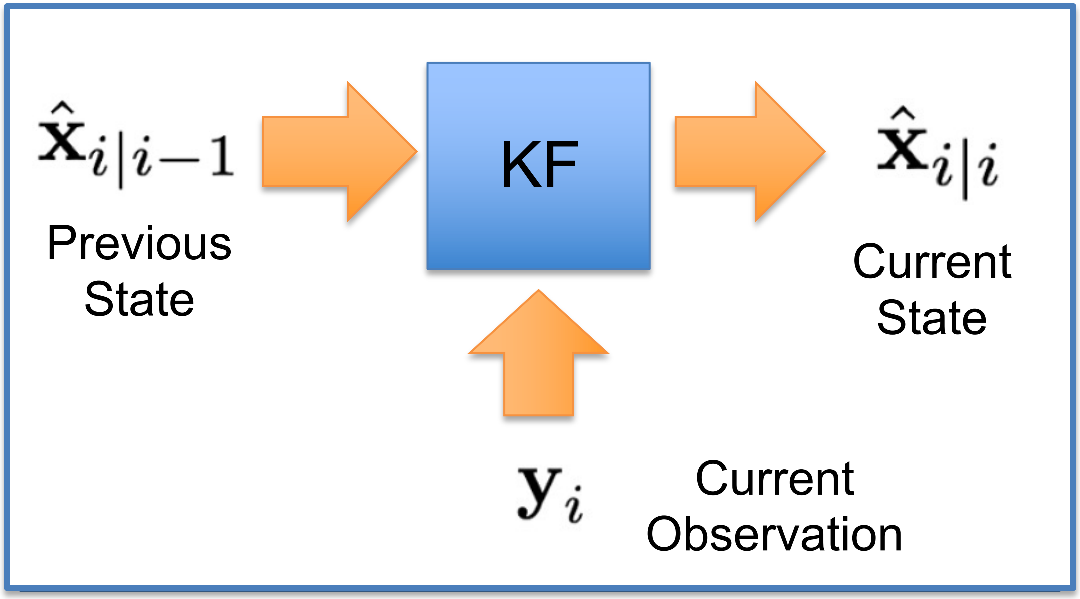
Measurement equation:

$$\mathbf{y}_i = H_i \mathbf{x}_i + \mathbf{v}_i$$

Model evolution equation:

$$\mathbf{x}_{i+1} = F_i \mathbf{x}_i + \mathbf{u}_i$$

Kalman Filter as solver



Inclusion of regularization terms

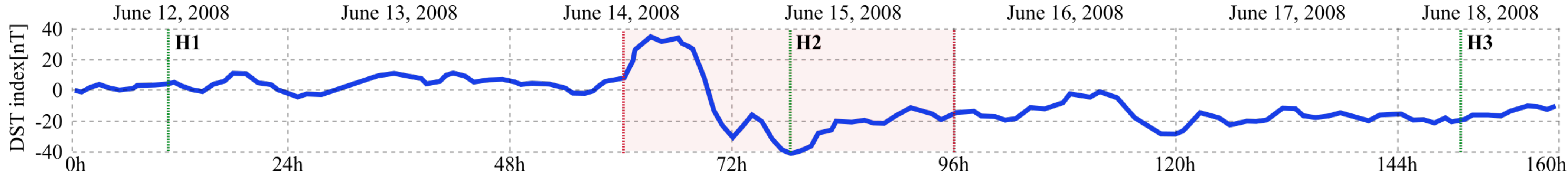
$$\begin{bmatrix} \mathbf{y}_i \\ 0 \end{bmatrix} = \begin{bmatrix} H_i \\ D_i \end{bmatrix} \mathbf{x}_i + \begin{bmatrix} \mathbf{v}_i \\ \mathbf{w}_i \end{bmatrix}$$

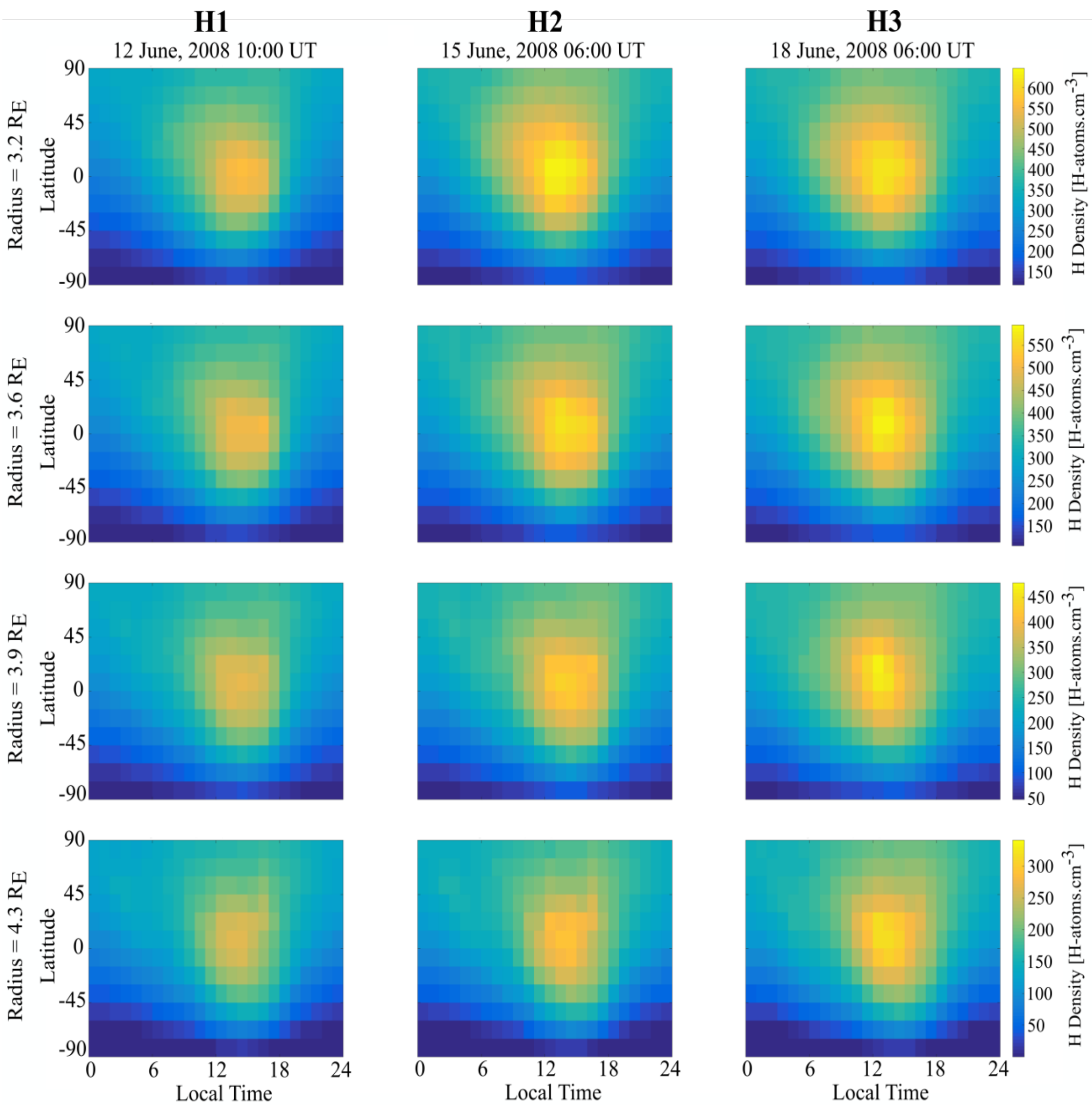
$$\mathbf{y}'_i = H'_i \mathbf{x}_i + v'_i$$

$$R'_i \triangleq \mathbb{E}[\mathbf{v}'_i (\mathbf{v}'_i)^T] = \begin{bmatrix} R_i & 0 \\ 0 & \lambda_i^{-1} I \end{bmatrix}$$

Dynamic tomographic estimation connected to the LMMSE estimation

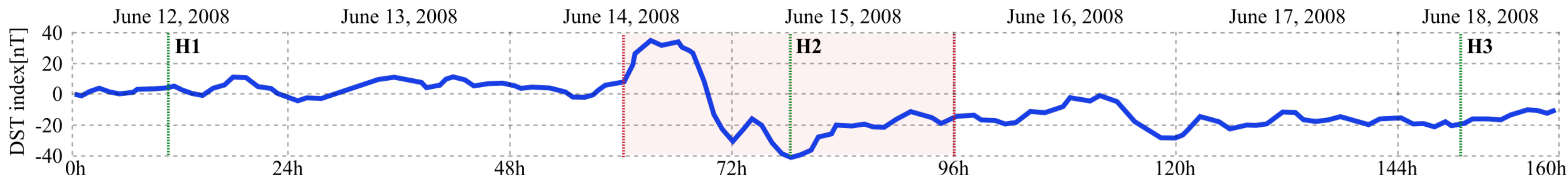
$$\hat{\mathbf{x}}^d_{i|i} = \underset{\mathbf{x}_i}{\operatorname{argmin}} ||\mathbf{y}'_i - H'_i \mathbf{x}_i||^2_{R'^{-1}_i} + ||\mathbf{x}_i - \hat{\mathbf{x}}_{i|i-1}||^2_{P^{-1}_{i|i-1}} + \lambda_\phi ||D_\phi \mathbf{x}_i||^2_2 + \lambda_\theta ||D_\theta \mathbf{x}_i||^2_2 + \lambda_r ||D_r \mathbf{x}_i||^2_2$$

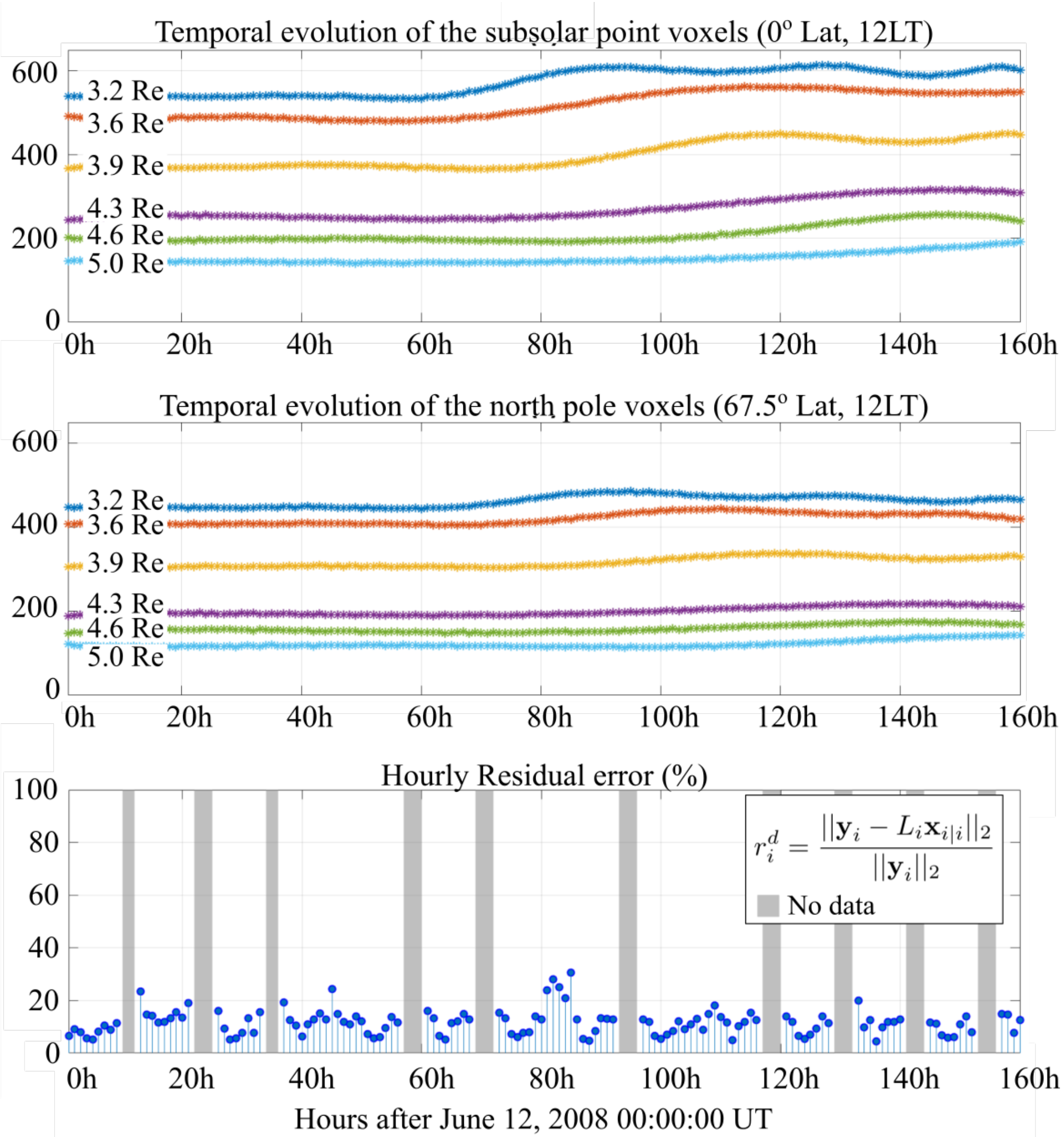




- ⊙ Using KF we have perform 160 dynamic reconstructions during the storm occurred in 15, June, 2008.
- ⊙ Hydrogen density enhancements has been seen as the storm develops. Such increments are then translate to higher altitudes with certain delay which suggest a vertical transport or upwelling.
- ⊙ Hydrogen densities at altitudes greater than 5Re remain constant. It suggest that the increment is related to the ballistic and escape populations.

Space Physics





⊙ Two voxel columns with the highest LOS coverage has been selected to analyze the temporal evolution of geocoronal H density.

⊙ In the subsolar point, simple calculations between 3.2Re and 3.9 Re profiles show a exospheric wind of ~60m/s.

⊙ Similar structure can be seen in those voxels near Geographic North Pole.

⊙ Hourly residual error is presented and has an average value of ~11%.

Space Physics

Difficulties working with data.

⦿ **Data cleaning:** TWINS data contains several values with non-physical interpretation for unknown reasons. *Possible solution*: Machine learning techniques to classify non-physical behavior in the acquired signal.

⦿**Computational resources**: Several tools enable us to reduce the computational time. Although Kalman Filter is a sequential procedure, the internal calculations (mainly matrix-vector operations) can easily be implemented in dedicated hardware such as GPU's, or with specialized libraries such as MKL (Math Kernel Library from Intel)

⦿ **Dynamic Storage**: Using both TWINS satellites during stereoscopic events will provide a great amount of data. The increase of spatial and even temporal resolution can be performed. However, increasing the number of voxel *N* involve the size increment of covariance matrices N^2 . Possible solution: Ensemble Kalman Filter (EnKF). It uses *L* samples of a the random process that could keep the internal statistical properties (variance). With L<<N, the used storage would be [L x N] instead of [N x N].

THANK YOU