Quick Comment on the Composition Equation

Eric Sutton

Continuity equation:
w/ altitude as vertical coordinate

Species *i*:

$$\frac{\partial \psi_i \rho}{\partial t} + \nabla \cdot \left(\psi_i \rho \vec{U}_i\right) = P_i - L_i$$

Total gas:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{U}\right) = 0$$

$$\frac{\partial \psi_i \rho}{\partial t} + \nabla \cdot \left[\psi_i \rho \left(\vec{U}_i - \vec{U}\right)\right] = P_i - L_i - \underbrace{\nabla \cdot \left(\psi_i \rho \vec{U}\right)}_{\psi_i \nabla \cdot \left(\rho \vec{U}\right) + \rho \vec{U} \cdot \nabla \psi_i}$$

$$\psi_i \nabla \cdot \left(\rho \vec{U}\right) \rightarrow -\frac{\partial \rho}{\partial t}$$

$$\therefore \quad \rho \frac{\partial \psi_i}{\partial t} = -\nabla \cdot \left(\psi_i \rho \vec{C}_i\right) + P_i - L_i - \rho \vec{U} \cdot \nabla \psi_i$$

Continuity equation:
w/ altitude as vertical coordinate

$$\rho \frac{\partial \psi_i}{\partial t} = -\nabla \cdot \left(\psi_i \rho \vec{C}_i\right) + P_i - L_i - \rho \vec{U}_H \cdot \nabla \psi_i - \rho W \frac{\partial \psi_i}{\partial z}$$

vs

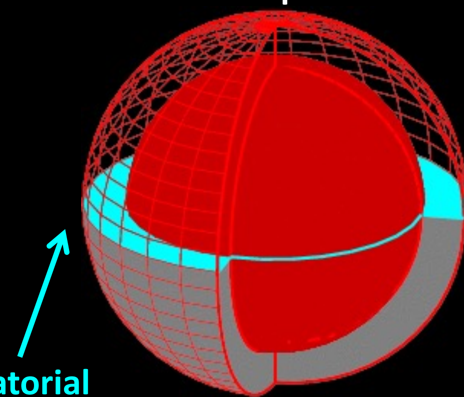
$$\rho \frac{\partial \psi_i}{\partial t} = -\nabla \cdot \left(\psi_i \rho \vec{C}_i\right) + P_i - L_i - \nabla\left(\psi_i \rho \vec{U}_H\right) - \frac{\partial}{\partial z}(\psi_i \rho W)$$

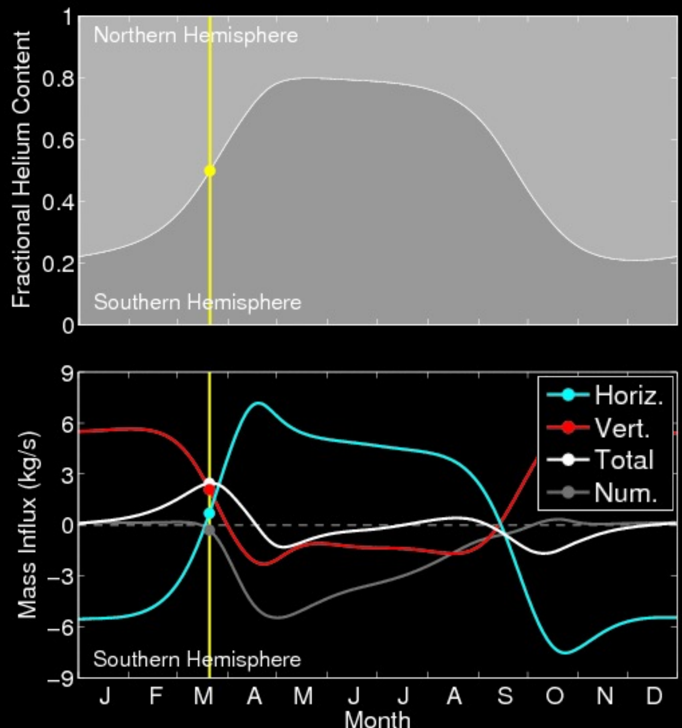
Transport Mechanisms
Numerical Experiment

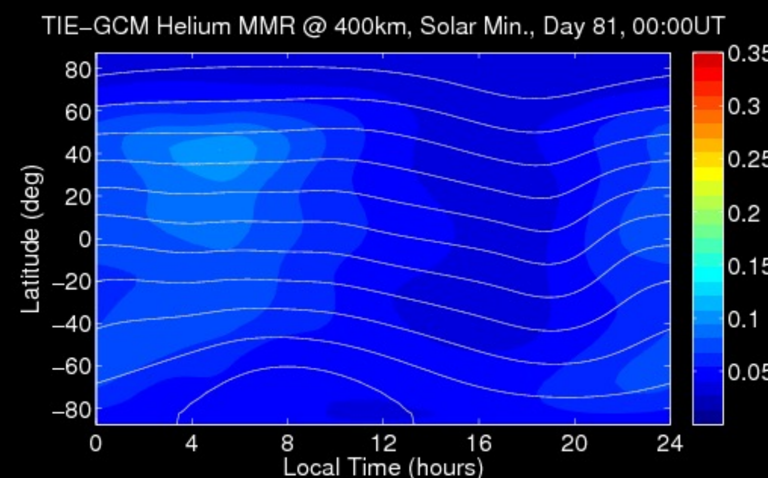
Hemispheric mass conservation equation:

$$\frac{\partial}{\partial t} M_{He} = \frac{p_0}{g}\int_S \psi_0 \omega_0 dS - \frac{p_1}{g}\int_S \psi_1 \omega_1 dS - \frac{1}{g}\int_V p\nabla \cdot (\psi \vec{V}) dV$$

Helium Mass Rate = **Vertical Mass Influx*** + **Horizontal Mass Influx**







*ω terms include background and diffusive vertical velocities

Transport Mechanisms

Numerical Experiment

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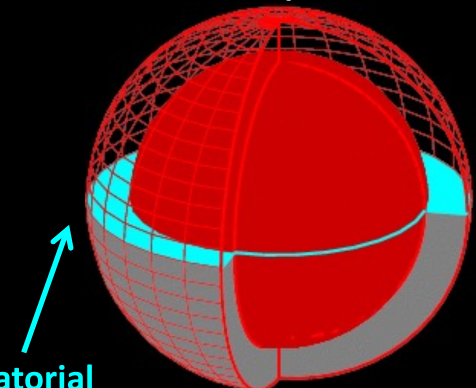

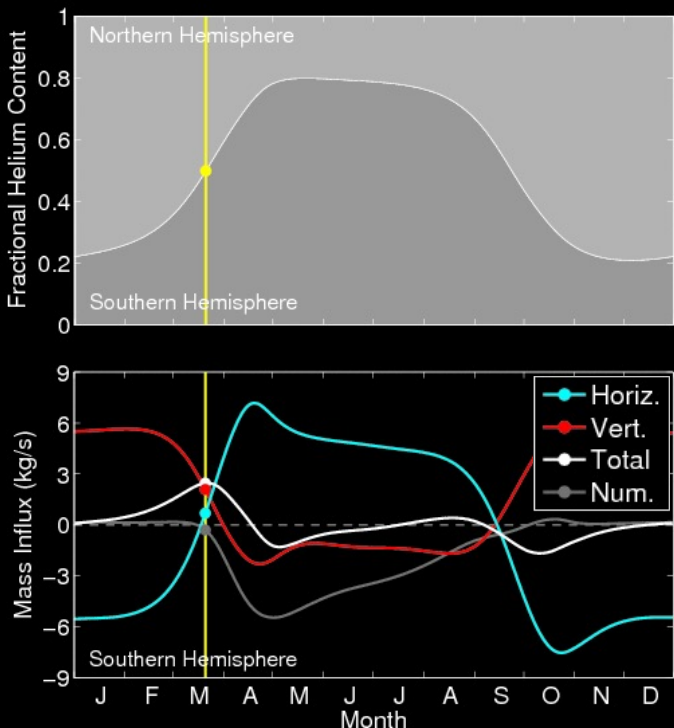

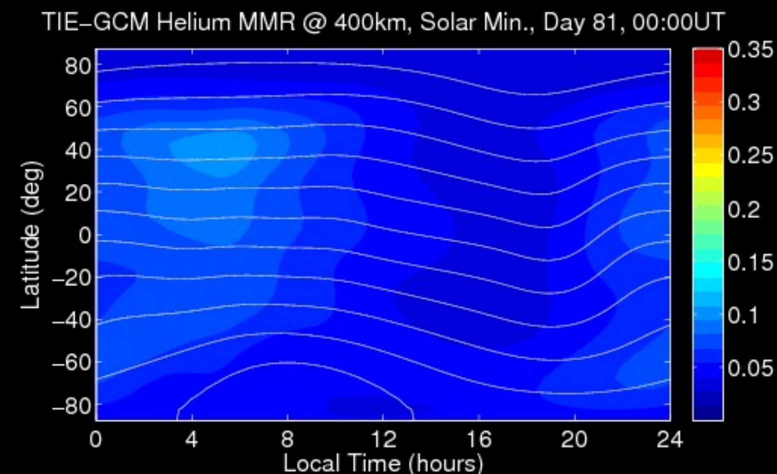

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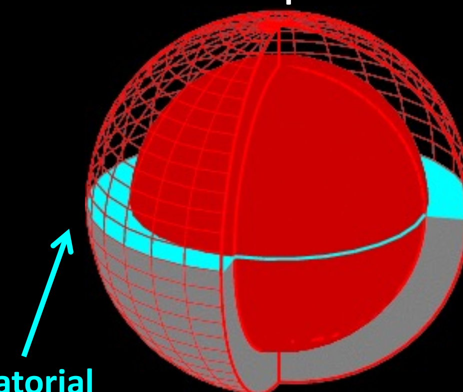
Transport Mechanisms

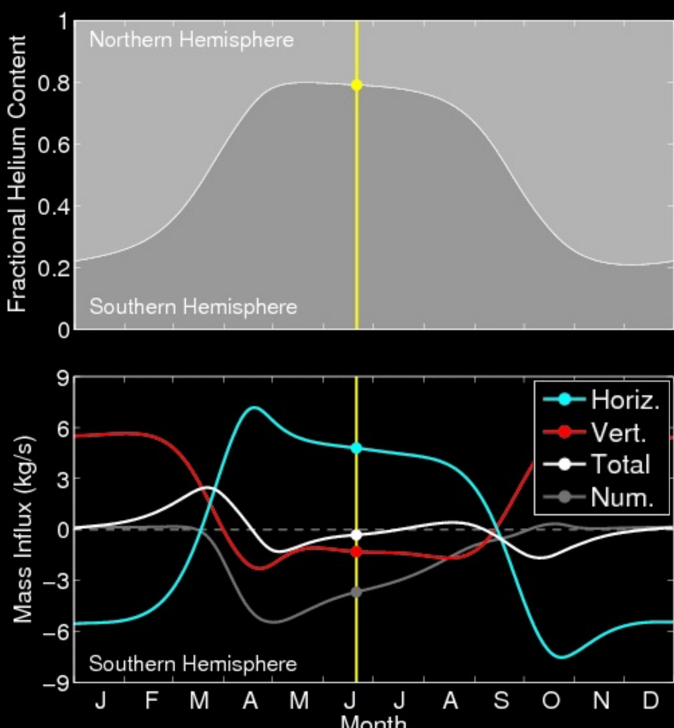
Numerical Experiment

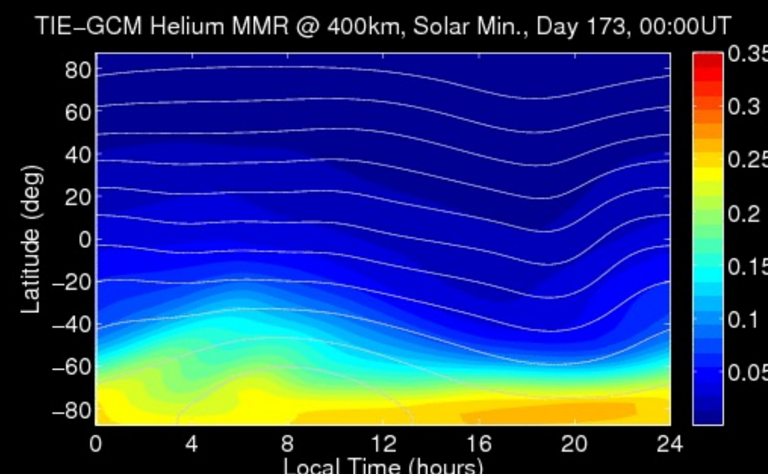
Hemispheric mass conservation equation:

$$\frac{\partial}{\partial t} M_{He} = \frac{p_0}{g} \int_S \psi_0 \omega_0 dS - \frac{p_1}{g} \int_S \psi_1 \omega_1 dS - \frac{1}{g} \int_V p \nabla \cdot (\psi \vec{V}) dV$$

Helium Mass Rate = **Vertical Mass Influx*** + **Horizontal Mass Influx**







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Continuity equation:
w/ log-pressure as vertical coordinate

Species i:

$$\frac{\partial \psi_i p}{\partial t} + \nabla \cdot \left(\psi_i p \vec{U}_i\right) = P_i - L_i$$

Total gas:

$$0 = \nabla \cdot \left(p\vec{U}\right)$$

$$p\frac{\partial \psi_i}{\partial t} + \nabla \cdot \left[\psi_i p\left(\vec{U}_i - \vec{U}\right)\right] = P_i - L_i - \nabla \cdot \left(\psi_i p \vec{U}\right)$$

$$\psi_i \nabla \cdot \left(p\vec{U}\right) + p\vec{U} \cdot \nabla \psi_i$$

$$0$$

$$\therefore \quad p\frac{\partial \psi_i}{\partial t} = -\nabla \cdot \left(\psi_i p \vec{C}_i\right) + P_i - L_i - p\vec{U} \cdot \nabla \psi_i$$

- Continuity equation overview
- Tracking flow across boundaries
  - Advection vs. divergence
- Role of ideal gas law vs. vertical coordinate
- Example of use