Reduced-order modeling for data assimilation in geospace applications

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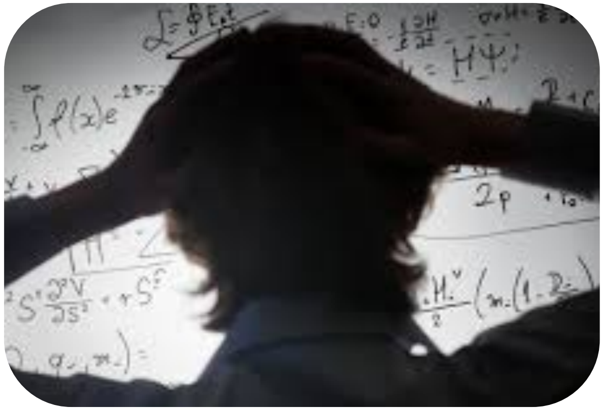
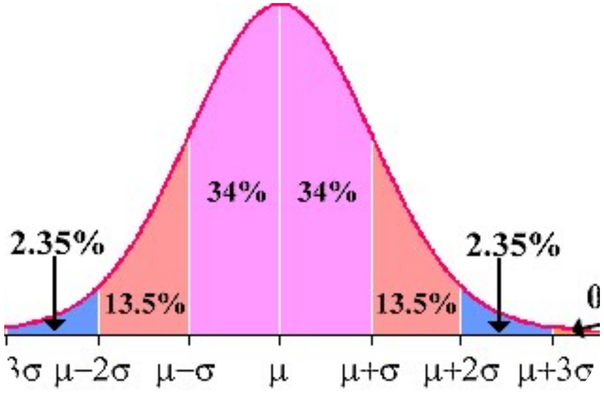
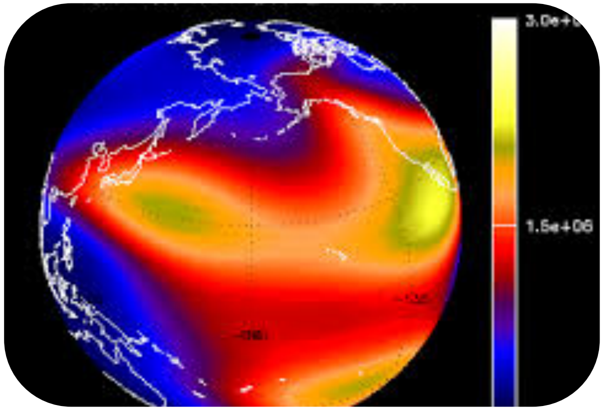
Paradigms of (Geospace) Science



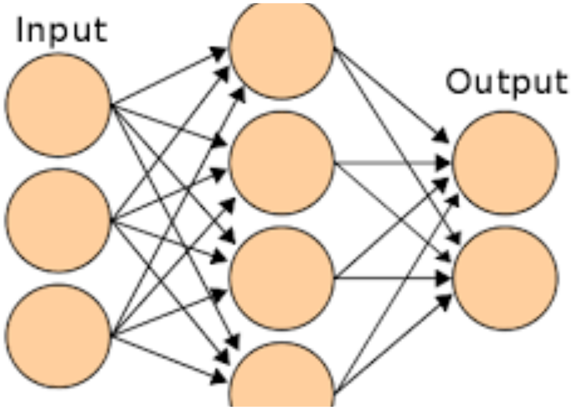
Theoretical



Empirical



Computational



Data-Driven

Case Study: Thermosphere-Ionosphere

Theoretical

- Navier-Stokes (NS)
- Based in Physical Laws
- (NS + Maxwell = MHD)
- No closed-form solutions: Numerical Solution Techniques

Empirical Models

- Empirical correlations between driver and state
- Simple and effective
- Poor prediction capabilities

Computational

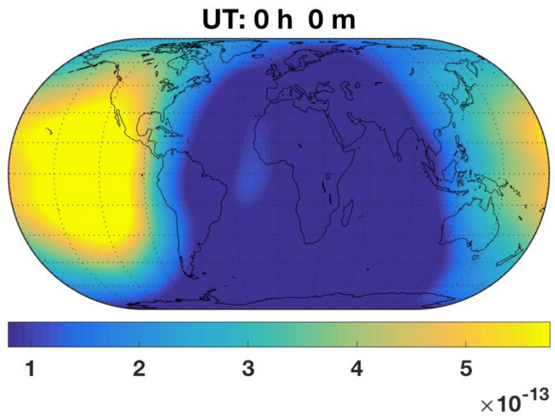
- Physical models with high fidelity
- Dynamical formulation with good potential for prediction
- Complexity

Data-Driven

- Union of statistics, applied mathematics, information and computer science
- Can be powerful with good methodology
- Non-trivial for extracting scientific information

Reduced Order Modeling methodology

- ***Prediction or Forecast is the holy grail!!***
- Not ideal to abandon heritage models for a data-only (black-box) methodology
  - Embed within them a wealth of knowledge
  - Dynamic (physical) models have inherent predictive capabilities



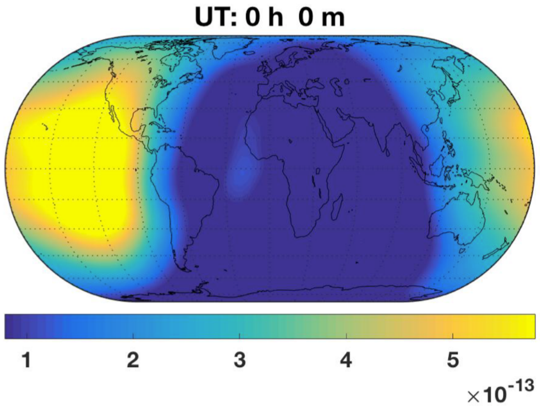
Chaos due to geomagnetic activity

- Use a dynamic systems formulation to address Next generation CEDAR science

Combines the paradigms for efficient and effective prediction and science extraction

Complexity

- Simplified model through efficient data compression
  - New *“Hermitian Space - Dynamic Mode Decomposition method” (Space Weather, 2018)* for compressing large amounts of data
  - Preserve predictive capabilities



12 years of TIE-GCM simulations

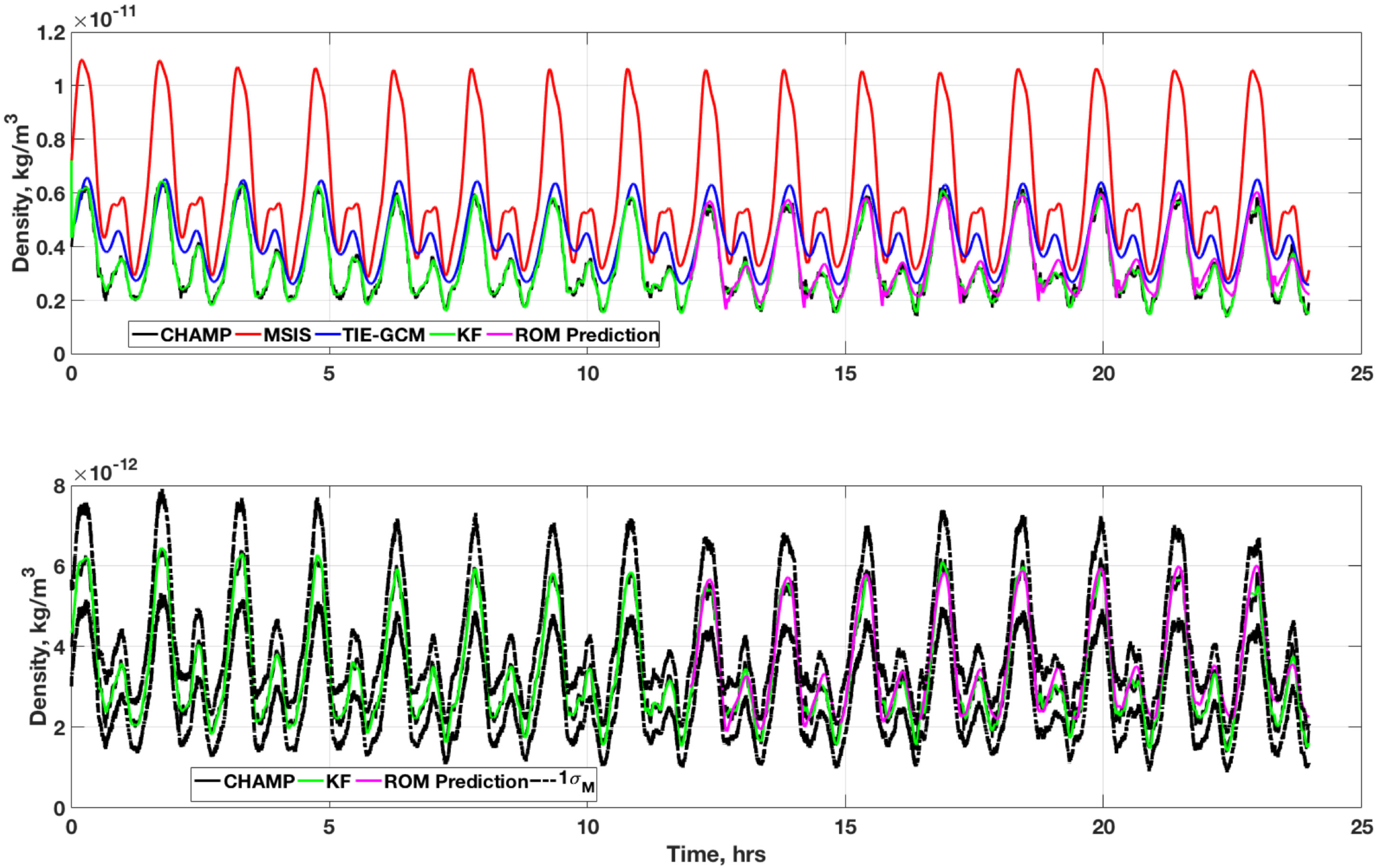
$$\mathbf{x}_{k+1} = \boldsymbol{A}\mathbf{x}_k + \boldsymbol{B}\mathbf{u}_k$$

- Dimensionality Reduction by projecting onto orthogonal subspace
  - The large number of degrees of freedom can be reduced using set of coherent structures or proper orthogonal decomposition (POD modes or EOFs)

$$\mathbf{x}_{k+1} = \boldsymbol{A}\mathbf{x}_k + \boldsymbol{B}\mathbf{u}_k \xrightarrow{\mathbf{z} = \boldsymbol{U}\mathbf{x}} \mathbf{z}_{k+1} = \widetilde{\boldsymbol{A}}\mathbf{z}_k + \widetilde{\boldsymbol{B}}\mathbf{u}_k$$

Complexity

- The methodology also provides simplified framework for data assimilation
  - *A new transformative framework for data assimilation and calibration of physical ionosphere-thermosphere models (Space Weather, 2018)*



- Model Error
- Time-Step
- Initialization
- Calibration

Complexity

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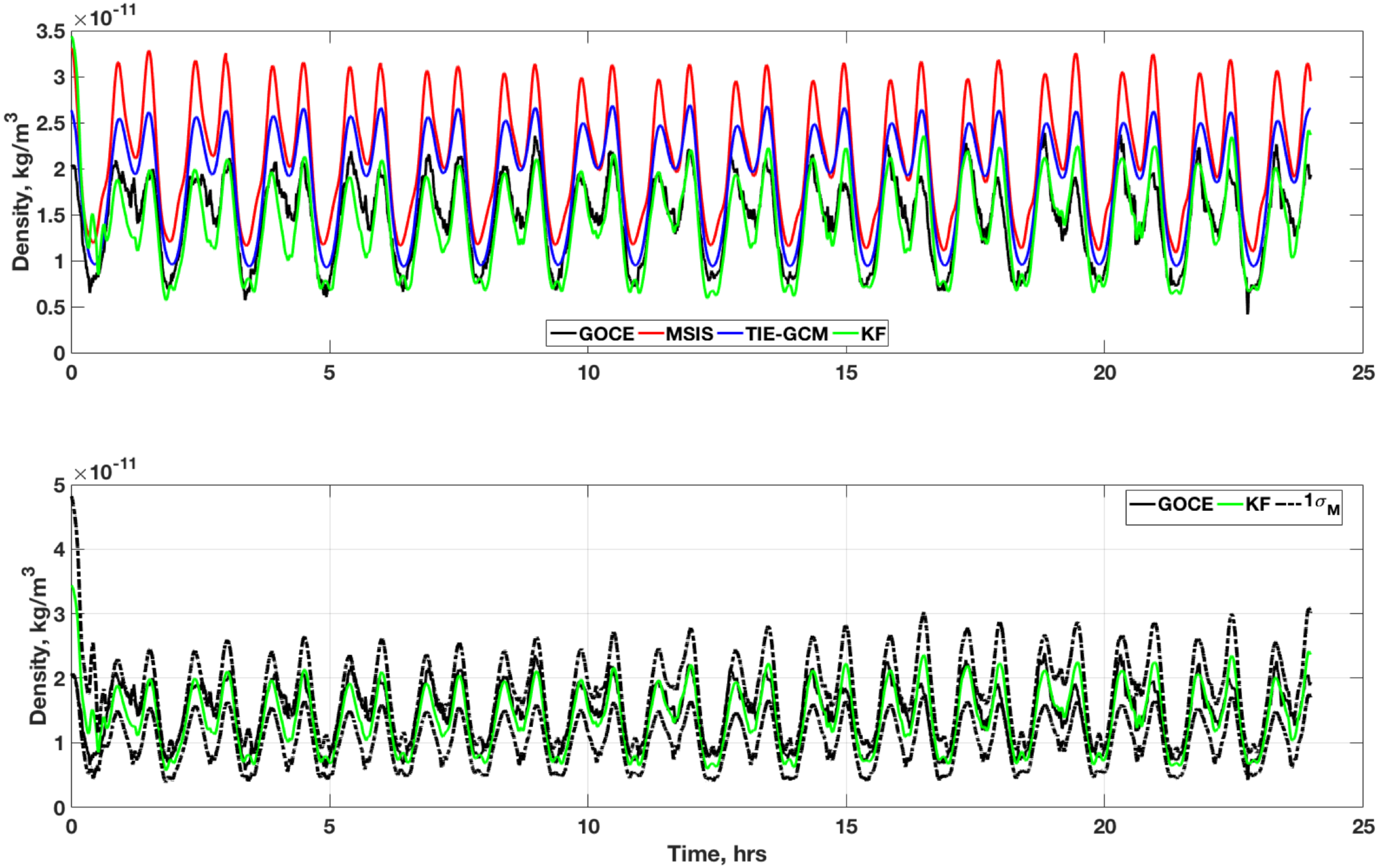
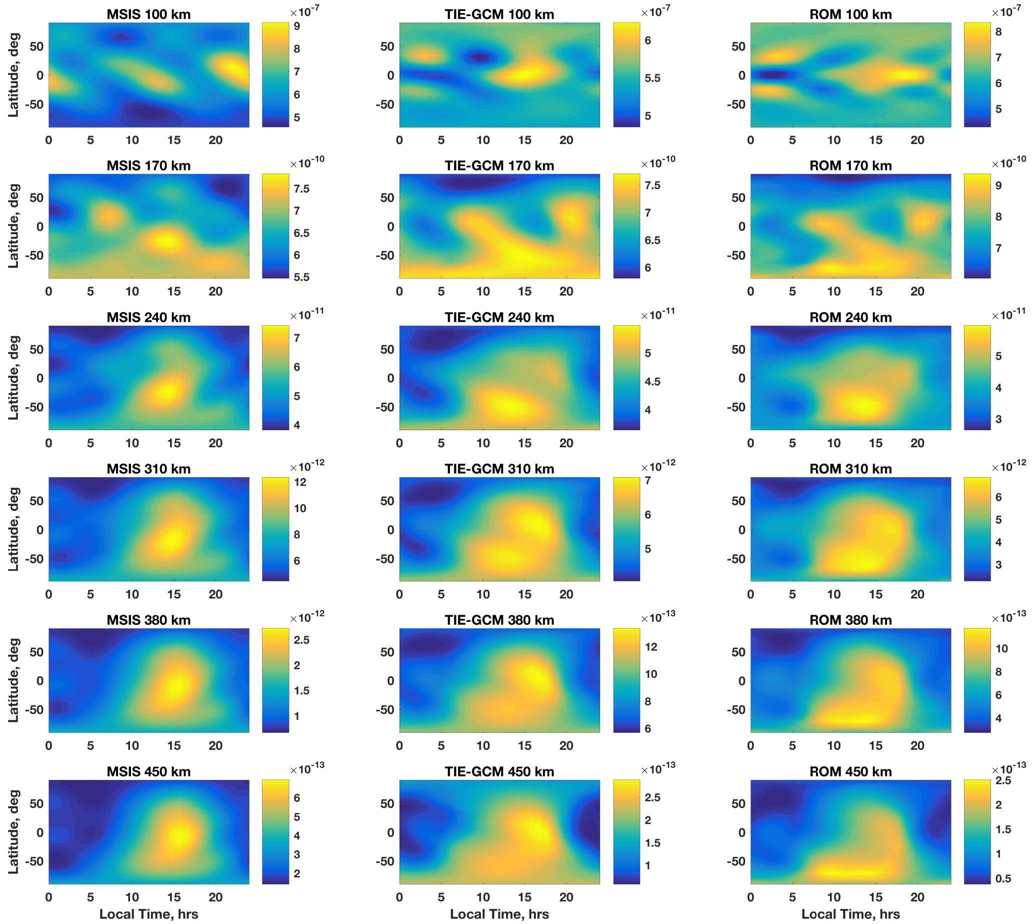
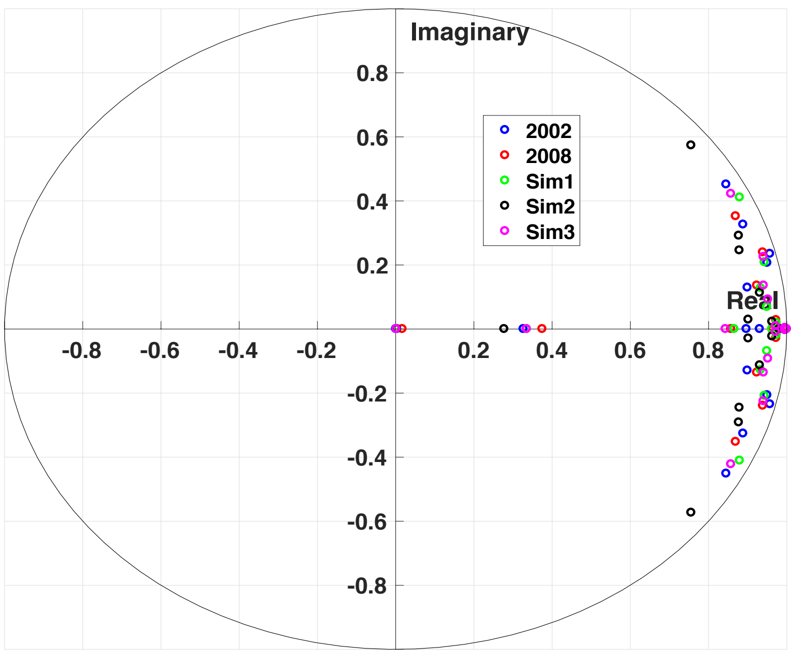


Science Insights

- The methodology also has promising potential for providing insights into the dynamics and coupling
  - Fine-tune the processes
  - Spectral decomposition of the dynamics

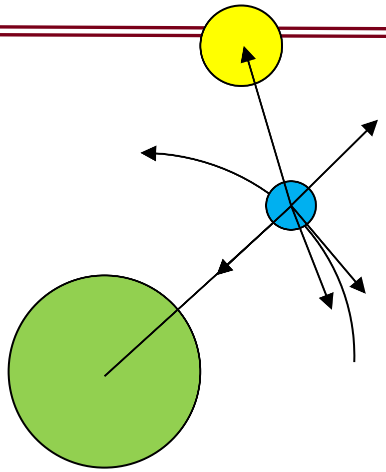


$\mathrm{eig}(\boldsymbol{A})$ or $\mathrm{eig}(\tilde{\boldsymbol{A}})$



Data Quality and Consistency

$$a_{drag} = a_{total} - a_{grav} - a_{rad}$$

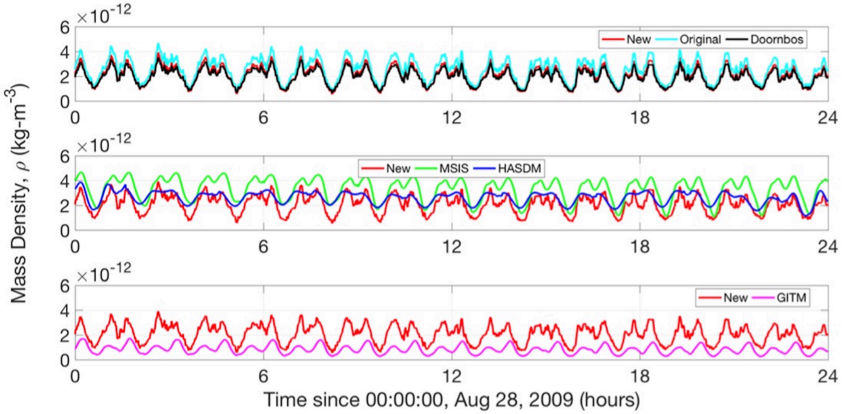


Drag Acceleration Measurements

+

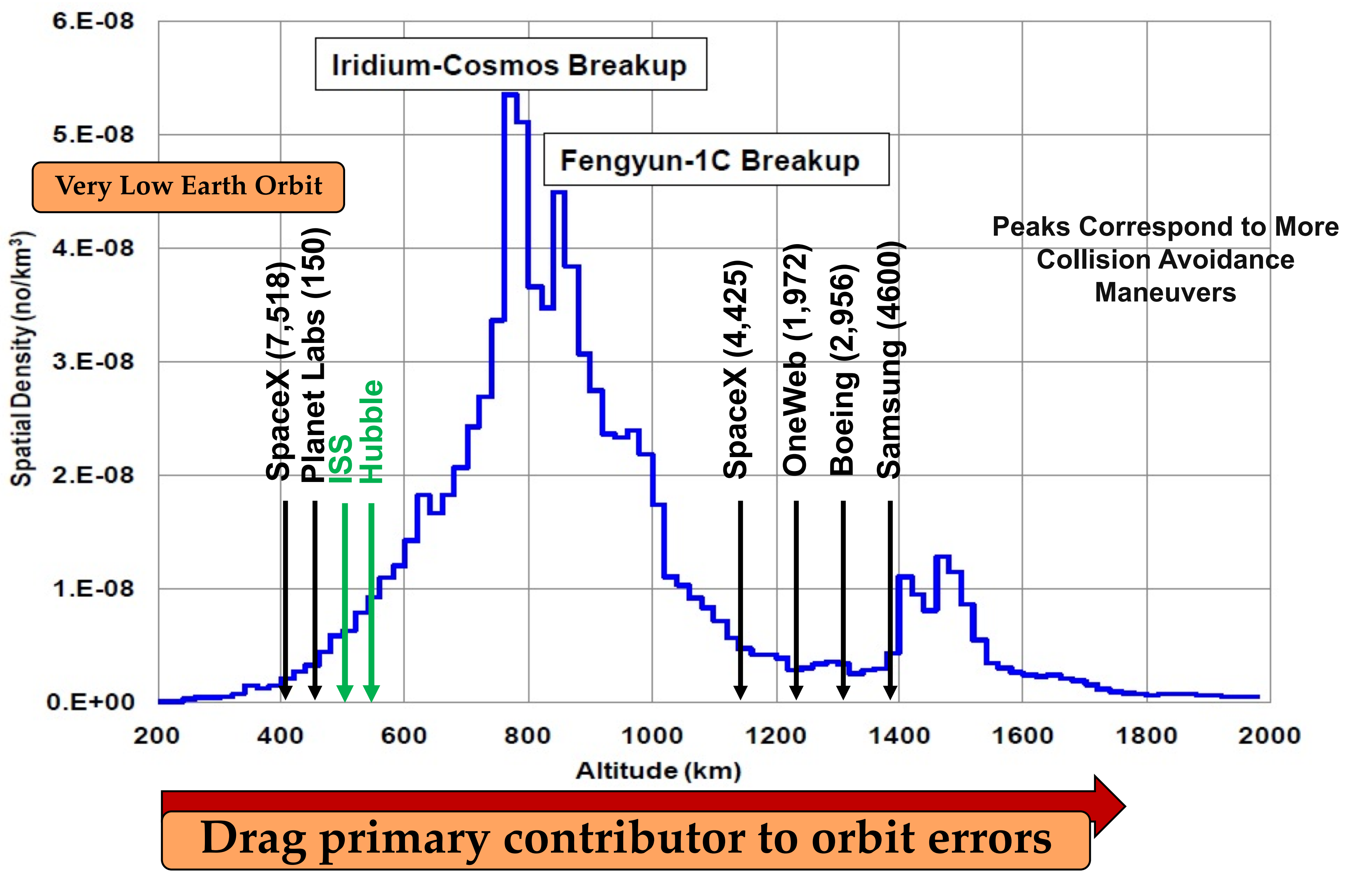
Composition and Temperature from (empirical model)

→ **Density Estimates**



The methodology also has the potential to provide self-consistent data by estimating the force model parameters in a dynamic formulation.

Data Availability



Thank you!!