k-fold Cross-Validation Applied to an Assimilative Mapping Analysis of SuperDARN and SuperMAG

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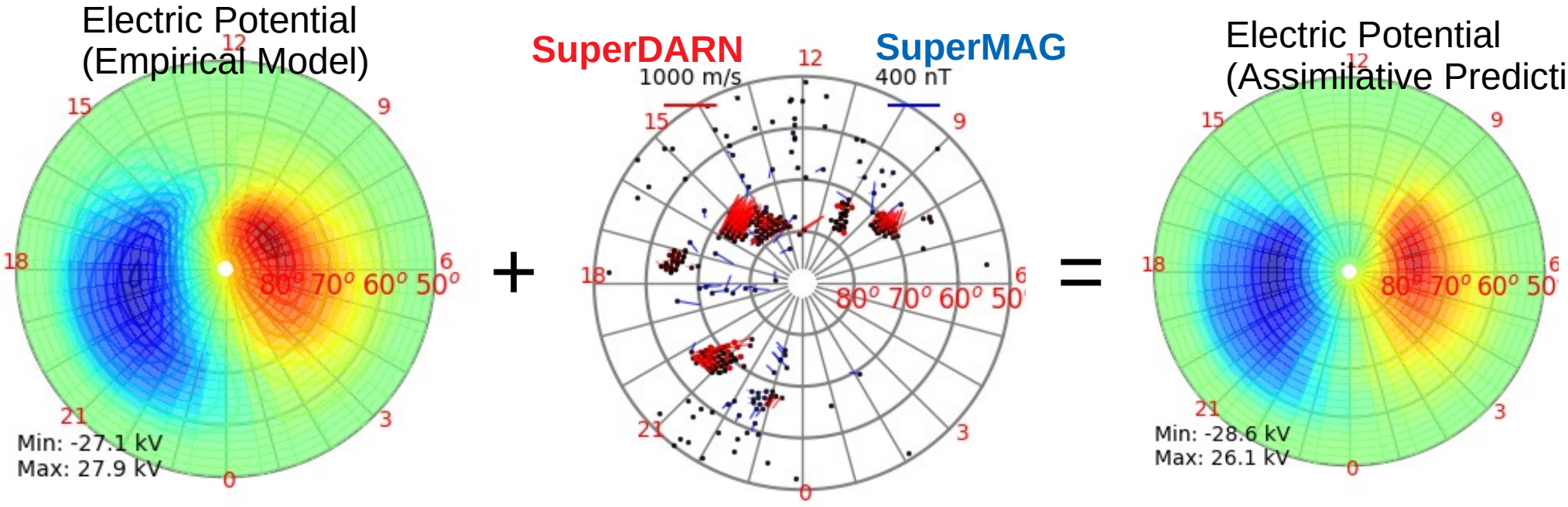
•Cross-validation is a robust approach to assess the goodness-of-fit of assimilative analysis to a data set that is independent of the data used in data assimilation

•We applied 4-fold cross validation to our assimilative analysis incorporating SuperDARN and SuperMAG observations

•We can also predict the observation variables (ion drift velocity for SuperDARN, magnetic pertubations for SuperMAG), instead of electric potential



Assimilative Mapping Algorithm

Background model (CS10 Electric Potential)

Background model evaluated at grid locations, *i*

$$\Phi_{CS10} = \Psi_{[n_i, 244]}\, x_b$$

Model is regressed onto basis functions (psi) to get initial state vector x_b

Observations (SuperMAG Magnetic Perturbation, SuperDARN Drift Velocity)

Observations at sampled locations, *j*

$$y \approx H_{[n_j, 244]}\, x$$

Linear operator **H** relates observation quantities (ground magnetic perturbation, ion drift velocity) to modeled quantity (potential)

Optimal Interpolation / Kalman Update

Calculate Gain

$$K_{[244, n_j]} = C_b H^T \left(H C_b H^T + C_r \right)^{-1}$$

Update State

$$x_a = x_b + K \left(y - H x_b \right)$$

Update Covariance

$$C_{a[244,244]} = \left(I - K H \right) C_b$$

Resulting Electric Potential and Error Covariance

$$\Phi_{OI} = \Psi_{[n_i, 244]}\, x_a \qquad COV\left(\Phi_{OI}\right)_{[n_i, n_i]} = \Psi C_a \Psi^T$$

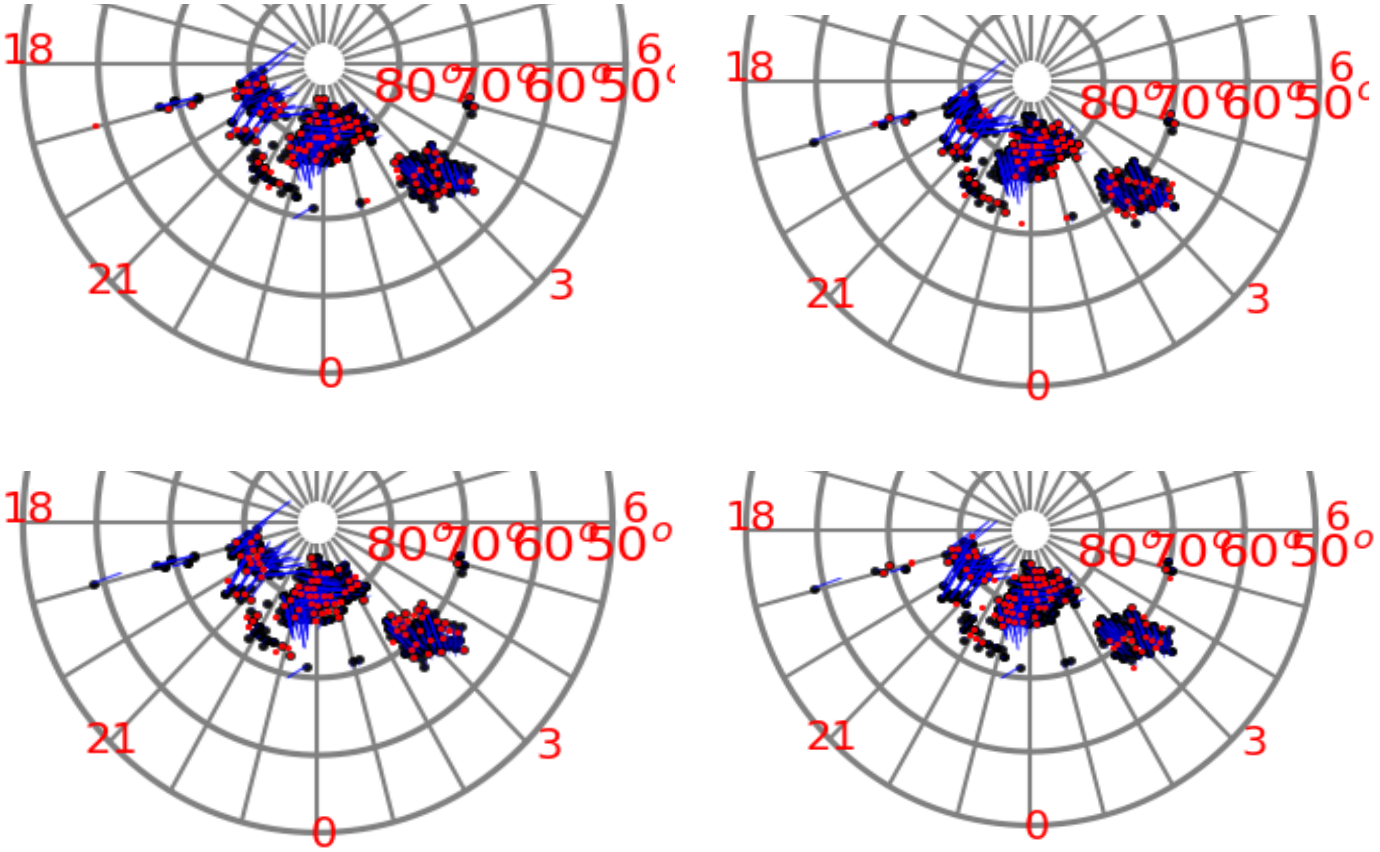
Kilcommons and Matsuo, 2018 (in prep), Cousins et al. 2013,2015, Richmond and Kamide, 1988

4-fold Cross Validation

1) Pick 75% of the available data and use it in the assimilative analysis

2) Obtain assimilative analysis results for magnetic perturbations and ion drift and compare to remaining 25% of the observations (calculate residual)

3) Repeat until have predicted every observation exactly once (4 times)



Four minutes of gridded SuperDARN data (~300 observations)

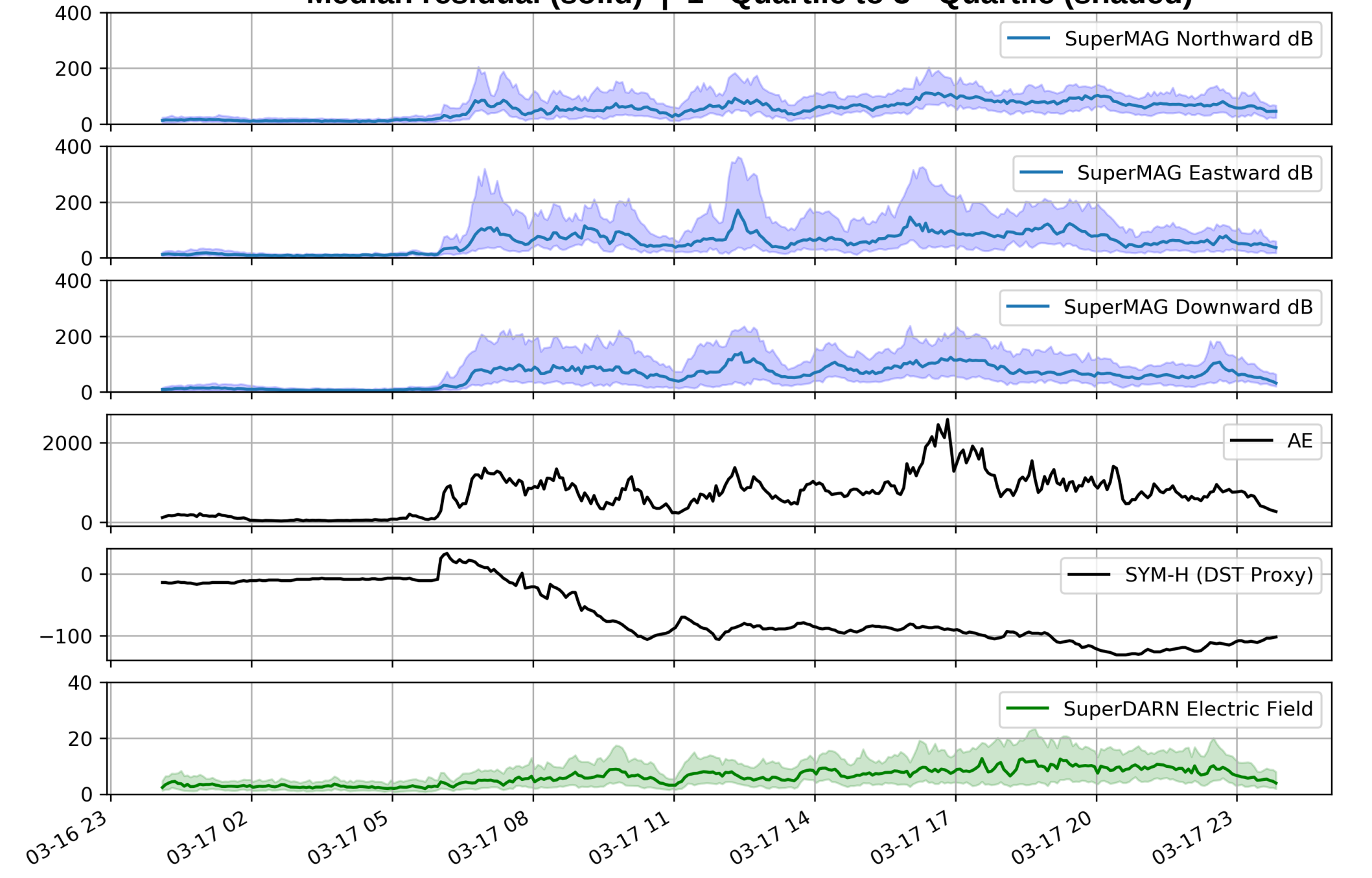
75% training (black)
25% test (red)

4 repetitions until all data in test set exactly once

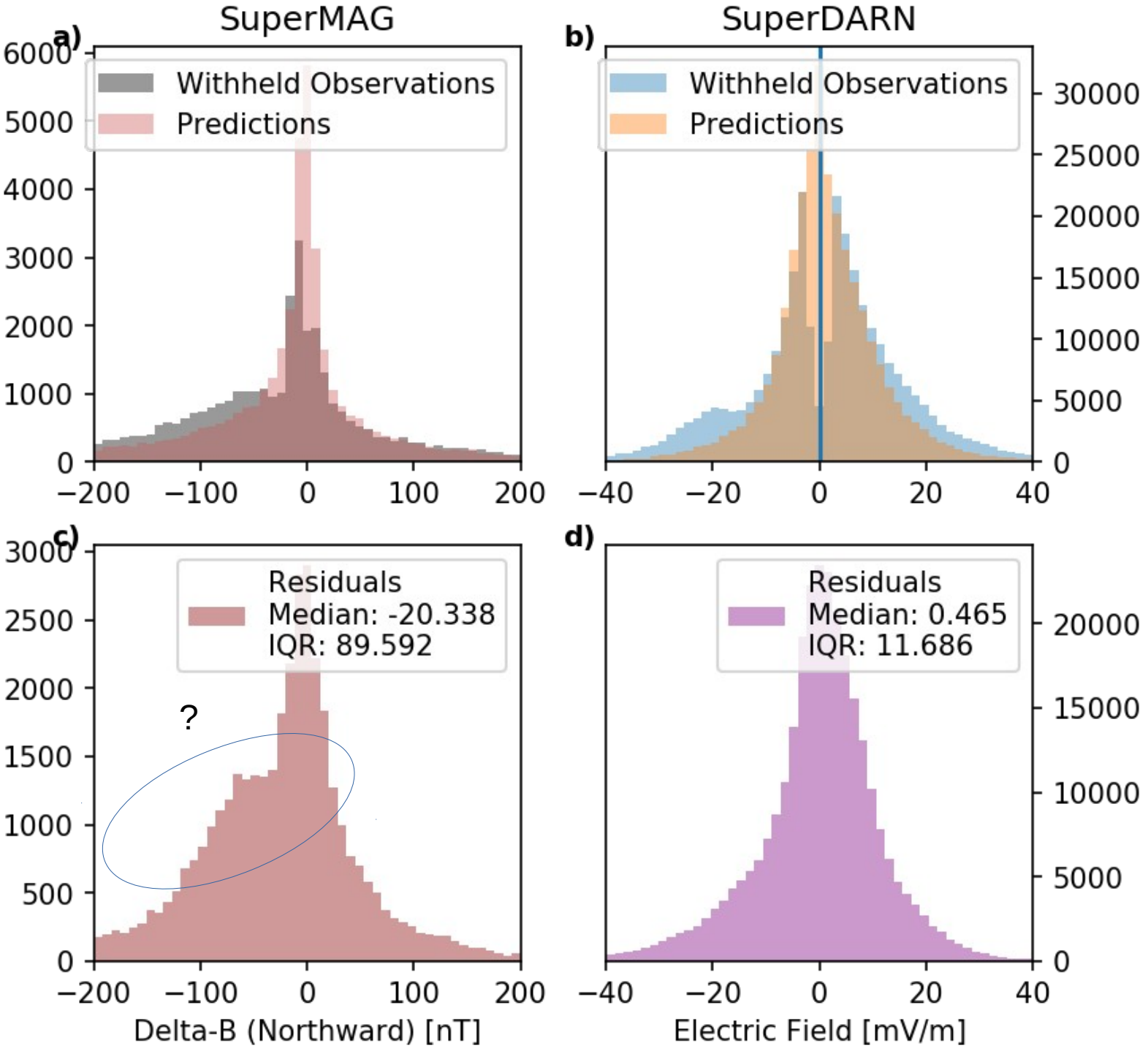
Showing SuperDARN only for simplicity, but same protocol applied to SuperMAG

4-fold Cross Validation

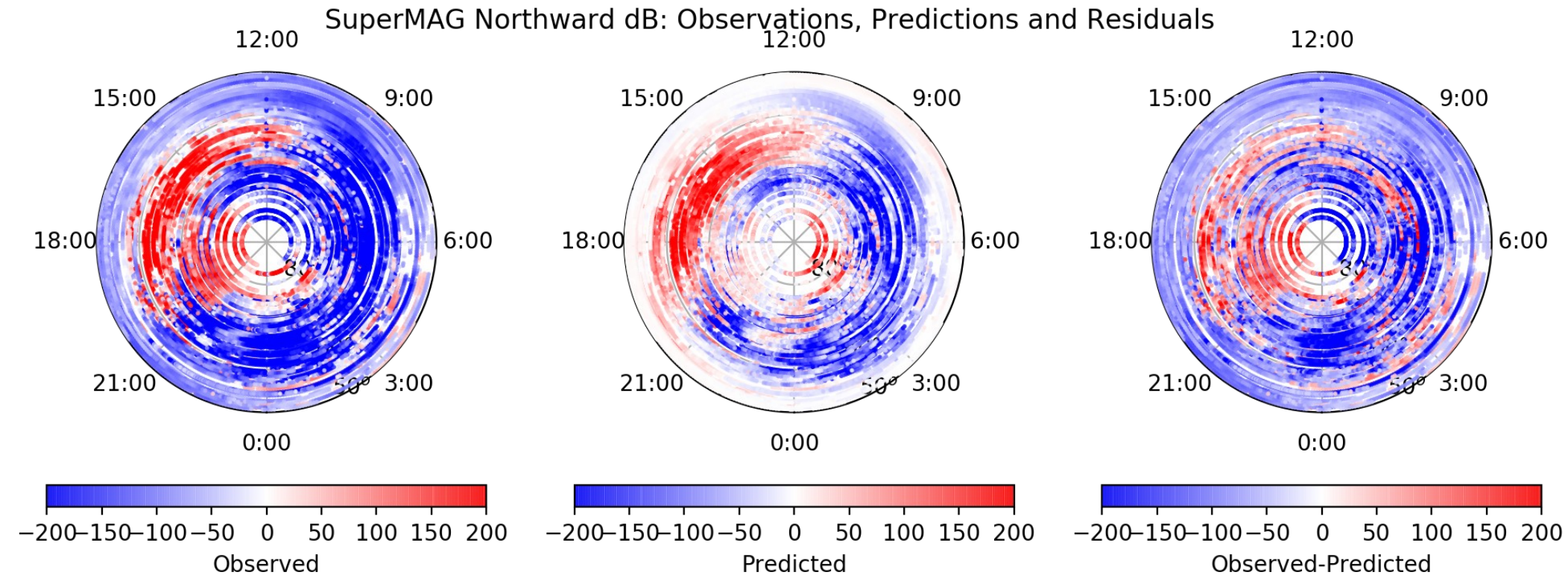
Results for 3-17-2013 (Strong Geomagnetic Storm)

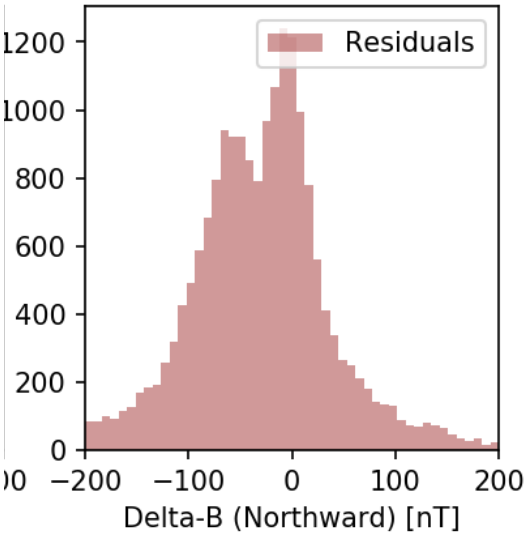


All Residuals from St. Patricks Day 2013

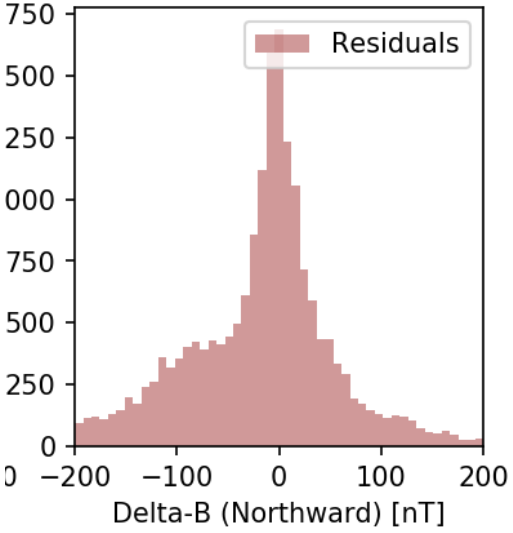


Locations of SuperMAG Residuals





(Dayside)



(Nightside)

Summary

- 4-fold cross validation applied to assimilative analysis incorporating SuperDARN and SuperMAG

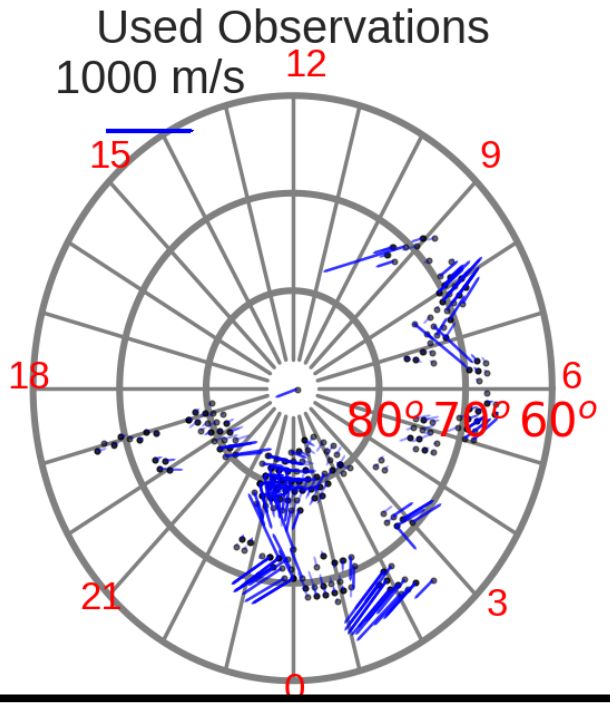
- Cross validation run for March 17, 2013, a strong geomagnetic storm

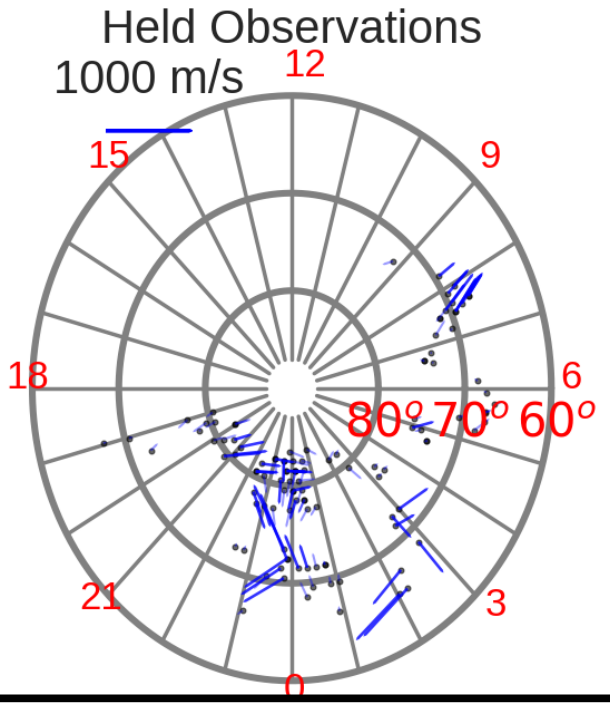
- **Residuals from strong storm reveal that assimilative analysis does not replicate all storm time dynamics in data (expected, for example limited resolution can not resolve mesoscale structures)**

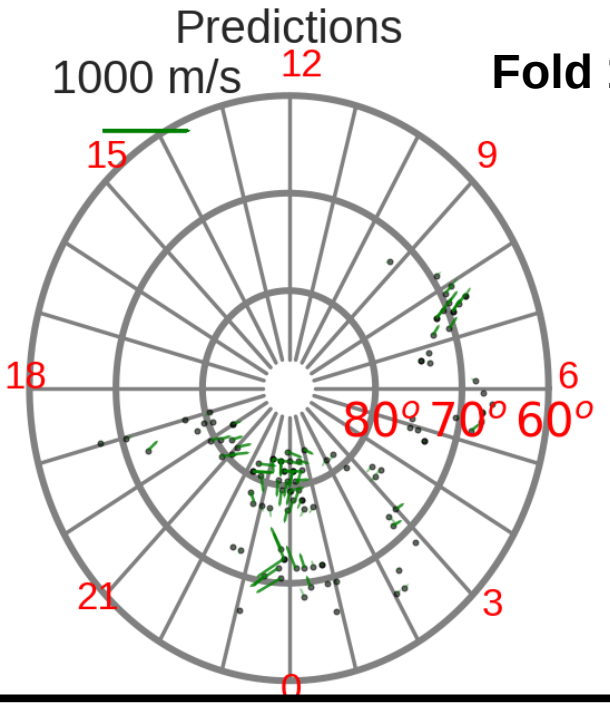
- **Cross validation suggests avenues for future work**

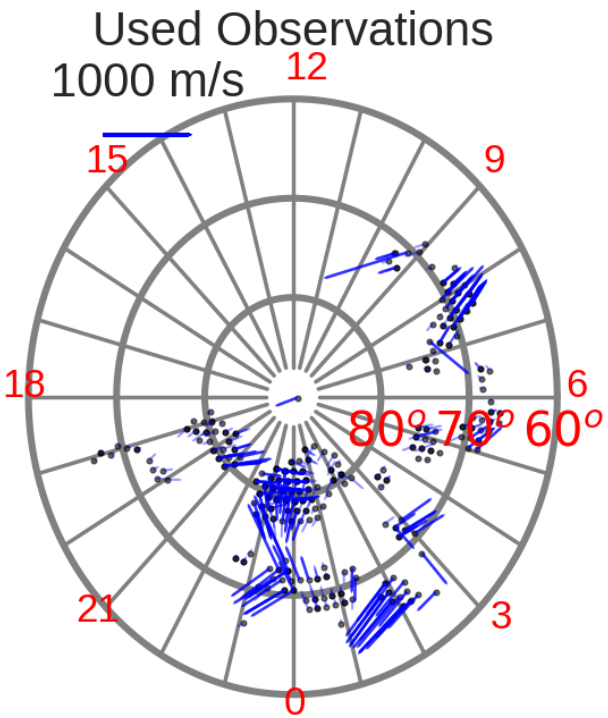
4-fold Cross Validation 4 Minutes of Data : 3-17-2013 (0:46-0:48)

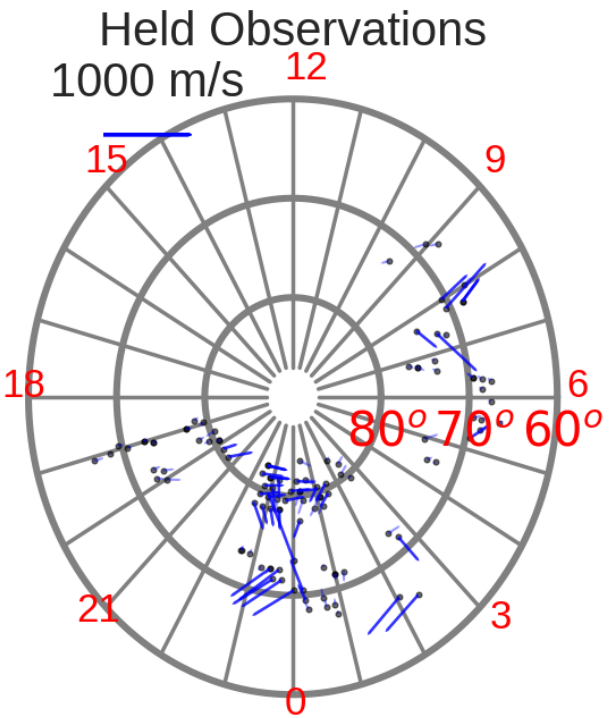
(*SuperMAG not shown | Only showing 2 of 4 folds*)

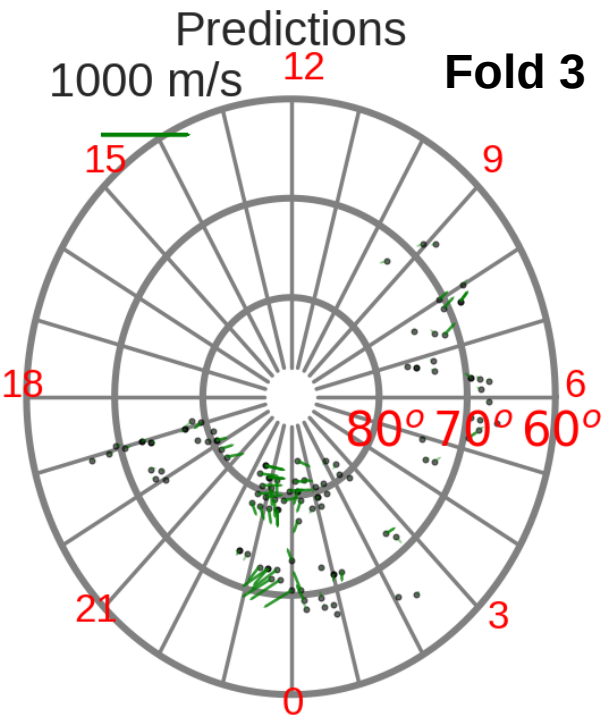






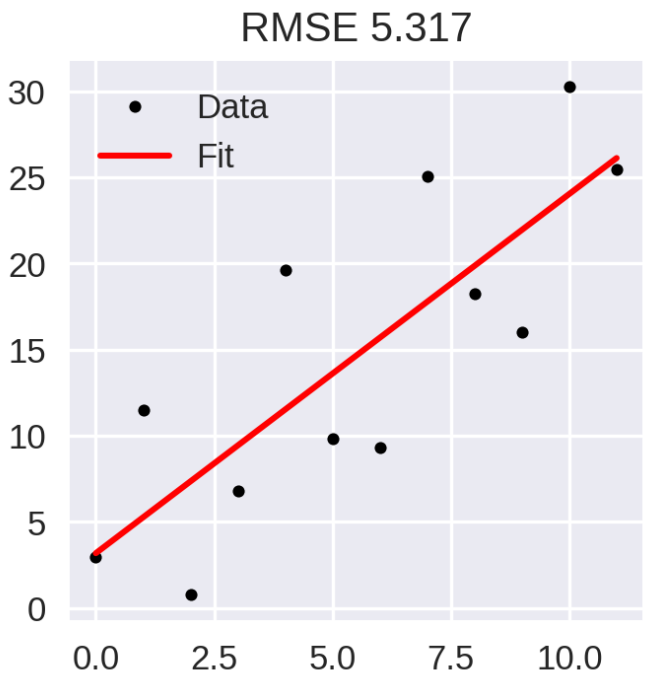


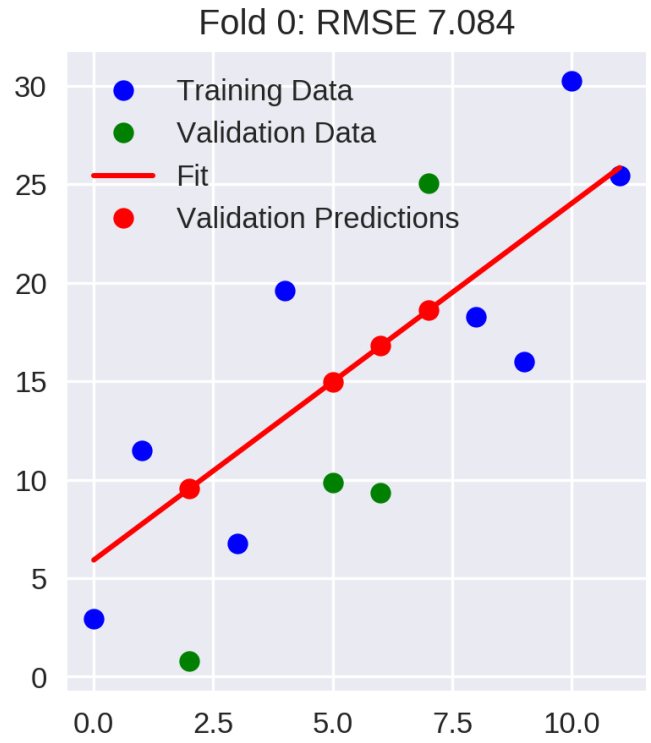


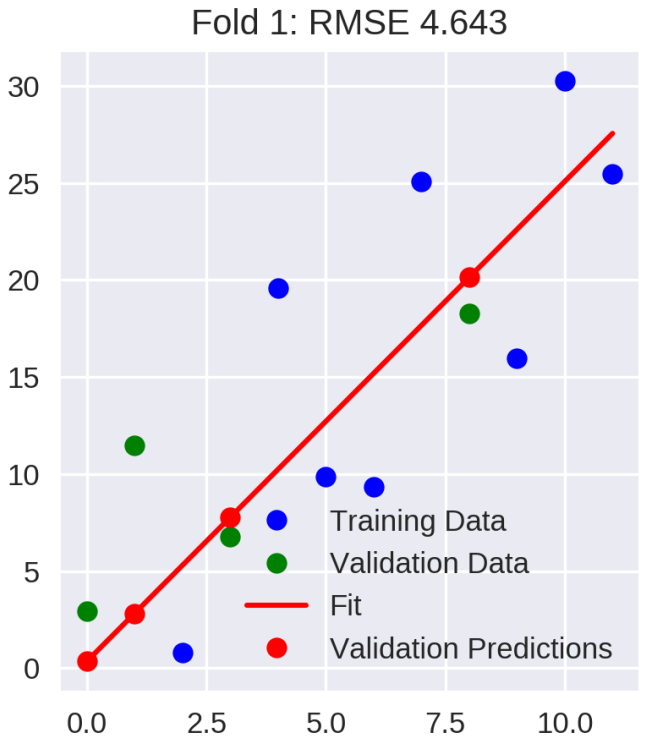


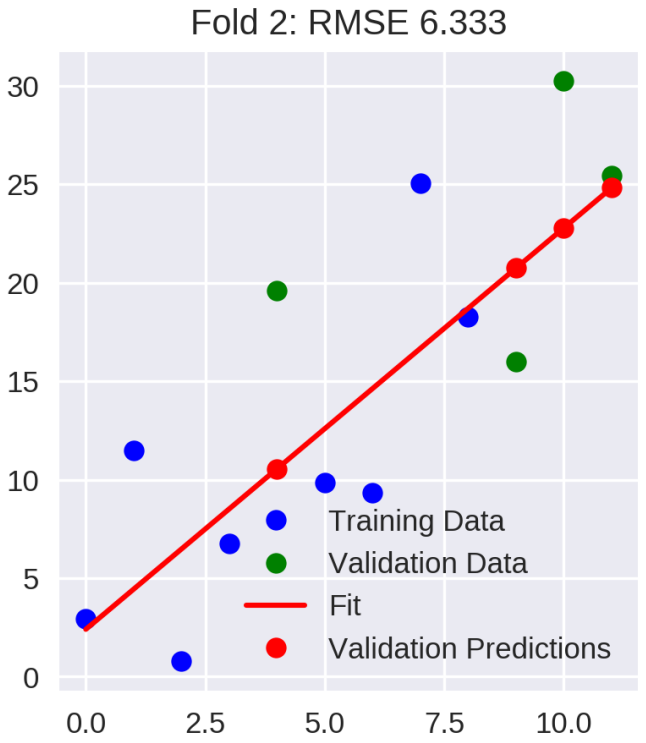
k-Fold Cross Validation (Example)

- Divide data into training set (8 points) and validation set (4 points)
- Fit the training data, compute error using only the validation data
- Do this *k* times (where *k* = *3* for this example)





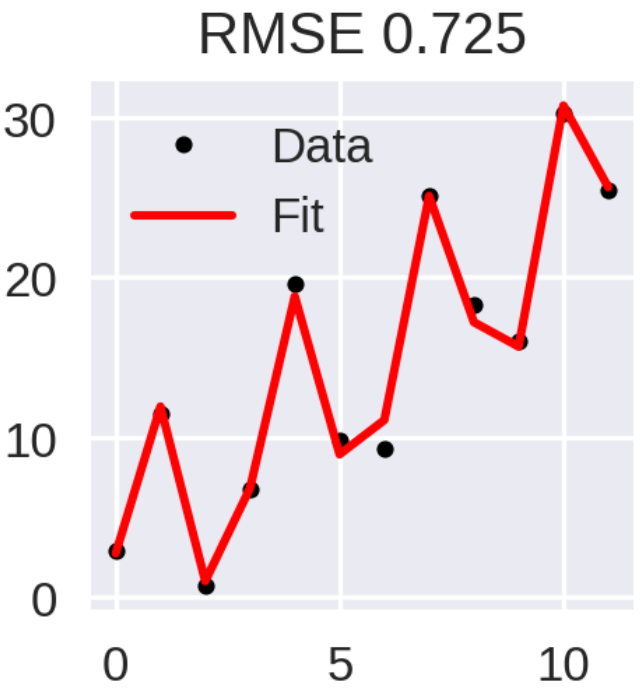


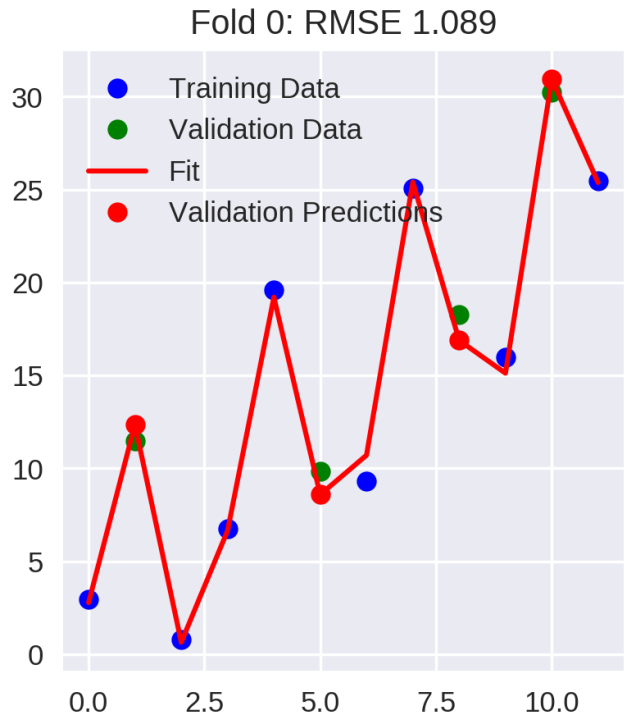


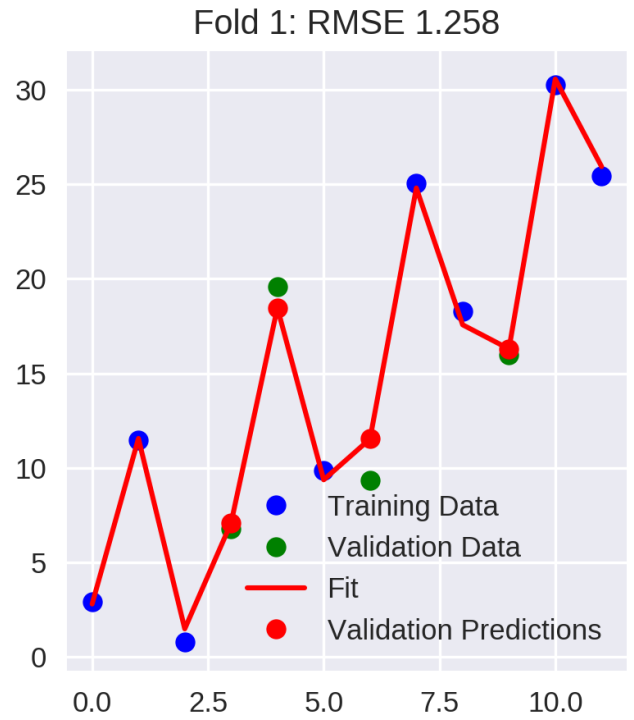
3-Fold Cross Validation

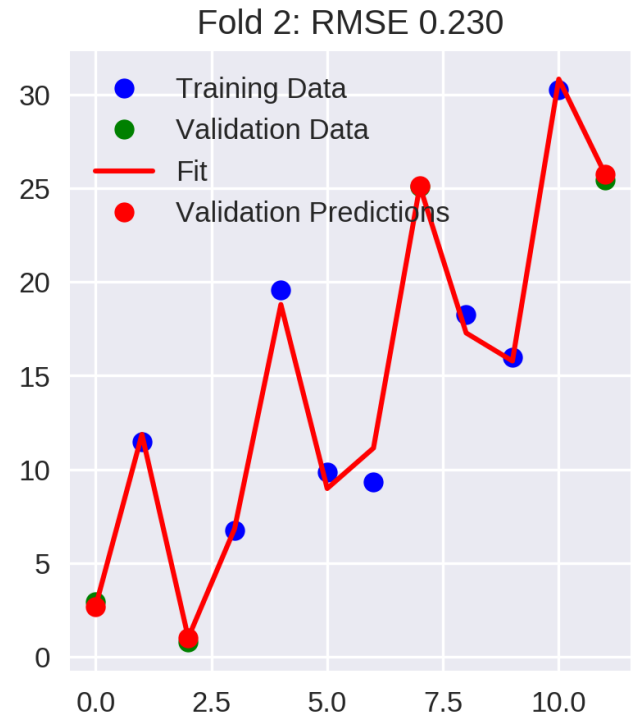
- Now we change our model, and use:
y = A*x + B*sin(x) + C (correct model)

- Not only are the RMSE smaller, the curves look identical in all three folds (model is robust)

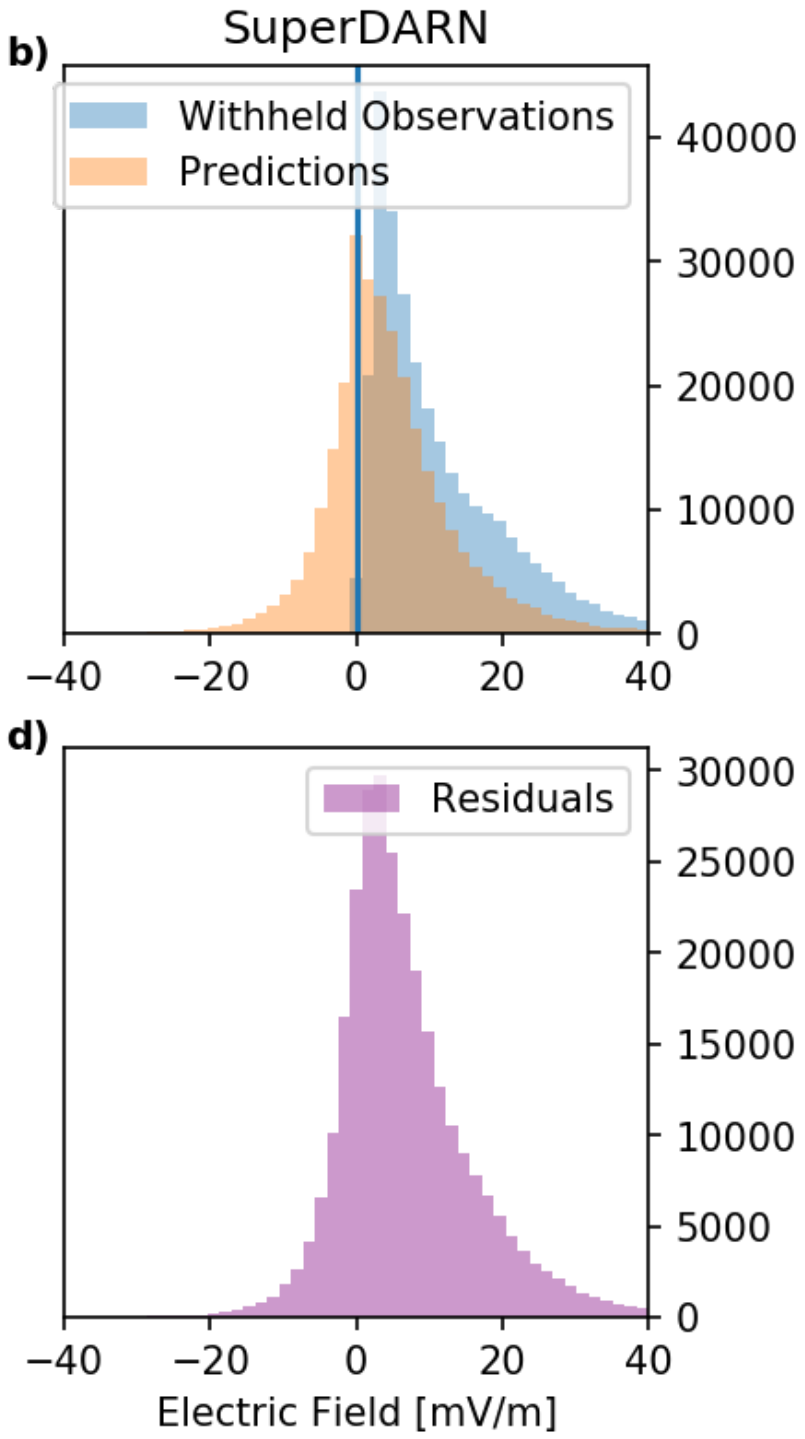




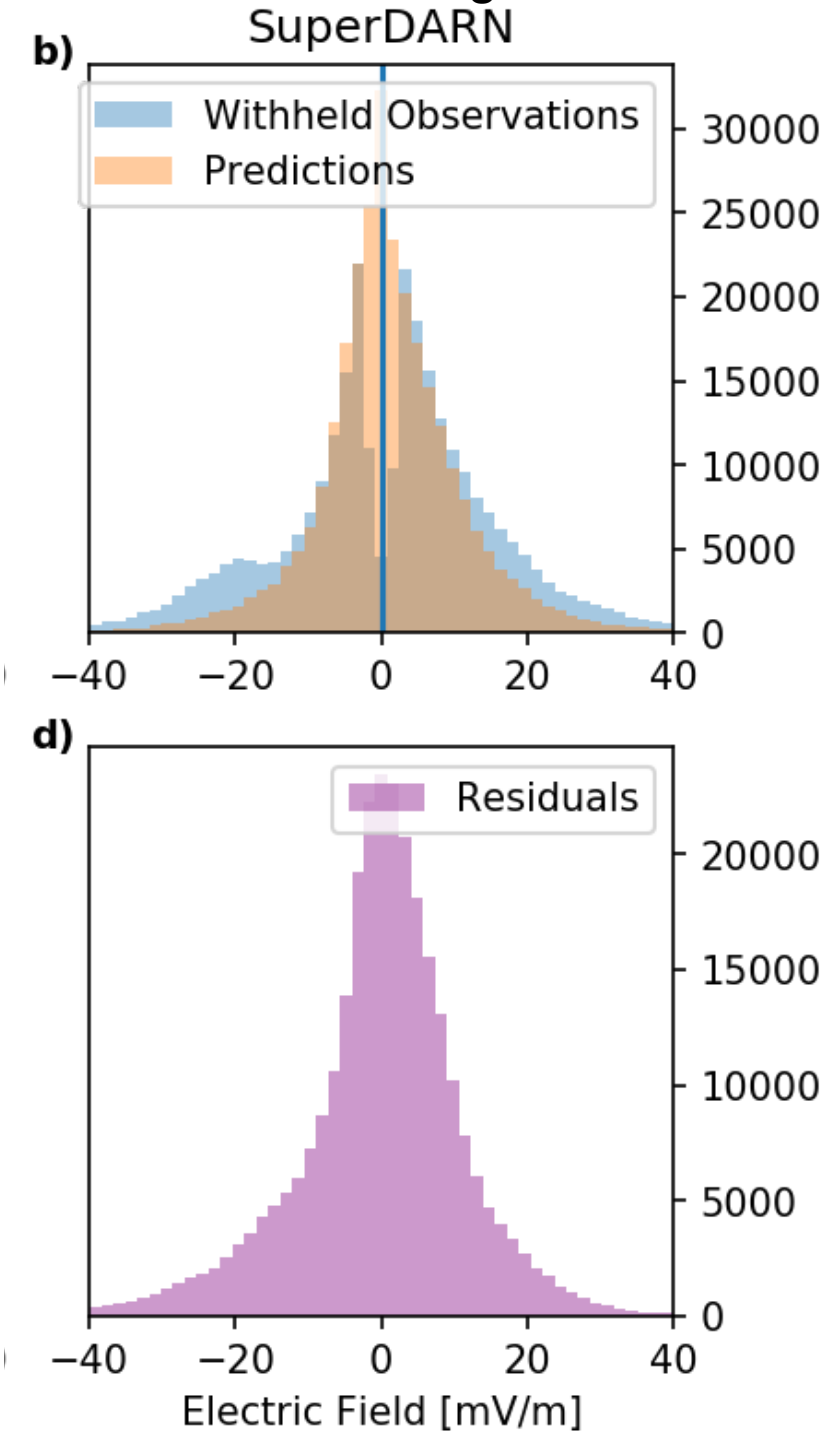




Positive Only SuperDARN Velocites

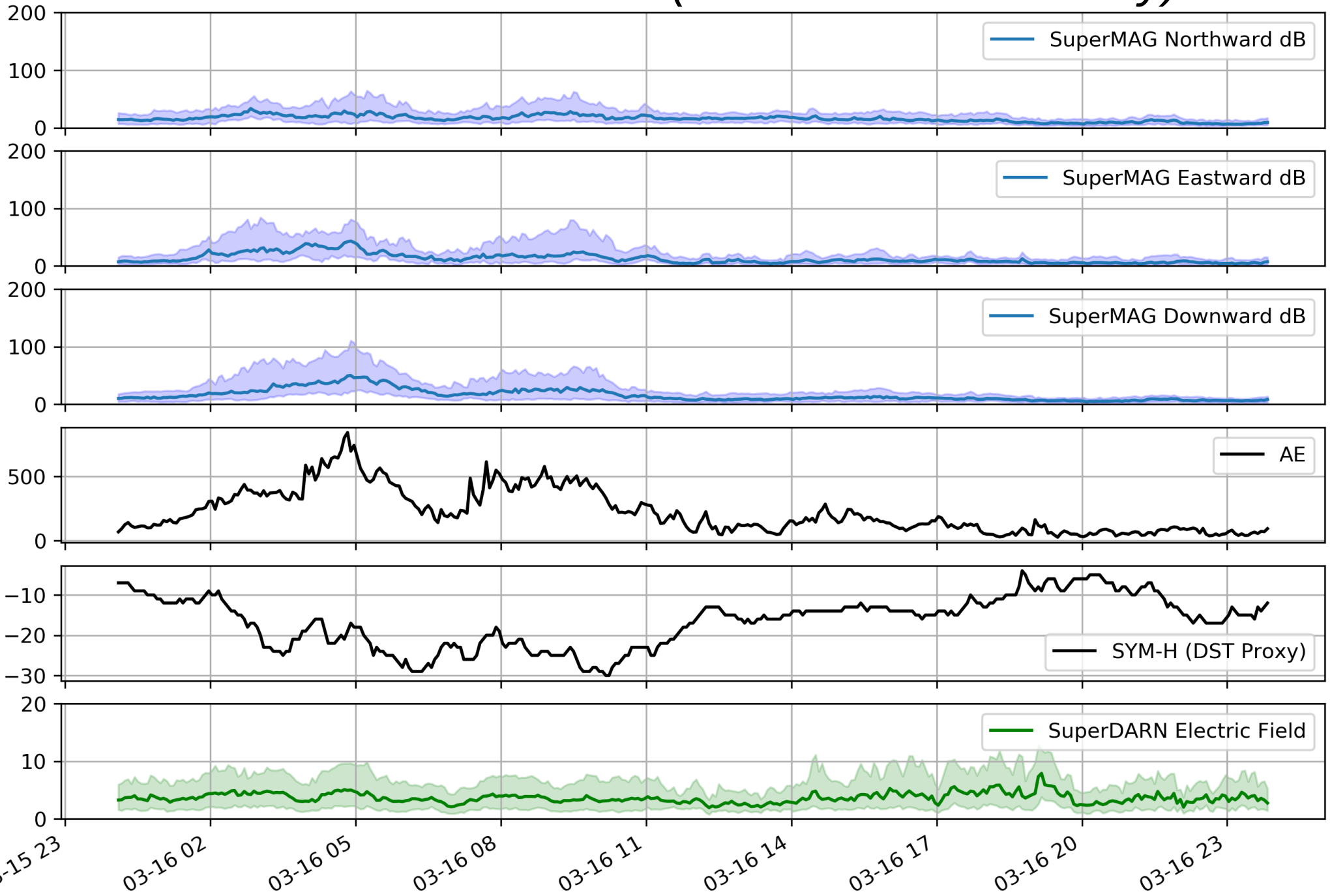


Azimuth<0 -> Negative



4-fold Cross Validation

Results for 3-16-2013 (Low/Moderate Activity)



Residuals for 3-16-2013 (Low/Moderate Activity)