



High-latitude Electric Field and Current Models

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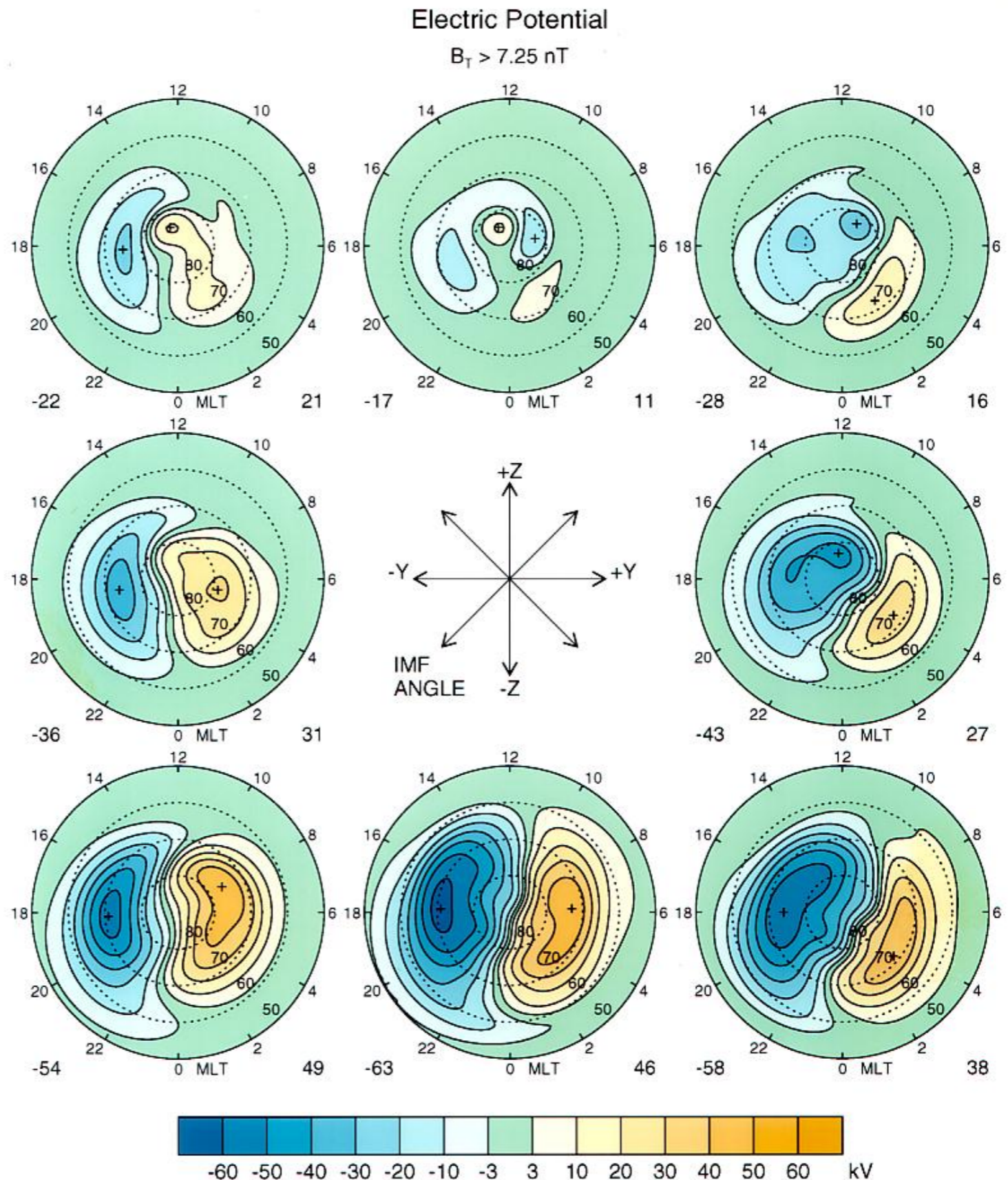
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Outline of empirical electric field and FAC model history

1995	First electric potential patterns derived from least-error fit of Dynamics Explorer-2 electric field measurements
1996	First version of the empirical electric potential model for any IMF input conditions
2001	Improved version with non-linear response. Separate, field-aligned current (FAC) model in same year.
2005a	Electric potential and FAC models combined together, with new ability to calculate Poynting flux/Joule heating.
2005b	Combined models revised to use Spherical Cap Harmonic Analysis (SCHA), still includes Poynting flux. Ground-level Geomagnetic prediction added
2013	New model for predicting geomagnetic field perturbations, using global, ground-based magnetometer measurements
2016	New FAC model under development, using data from Øersted, CHAMP and Swarm satellites.

The first electric potential maps (1995) used a least-error fit of spherical harmonic coefficients to derive the potential patterns from the sparse and randomly distributed measurements. The passes were sorted into “bins” by IMF magnitude, clock angle, and dipole tilt angle for each fit. The low-latitude boundary, where the potential is zero, is fixed at 45°



$$\Phi(\Lambda, \varphi) = \sum_{l=0}^8 \sum_{m=0}^{\text{Min}(3,l)} P_l^m(\cos \Lambda) (A_l^m \cos m\varphi + B_l^m \sin m\varphi)$$

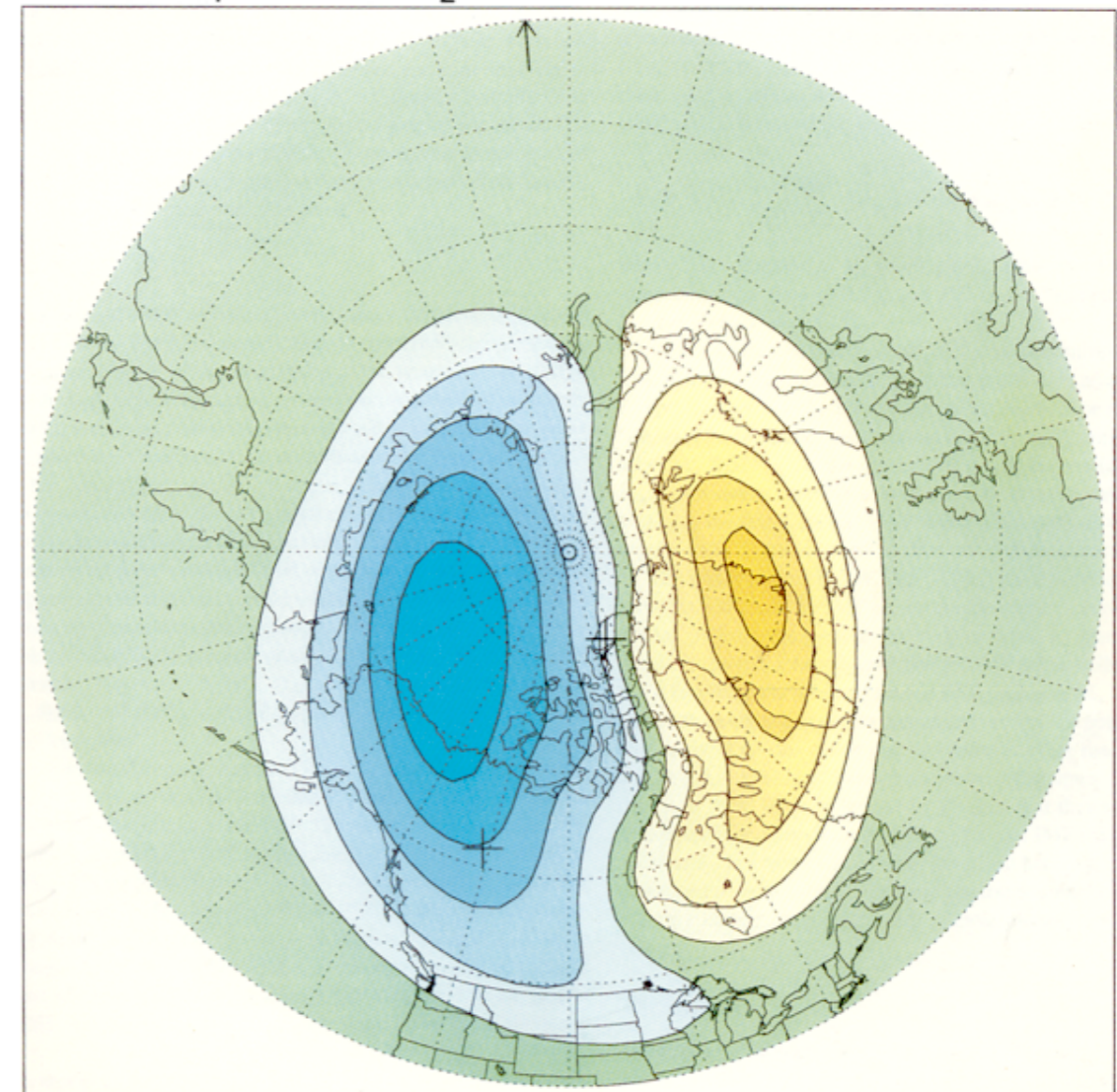
$$B_{11}(\omega) = \sum_{n=0}^4 (C_n \cos n\omega + D_n \sin n\omega)$$

$$C_{B_{lm}n} = R_0 + R_1 B_T + R_2 \sin \mu + R_3 V_{SW}$$

The next model version (1996) could create a potential map for any arbitrary IMF, using a relatively simple multiple linear regression fit .

The spherical harmonic coefficients from the sorted, "binned" patterns, along with their averaged IMF values, were used as the inputs to the model construction.

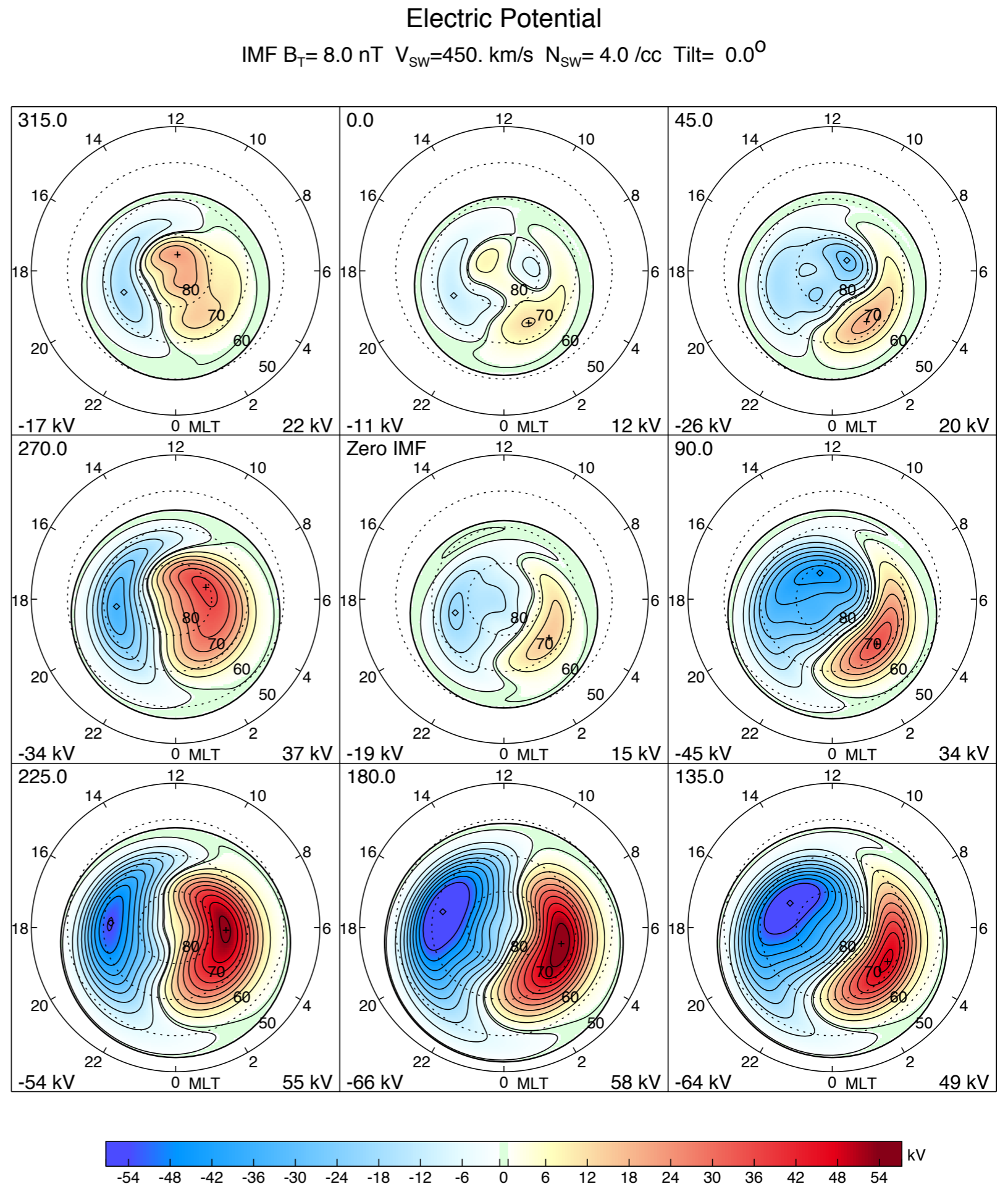
Ionospheric Electric Potential 06/18/95 6.7 UT
IMF $B_y = -1.9$ nT $B_z = -7.9$ nT SW Vel= 350.0 km/sec



A 2001 version had a non-linear variation with IMF magnitude, and used a variable-size, low-latitude boundary.

The most recent, 2005 version of electric potential model (shown here) had these changes:

- Uses “spherical cap harmonic analysis” (SCHA)
- Improved “saturation” response for large IMF magnitudes
- IMF propagation delays use “tilted phase-front” timing
- Modified the low-latitude boundary variations



In the 2nd 2005 version, a least-squares solution for all coefficients is done in one step, rather than using intermediate fits from passes grouped by IMF.

$$\psi(\Lambda, \varphi) = \sum_{k=0}^{12} \sum_{m=0}^{2 < k} P_{nk(m)}^m (\cos \Lambda) (g_k^m \cos m\varphi + h_k^m \sin m\varphi)$$

$$g_k^m = c_0 + c_1 E(B_T V_{SW}) + c_2 \sin(t) + c_3 \sin^2(t) + c_4 P_{SW} +$$

$$c_5 E(B_T V_{SW}) \cos(\theta) + c_6 \sin(t) \cos(\theta) + c_7 \sin^2(t) \cos(\theta) + c_8 P_{SW} \cos(\theta) +$$

$$c_9 E(B_T V_{SW}) \sin(\theta) + c_{10} \sin(t) \sin(\theta) + c_{11} \sin^2(t) \sin(\theta) + c_{12} P_{SW} \sin(\theta) +$$

$$c_{13} E(B_T V_{SW}) \cos(2\theta) + c_{14} E(B_T V_{SW}) \sin(2\theta)$$

A non-linear “saturation” response is obtained by an exponential function of the interplanetary electric field:

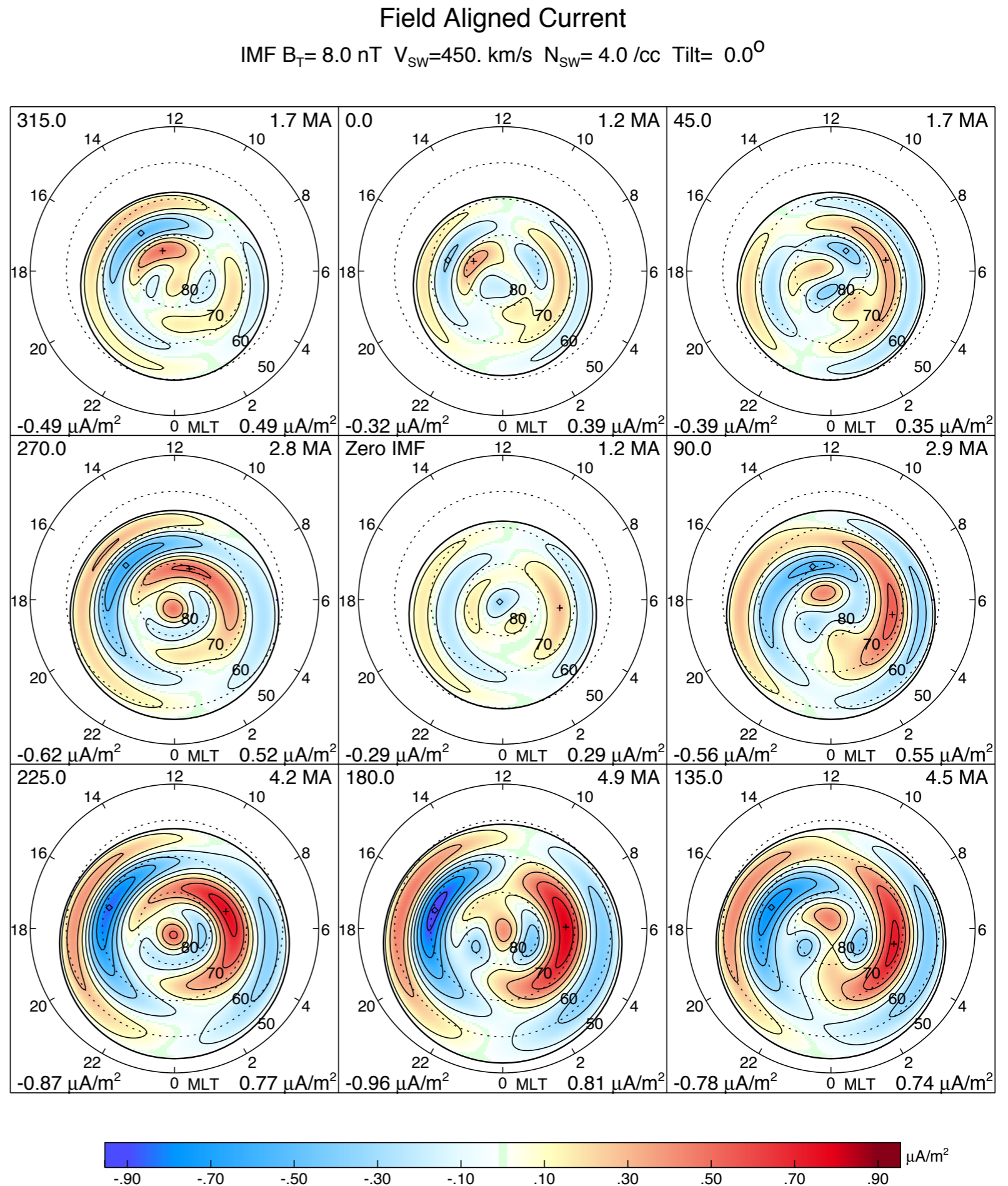
$$E(B_T V_{SW}) = \left(1 - \exp(-1.33 B_T V_{SW})\right) (B_T V_{SW})^{0.47}$$

2005 model merged in the field-aligned currents, developed in 2001. FAC obtained from the 2D surface Laplacian operating on the potential:

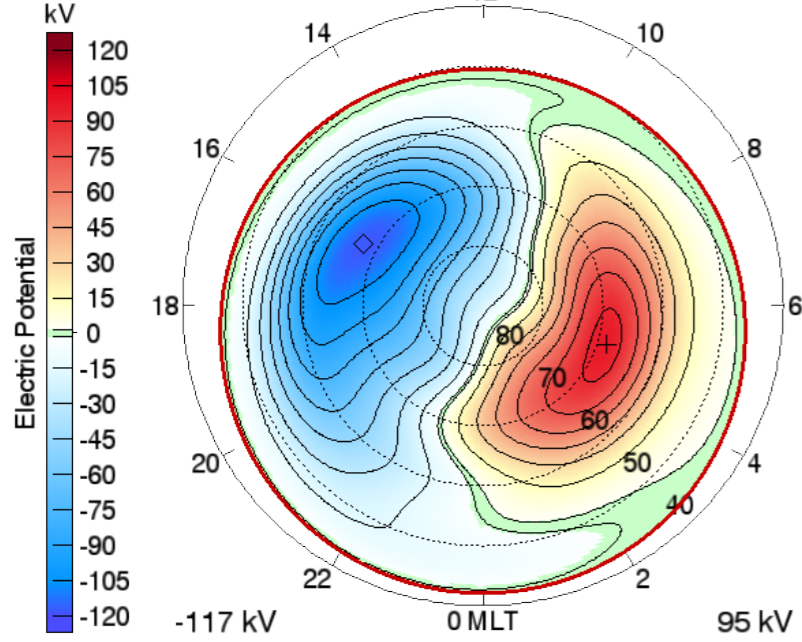
$$\Delta \mathbf{B} = \hat{\mathbf{r}} \times \nabla_S \psi$$

$$J_{\parallel} = \nabla_S^2 \psi / \mu_0$$

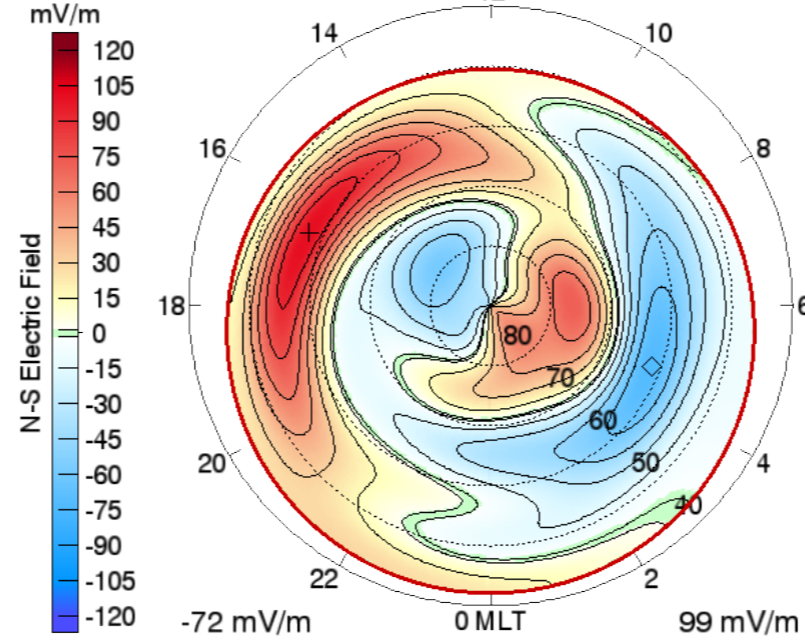
The ionospheric conductivity variations are implicitly included in the magnetic field measurements.



8 Nov. 2004
06:00 UT

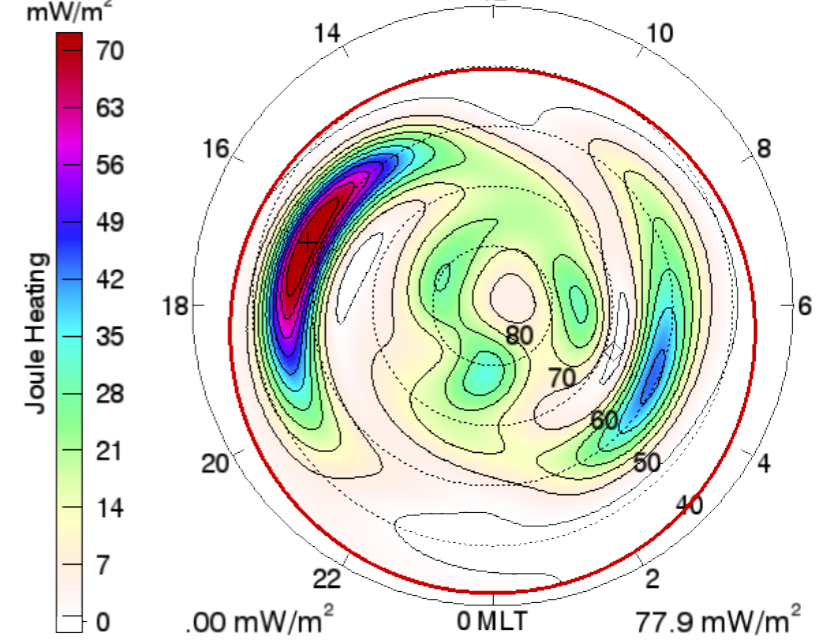


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$$\mathbf{E} = -\nabla_S \Phi$$

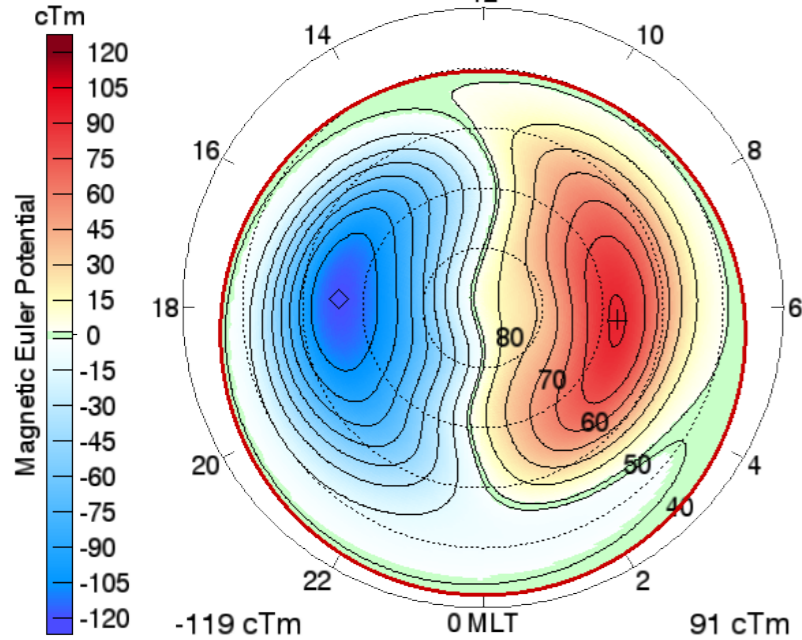
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$$\mathbf{S} = \mathbf{E} \times \Delta \mathbf{B} / \mu_o$$

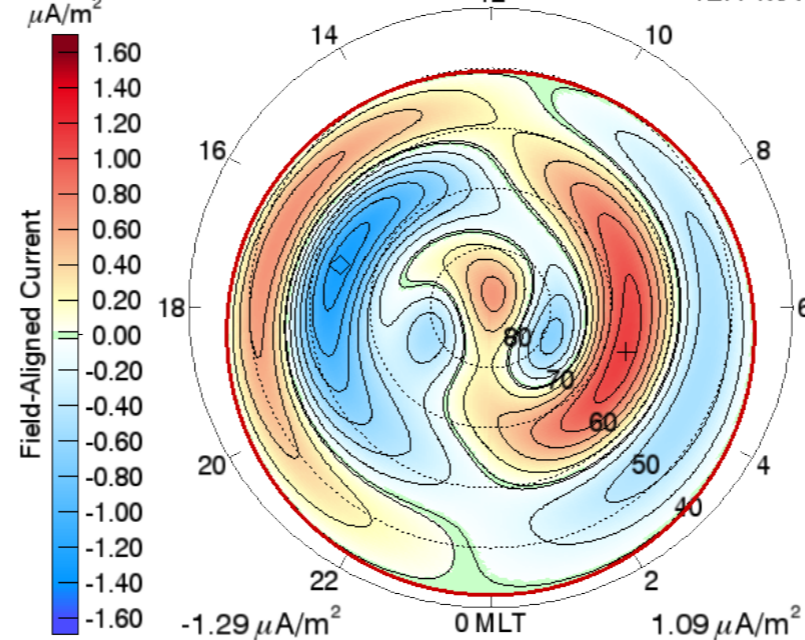
The magnetic potentials in the FAC model became even more useful in combination with the electric potentials to obtain the Poynting flux

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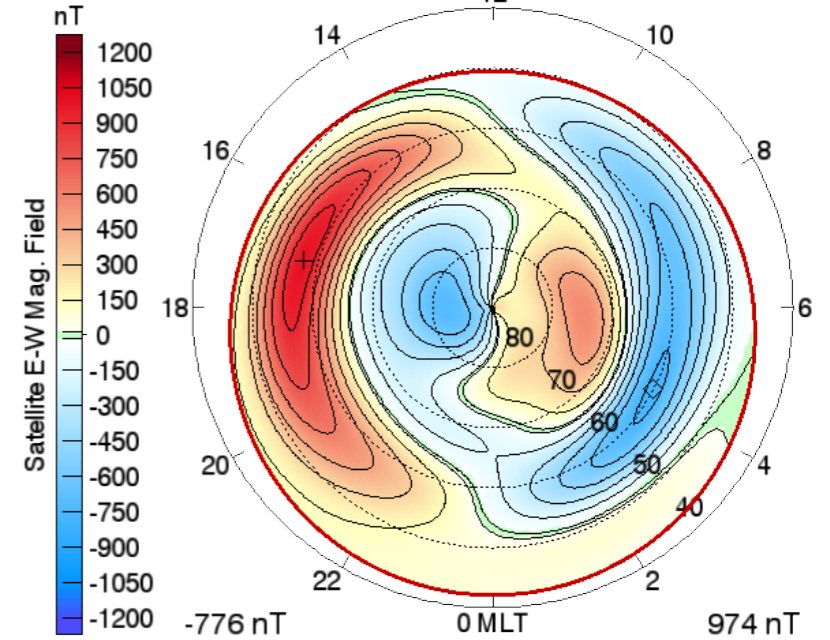
$$J_{\parallel} = \nabla_S^2 \psi / \mu_o$$

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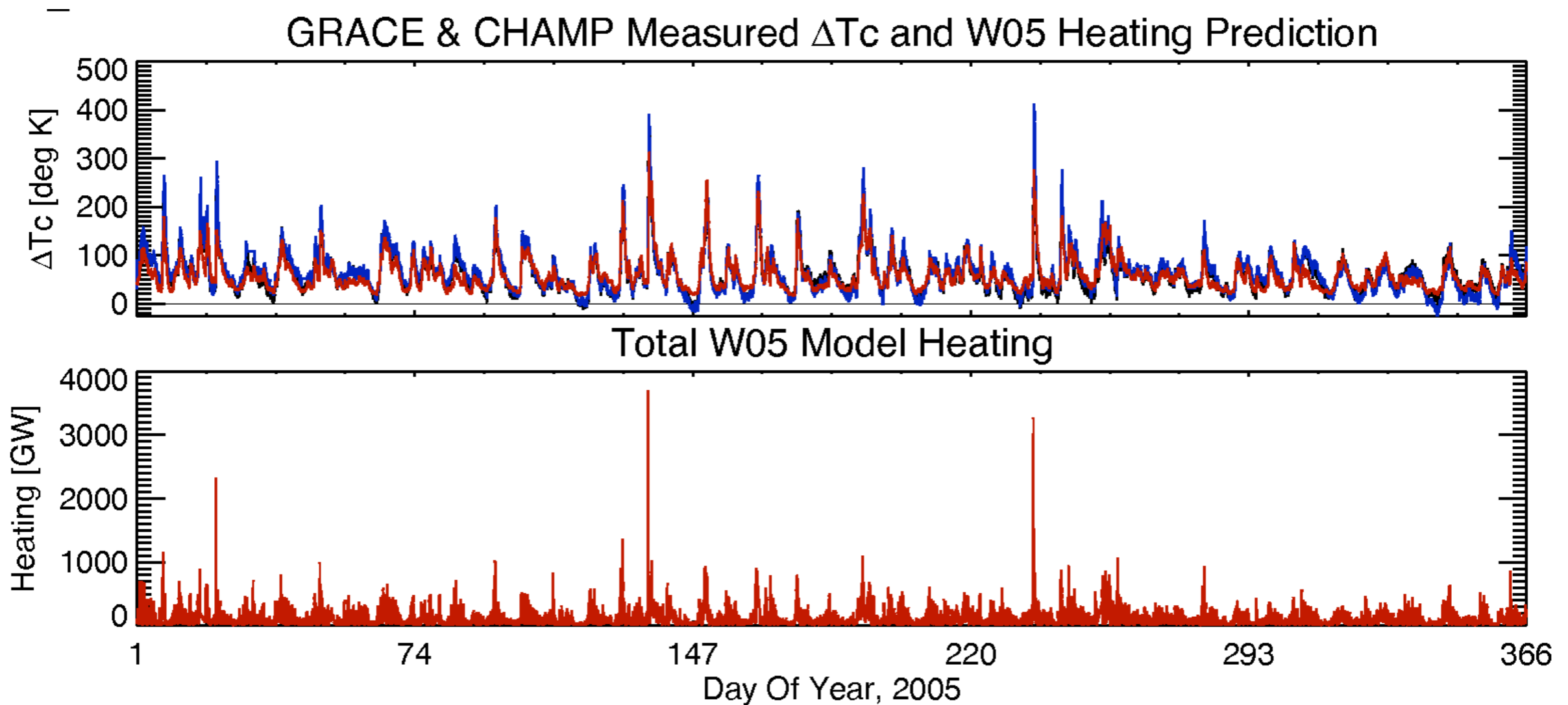


$$\Delta \mathbf{B} = \hat{\mathbf{r}} \times \nabla_S \psi$$

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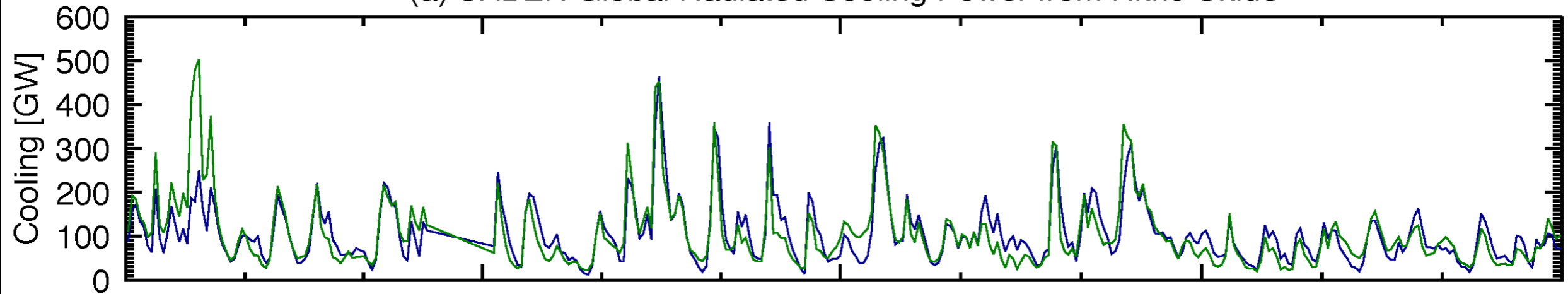


In a comparison with the temperature changes in the JB2008 model, derived from CHAMP and GRACE, predicted and measured ΔT_c have ≈ 0.9 correlation for 2002 - 2006. Red line is ΔT_c prediction from W05, blue and black lines are measured. The W05 model provides more than enough Joule heating.

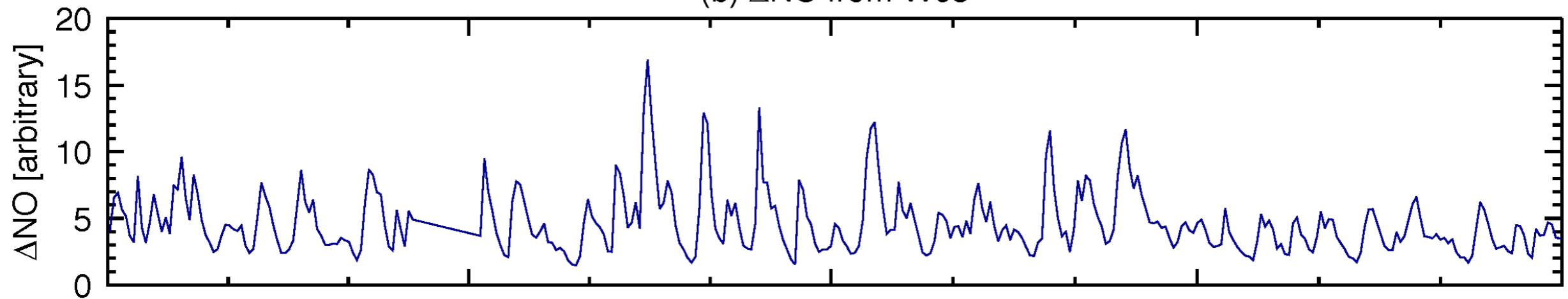


The SABER measurements agree very well with the predicted values.
Correlation for the year 2005 is 0.85, 0.91 in 2004.
Green: SABER measurements. Blue: $\text{NO}(W05) * 29.16 - 30.4$

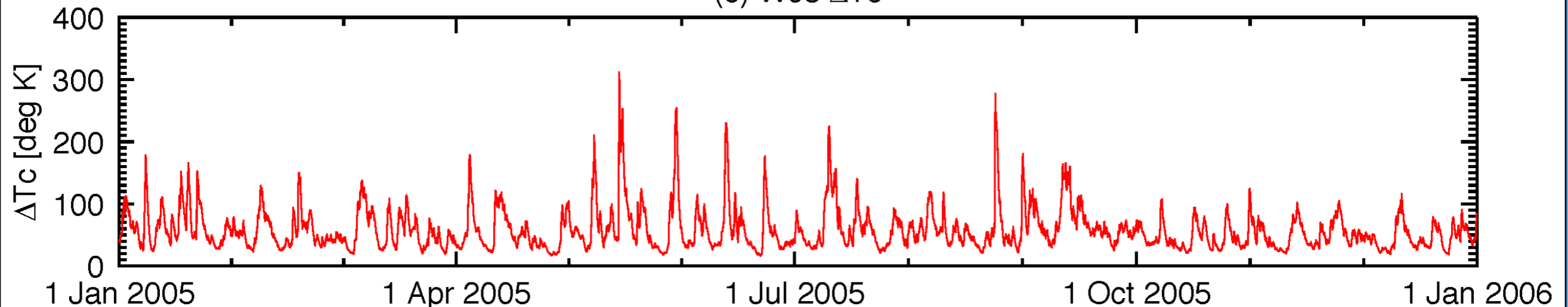
(a) SABER Global Radiated Cooling Power from Nitric Oxide



(b) ΔNO from W05

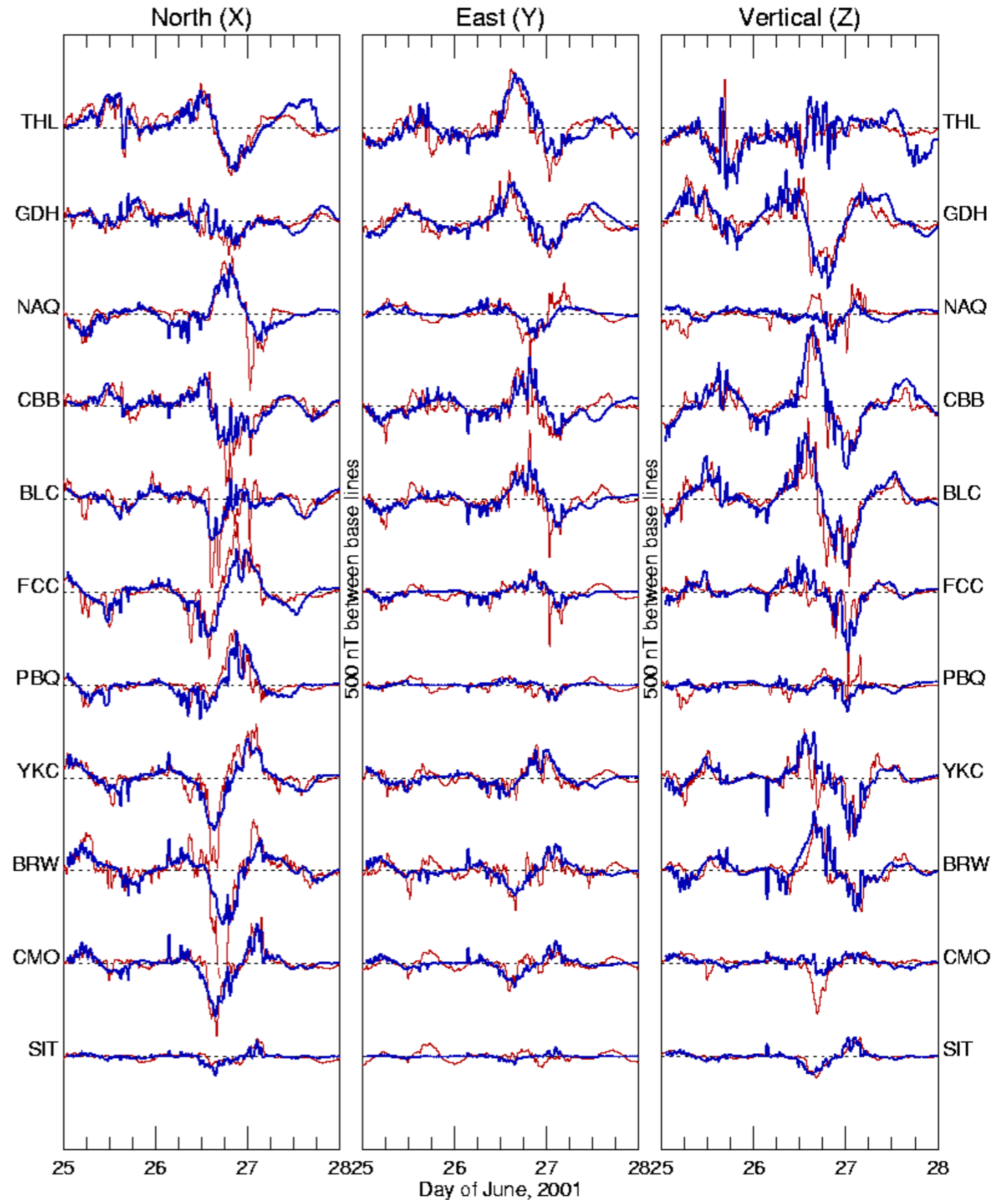
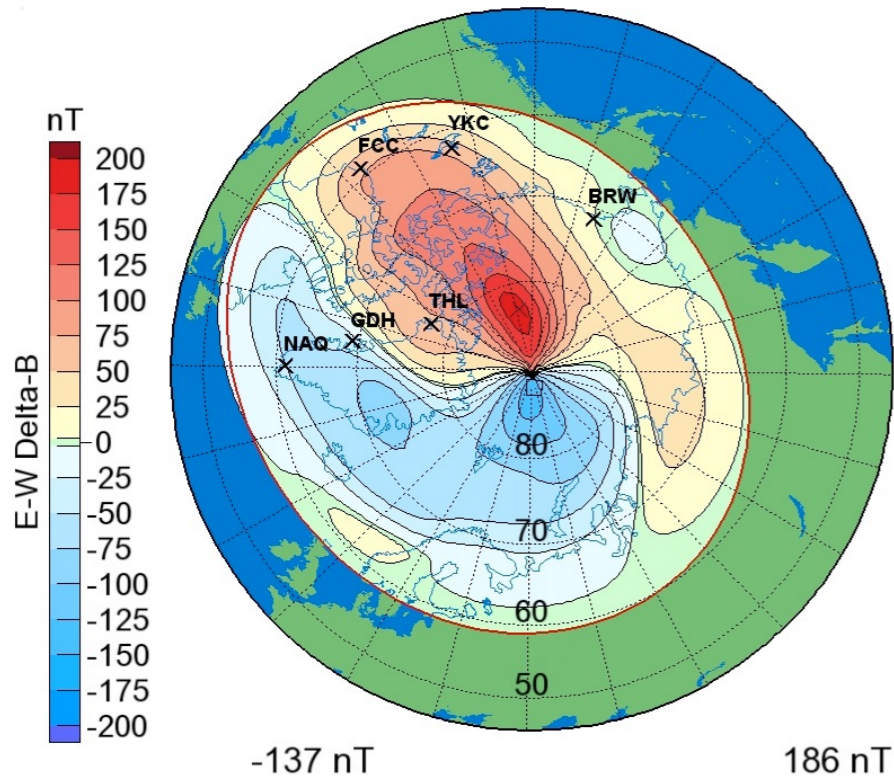
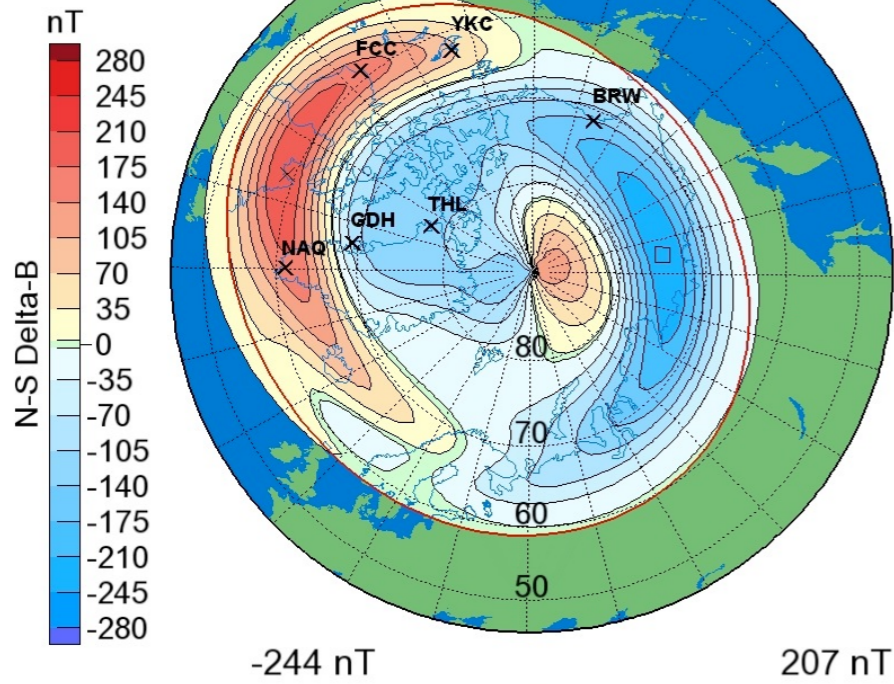


(c) W05 ΔT_c



ΔB from 2005 model

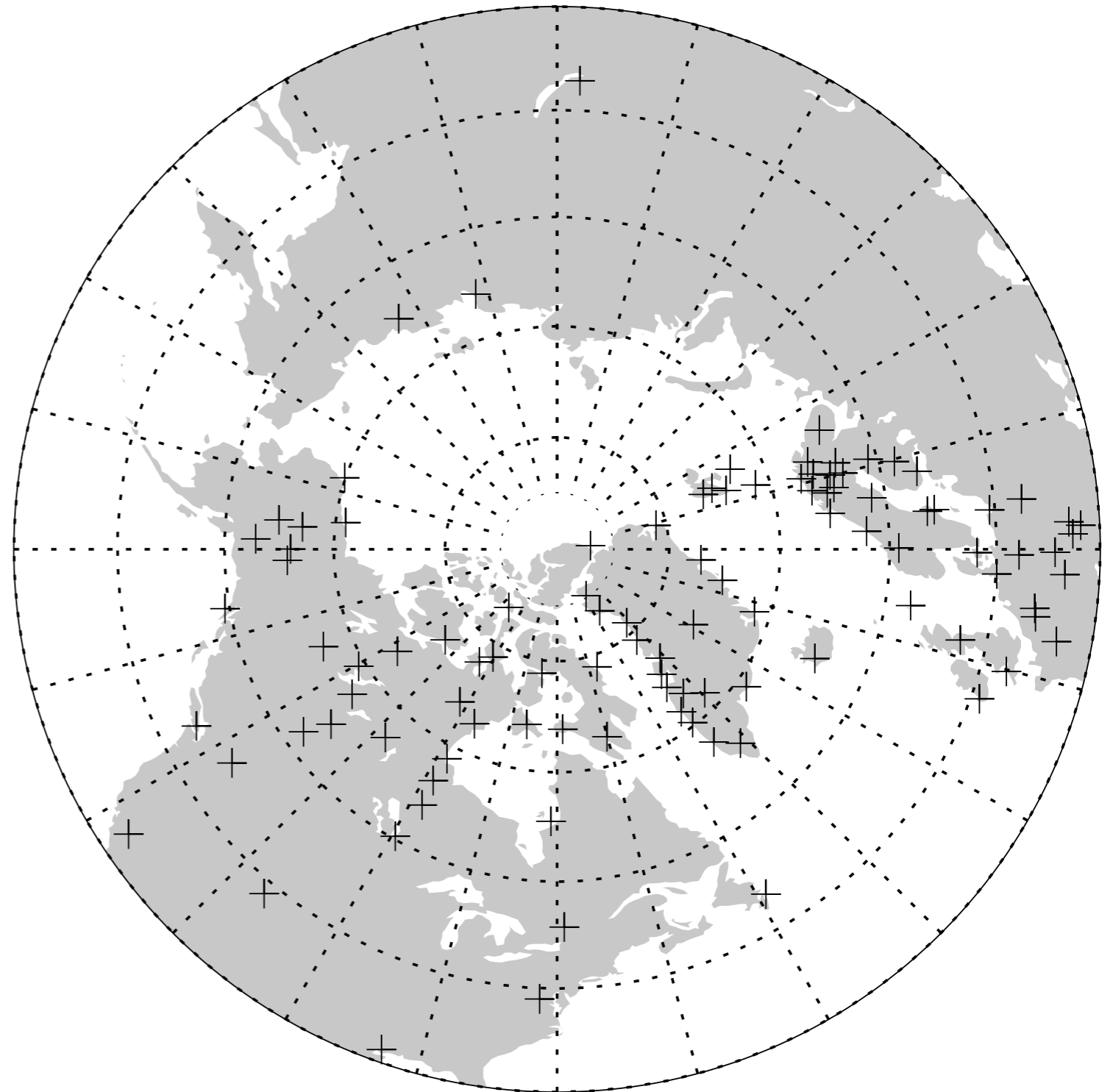
26 June 2001
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In order to obtain better geomagnetic predictions, the 2013 model is based entirely on ground-level magnetometer measurements and the IMF.

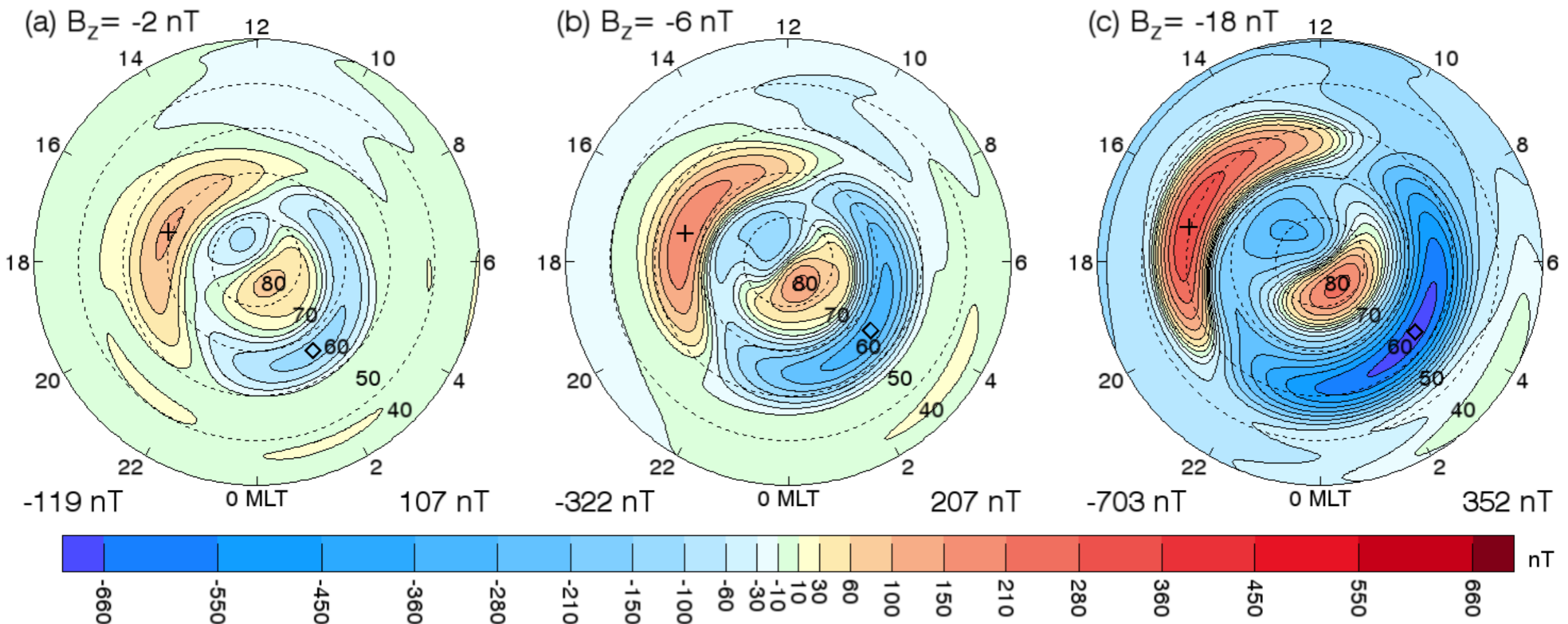
The data are from >140 magnetometer stations in Northern hemisphere, over an 8-year period (1998-2005 , solar wind velocity, IMF, and F10.7.

Effects of conductivity variations and induced, underground currents are implicitly included.



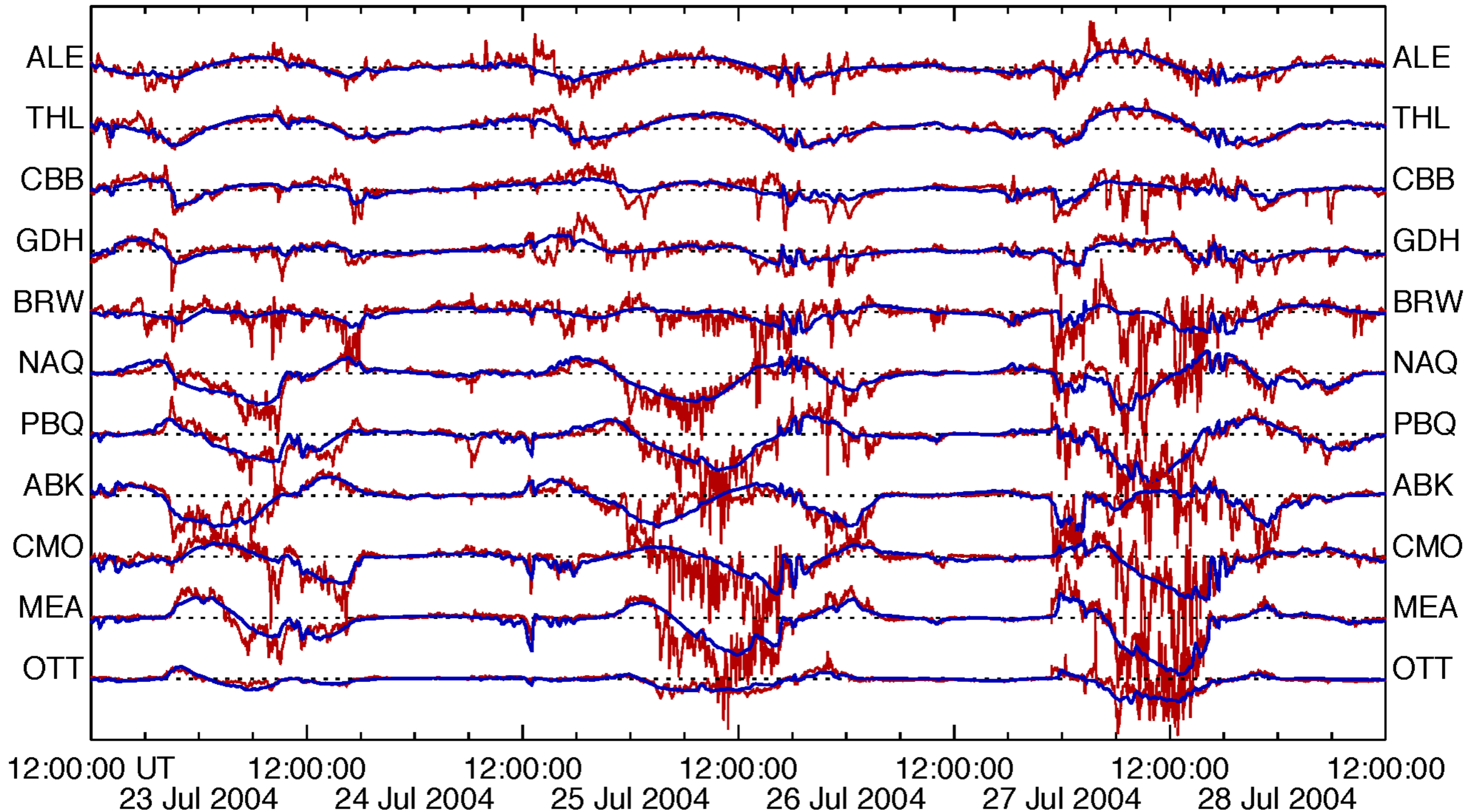
Geomagnetic North-South component of ΔB , shown for IMF B_z values -2, -6, and -18 nT

The values become more negative at low latitudes, indicating that the effects of the ring current (Dst) are included in the statistical averages.

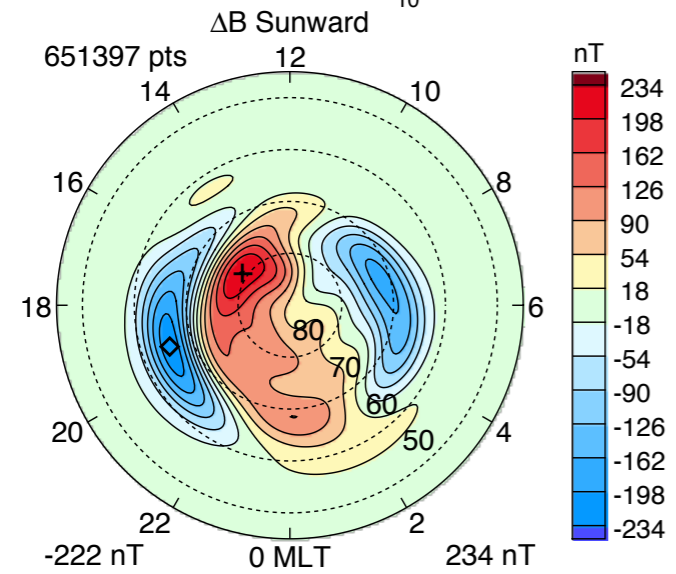


The model does very well at prediction of ΔB levels; not so well on the superposed, random, higher frequency variations. These and substorms could be added in the future.

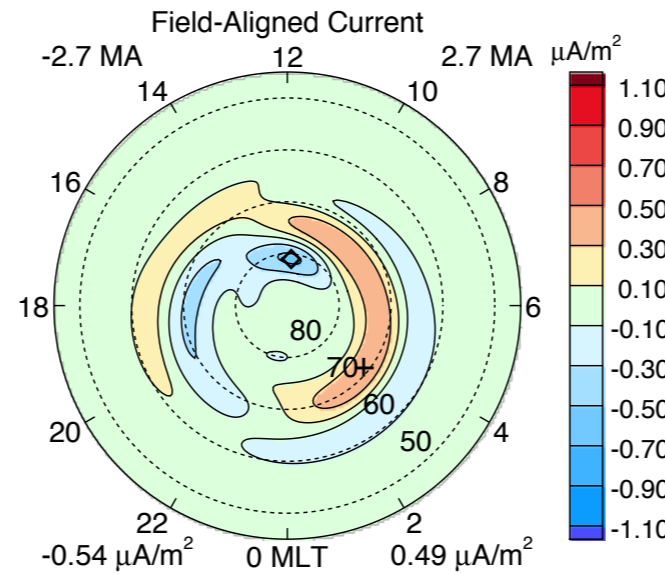
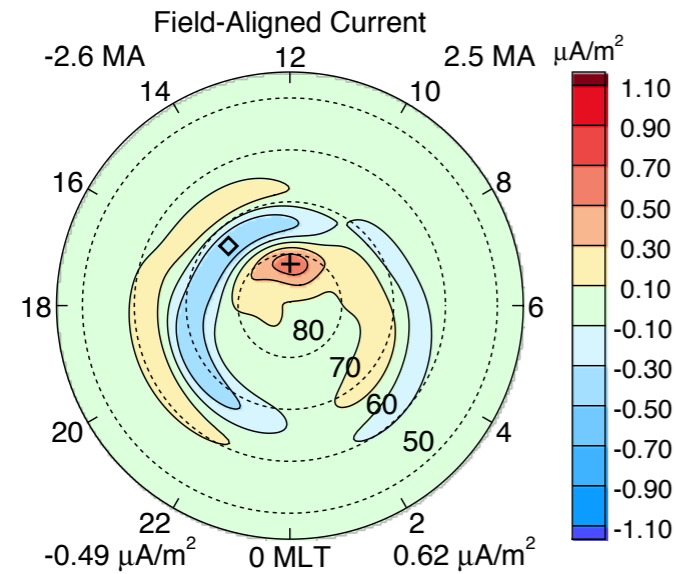
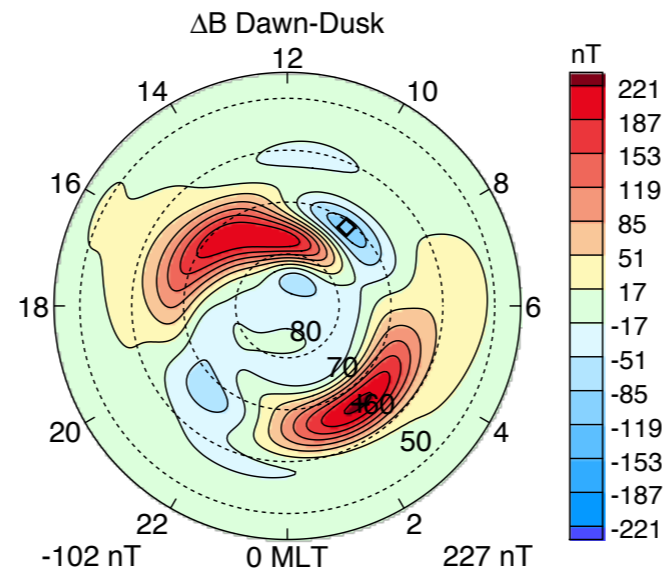
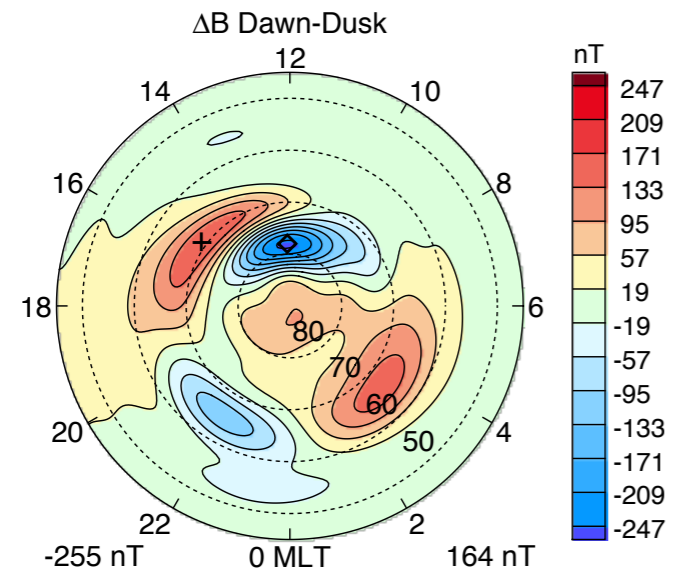
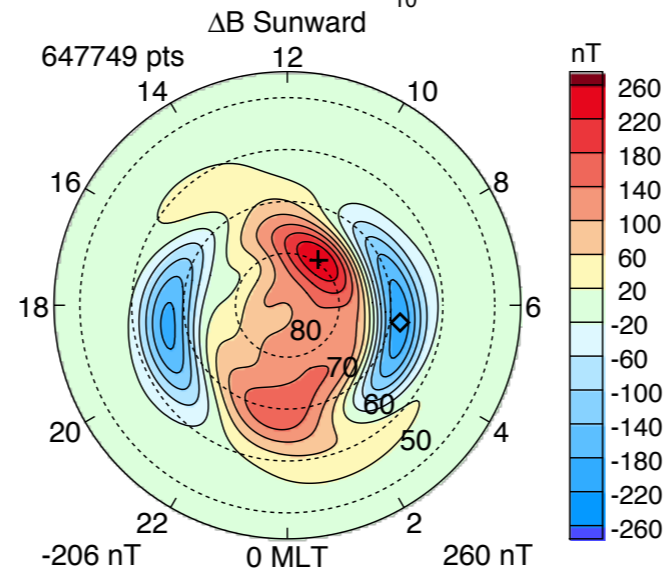
ΔB North-South (X), 1200 nT between base lines



ΔB Fit, SWE=04400±0600 μ V/m @270°±23°
Tilt=000°±12° F₁₀=160±070



ΔB Fit, SWE=04400±0600 μ V/m @090°±23°
Tilt=000°±12° F₁₀=160±070



New FAC model using data from Øersted, CHAMP and Swarm satellites. Examples of FAC reconstruction at two IMF clock angles are shown.

Two orthogonal components of ΔB are fit using Spherical Cap Harmonic Analysis (SCHA), then FAC obtained from

$$\mu_0 \mathbf{J} = \nabla \times \Delta \mathbf{B}$$

Test version EW16 FAC model at eight IMF clock angles

