



# Horizontal Wind Model (HWM)

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**Work Supported by the  
Chief of Naval Research**

Motivation

Formulation

Parameter Estimation

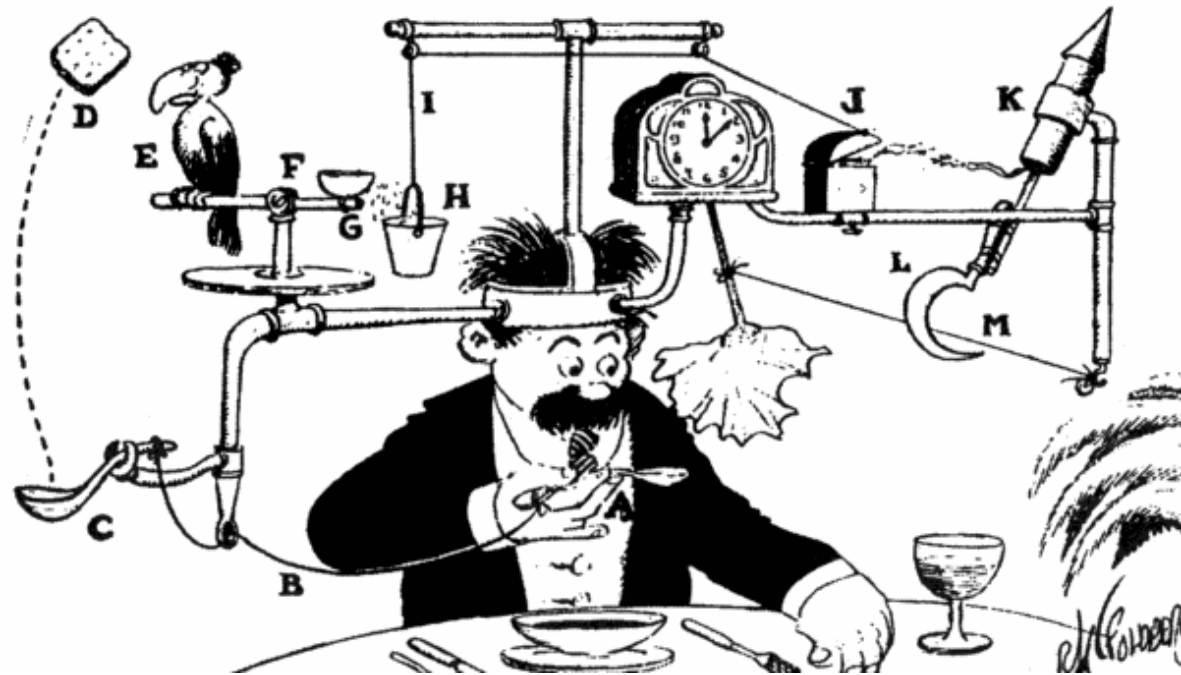
Data Sets

Statistical  
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Sometimes simple is preferable to complicated.

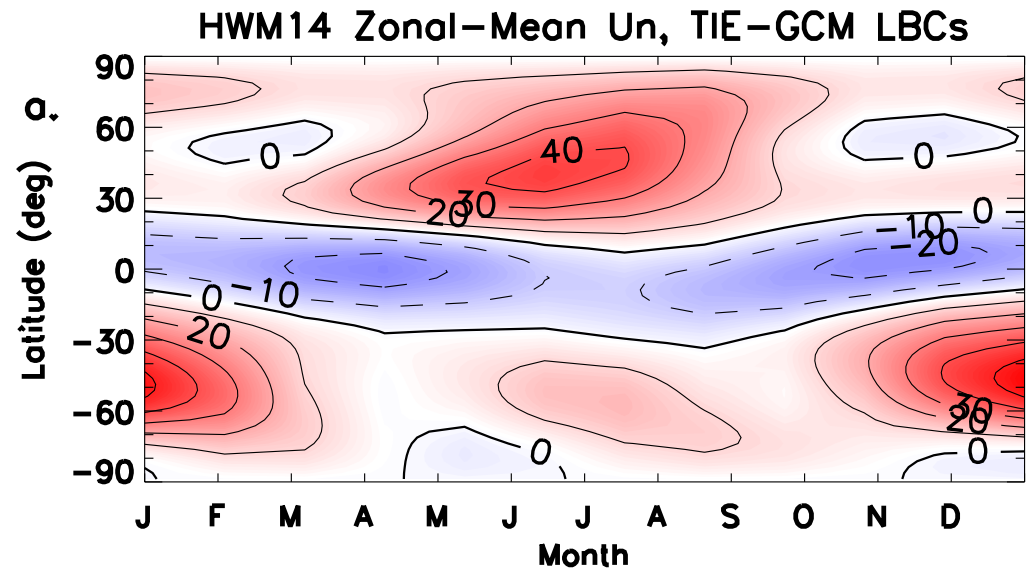


# Motivation

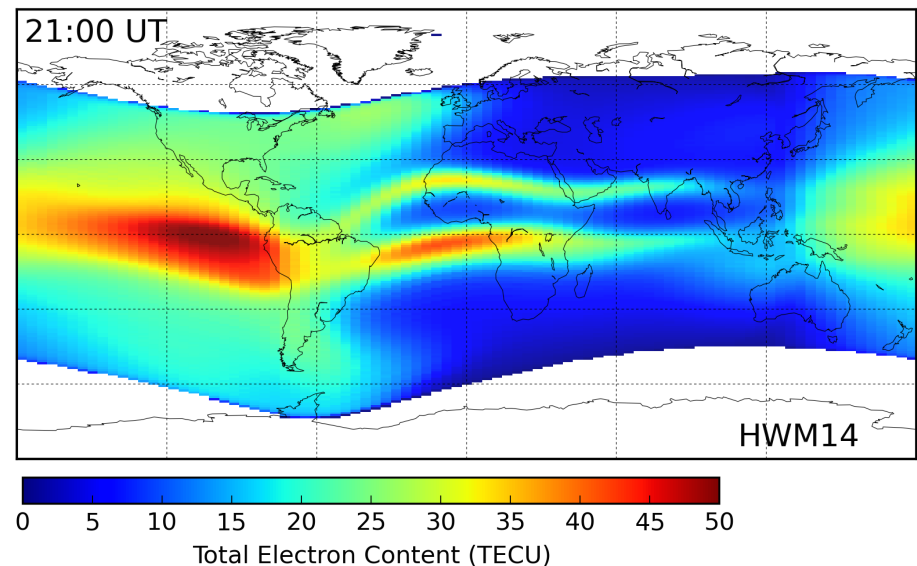
With the increasing complexity of coupled whole atmosphere models it is important to have an accurate observationally derived climatological specification of the atmosphere's wind fields.

HWM is able to provide a reasonable representation the variations of middle- and upper-atmosphere winds because they are predominantly driven by *in situ* solar heating under the periodic cyclical influence of the earth's rotation, tilt, and orbit around the sun.

When and where appropriate, HWM reduces the computational complexity of theoretical and applied calculations by avoiding the need to simultaneously compute the wind fields from first principles.



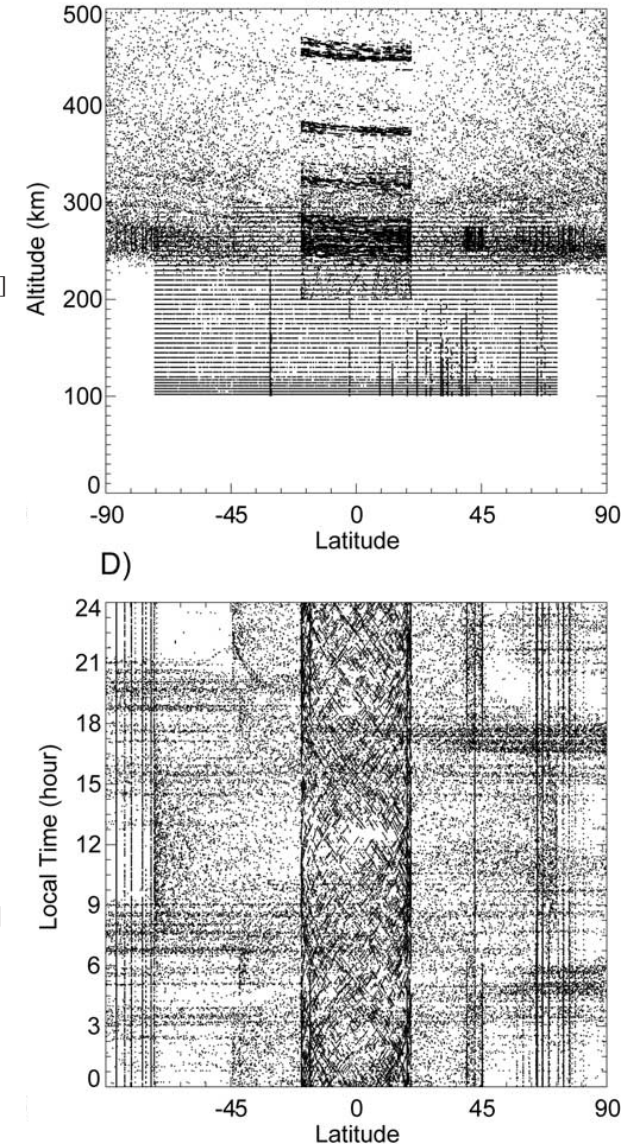
SAMI3 Total Electron Content



# Observational Data

Instrument	Location	Height (km)	Years	Local Time	Days	Data Points	Reference
<i>Satellite</i>							
AE-E NATE <sup>a</sup>	±18.0°N	220–400	1975–1979	both	799	200,500	<i>Spencer et al.</i> [1973]
DE 2 WATS <sup>b</sup>	±89.0°N	200–600	1981–1983	both	536	391,500	<i>Spencer et al.</i> [1981]
DE 2 FPI <sup>c</sup>	±89.0°N	250	1981–1983	both	308	47,600	<i>Hays et al.</i> [1981]
UARS HRDI	±72.0°N	50–115	1993–1994	day	834	30,100,000	<i>Hays et al.</i> [1993]
UARS WINDII 5577 Å	±72.0°N	90–300	1991–1996	day	949	24,672,000	<i>Shepherd et al.</i> [1993]
UARS WINDII 6300 Å	±42.0°N	200–300	1991–1996	night	243	2,237,942	<i>Shepherd et al.</i> [1993]
<i>Sounding Rocket</i>							
Falling Sphere	8°S–60°N	8–98	1969–1991	both	1,186	96,205	<i>Schmidlin et al.</i> [1985]
Rocketsonde	38°S–77°N	2–90	1969–1991	both	5,082	843,000	<i>Schmidlin et al.</i> [1986]
TMA	31°S–70°N	59–277	1956–1998	both	276	92,792	<i>Larsen</i> [2002]
<i>Fabry-Perot Interferometer</i>							
Arecibo	18.4°N, 66.8°W	250	1980–1999	night	473	14,198	<i>Burnside and Tepley</i> [1989]
Arequipa	16.2°S, 71.4°W	250	1983–2001	night	1048	32,238	<i>Meriwether et al.</i> [1986]
Arrival Heights	77.8°S, 116.7°E	250	2002–2005	night	535	54,214	<i>Hernandez et al.</i> [1991]
Halley Bay	75.5°S, 26.6°W	250	1988–1998	night	799	82,614	<i>Crickmore et al.</i> [1991]
Millstone Hill	42.6°N, 71.5°W	250	1989–2002	night	1,770	68,333	<i>Sipler et al.</i> [1982]
Mount John	44.0°S, 170.4°E	89, 96, 250	1991–1996	night	560	2,660	<i>Hernandez et al.</i> [1991]
Søndrestrom	67.0°N, 51.0°W	250	1984–2004	night	1,223	69,734	<i>Killeen et al.</i> [1995]
South Pole <sup>d</sup>	90.0°S	86, 250	1989–1999	night	1,091	163,044	<i>Hernandez et al.</i> [1991]
Svalbard <sup>c</sup>	78.2°N, 15.6°E	250	1980–1983	night	44	7,472	<i>Smith and Sweeny</i> [1980]
Thule	76.5°N, 68.4°W	250	1987–1989	night	172	21,500	<i>Killeen et al.</i> [1995]
Resolute Bay	74.7°N, 94.9°E	250	2003–2005	night	166	5,299	<i>Wu et al.</i> [2004]
Watson Lake	60.1°N, 128.6°W	250	1991–1992	night	135	28,000	<i>Niciejewski et al.</i> [1996]
<i>Incoherent Scatter Radar<sup>e</sup></i>							
Arecibo	18.3°N, 66.8°W	100–170	1974–1987	day	149	30,600	<i>Harper</i> [1977]
Chatanika	65.1°N, 147.4°W	90–130	1976–1982	day	97	38,721	<i>Johnson et al.</i> [1987]
European Incoherent Scatter	69.6°N, 19.2°E	100–120	1985–1987	day	29	2,900	<i>Williams and Virdi</i> [1989]
Millstone Hill	42.6°N, 71.5°W	120–400	1983–1987	both	142	23,536	<i>Salah and Holt</i> [1974]
Søndrestrom	67.0°N, 50.9°W	150–400	1983–1987	both	146	19,600	<i>Wickwar et al.</i> [1984]
St. Santin <sup>f</sup>	44.6°N, 2.2°E	90–165	1973–1985	day	256	18,382	<i>Amayenc</i> [1974]
<i>Medium-Frequency Radar<sup>g</sup></i>							
Adelaide	34.5°S, 138.5°E	60–98	2001–2004	both	834	481,634	<i>Vincent and Lesicar</i> , 1991
Bribe Island	28.0°S, 153.0°W	60–98	1995	both	280	184,176	<i>Reid</i> [1987]
Davis	68.6°S, 78.0°E	50–100	2001–2004	both	730	526,160	<i>Vincent and Lesicar</i> [1991]
Poker Flat	65.1°N, 147.5°W	44–108	1979–1985	both	1857	2,746,684	<i>Murayama et al.</i> [2000]
Wakkanai	45.4°N, 141.8°E	50–108	1998–2003	both	1538	1,874,672	<i>Murayama et al.</i> [2000]
Yamagawa	31.2°N, 130.6°E	60–98	1998–2003	both	1593	1,040,042	<i>Murayama et al.</i> [2000]
<i>Wind and Temperature Lidar</i>							
Fort Collins	40.6°N, 105.1°W	75–115	2002–2002	both	244	93,288	<i>She et al.</i> [2004]
<i>Numerical Weather Prediction Analysis<sup>h</sup></i>							
NOAA GFS Analysis	Global	0–35	2002–2007	both	1520	–	<i>Kalnay et al.</i> [1990]
NASA GEOS4 Analysis	Global	0–55	2002–2007	both	1520	–	<i>Bloom et al.</i> [2005]

Table 1. HWM07 Observational Database Summary



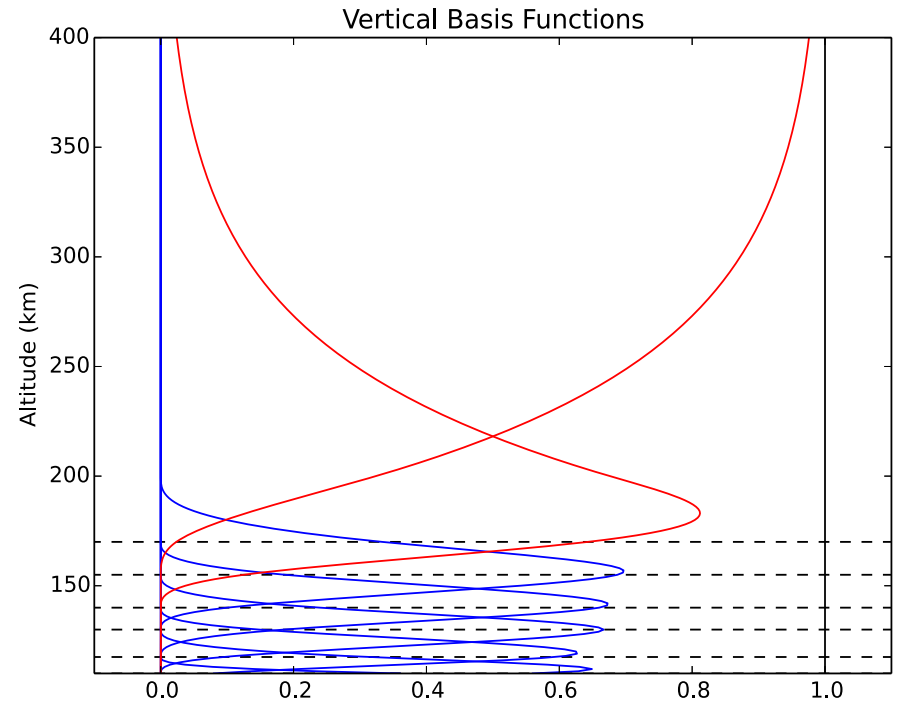
# Mathematical Formulation

$$\begin{aligned}
 u(\tau, \delta, \theta, \phi) &= \sum_{n=0}^N \sum_{s=0}^S \Psi_1(\tau, \theta, s, n) \\
 &+ \sum_{s=0}^S \sum_{l=1}^L \sum_{n=0}^N \Psi_2(\tau, \delta, \theta, s, l, n) \\
 &+ \sum_{s=0}^S \sum_{m=1}^M \sum_{n=0}^N \Psi_3(\tau, \phi, \theta, s, m, n)
 \end{aligned}$$

$$\begin{aligned}
 \Psi_1(\tau, \theta, s, n) &= -C_r^{s,n} \cdot \sin(n\theta) \cdot \cos(s\tau) \\
 &+ C_i^{s,n} \cdot \sin(n\theta) \cdot \sin(s\tau)
 \end{aligned}$$

$$\begin{aligned}
 \Psi_2(s, l, n, \tau, \delta, \theta) &= C_{a_r}^{s,l,n} \cdot V_n^l(\theta) \cdot \cos(l\delta) \cdot \cos(s\tau) + \\
 &C_{a_i}^{s,l,n} \cdot V_n^l(\theta) \cdot \sin(l\delta) \cdot \cos(s\tau) + \\
 &B_{a_r}^{s,l,n} \cdot W_n^l(\theta) \cdot \cos(l\delta) \cdot \cos(s\tau) + \\
 &B_{a_i}^{s,l,n} \cdot W_n^l(\theta) \cdot \sin(l\delta) \cdot \cos(s\tau) + \\
 &C_{b_r}^{s,l,n} \cdot V_n^l(\theta) \cdot \cos(l\delta) \cdot \sin(s\tau) + \\
 &C_{b_i}^{s,l,n} \cdot V_n^l(\theta) \cdot \sin(l\delta) \cdot \sin(s\tau) + \\
 &B_{b_r}^{s,l,n} \cdot W_n^l(\theta) \cdot \cos(l\delta) \cdot \sin(s\tau) + \\
 &B_{b_i}^{s,l,n} \cdot W_n^l(\theta) \cdot \sin(l\delta) \cdot \sin(s\tau),
 \end{aligned}$$

$$U(\tau, \delta, \theta, \phi, z) = \sum_j \beta_j(z) u_j(\tau, \delta, \theta, \phi)$$



$$V_n^l(\theta) = \frac{1}{\sqrt{n(n+1)}} \frac{d}{d\theta} P_n^l(\theta)$$

$$W_n^l(\theta) = \frac{1}{\sqrt{n(n+1)}} \frac{m}{\cos(\theta)} P_n^l(\theta)$$

$$u: \{C_r, C_i, B_r, B_i\} \leftrightarrow -v: \{B_r, B_i, -C_r, -C_i\}$$

$$w_{\text{los}} = u \sin \varphi + v \cos \varphi$$

# Parameter Estimation

7 x 10<sup>6</sup> rows  
(observations)

24,000 columns  
(parameters)

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

$$\begin{array}{cccccccc}
 \beta_0 \Psi_0 & \beta_1 \Psi_1 & \beta_2 \Psi_2 & \beta_3 \Psi_3 & & & & \\
 & \beta_1 \Psi_1 & \beta_2 \Psi_2 & \beta_3 \Psi_3 & \beta_4 \Psi_4 & & & \\
 & & \beta_2 \Psi_2 & \beta_3 \Psi_3 & \beta_4 \Psi_4 & \beta_5 \Psi_5 & & \\
 & & & \ddots & & & & \\
 & & & & \beta_{j+1} \Psi_{j+1} & \beta_{j+1} \Psi_{j+1} & \beta_{j+1} \Psi_{j+1} & \beta_{j+1} \Psi_{j+1} \\
 & & & & & \ddots & & \\
 & & & & & & \beta_{24} \Psi_{24} & \beta_{25} \Psi_{25} & \beta_{26} \Psi_{26} & \beta_{27} \Psi_{27} \\
 & & & & & & & \beta_{25} \Psi_{25} & \beta_{26} \Psi_{26} & \beta_{27} \Psi_{27} & \beta_{28} \Psi_{28} \\
 & & & & & & & & \beta_{26} \Psi_{26} & \beta_{27} \Psi_{27} & \beta_{28} \Psi_{28} & \beta_{29} \Psi_{29}
 \end{array}$$

$$\begin{array}{c}
 \mathbf{m}_0 \\
 \mathbf{m}_1 \\
 \mathbf{m}_2 \\
 \mathbf{m}_3 \\
 \mathbf{m}_4 \\
 \vdots \\
 \mathbf{m}_j \\
 \vdots \\
 \mathbf{m}_{26} \\
 \mathbf{m}_{27} \\
 \mathbf{m}_{28} \\
 \mathbf{m}_{29}
 \end{array}
 \times
 \begin{array}{c}
 \mathbf{d}_0 \\
 \vdots \\
 \mathbf{d}_{22} \\
 \mathbf{d}_{23} \\
 \mathbf{d}_{24} \\
 \mathbf{d}_{25} \\
 \mathbf{d}_{26}
 \end{array}
 =$$

Iterative linear least-squares optimal estimation procedure (e.g. Rodgers *et al.*, 2000]

$$\mathbf{m}_{n+1} = \mathbf{m}_n + [\mathbf{G}^T \mathbf{S}_\epsilon^{-1} \mathbf{G} + \mathbf{S}_n^{-1}]^{-1} \mathbf{G}^T \mathbf{S}_\epsilon [\mathbf{d} - \mathbf{G}\mathbf{m}_n]$$

$$\mathbf{S}_{n+1} = [\mathbf{G}^T \mathbf{S}_\epsilon^{-1} \mathbf{G} + \mathbf{S}_n^{-1}]^{-1}$$

$\mathbf{S}_n$  Parameter Covariance

$$\mathbf{S}_\epsilon = \text{diag} \| 1/\sigma_i^2 \|$$

~10 to 60 m/s  
 $\sigma_i = 37.5$  m/s

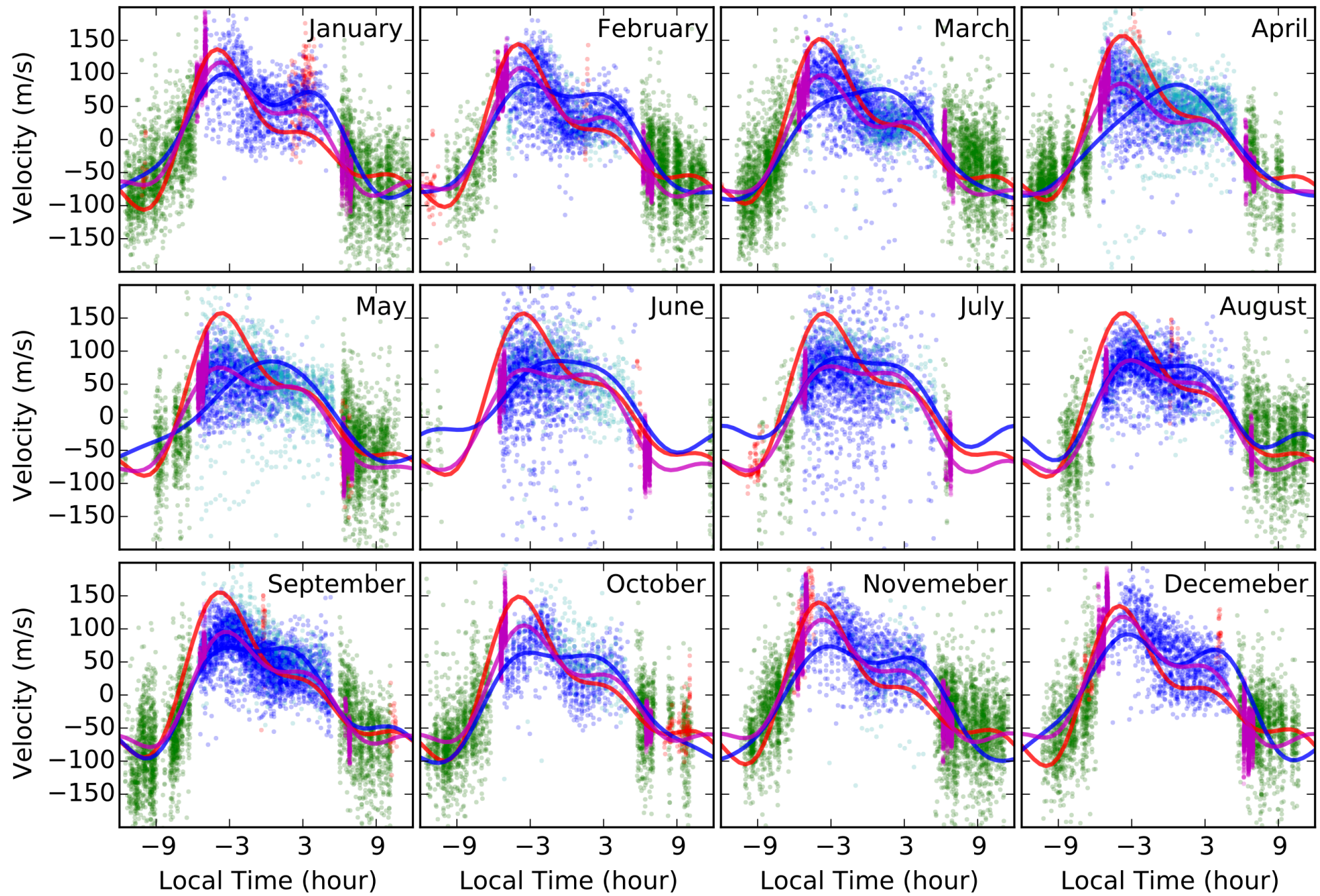
Convergence after approximately ten iterations with  
2 x 10<sup>6</sup> observation each

# Statistical Performance Measures

Additional statistics provided in Drob *et al.*, (2014)

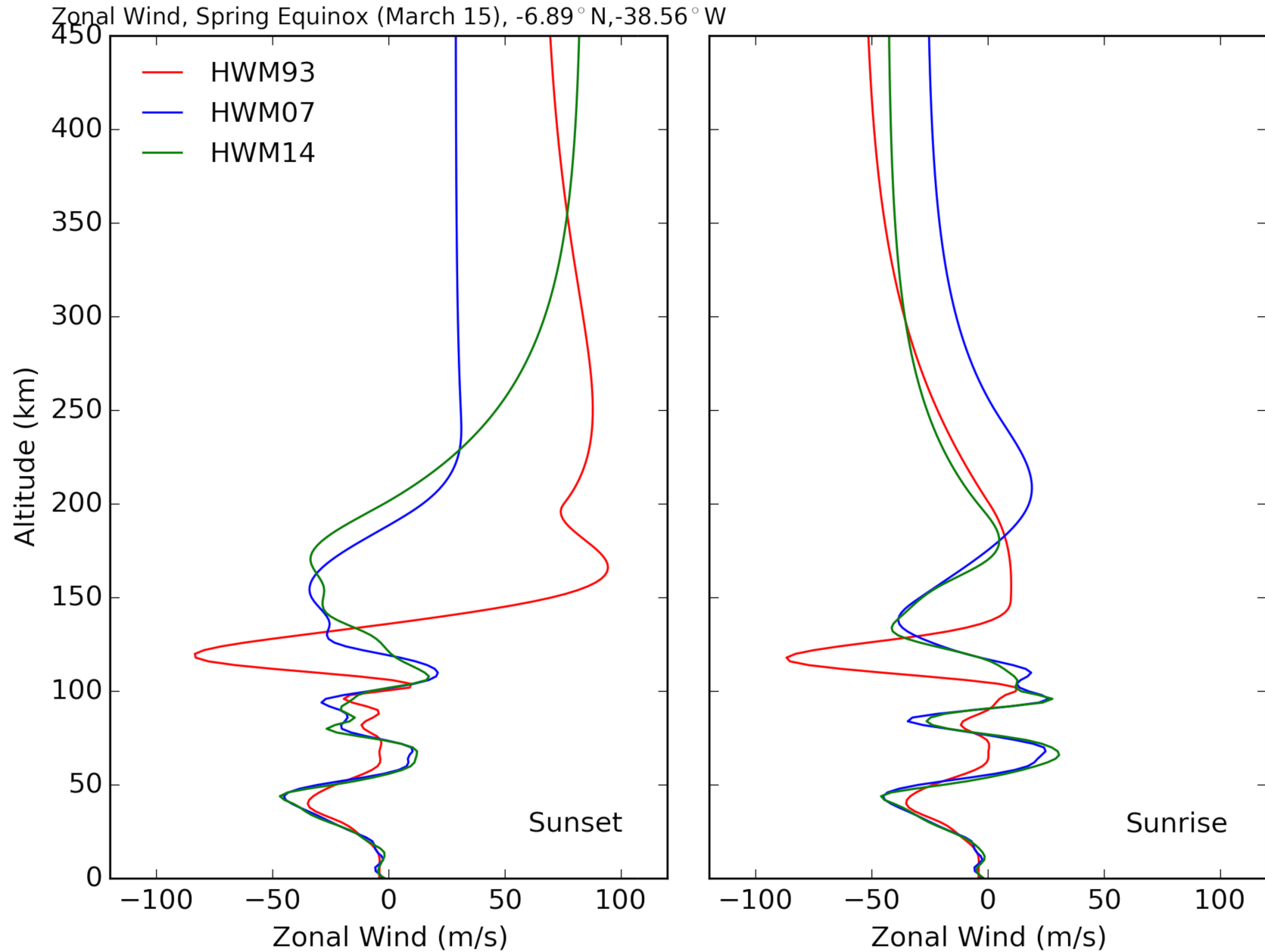
	$N$	$\beta_{obs}$	$\beta_{93}$	$\beta_{07}$	$\beta_{14}$	$\sigma_{obs}$	$\sigma_{93}^{rms}$	$\sigma_{07}^{rms}$	$\sigma_{14}^{rms}$
Line-of-Sight, Fabry-Perot Interferometer									
Arrival Heights	138690	-14.56	-13.19	-0.59	-6.74	103.55	101.06	68.60	69.07
Resolute Bay	17377	0.41	-0.83	2.17	0.52	107.91	65.74	71.30	67.98
Arecibo	8051	11.62	-2.59	-9.25	-5.58	34.09	33.97	35.54	30.84
Millstone Hill	7503	-20.05	-9.76	-12.50	-9.92	59.95	48.42	47.71	45.79
Søndrestrøme	3730	2.97	-15.97	-27.72	-8.82	110.57	87.24	93.62	81.23
Cross-Track, Satellite									
AE-E NATE	57428	7.74	7.76	5.31	1.81	68.73	56.58	55.40	51.40
GOCE	573672	95.79	19.58	40.46	6.81	48.83	50.41	60.60	36.05
Zonal, Fabry-Perot Interferometer									
Arecibo	79108	36.77	12.65	-8.04	-6.19	52.66	53.07	47.59	44.71
Arequipa	99198	84.48	-25.81	-3.84	0.74	71.66	78.52	72.13	68.83
Halley Bay	91245	14.85	15.51	-5.13	-9.07	93.81	93.29	66.44	65.35
Jicamarca	1901	48.33	-43.89	-23.19	-7.74	59.29	79.79	64.31	59.54
Millstone Hill	68175	22.12	28.37	-9.74	-9.62	70.32	68.29	63.56	63.21
Mount John	1949	35.17	-1.38	-8.55	1.70	70.81	49.92	56.79	54.97
Movil	24227	63.54	-26.99	-9.16	4.39	55.55	60.72	56.49	48.35
Psigah	12610	35.01	15.84	-27.40	-13.64	57.67	51.25	56.36	51.32
Renoir	12483	57.23	-34.74	-6.05	-0.71	44.94	63.27	46.09	40.32
Søndrestrøme	10442	-15.31	20.62	-7.31	-3.94	102.68	114.75	97.96	104.09
Svalbard	1353	-40.74	8.83	-3.57	-9.32	129.08	140.66	125.08	129.76
Thule	15643	2.78	27.58	-1.95	-7.45	150.12	133.50	118.98	118.30
Watson Lake	4979	-20.23	52.09	13.47	5.69	67.85	86.19	68.89	64.96
Poker Flat	450925	-14.35	-4.16	0.49	-9.09	70.35	65.00	72.56	60.30
Zonal, Satellite/Rocket									
DE2 WATS	7233	-19.95	1.00	-6.92	-6.52	125.73	88.99	89.17	94.83
TMA	2774	9.40	-2.02	6.37	-5.82	53.60	57.35	34.79	33.21
UARS WINDII 557.7 nm	415238	-39.77	-8.32	-4.37	-5.19	72.22	72.03	59.50	59.31

# Statistical Performance Measures





# Observed Morphology



# Programming Tips

Official HWM14 source <http://onlinelibrary.wiley.com/doi/10.1002/2014EA000089/full>  
supplemental information: [ess224-sup-0002-supinfo.tgz](http://onlinelibrary.wiley.com/doi/10.1002/2014EA000089/full)

Please read the 'README file' and the Hedin *et al.*, (1994), Drob *et al.*, (2007), and Drob *et al.* (2014) papers.

```
subroutine hwm14(iyd,sec,alt,glat,glon,stl,f107a,f107,ap,w)
  implicit none
  integer(4),intent(in)  :: iyd      ! year and day as yyddd
  real(4),intent(in)    :: sec      ! ut(sec)
  real(4),intent(in)    :: alt      ! altitude(km)
  real(4),intent(in)    :: glat     ! geodetic latitude(deg)
  real(4),intent(in)    :: glon     ! geodetic longitude(deg)
  real(4),intent(in)    :: stl      ! not used !!!
  real(4),intent(in)    :: f107a    ! not used !!!
  real(4),intent(in)    :: f107     ! not used !!!
  real(4),intent(in)    :: ap(2)   ! ap(1) not used !!!, ap(2) current 3hr
  real(4),intent(out)   :: w(2)    ! w(1) +northward, w(2) +eastward, (m/s)
```

HWM14 compiles with Python Numpy's f2py script without any code modifications.

```
%f2py -c -m hwm hwm14.f90 -fcompiler=gnu95
```

```
import hwm
```

```
[u,v] = hwm.hwm14(iyd,sec,alt,glat,glon,stl,f107a,f107,ap=[-1,-1])
```

```

subroutine generic()

  use qwm, only content, wavefactor, tidefactor ! <= switches and factors
  implicit none

  ! some code ..

  ap(1:2) = -1           ! Disturbance Wind Model off

  ! some code ..

  content(2:3) = .false. ! Stationary waves and tides off (zonal mean only)

  ! some code ..

  content(1) = .false.   ! Zonal means off
  content(3) = .true.    ! Only the migrating tides now

  ! some code ..

  tidefactor(3) = 0.0    ! Turn off the terdiurnal tide
  tidefactor(2) = 2.0    ! Scale the migrating semidiurnal tide by x2

  ! some code ..

end subroutine generic

```

Python Module (via Numpy's f2py script)

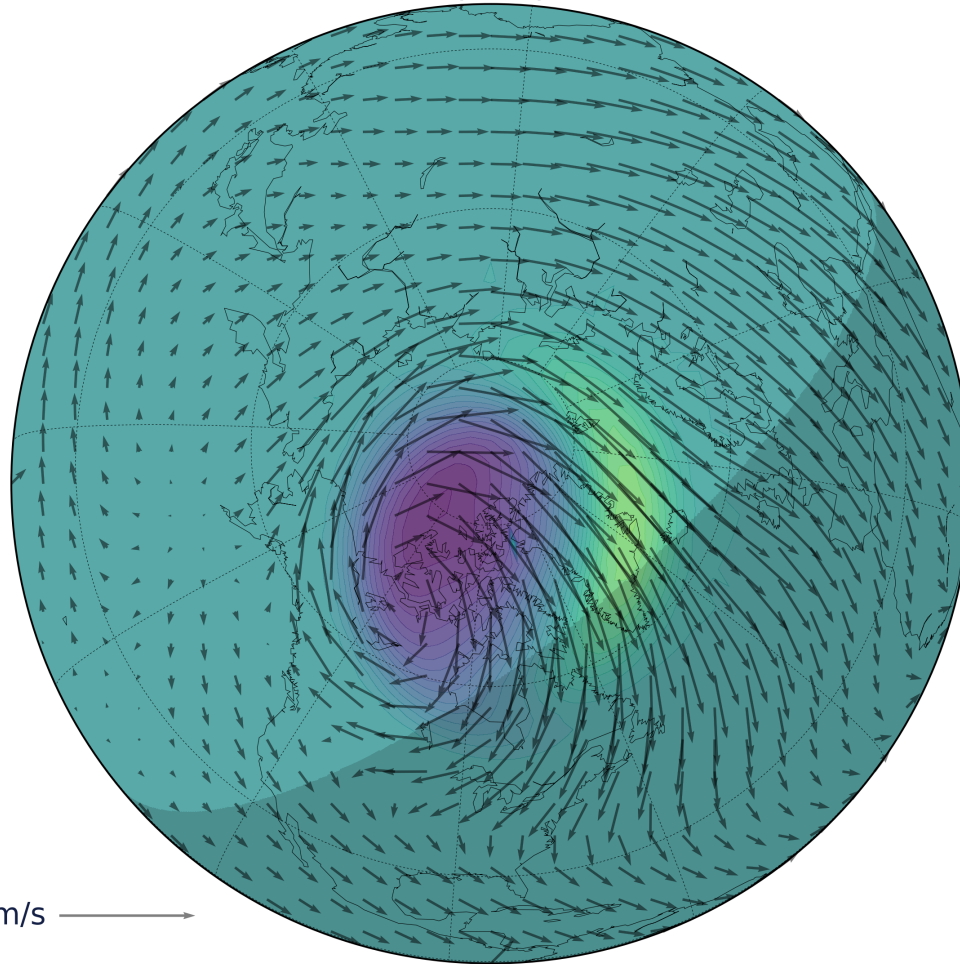
```

hwm.qwm.content[1:3] = -1 # 0 = .false. -1 = .true. Python to Fortran
hwm.qwm.tidefactor[0] = 1.5 # Scale the diurnal tide python[n] = fortran(n+1)

```

# Interactive Demo

06/21/2016 (173) 03:00 UTC



HWM14

Wiemer05

353 m/s →

Day	<div style="width: 150px; height: 10px; background-color: orange;"></div>	172
Altitude	<div style="width: 230px; height: 10px; background-color: orange;"></div>	275
UT (hour)	<div style="width: 50px; height: 10px; background-color: orange;"></div>	3
Ap	<div style="width: 20px; height: 10px; background-color: orange;"></div>	21

$\Phi$ (deg)	<div style="width: 100px; height: 10px; background-color: orange;"></div>	-90
$B_T$ (nT)	<div style="width: 50px; height: 10px; background-color: orange;"></div>	5
$\rho$ ( $\#/cm^3$ )	<div style="width: 150px; height: 10px; background-color: orange;"></div>	9
$V$ (km/s)	<div style="width: 200px; height: 10px; background-color: orange;"></div>	468