Conductance and Conductivity…

…in the Wave Theory of the Ionosphere…

…How much is left of Electrostatics?

Russell Cosgrove

Traditional MI Coupling

- What is the justification for such a simple ionospheric model?

$\boldsymbol{B}_0$-lines

Incident and Reflected Alfven Waves

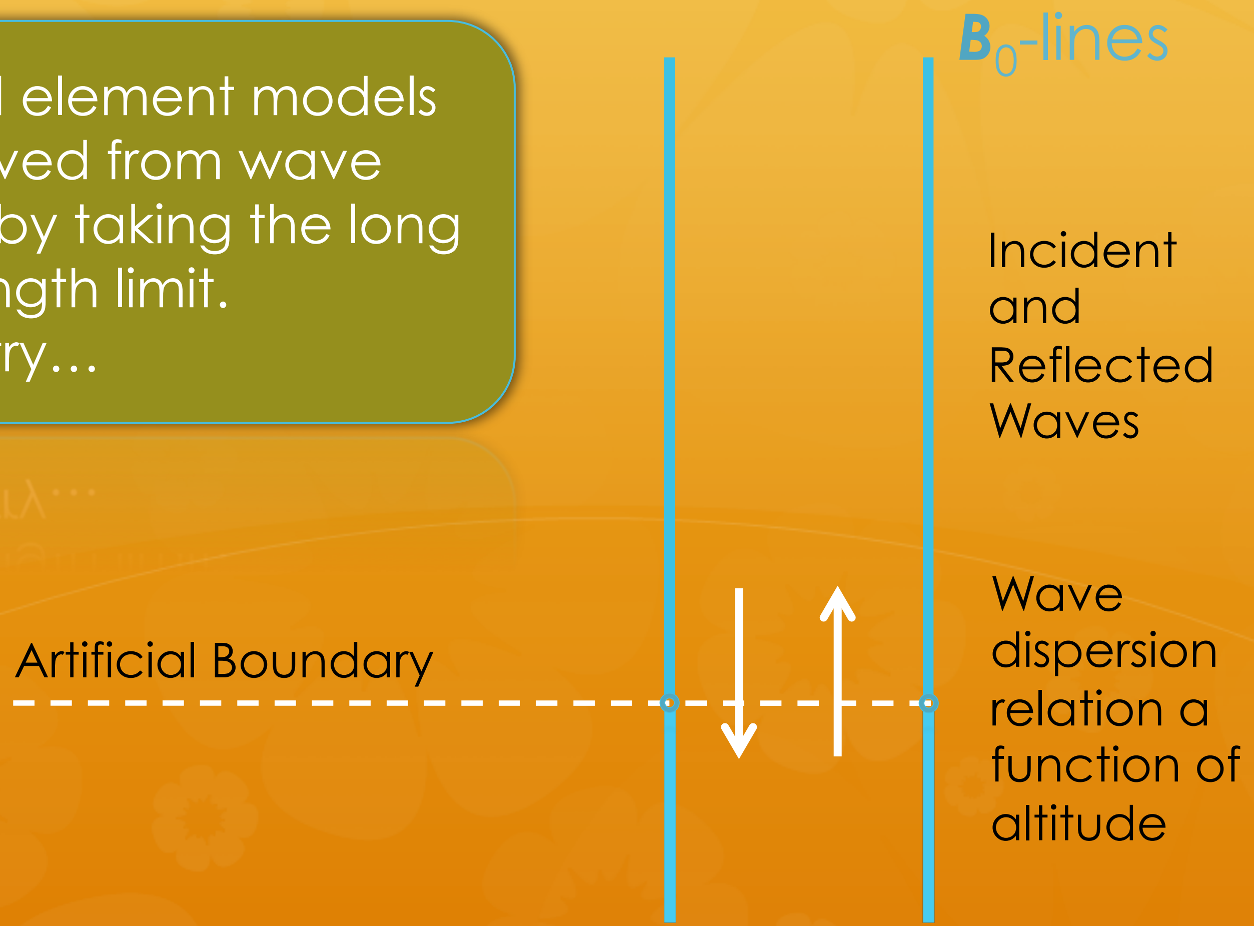
Wave Model for Magnetosphere

Lumped Element Model for Ionosphere

Resistor: the Ionospheric "conductance" derived from the electrostatic assumption

Actual MI Coupling

- Lumped element models are derived from wave models by taking the long wavelength limit.
- So let's try…



The Gap in Knowledge

- The usual wave dispersion relations involve inappropriate approximations.
- We need full inclusion of collisional effects to address conductivity.
- Apparently, nobody has ever verified that electrostatic theory can be recovered from the wave theory.
- Is it obvious? No!
- We don't know the wavelengths of the propagating modes!
- We don't know the dissipation scale lengths!
- We don't know the polarizations!

So it's either Continue Assuming Everything is Good…or Derive/Solve the Dispersion Relation…

- Actually, turns out we don't really need to derive anything.
- Modern computers allow exact computation.

Just Find the Eigenvectors!

Two fluid equations

$$
\begin{aligned}
\frac{\partial \vec{v}_e}{\partial t} &= -\Omega_e \left(\frac{\vec{E}}{B} + \vec{v}_e \times \hat{b} \right) - \nu_{en} \left(\vec{v}_e - \vec{v}_n \right) \\
&- \nu_{ei} \left(\vec{v}_e - \vec{v}_i \right) - \left(\vec{v}_e \cdot \vec{\nabla} \right) \vec{v}_e \\
\frac{\partial \vec{v}_i}{\partial t} &= \Omega_i \left(\frac{\vec{E}}{B} + \vec{v}_i \times \hat{b} \right) - \nu_{in} \left(\vec{v}_i - \vec{v}_n \right) \\
&- \nu_{ie} \left(\vec{v}_i - \vec{v}_e \right) - \left(\vec{v}_i \cdot \vec{\nabla} \right) \vec{v}_i \\
\epsilon_0 \frac{\partial \vec{E}}{\partial t} &= -en \left(\vec{v}_i - \vec{v}_e \right) + \mu_0^{-1} \vec{\nabla} \times \delta \vec{B} \\
\frac{\partial \delta \vec{B}}{\partial t} &= -\vec{\nabla} \times \vec{E} \qquad (1)
\end{aligned}
$$

- Write as matrix eqn.
- $\frac{\partial Y}{\partial t} + \underline{M} Y = F$
- Fourier Transform and Linearize
- $-\omega X + \underline{H} X = 0$
- Get eigenvectors/values

The Alfven wave is the lowest frequency: it works down to DC, and should take most of the energy

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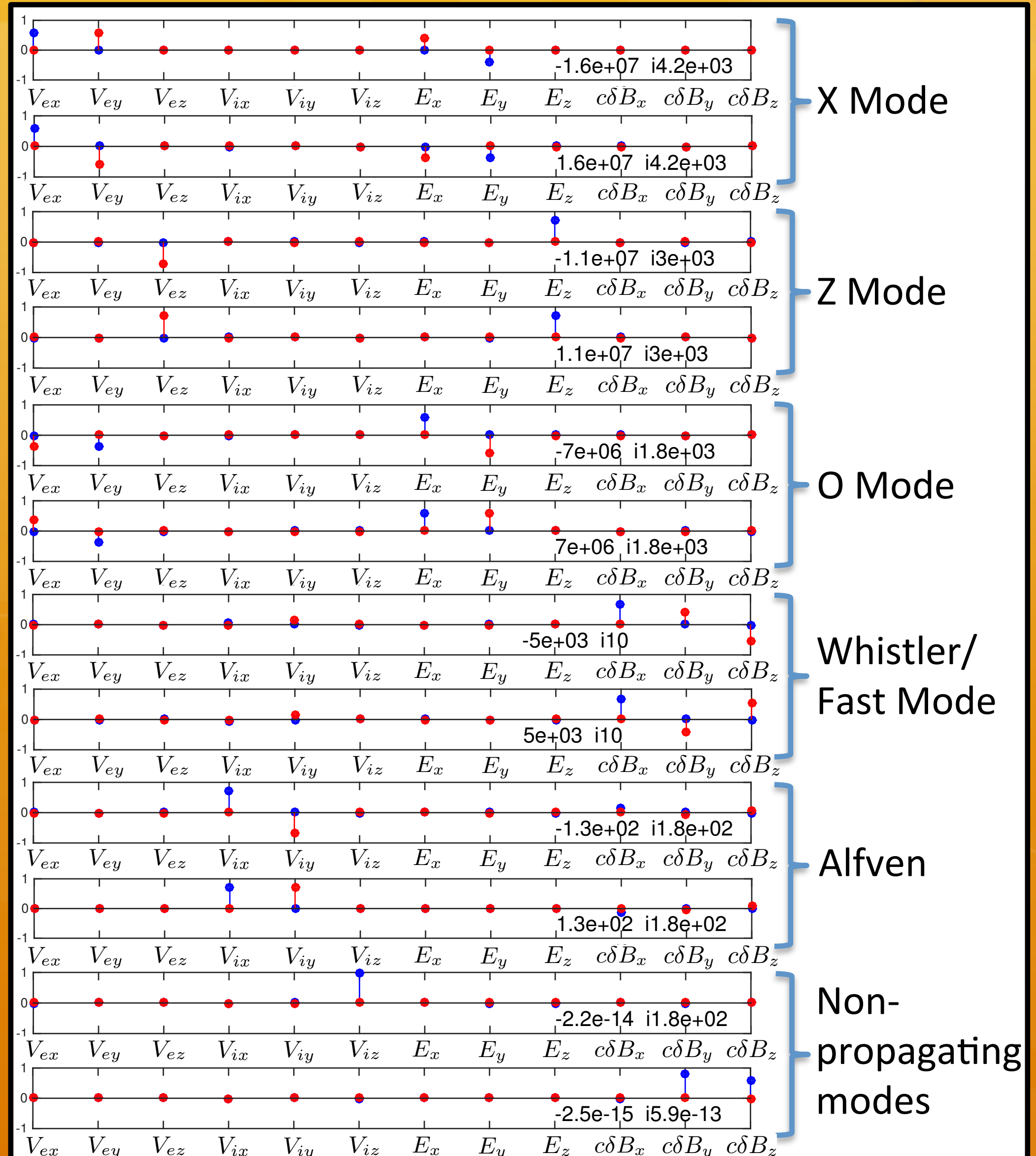
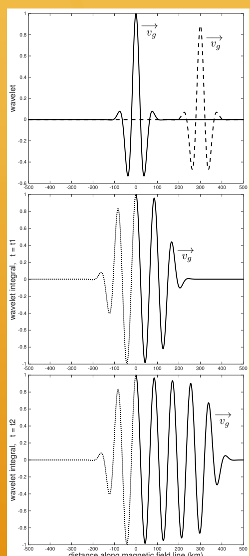


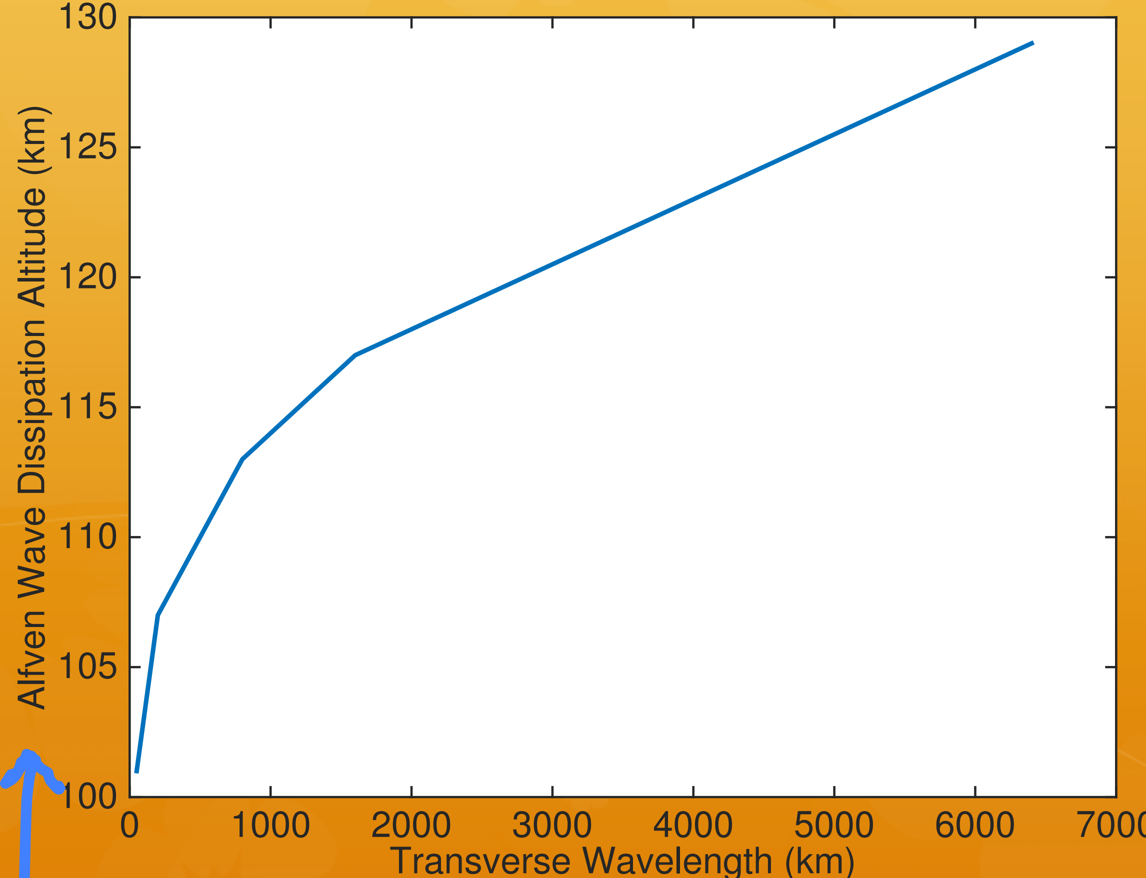
Results: Alfven Wave Penetration Depth

- Alfven wave packet travels with group velocity



- Multiply group velocity by dissipation time scale
- Gives the dissipation scale length
- When dissipation scale length is 10% of altitude

Only short wavelength Alfven waves get down to lower E region!



Mode conversion to Whistler mode required for large-scale E field to map to lower E region!

Results: At the Auroral Arc Scale

Parallel Wavelengths are short!

λ_y (km)	λ_z (km)	z_{dis} (km)	alt. (km)
1	13.6	84.3	99
1	40.0	629.9	109
1	1648.4	78.1	119
10	13.8	84.6	99
10	29.7	5,062	109
10	80.3	2.5×10^4	119
50	24.4	3.4	99
50	41.2	721.3	109
50	83.4	9.2×10^4	119

Perpendicular Wavelength (λ_y)

Dissipation Scale Length (z_{dis})

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Results: Conductivity

Derive input admittance to thin slab by superposing incident and reflected waves

Zero conductivity below

$$\tilde{\mathbf{E}} = [\hat{x}E_{1x}\cosh(ik_z z) + \hat{y}E_{1y}\cosh(ik_z z) - \hat{z}E_{1z}\sinh(ik_z z)]\, 2e^{i(\omega t - k_y y)}$$

$$\tilde{\mathbf{B}} = [-\hat{x}B_{1x}\sinh(ik_z z) - \hat{y}B_{1y}\sinh(ik_z z) + \hat{z}B_{1z}\cosh(ik_z z)]\, 2e^{i(\omega t - k_y y)}.$$

$$Y_O = \left.\frac{J_z}{\vec{\nabla}\cdot\tilde{\mathbf{E}}_\perp}\right|_{z=-l} = \frac{-iB_{1x}}{\mu_0 E_{1y}}\tan(k_z l) \simeq \frac{-iB_{1x}k_z}{\mu_0 E_{1y}} l, \quad \text{and}$$

$$\sigma_{Pw} \triangleq Y_O/l|_{l\to 0} = \frac{-iB_{1x}k_z}{\mu_0 E_{1y}}.$$

Wave Pedersen

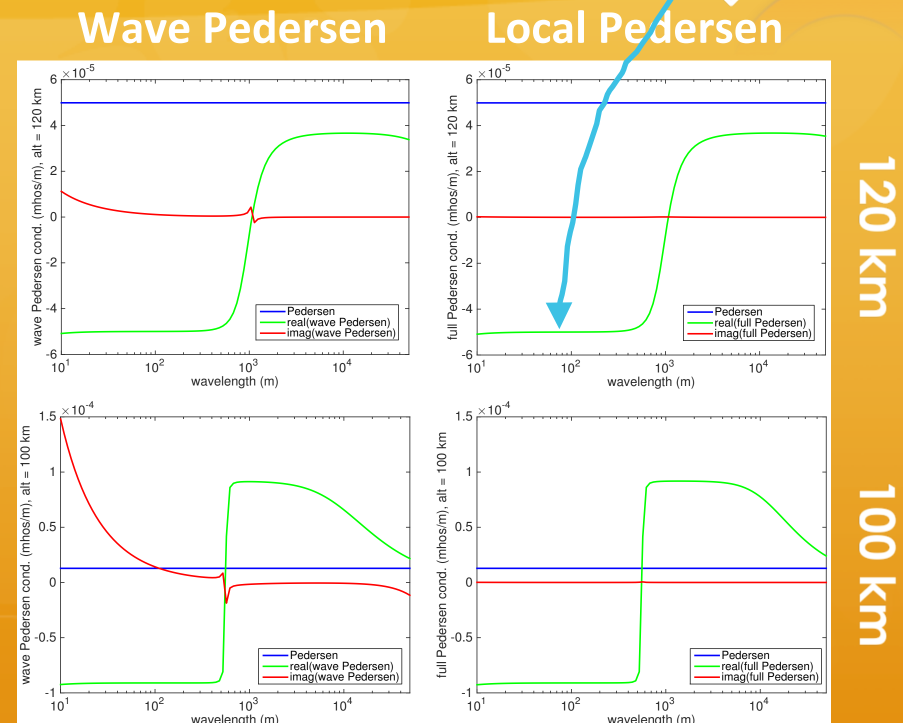
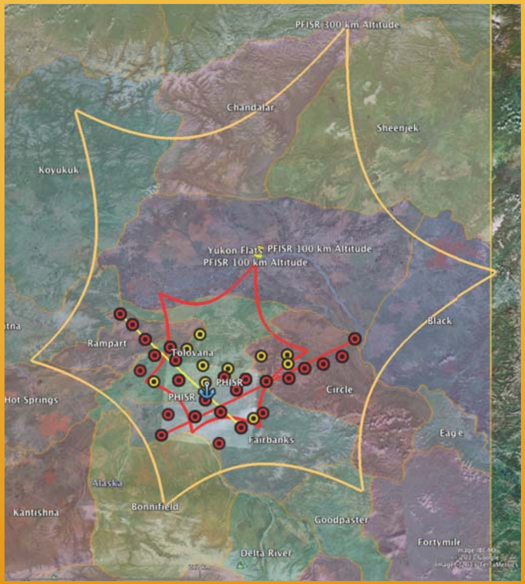


Figure 16: The wave Pedersen conductivity σ_{Pw} (left column, green and red lines), compared with the full Pedersen conductivity $\sigma_P(\omega)$ evaluated using the full complex ω (right column, green and red lines, from equation (30)), and the standard zero-frequency Pedersen conductivity (blue lines) at altitudes of 120 km (top row) and 100 km (bottom row). The phase velocity $V_p = 40$ m/s was used to derive the real part of ω to be matched by the Alfvén wave.

ω has a large imaginary part; it's not close to zero!

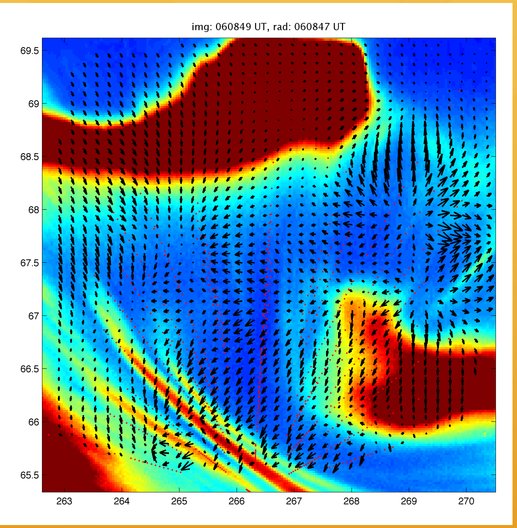
$\sigma_P(0)$ is a constant (blue lines).
$\sigma_P(\omega)$ is far from constant!
At short wavelengths $\sigma_P(\omega)$ has a negative real part!!

Poker Flat Experiment: Combined AMISR and MT Sensor Array

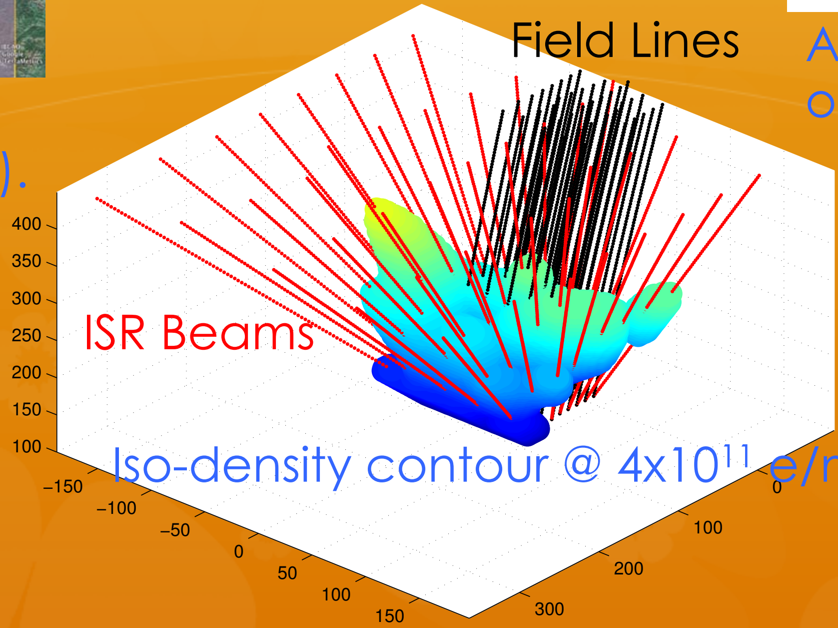


Array of MT sensors (mag + ground E-field). Collaboration with Geophysicist Adam Schultz, Oregon State U.

- Can we predict the ground magnetic deflection from AMISR plus ground conductivity?



AMISR Imaged E-field overlaid with Green Line



Iso-density contour @ 4×10^{11} e/m^3

Conclusions

- Alfven wave theory in the ionosphere does not reduce to electrostatics:
  - Short wavelengths compared to E-F spacing.
  - Dissipation scale length doesn't match.
  - Pedersen conductivity strongly effected.
- Efficient mode coupling between Alfven and Whistler waves is required to salvage E field mapping at long wavelengths.
- At the Arc scale (and smaller) there seems no way to recover Electrostatics. Looks like Wrong Assumption!
- Joint AMISR-Magnetotelluric data analysis will shed some light.

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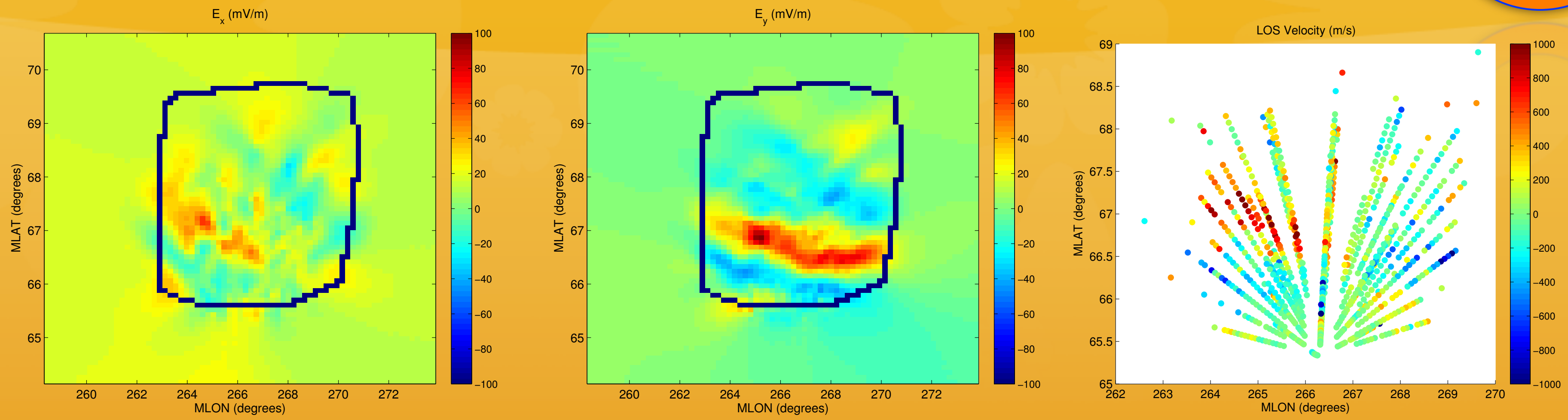
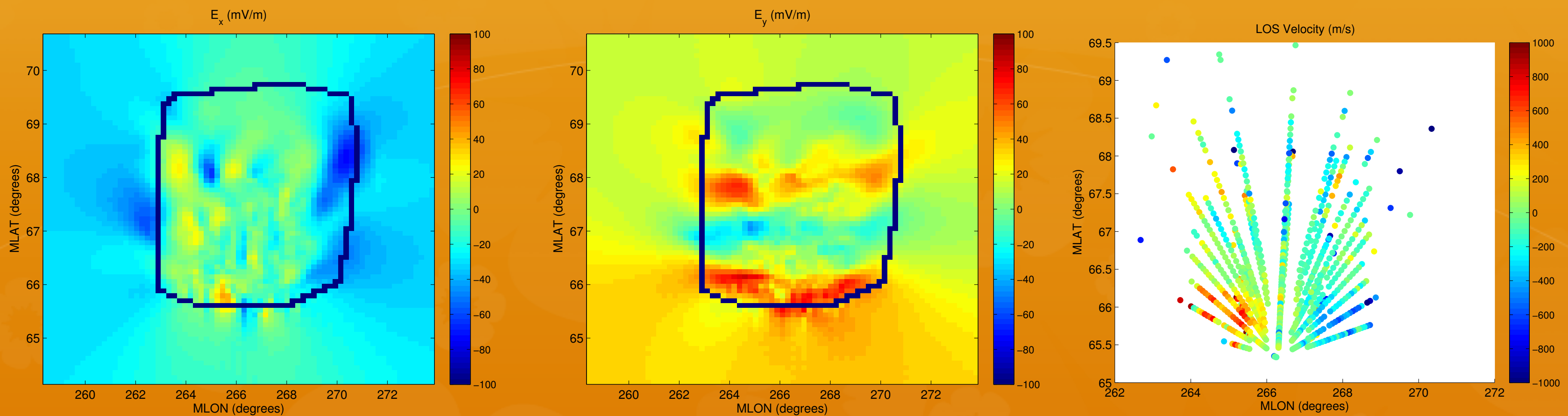
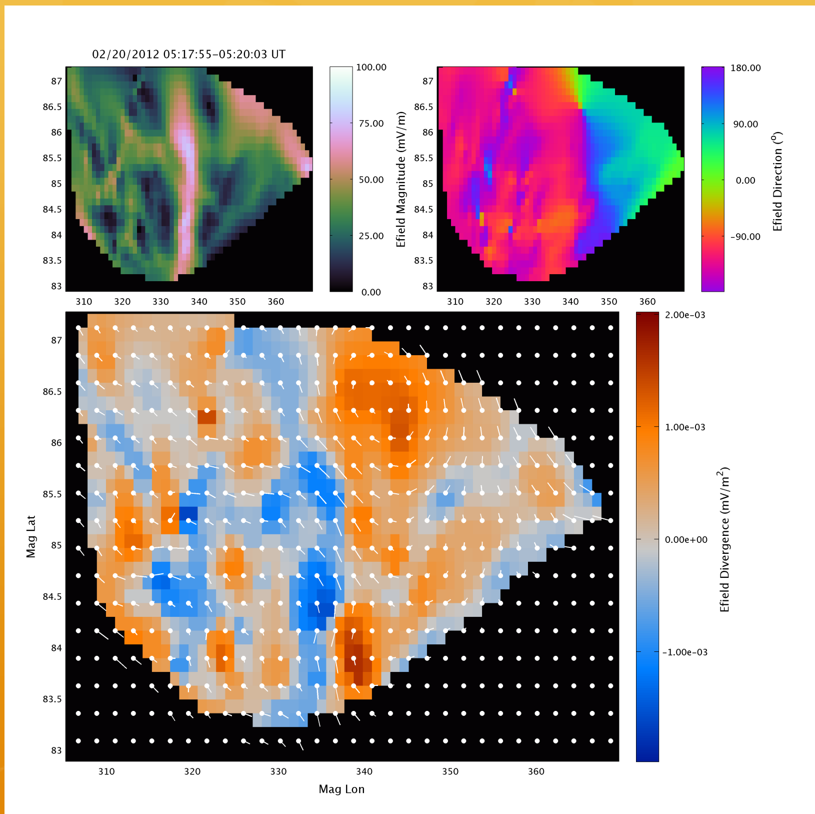
Two Auroral Arc Examples

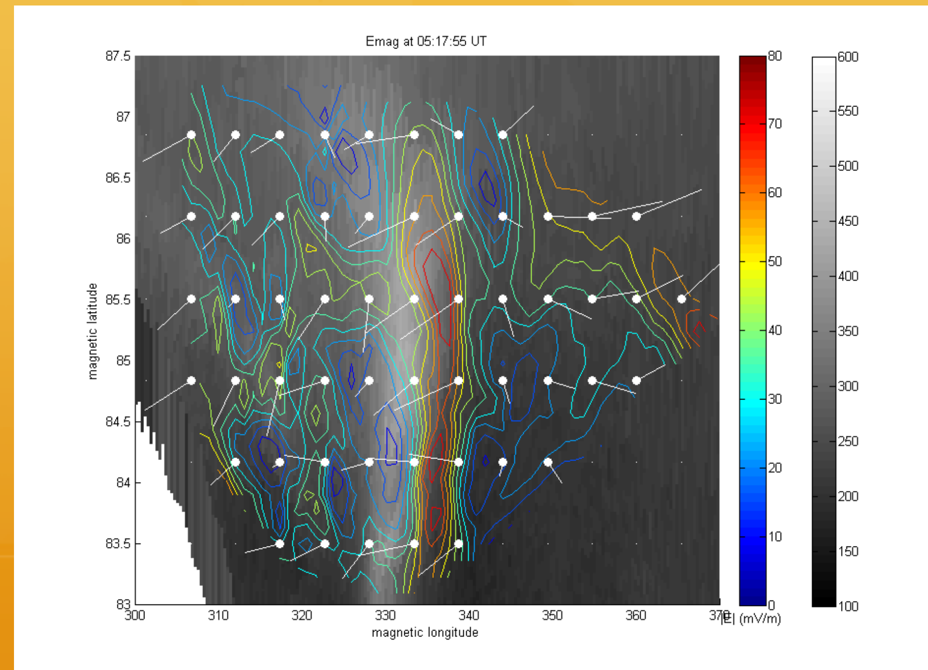


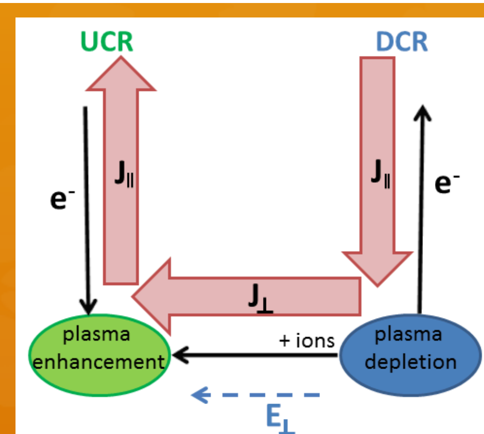


Comparison to Optical Arc

Courtesy Hanna Dahlgren, Gareth Perry





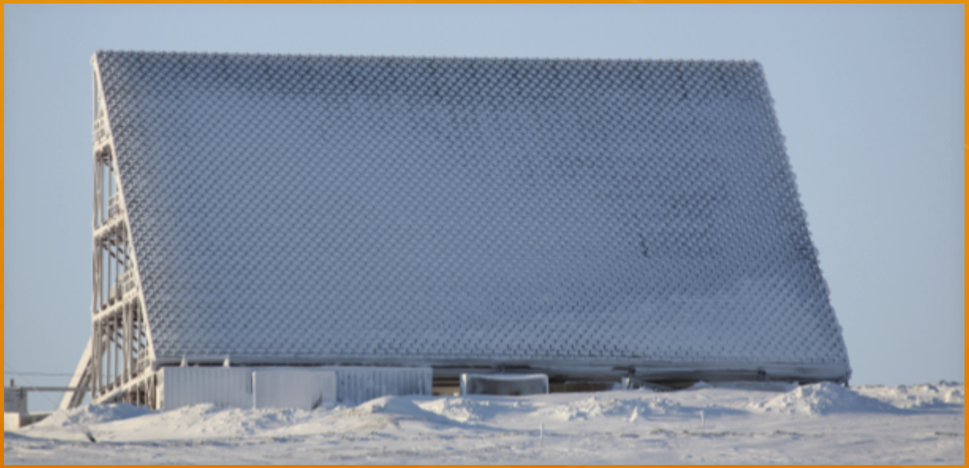


Negative divergence at position of optical arc = precipitating electrons!

Fitting Electric Field to Multibeam Incoherent Scatter Radar Data

Poker Flat Incoherent Scatter Radar, Poker Flat, Alaska

Courtesy of Michael Nicolls,
Russell Cosgrove, and
Hasan Bahcivan



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