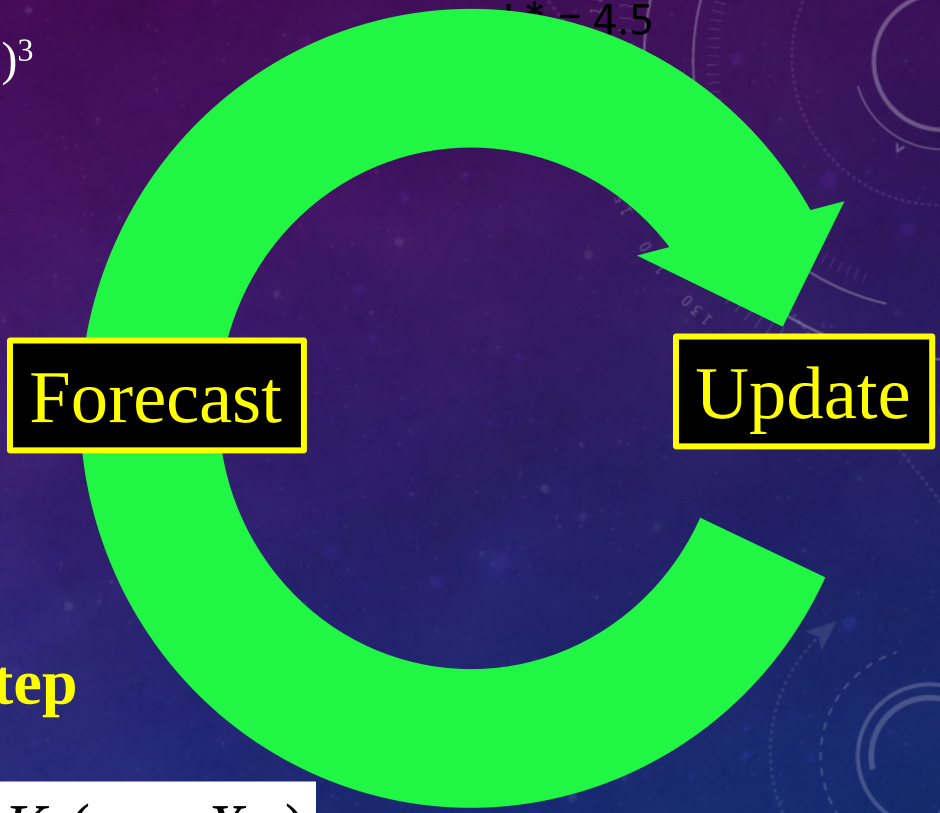
DATA ASSIMILATION OF RADIATION BELT ELECTRONS USING MULTI-POINT OBSERVATIONS AND THE VERB CODE: *LESSONS LEARNED AND FUTURE DEVELOPMENTS*

ADAM C KELLERMAN, Y. Y. SHPRITS, D. A. KONDRASHOV,

T. PODLADCHIKOVA, A. Y. DROZDOV, H. ZHU

Kalman Filter and Errors

X: State vector, PSD $(c/(cm.MeV))^3$
M: Model matrix (VERB code)
P: State error covariance matrix
Q: Model covariance matrix.
y: PSD measurements
K: Kalman Gain
R: Measurement error



Forecast Step:

$$X_f = M_t X_{t-1}$$

$$P_f = M_t P_{t-1} M_t^T + Q_t$$

Update Step

$$X_a = X_f + K_t (y_t - X_f)$$

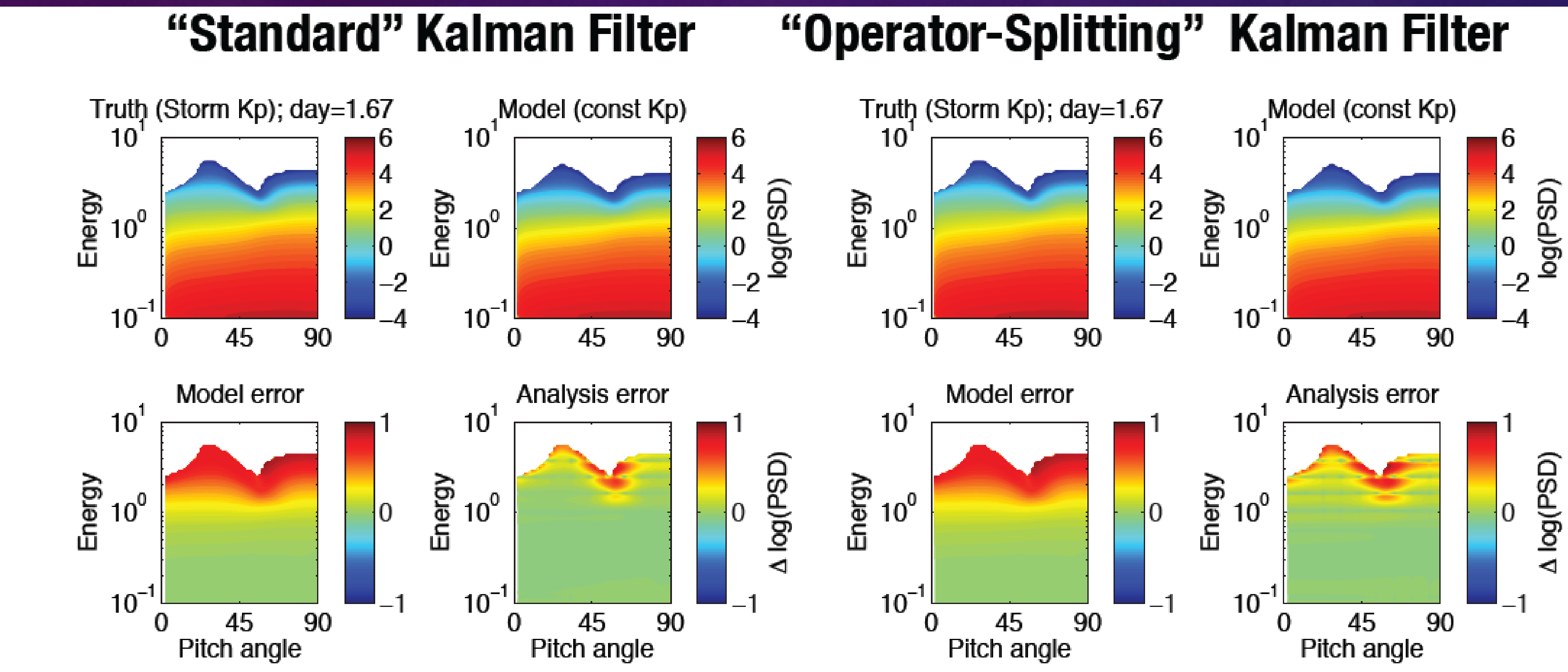
$$K_t = P_f (P_f + R_t)^{-1}$$

$$P_a = (I - K_t) P_f$$

Operator Splitting in 2D

$$X_t^f = M_{t-1} X_{t-1}^f$$

$$X_t^f = M_{t-1_\alpha} M_{t-1_E} X_{t-1}^f$$

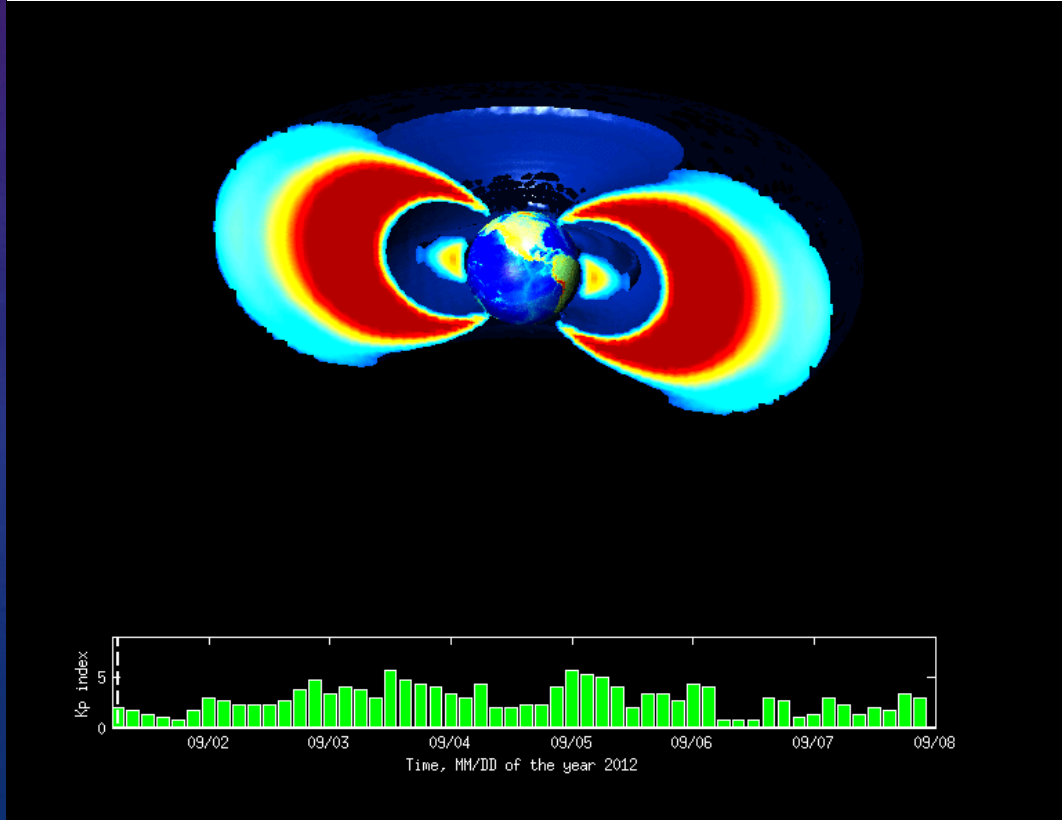


VERB 2.0

In this study, we use the Versatile Electron Radiation Belt (VERB) code

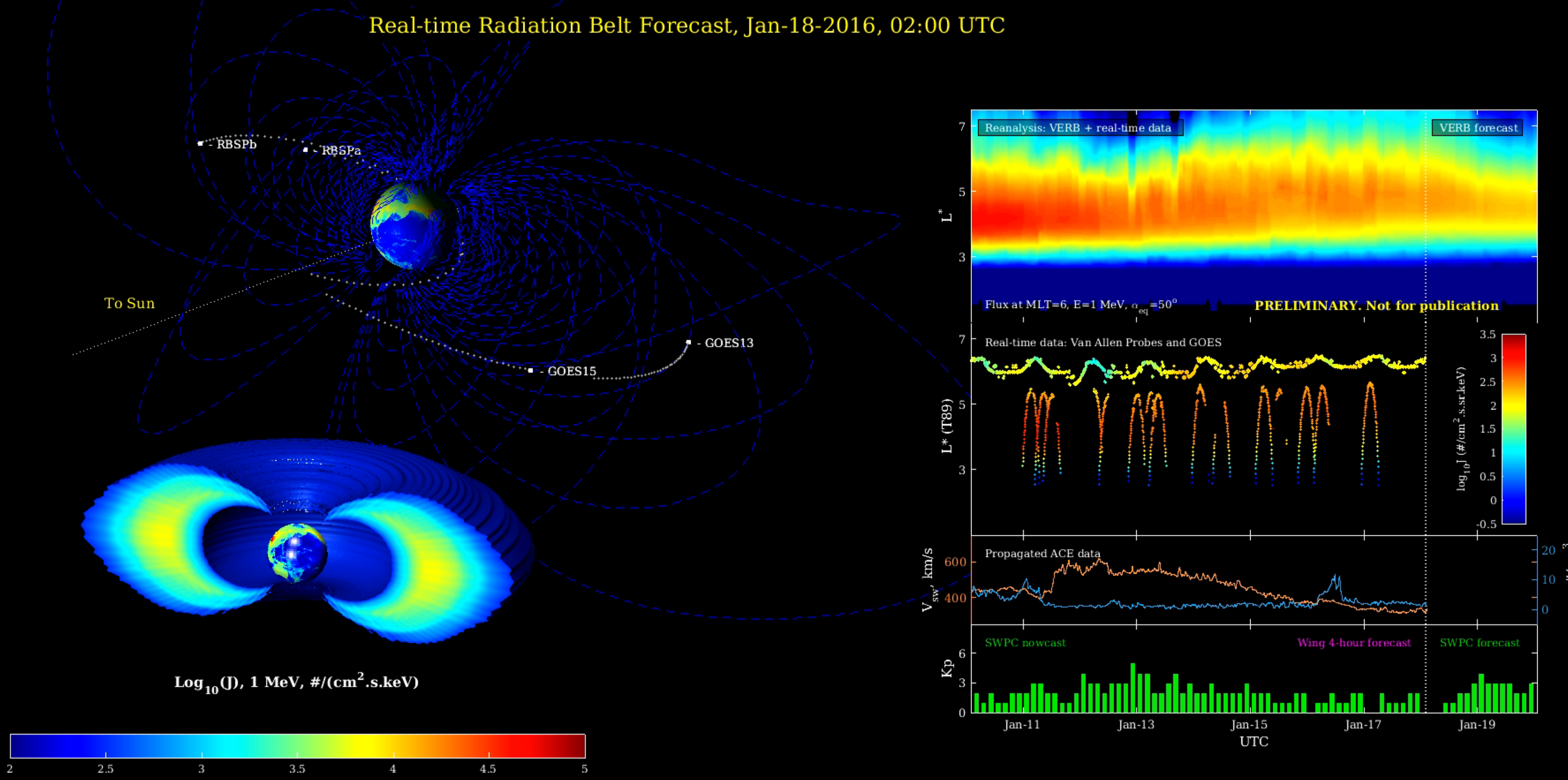
Diffusion code that solves the Fokker-Planck equation for electron phase space density (PSD)

$$\begin{aligned}
\frac{\partial f}{\partial t} &= L^2 \frac{\partial}{\partial L}\Big|_{\mu,J} \frac{1}{L^2} D_{LL} \frac{\partial f}{\partial L}\Big|_{\mu,J} \\
&+ \frac{1}{p^2}\frac{\partial}{\partial p}\Big|_{\alpha_0,L} p^2 D_{pp} \frac{\partial f}{\partial p}\Big|_{\alpha_0,L} + \\
&+ \frac{1}{T(\alpha_0)\sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0}\Big|_{p,L} T(\alpha_0)\sin(2\alpha_0) D_{\alpha_0\alpha_0} \frac{\partial f}{\partial \alpha_0}\Big|_{p,L}.
\end{aligned}$$



Shprits et al, 2013, Nature Physics

Forecast Example – Computed 1 MeV flux in T89



Some important lessons learned and future developments

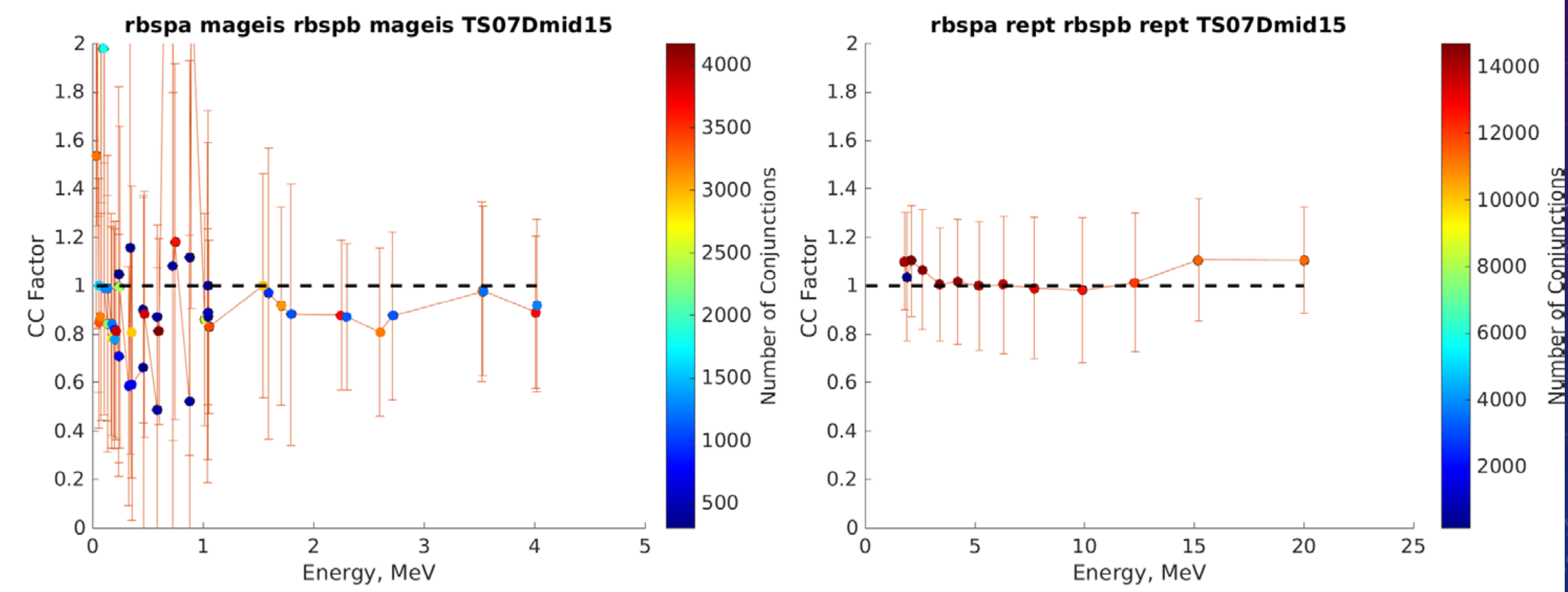
1. Operator splitting may be used to approximate a full 3D Kalman filter approach.
2. Statistical errors do not result in significant differences in model forecast performance – in future we require an activity dependent approach, and perhaps more attention to spatial dependence.
3. With the current framework we can not determine innovation, and interpretation of the 3 sets of covariance matrices is difficult – we are limited to improving the model through statistical error analysis and physical understanding
4. The method is however suitable for fast operation and performs quite well in an operational framework

Future:

1. Introduce 2D Kalman filtering in pitch angle and energy – this should allow us to improve our wave-particle interaction models
2. Implement more sophisticated ways of accounting for errors and test

Error Analysis – Models and Data

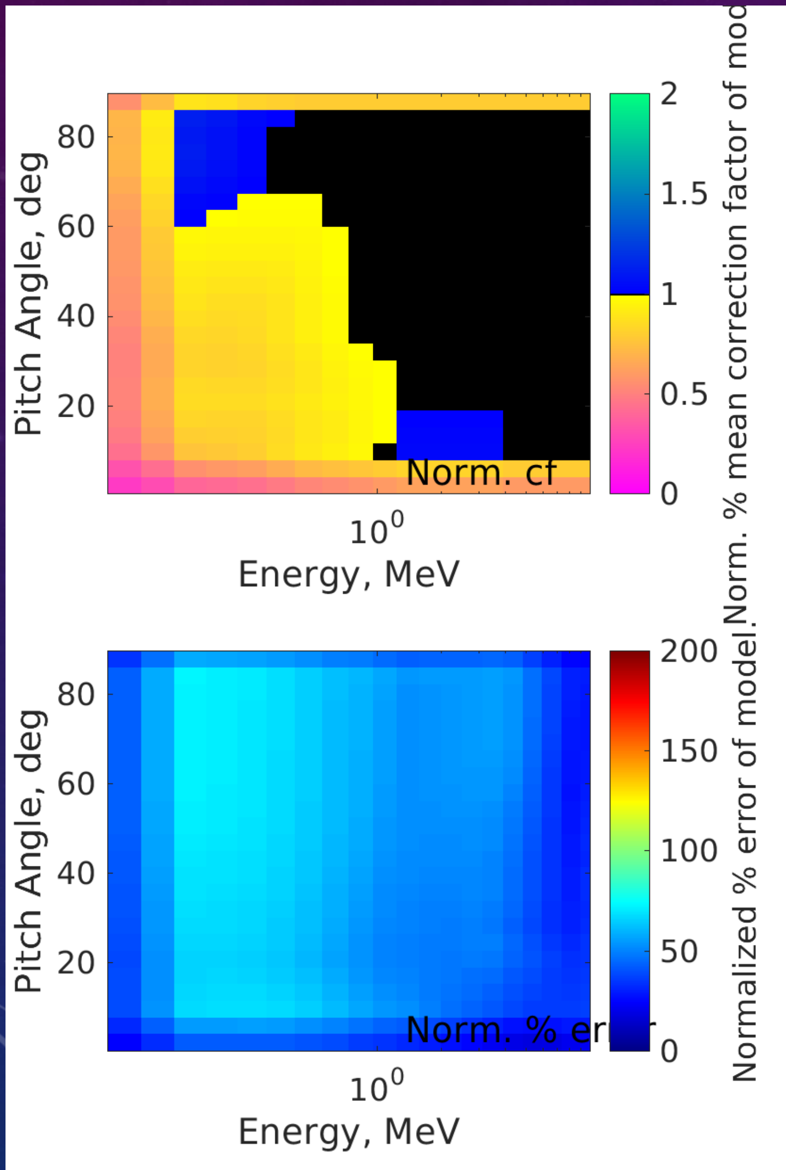
We have computed errors in electron PSD using several
B-field models and a PSD matching technique



Error Analysis – Models and Data

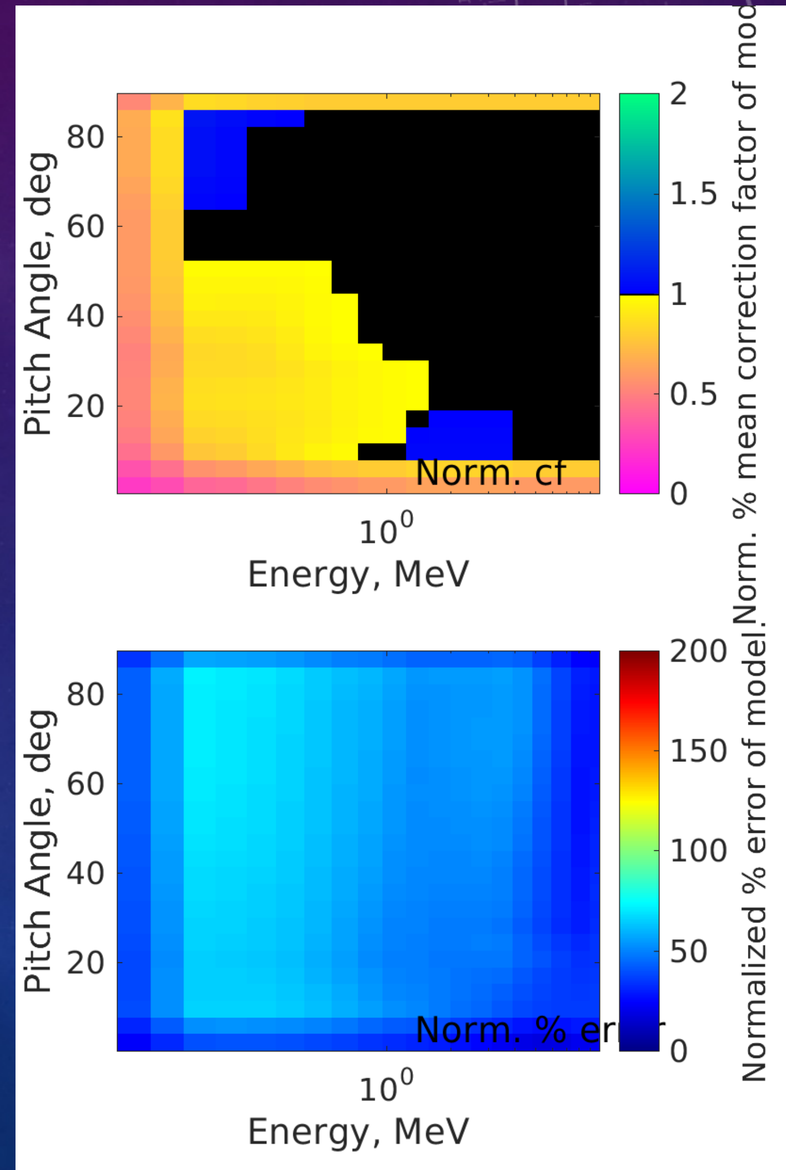
Old Diffusion Coefficients

See *Subbotin et al.*, [2010] JGR



New Diffusion Coefficients

Hiss - *Spasojevic et al,* [2015] and Orlova et al, [2016]



Validation metrics

$$PE = 1 - \frac{\sum_{i=1}^{N}(m_i - p_i)^2}{\sum_{i=1}^{N}(m_i - \langle m_i \rangle)^2}.$$

$$\begin{aligned} SS &= \frac{PE_{Model} - PE_{Persist}}{1 - PE_{Persist}} \\ &= \frac{\sum_{i=1}^{N}(m_i - m_{i-1})^2 - \sum_{i=1}^{N}(m_i - p_i)^2}{\sum_{i=1}^{N}(m_i - m_{i-1})^2}. \end{aligned}$$

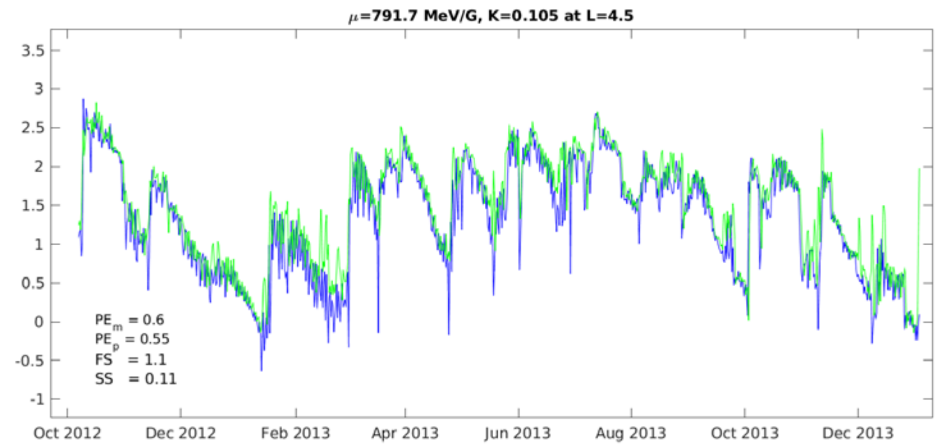
$$\begin{aligned} FS &= \frac{PE_{Model}}{PE_{Persist}} \\ &= \frac{\sum_{i=1}^{N}(m_i - \langle m_i \rangle)^2 - \sum_{i=1}^{N}(m_i - p_i)^2}{\sum_{i=1}^{N}(m_i - \langle m_i \rangle)^2 - \sum_{i=1}^{N}(m_i - m_{i-1})^2}. \end{aligned}$$

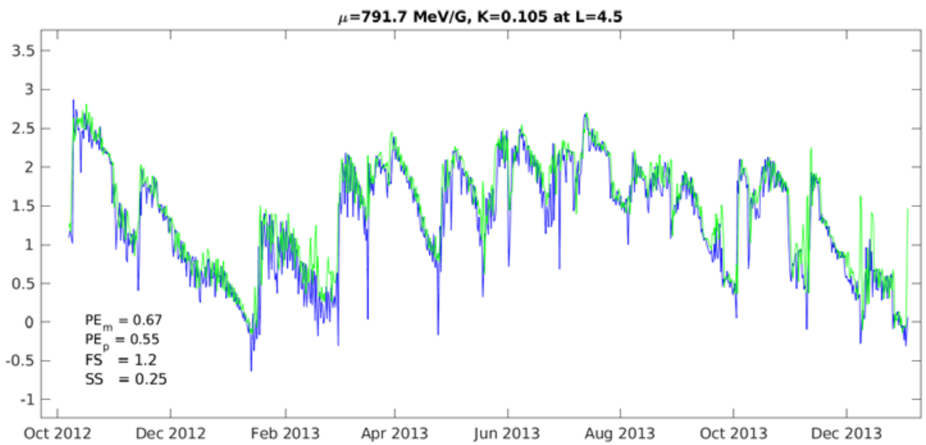
Forecast Performance

Old Diffusion Coefficients

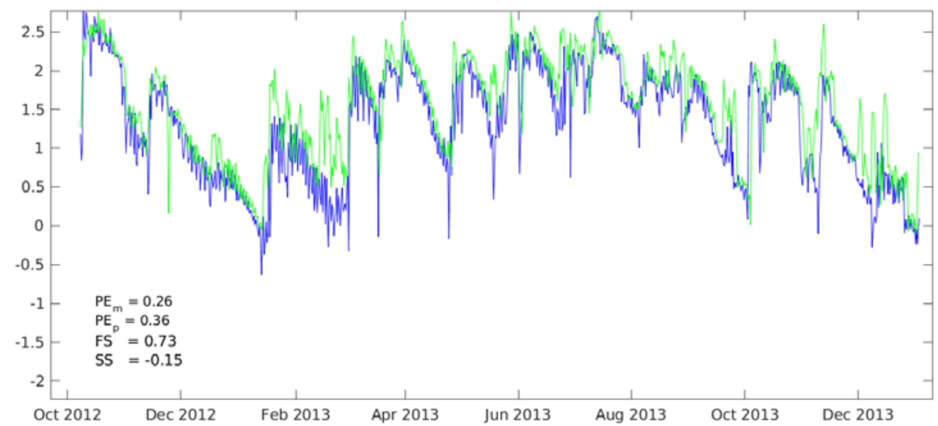
New Diffusion Coefficients

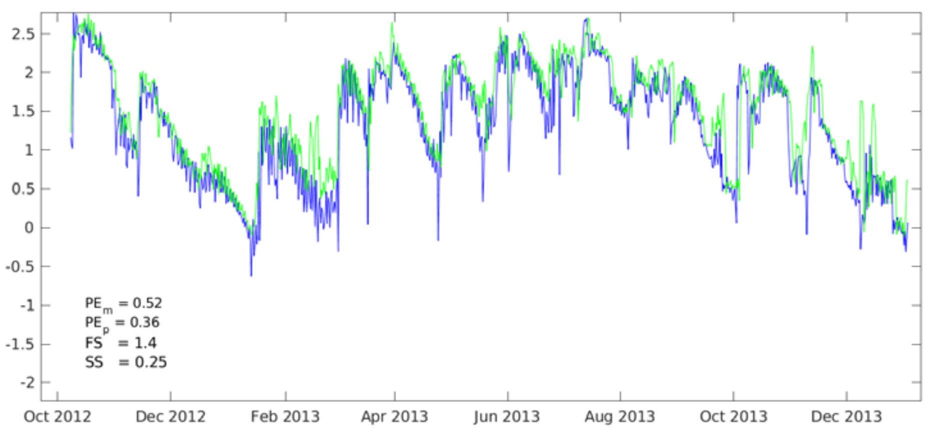
PSD in a 15 min interval, one day ahead



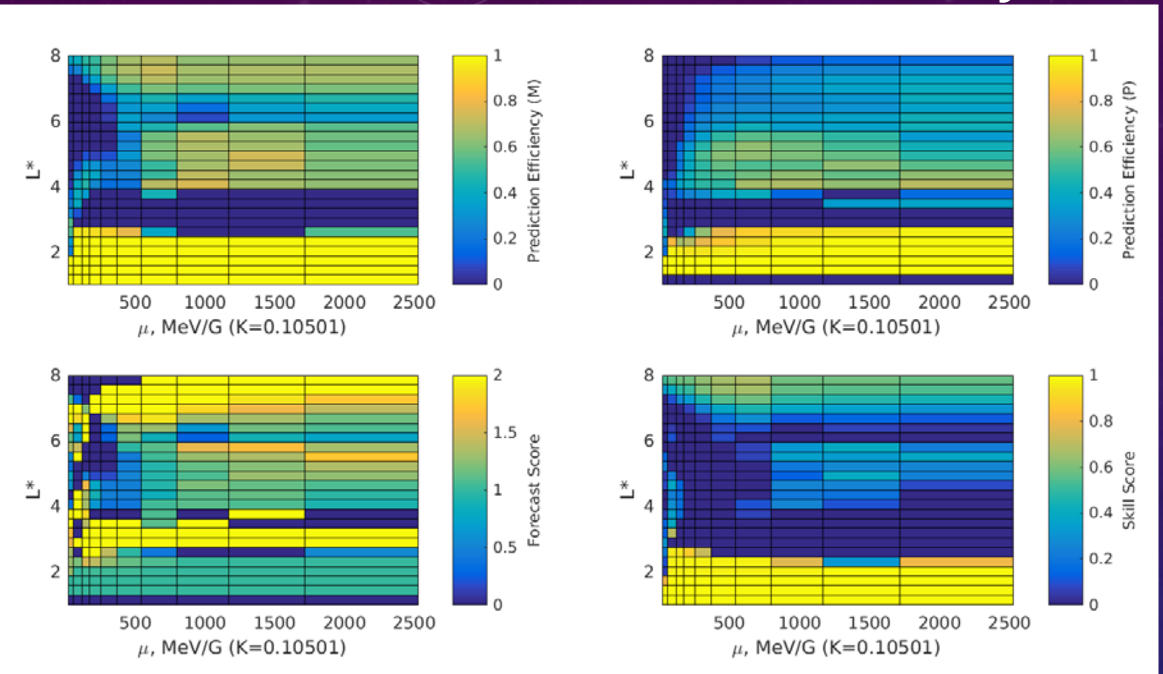


PSD in a 15 min interval, two days ahead

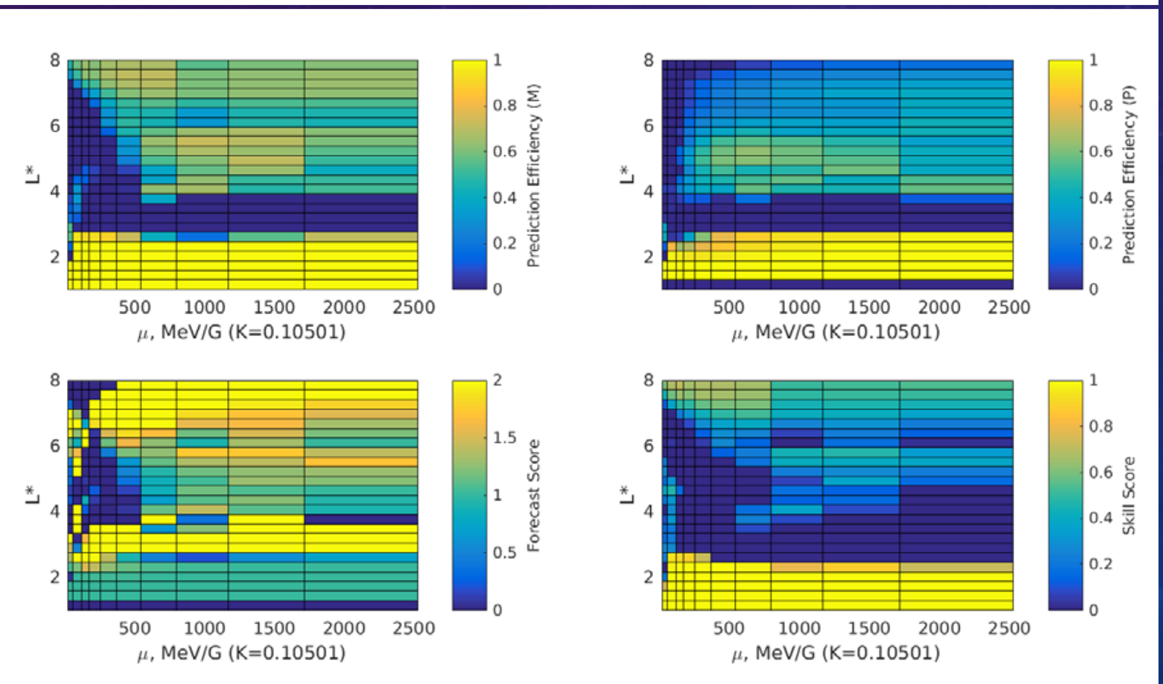




Performance Analysis – one day

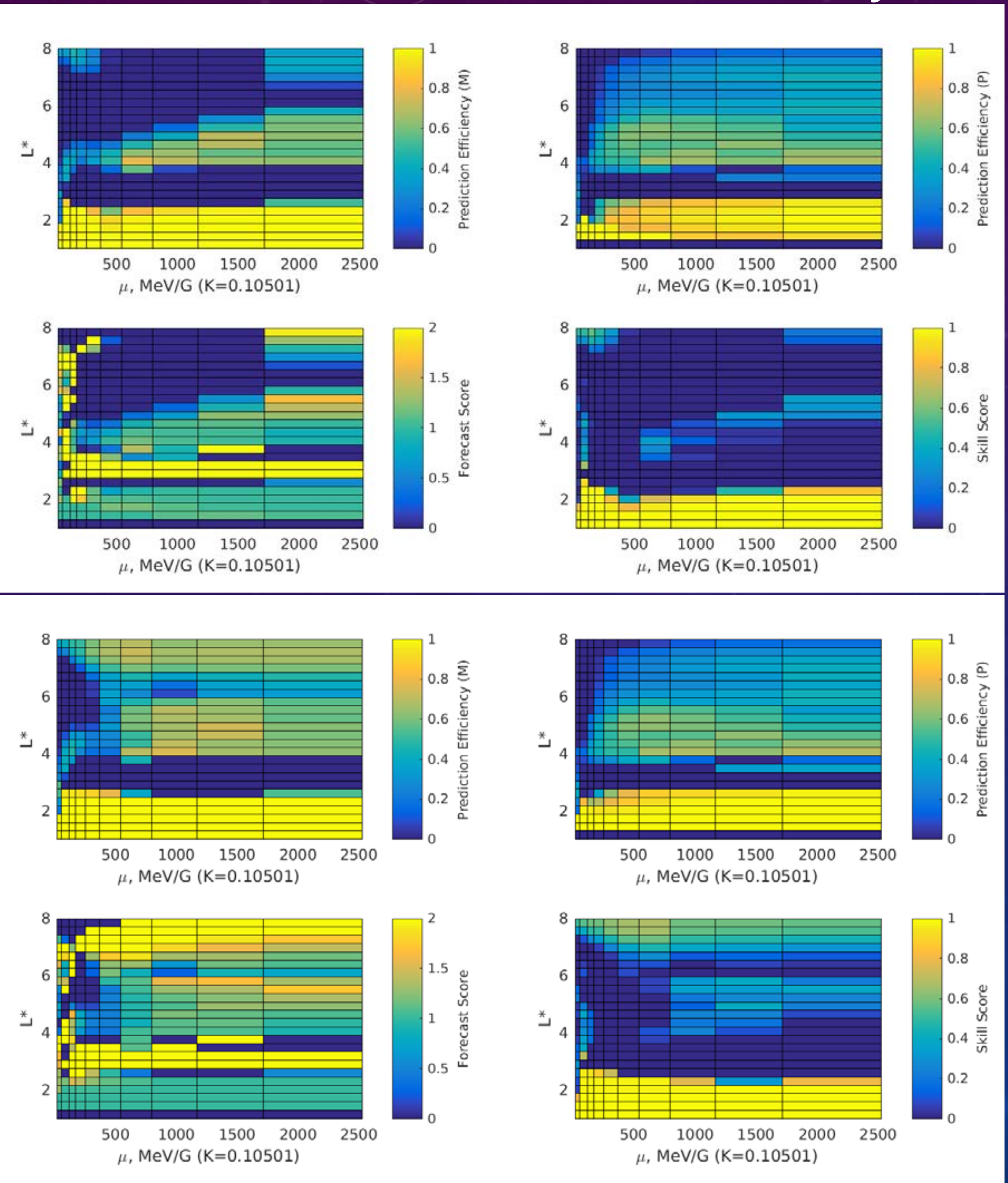


Equal errors



Statistical errors

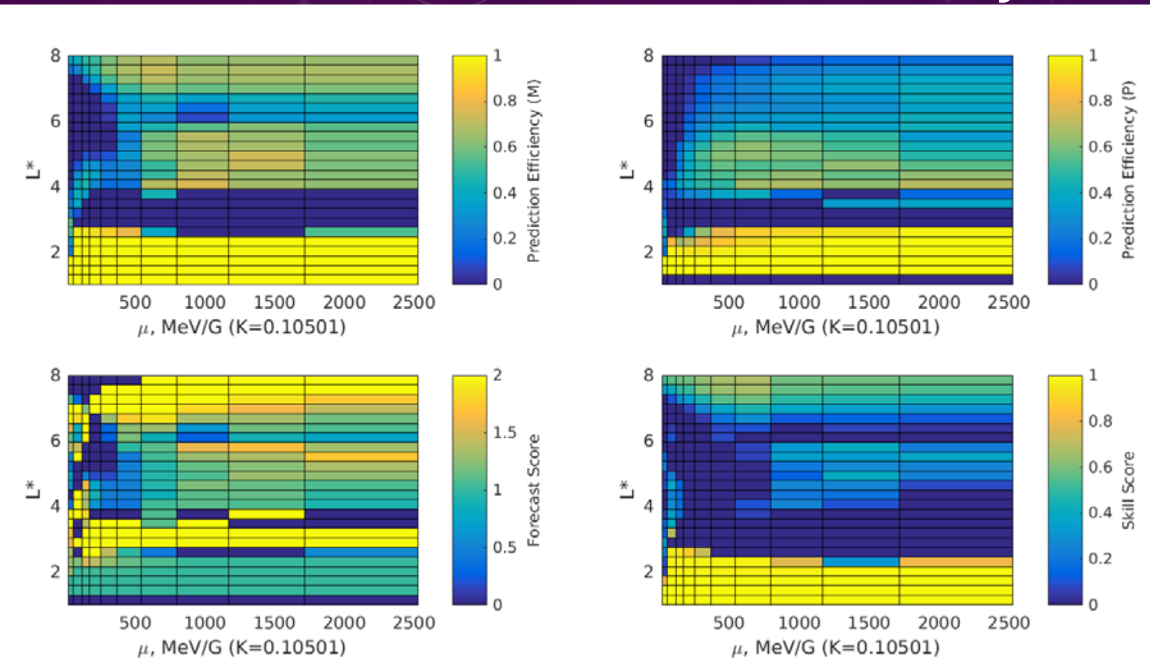
Performance Analysis – one day



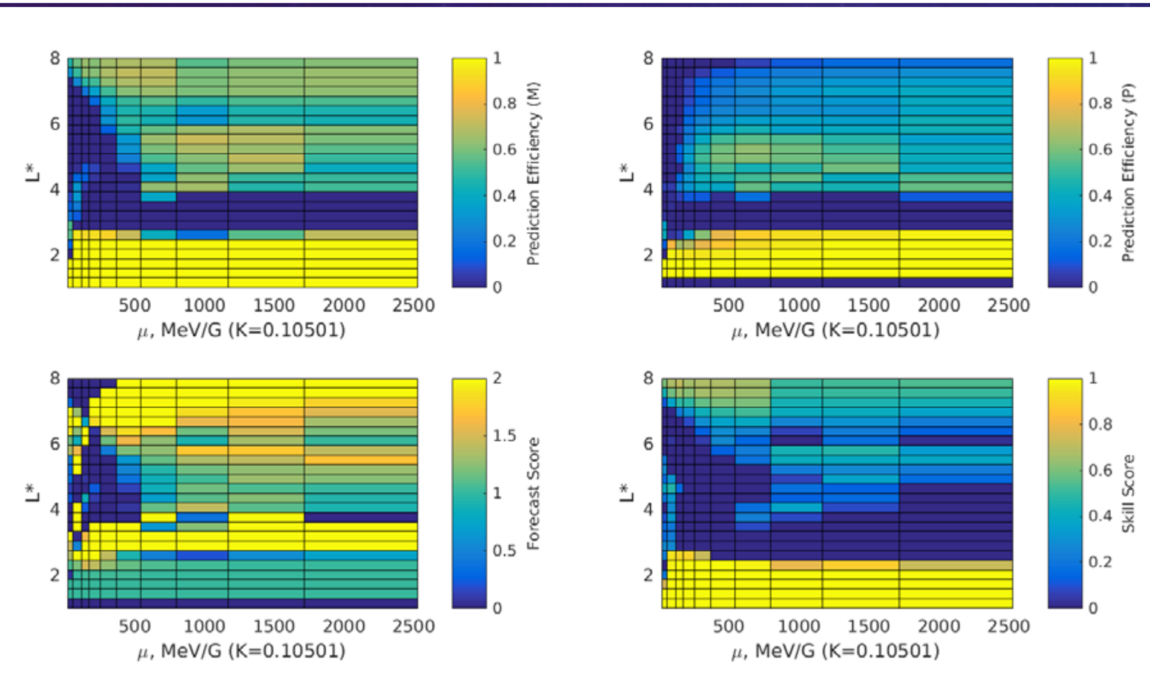
Old Dxx – Equal errors

New Dxx – Equal errors

Performance Analysis – one day

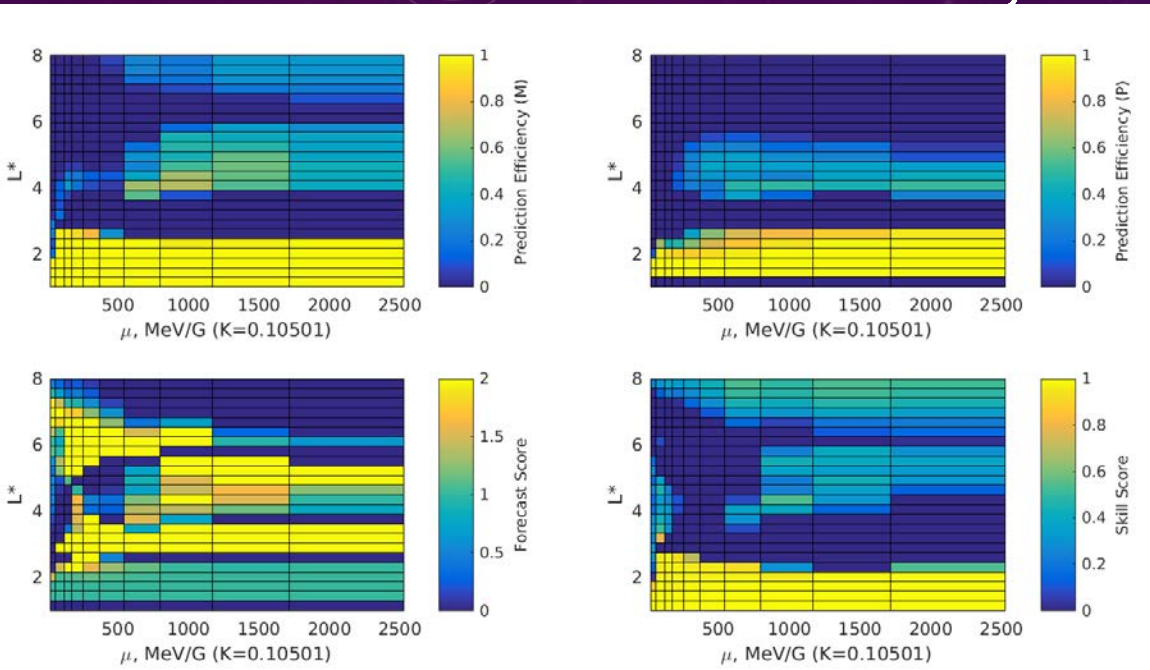


New Dxx – Equal errors

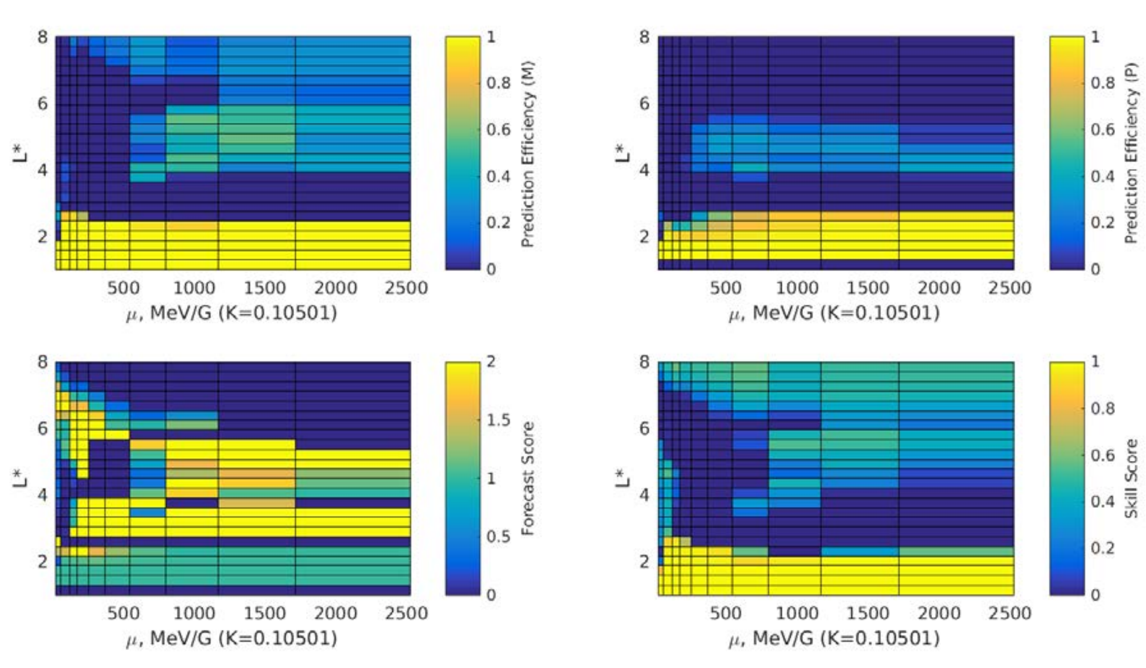


New Dxx – statistical errors

Performance Analysis – two day



New Dxx – Equal errors



New Dxx – statistical errors