Adaptive technique for high-latitude conductivity covariance refinement

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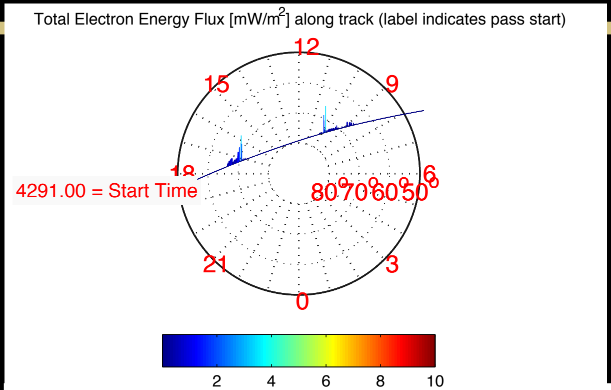
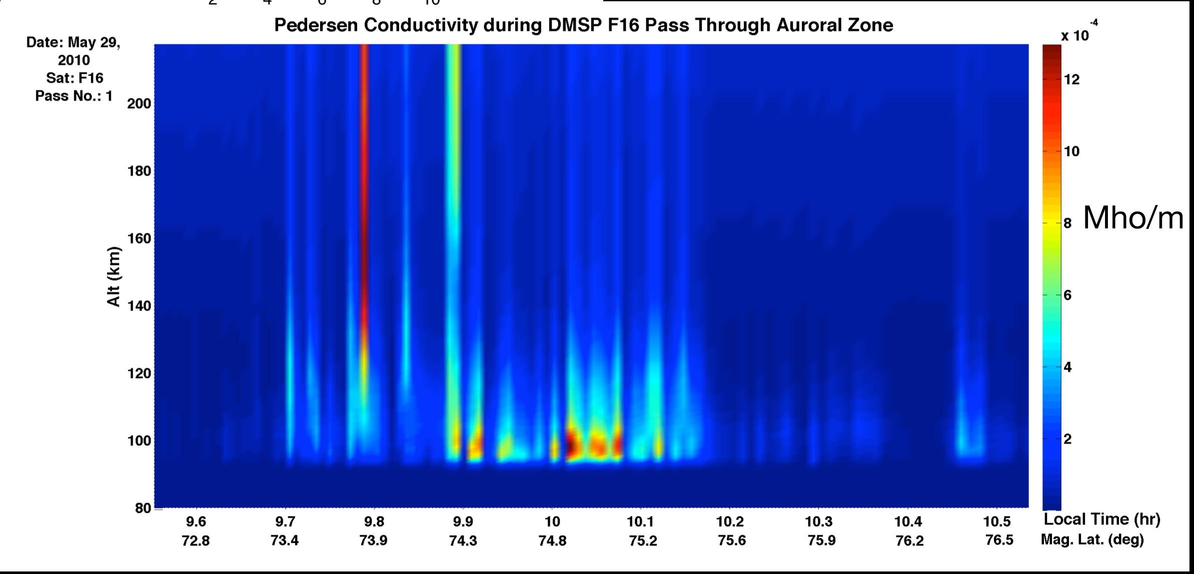
Overview

- Outline
  - Conductivity calculations
  - Methodology
  - Discussion of improvements moving forward
- What we hope to accomplish
  - Devise self-consistent scheme to give conductivity gradients from satellite observations
  - Enable 3D studies of the high-latitude ionosphere





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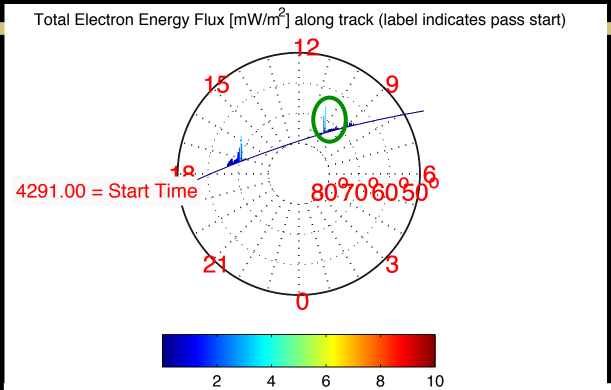

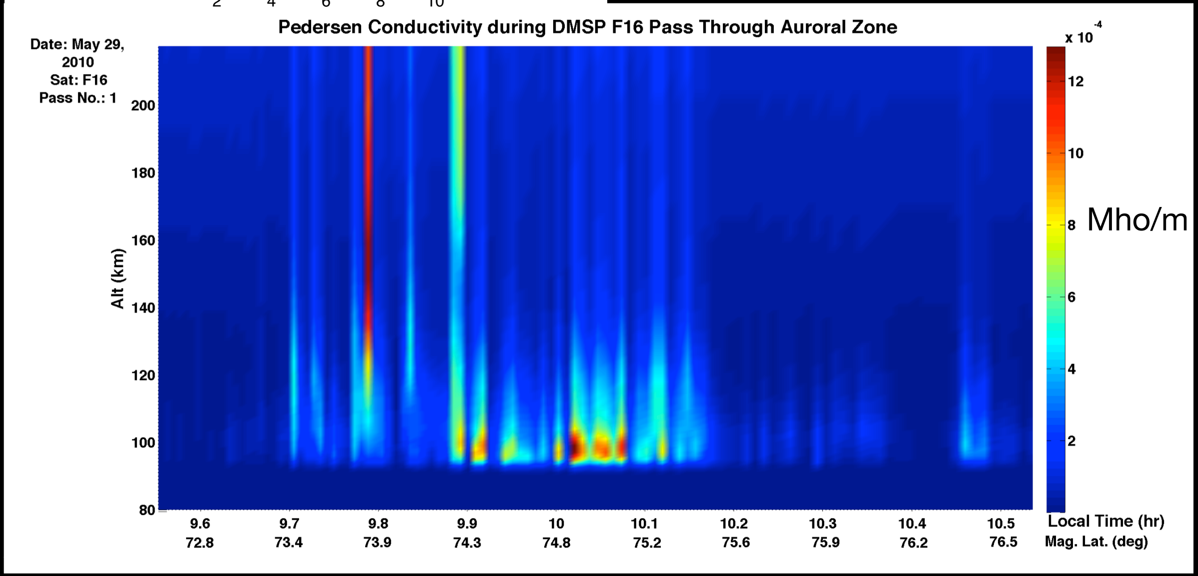

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Calculating Conductivity

Ionization sources:

- DMSP particle precipitation poleward of 45° mag. lat., separated by hemisphere
- EUVAC model of solar irradiance

Transport and chemistry calculations:

- Two-stream electron transport code (*Banks and Nagy* [1970], *Nagy and Banks* [1970], *Banks et al.* [1974])
- Elastic collisions with O, N_2, and O_2; Inelastic collisions leading to excitation and ionization
- Energy redistribution in 190 bin energy grid (0.25 eV – 49 keV)

GLobal AirglOW model (*Solomon et al.* [1988])

$$\sigma_P = \frac{q_e}{B}\left[N_{O^+}\frac{r_{O^+}}{1+r_{O^+}^2} + N_{O_2^+}\frac{r_{O_2^+}}{1+r_{O_2^+}^2} + N_{NO}\frac{r_{NO}}{1+r_{NO}^2} + N_e\frac{r_e}{1+r_e^2}\right]$$

$$\sigma_H = \frac{q_e}{B}\left[-N_{O^+}\frac{1}{1+r_{O^+}^2} - N_{O_2^+}\frac{1}{1+r_{O_2^+}^2} - N_{NO}\frac{1}{1+r_{NO}^2} + N_e\frac{1}{1+r_e^2}\right]$$

Methodology

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Following methodology has been applied to electric fields by *Matsuo et al.* [2002, 2005] and *Cousins et al.* [2013a, 2013b]

Extensive set of observations:
-> Mean field yields background conductivity field
-> EOFs give variation from background field

$$\mathbf{x}_b = \bar{\Sigma}$$

EOFs used as bases whose coefficients are updated with new observations; background covariance matrix built from these

$$\mathbf{P}_b = \Psi E\left[\alpha \cdot \alpha^T\right]\Psi^T$$

$$\alpha = \text{EOF coefficients}$$

Background covariance parameterized, estimated at each analysis time step

$$\mathbf{P}_b \simeq \mathbf{P}_b(\zeta)$$

$$\text{diag}(\mathbf{P}_b) \simeq \zeta_{b1} v^{-\zeta_{b2}}$$

$$v = \text{no. of bases}$$

In method of Dee [1994]

Parameters estimated using maximum likelihood method in innovation vector space by matching innovation vector to covariance model at each analysis step

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New application: Whole process applied at discrete levels of altitude for conductivity

Outstanding Issues

Address shortcomings in ionospheric modeling (namely 2-D and Maxwellian distribution assumptions)

- Already done

Conductivity is not directly observed

Completed ↑

Global picture of height-dependent conductivity

Outstanding issues ↓

Sparsity of observations

- Constrained to DMSP paths

Stability of estimation process

- Uniqueness
- Identifiability of parameters

AMIE – relationship among electrodynamic variables

Inverse procedure to infer maps of

$$\vec{E}, \Phi, \vec{I}_\perp, \vec{J}_{||}, \Delta\vec{B}$$

From observations of

$\vec{E}$ IS or HF radar, Satellites

$\vec{I}_\perp$ IS radar

$\vec{J}_{||}$

$\Delta\vec{B}$ Satellite or ground-based magnetometers

linear relationship (for a given $\underline{\underline{\Sigma}}$)

$$F(\vec{E}) = \Phi, \vec{I}_\perp, \vec{J}_{||}, \Delta\vec{B}$$

$$\vec{E} = -\nabla\Phi$$

$$\vec{I} = \underline{\underline{\Sigma}} \cdot \vec{E}$$

$$\vec{J}_{||} = \nabla \cdot \vec{I}_\perp$$

$$\vec{I}_\perp, \vec{J}_{||} \leftrightarrow \Delta\vec{B}$$

Biot-Savart's law

Slide source: Tomoko Matsuo

Concluding Remarks

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Self-consistent procedure to determine conductivity profiles in high-latitude ionosphere

- Provides better starting point for electrodynamic calculations

Addressing shortcomings in ionospheric modeling (namely 2-D and Maxwellian distribution assumptions)

Open questions:

- Global coverage
- Limited observations
- Performance of the inversion procedure