

# Tomographic estimation of exospheric hydrogen density distributions

Gonzalo Cucho-Padin, Lara Waldrop

University of Illinois at Urbana Champaign, Urbana, IL

e-mail: gac3@illinois.edu



## Abstract

For the past decade, the Lyman-alpha detectors (LADs) onboard NASA's **Two Wide-angle Imaging Neutral-atom Spectrometers (TWINS)** mission have obtained routine measurements of solar Lyman-alpha photons (121.6nm) resonantly scattered by atomic hydrogen (H) in the terrestrial exosphere. This data has been used to derive global, 3D models of exospheric H density beyond 3 Re, which are needed to understand the influence of solar activity and ion-neutral coupling throughout the terrestrial environment as well as to assess its long-term evolution through atmospheric escape. These historical models are based on parametric fitting of assumed functional forms which may be inaccurate or invalid, thus limiting confidence in conclusions drawn from analysis of the resulting distributions. In this work, we present a new means of global, 3D reconstruction of exospheric H density through a tomographic inversion of the scattered H Lyman-alpha emission. Our approach avoids the conventional dependence on ad hoc and arbitrary parametric formulations and enables a more accurate characterization of the global structure of this region.

## Introduction

**MOTIVATION:** Knowledge of the 3D structure of the H geocorona is necessary both to understand its role as a dynamic buffer against the solar-driven environment of interplanetary space as well as assess the rate of its permanent escape from Earth's gravity through evaporation.

Hydrogen atoms (H) in Earth's upper exosphere resonantly scatters solar Lyman-alpha (121.567nm) radiation, creating the ultraviolet optical signature known as the H geocorona.

In the **optically thin** region, located beyond ~3Re, the density number of H atoms is relatively low, scattering events can be assumed to occur exactly once.

The mathematical relationship between TWINS LAD's measurements and the neutral hydrogen atoms ( $N_H$ ) in the exosphere is given by:

$$F_p = \frac{g^*}{10^6} \int_0^{L_{max}} N_H(l) I(\alpha) dl + F_{IP}$$

Eq. 1:  $g^*$  is the local g-factor,  $N_H$  is the hydrogen density,  $I(\alpha)$  is the phase function,  $F_{IP}$  is the interplanetary glow and  $F_p$  is the intensity measured by the LAD's.  $F_p$  and  $F_{IP}$  in Rayleighs

Previous methods that estimate the 3D hydrogen density distributions have been based on **parametric fitting** of assumed functional forms involving spherical harmonics expansions [Bailey et al., 2011] and [Hodges, 1994].

The tomographic inversion results reported here are based on one day of data acquired by the LAD1/2 onboard the TWINS 1 satellite -- the same interval that was used by [Bailey et al., 2011] to obtain parametric estimates of H density.

The algebraic analysis of eq. 1 as well as the correct discretization of the space beyond 3Re enable us to formulate an inverse problem that can be solved through regularization techniques.

## Data

**TWINS mission and instrumentation:** It is comprised of two satellites (TWINS1/2) which enable stereoscopic sensing of the magnetosphere and exosphere. Each spacecraft is approximately nadir pointing and is placed in a Molniya orbit with 63.4° inclination and 7.2 Re apogee. Each satellite has two Lyman-alpha detectors (LAD1/2) and acquire data within 1-minute time resolution. The data used in this project was obtained from the NASA server for 11 June 2008.

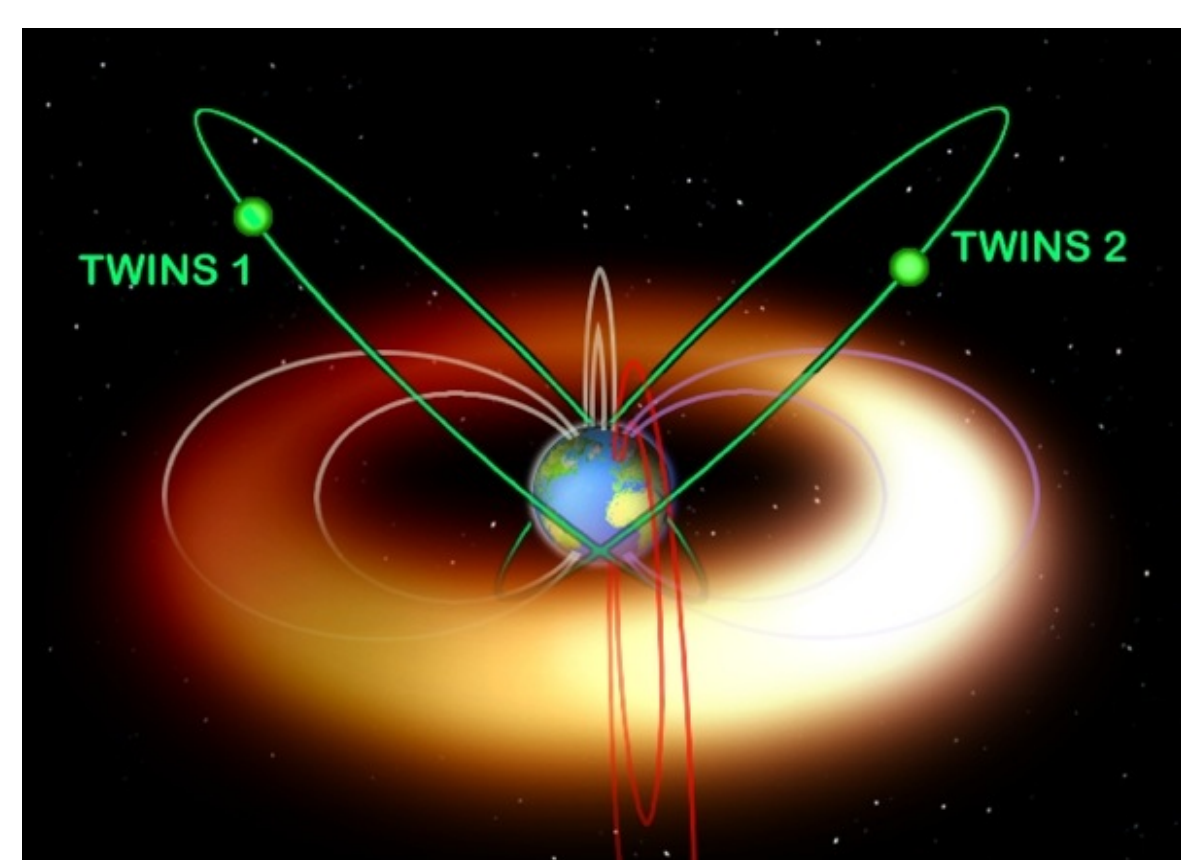


Fig. 1: Source: <http://twins.swri.edu>.

## Methodology: Data pre processing stage and tomographic approach

### Step 1 Data filtering 1

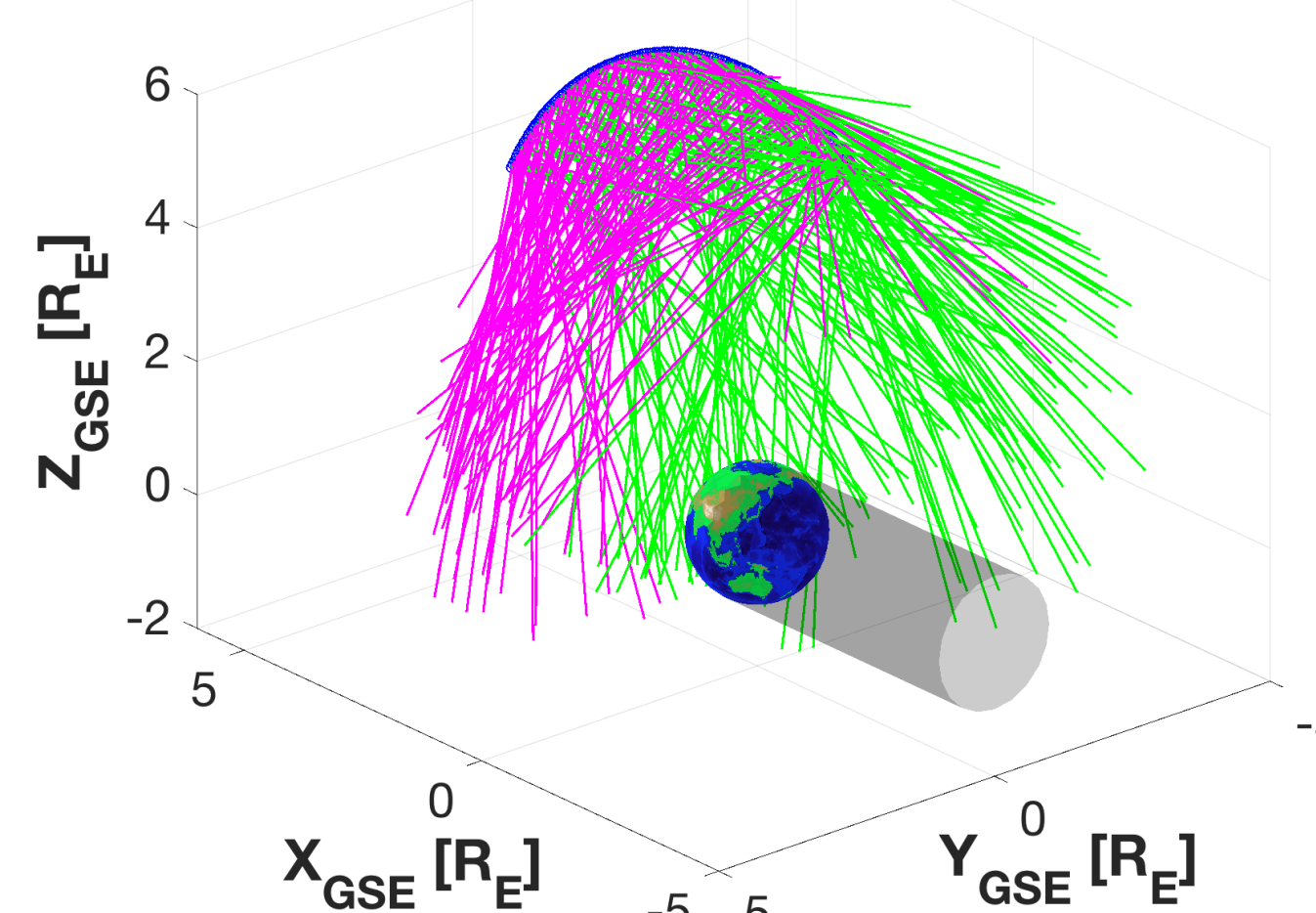
Data Pre-processing stage:

Fig. 2 To ensure validity of the linear emission model, we omit data along LOS's which:

(1) pointing towards the sun,

(2) cross the Earth's shadow,

(3) pass within 2RE of Earth's surface.

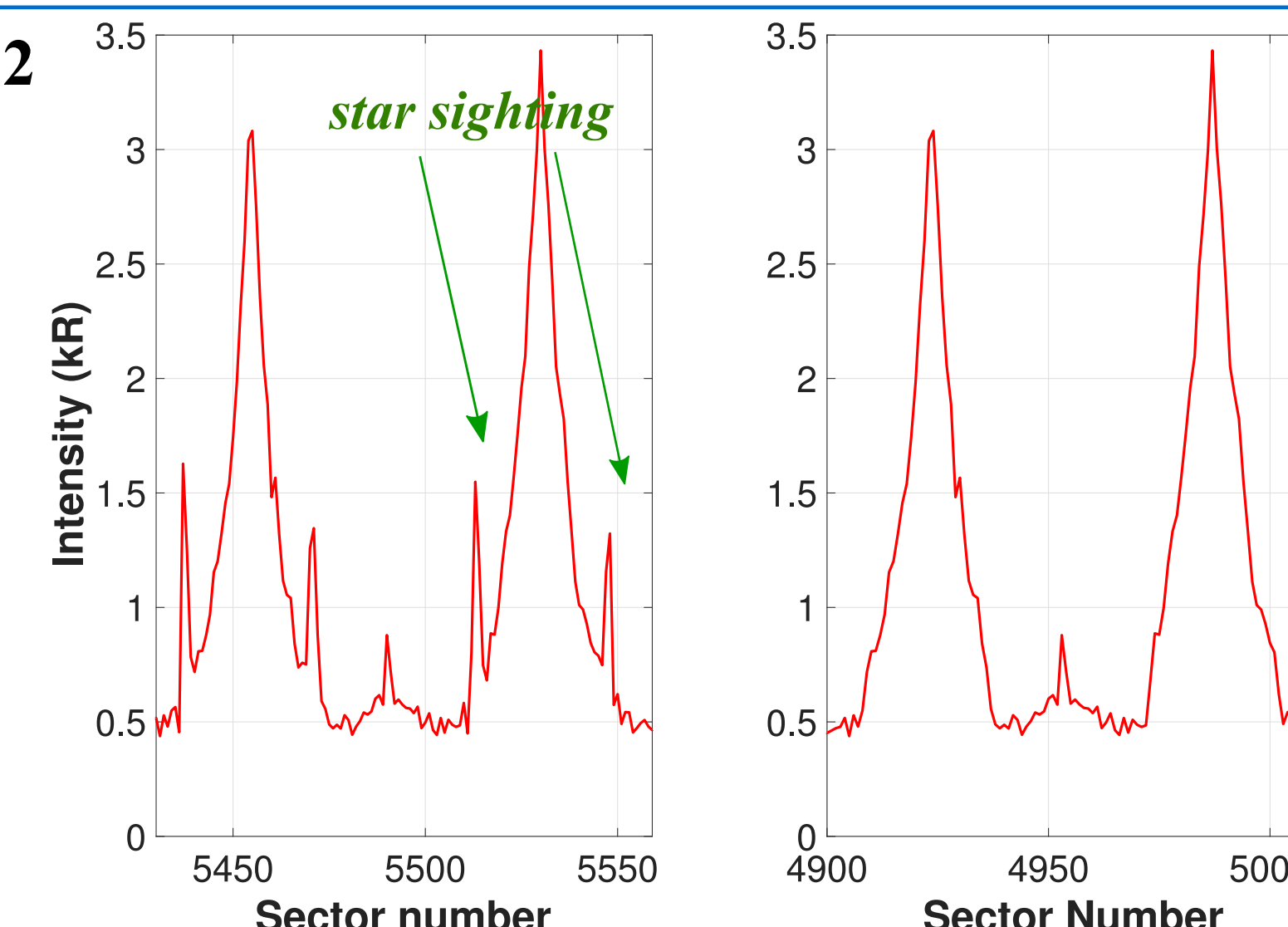


### Step 2 Data filtering 2

Data Pre-processing stage:

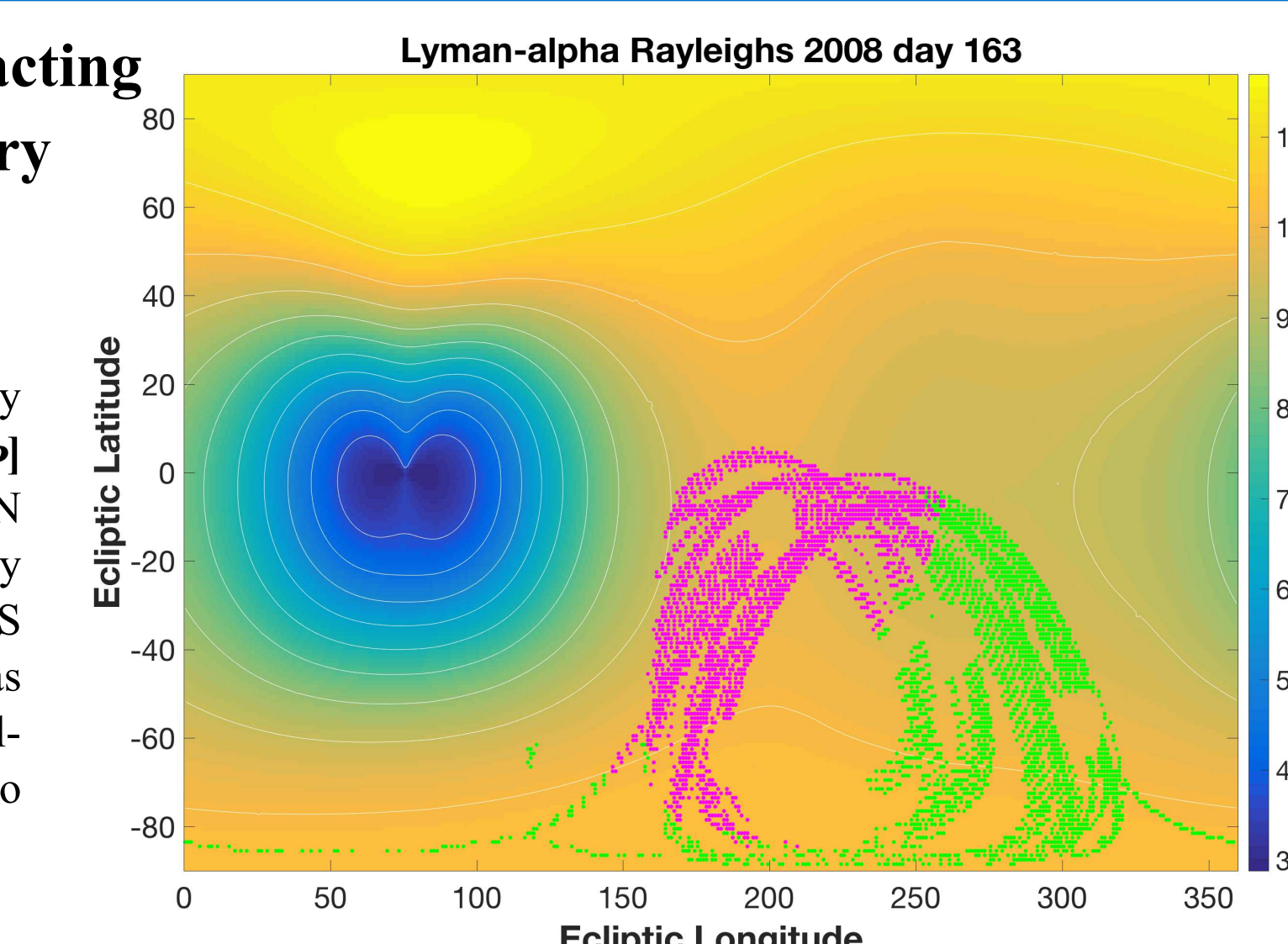
Fig.3: O and B type stars can be source of stellar contamination for LAD's. We have identified stellar emission signatures and removed contaminated data

In this project:  
Total number of LOS: 13063 from LAD1/2



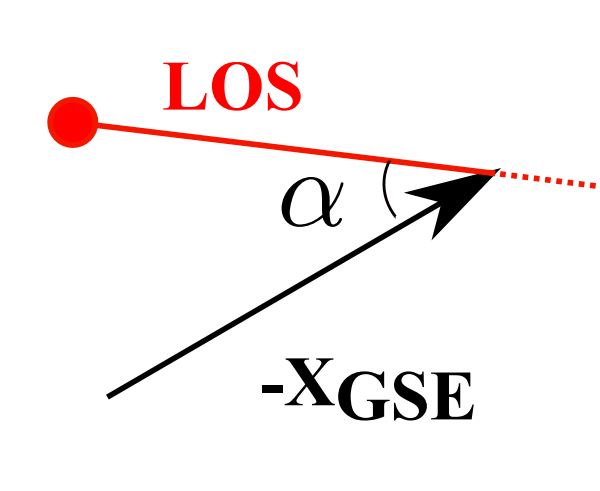
### Step 3 Subtracting the Interplanetary background

Fig. 4 The interplanetary background [FIP] derived from SWAN measurements for day 163 year 2008. LOS from both LADs has been located in the all-sky map in order to subtract its [FIP].



### Step 4 Calculating the photon scattering intensity by H atoms

Fig. 5: The Lyman-alpha photons are scattered anisotropically by H atoms. Its angle-dependant intensity was studied by [Brandt et al., 1959].



$$I(\alpha) = 1 + \frac{1}{4} \left( \frac{2}{3} - \sin^2 \alpha \right)$$

### Step 5 Calculating the g\* factor

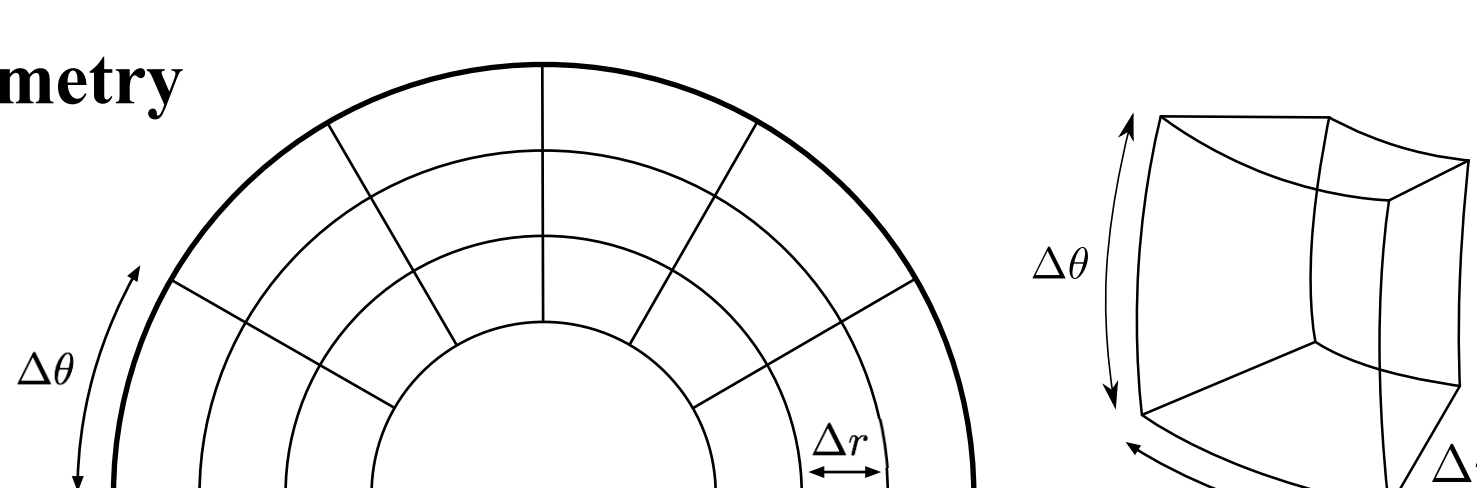
The g-factor or Lyman-alpha resonant scattering rate is defined in terms of (1) the spectral density at the center of the solar Ly- $\alpha$  line flux [F] (2) the inter-state transition probability of H atoms.

TIMED/SEE measurements of solar Ly- $\alpha$  line flux [F], scaled to line center using empirical relationships, were used to calculate the g\*-factor

### Step 6 Setting up the geometry

Tomographic Approach:

Fig.6 (1) Discretize region into J spherical voxels. (2) Project unknown density function onto J orthonormal basis functions. (3) Rewrite ith measurement of intensity and cast measurement ensemble a matrix equation.



$$N_H(r) = \sum_{j=1}^J x_j N_{Hj}(r)$$

$$y_i(r_i, \hat{n}_i) = \sum_{j=1}^J \left[ \int_0^{L_{max}} \frac{g^*}{10^6} N_{Hj}(r') I(\hat{n}_i) dl \right] x_j$$

$$y = Lx + w \text{ Poisson noise}$$

In this project:  
Range of analysis: [3-15] RE  
# of radial sections: 26  
# of azimuthal sections: 24  
# of polar sections: 12

### Step 7 Inverse problem and Regularization

Since the observation matrix  $L$  is not a full rank, a regularization technique must be used to solve the system. The selected regularization method is presented in the following equations:

$$\Phi(x) = \|Lx - y\|_2^2 + \lambda R R P E(x)$$

$$\lambda R R P E(x) = \lambda_r \|x\|_{D_r} + \lambda_\phi \|x\|_{D_\phi} + \lambda_\theta \|x\|_{D_\theta}$$

$$D_r \rightarrow \partial^2 / \partial r^2$$

$$D_\phi \rightarrow \partial / \partial \phi$$

$$D_\theta \rightarrow \partial / \partial \theta$$

In this project:

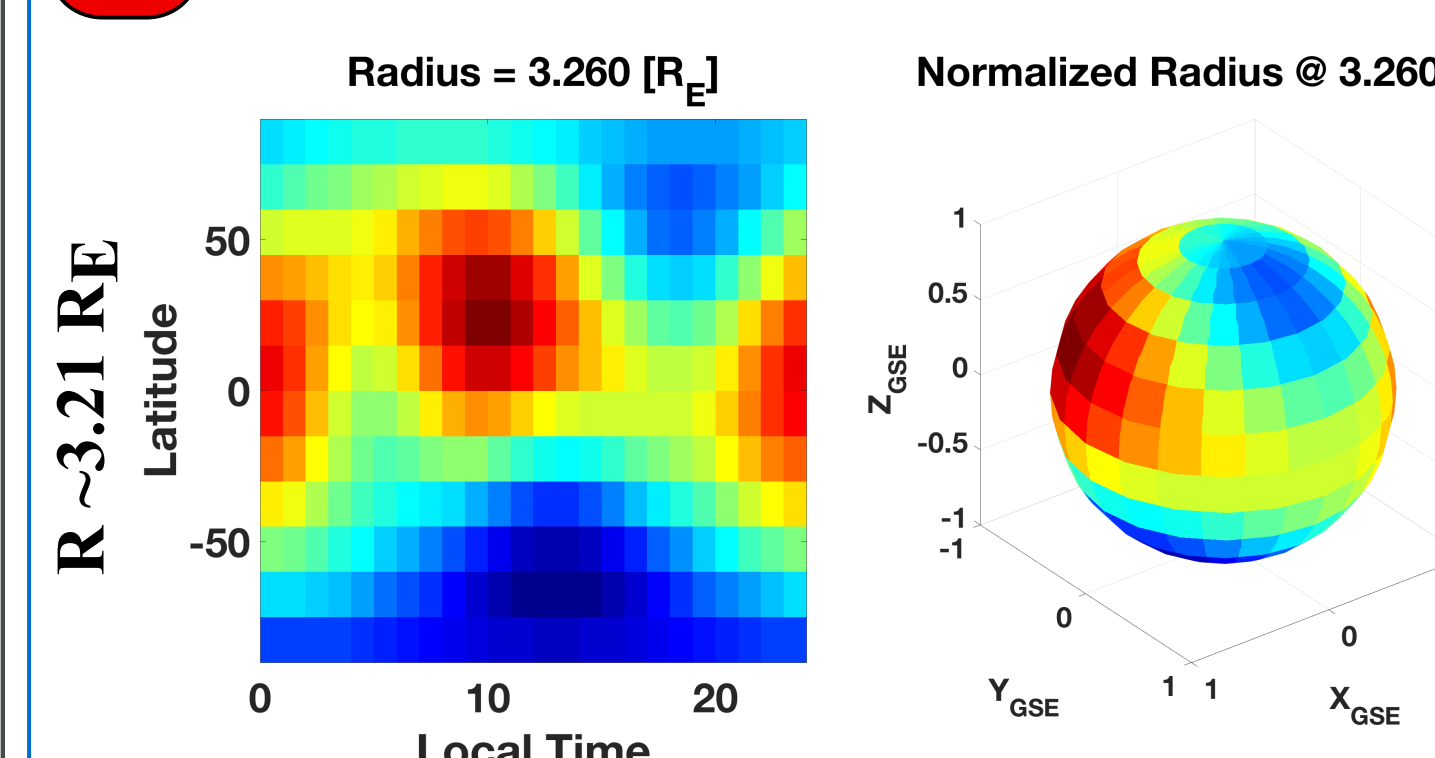
$$\lambda_r = 10$$

$$\lambda_\phi = 12.38$$

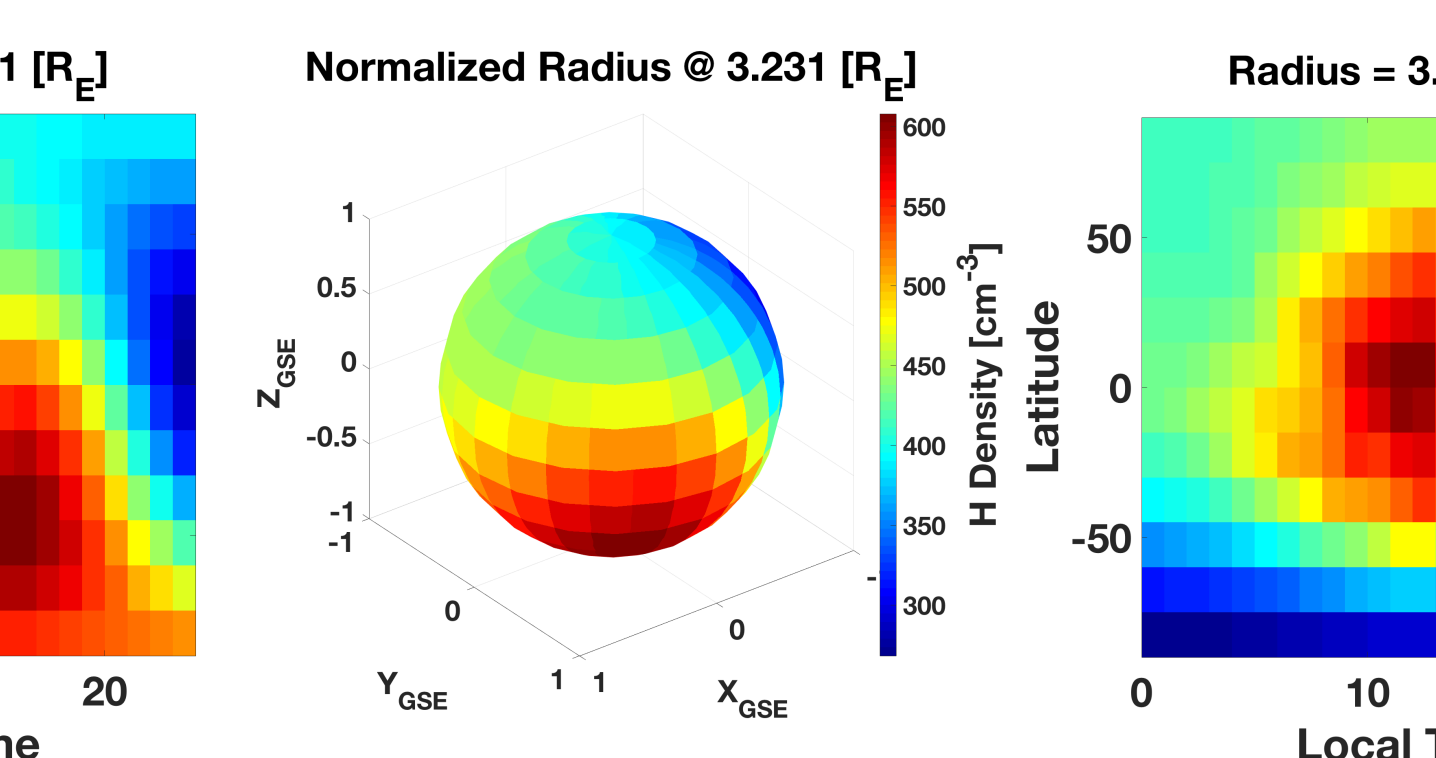
$$\lambda_\theta = 5.01$$

## Reconstruction results

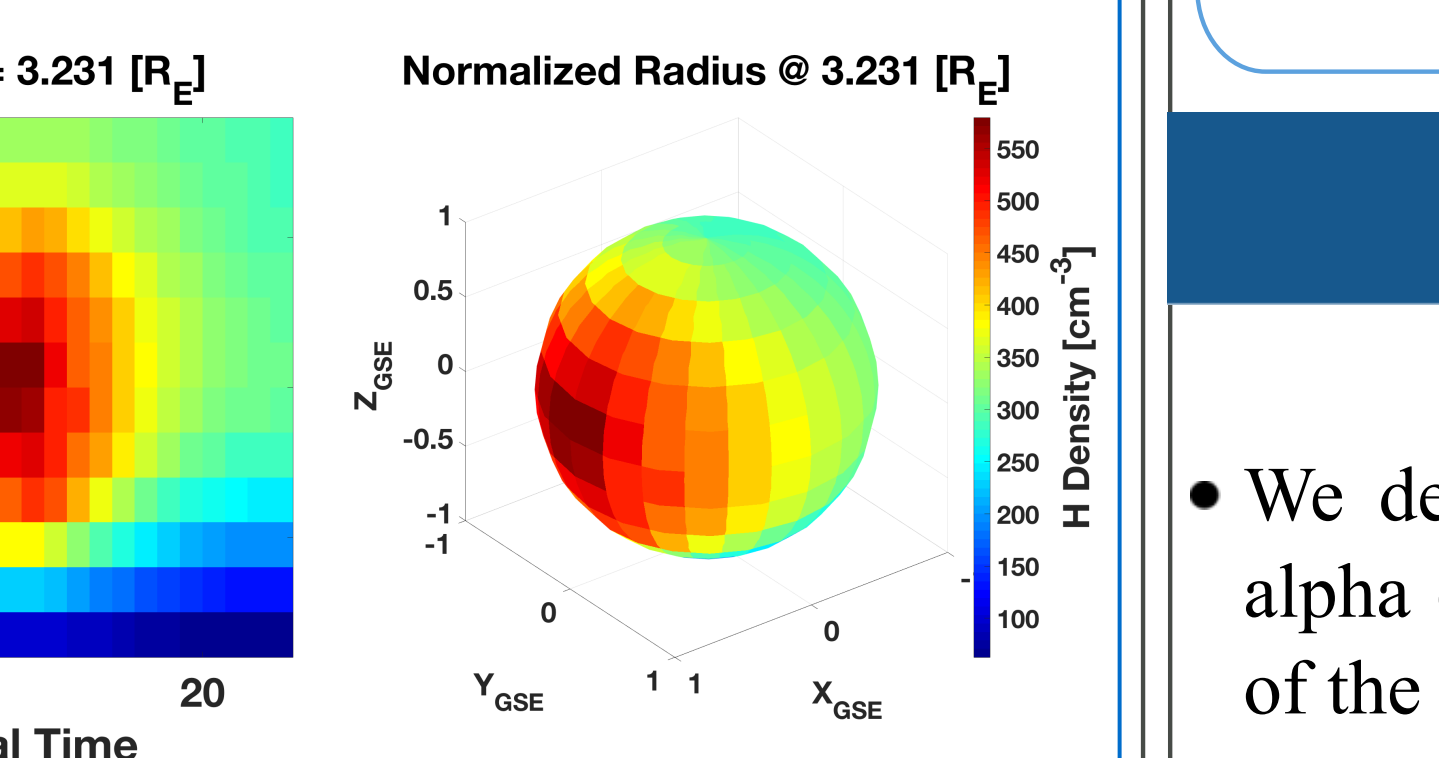
### R.1 Monte Carlo theoretical evaluation



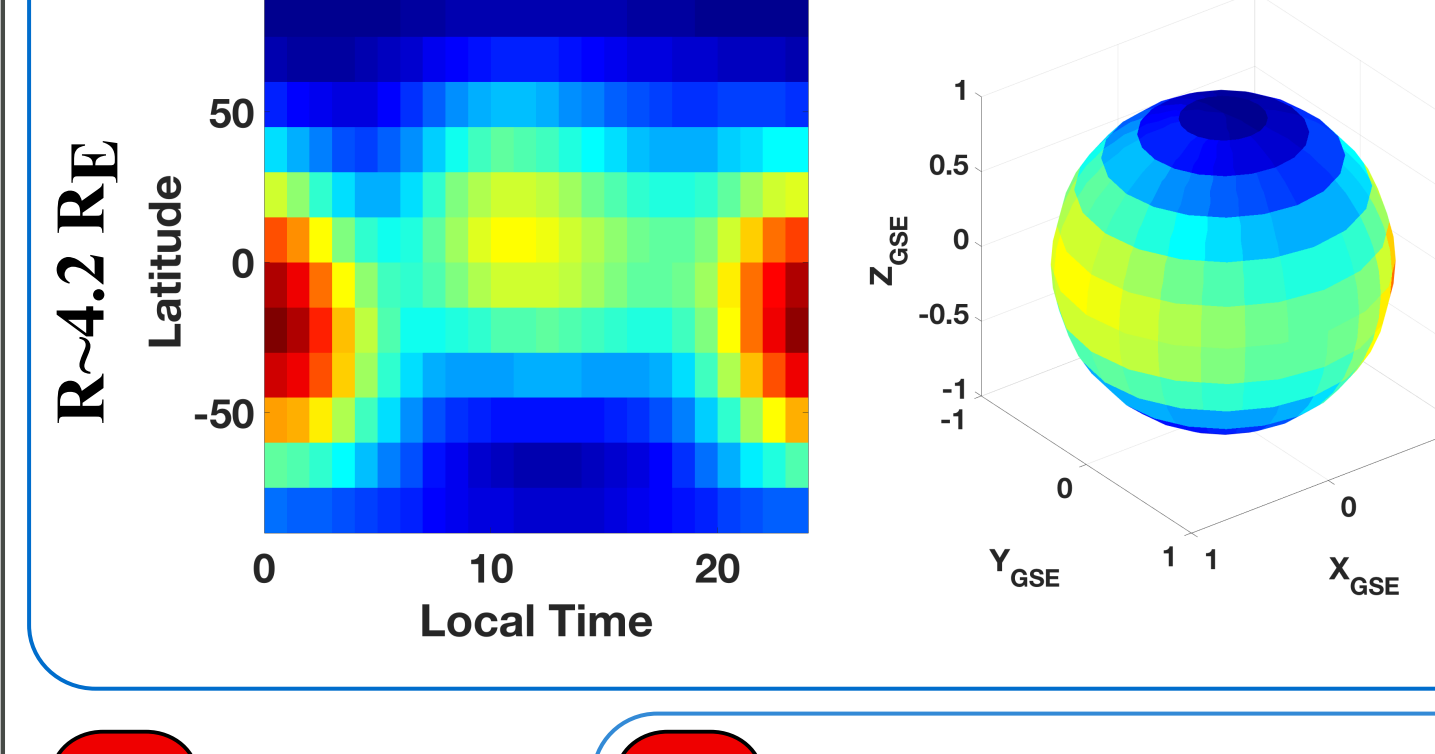
### Parametric fitting



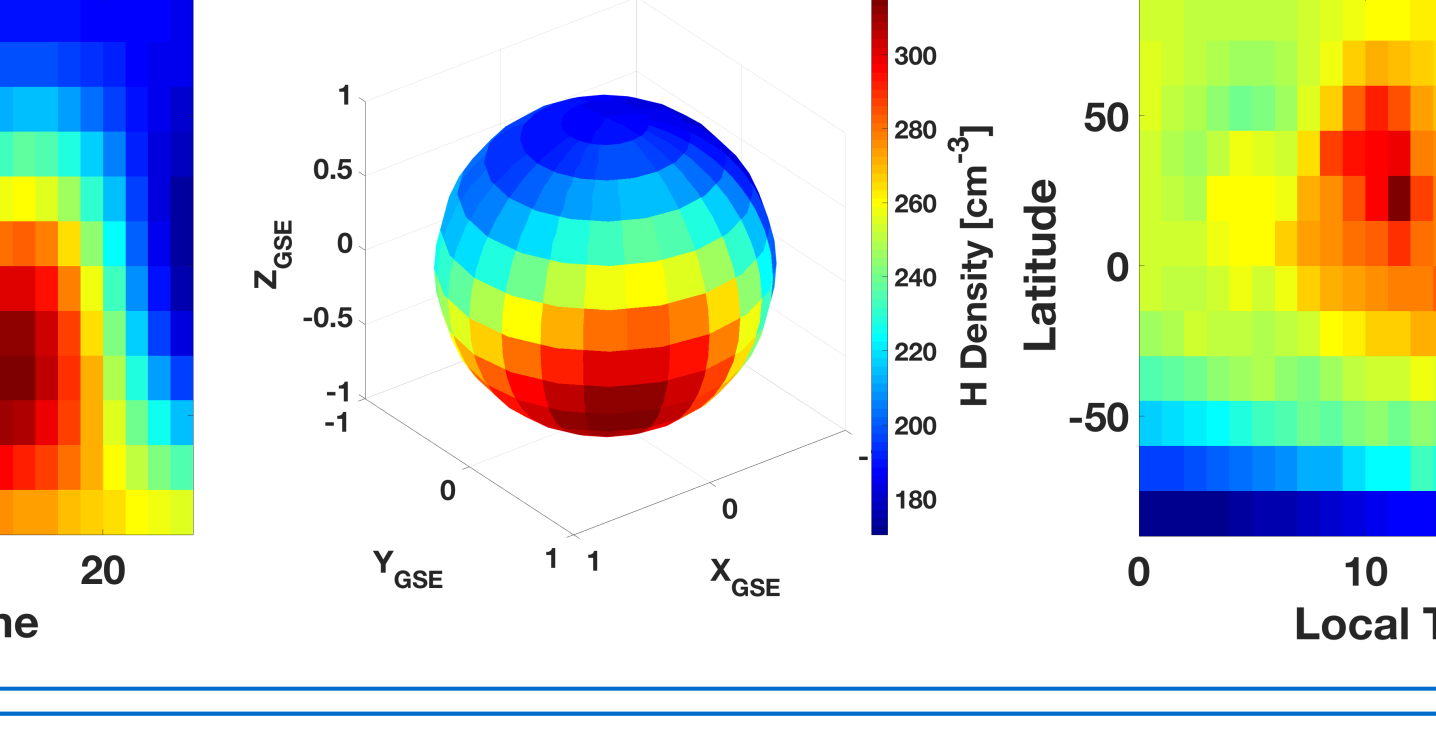
### Tomographic inversion



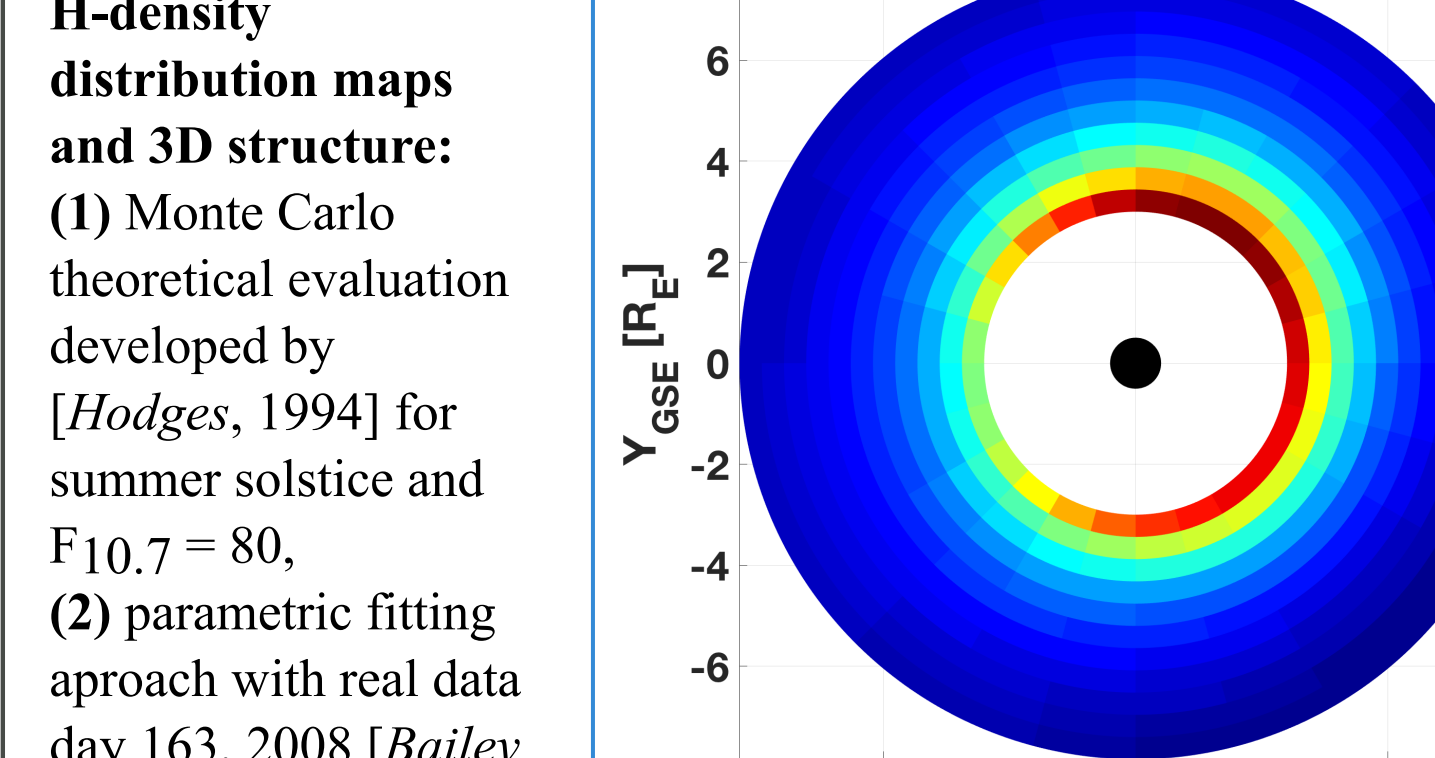
### R.2 Parametric fitting, Ecliptic plane



### R.3 H profile for Dawn, H profile for Dusk, H profile for Noon



### R.2 Tomographic inversion, Ecliptic plane



### R.3 Pole-to-pole sectors for Noon, Dawn and Dusk

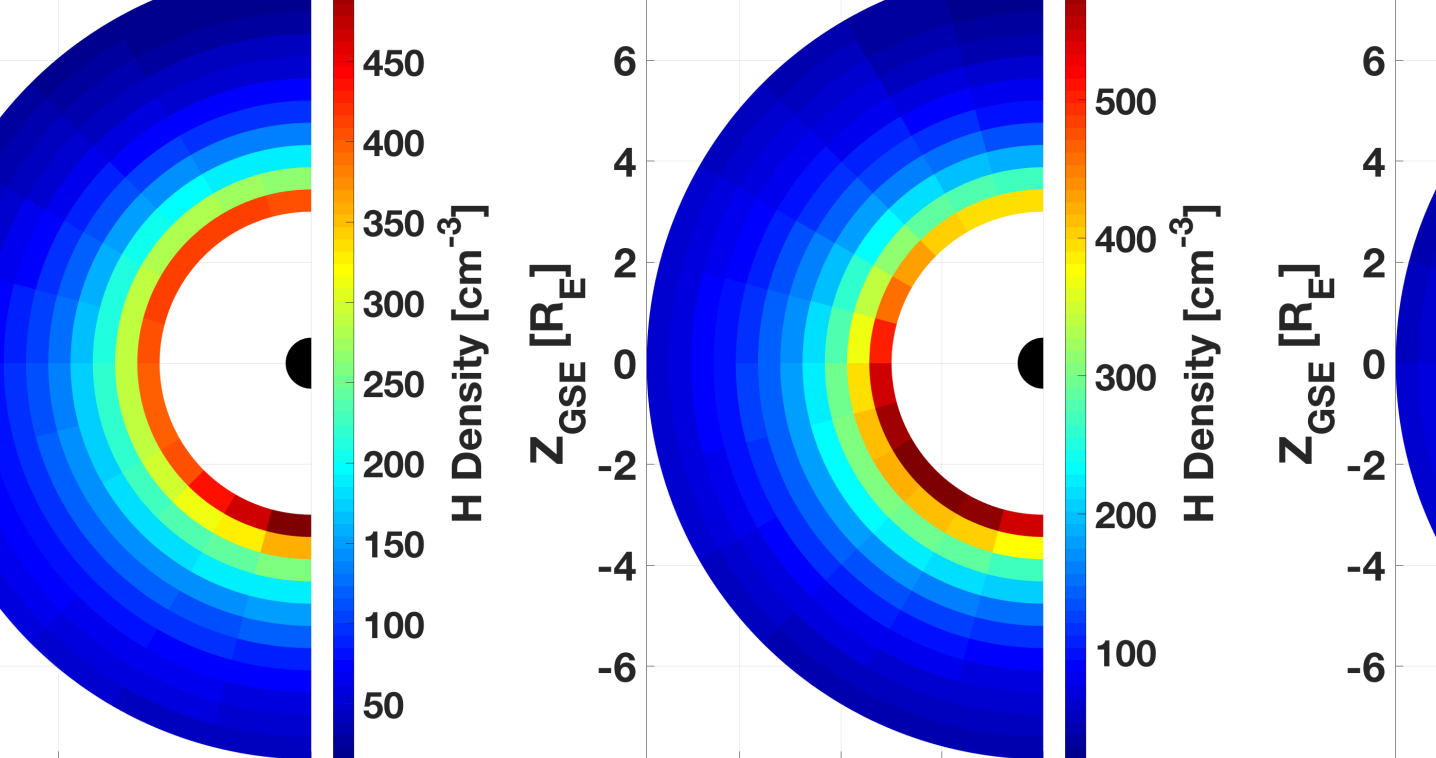
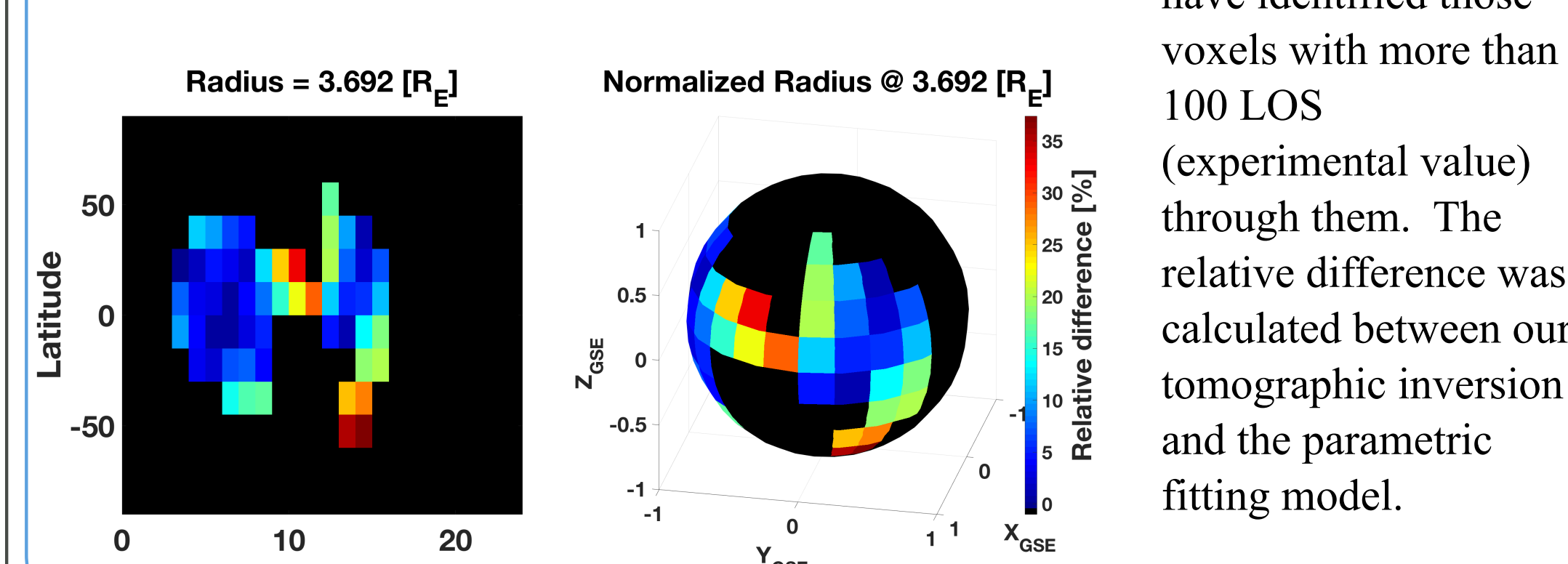
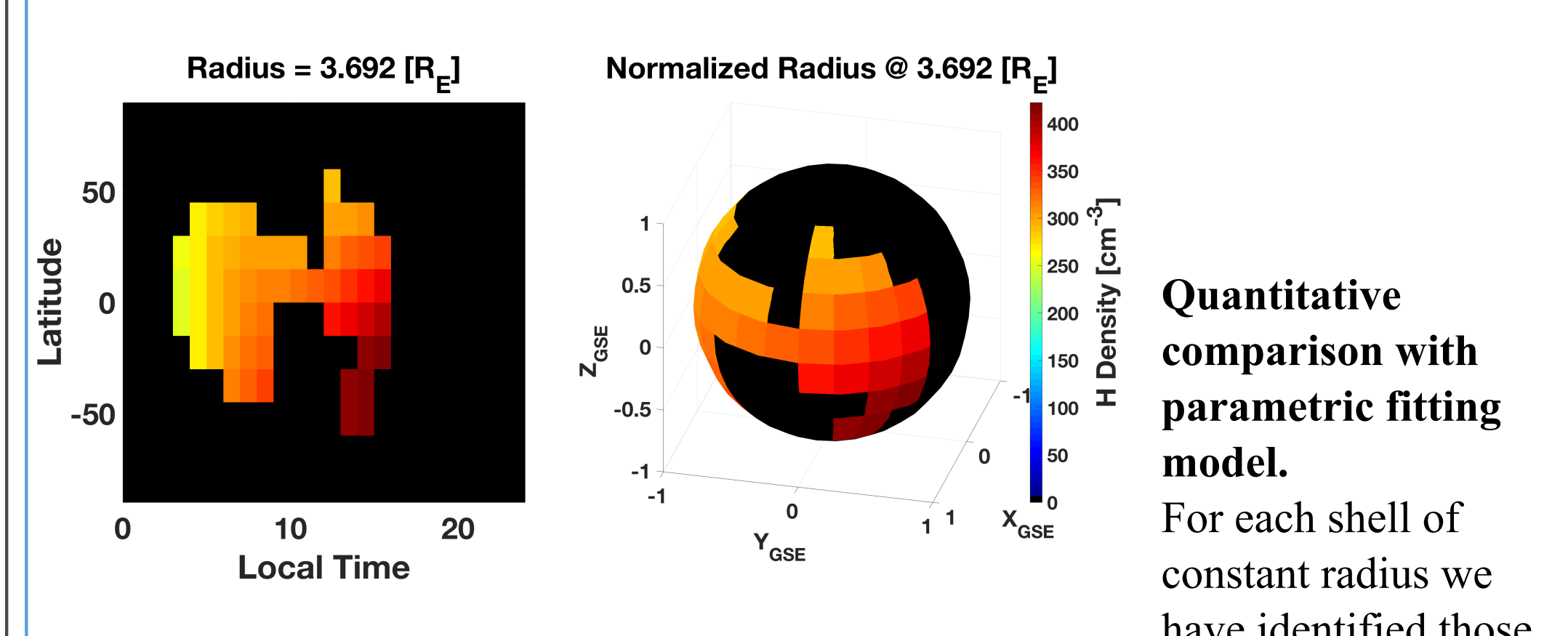
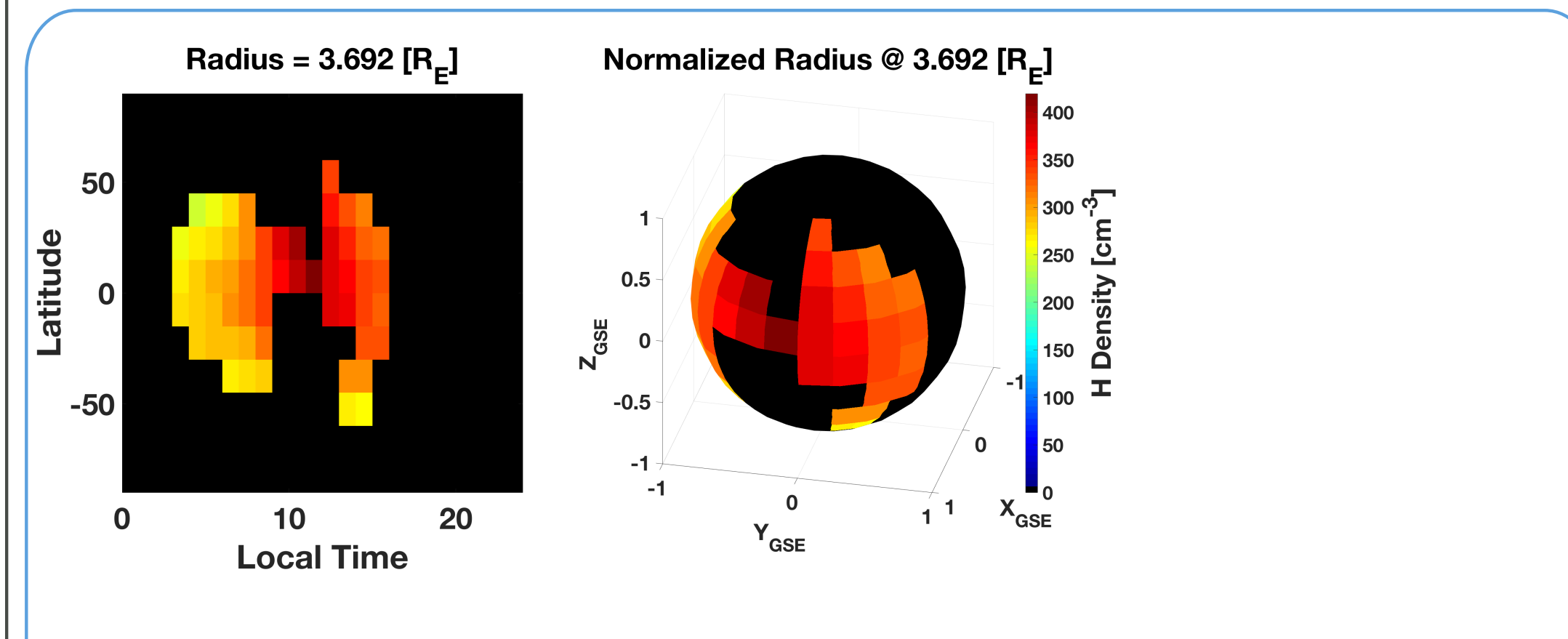
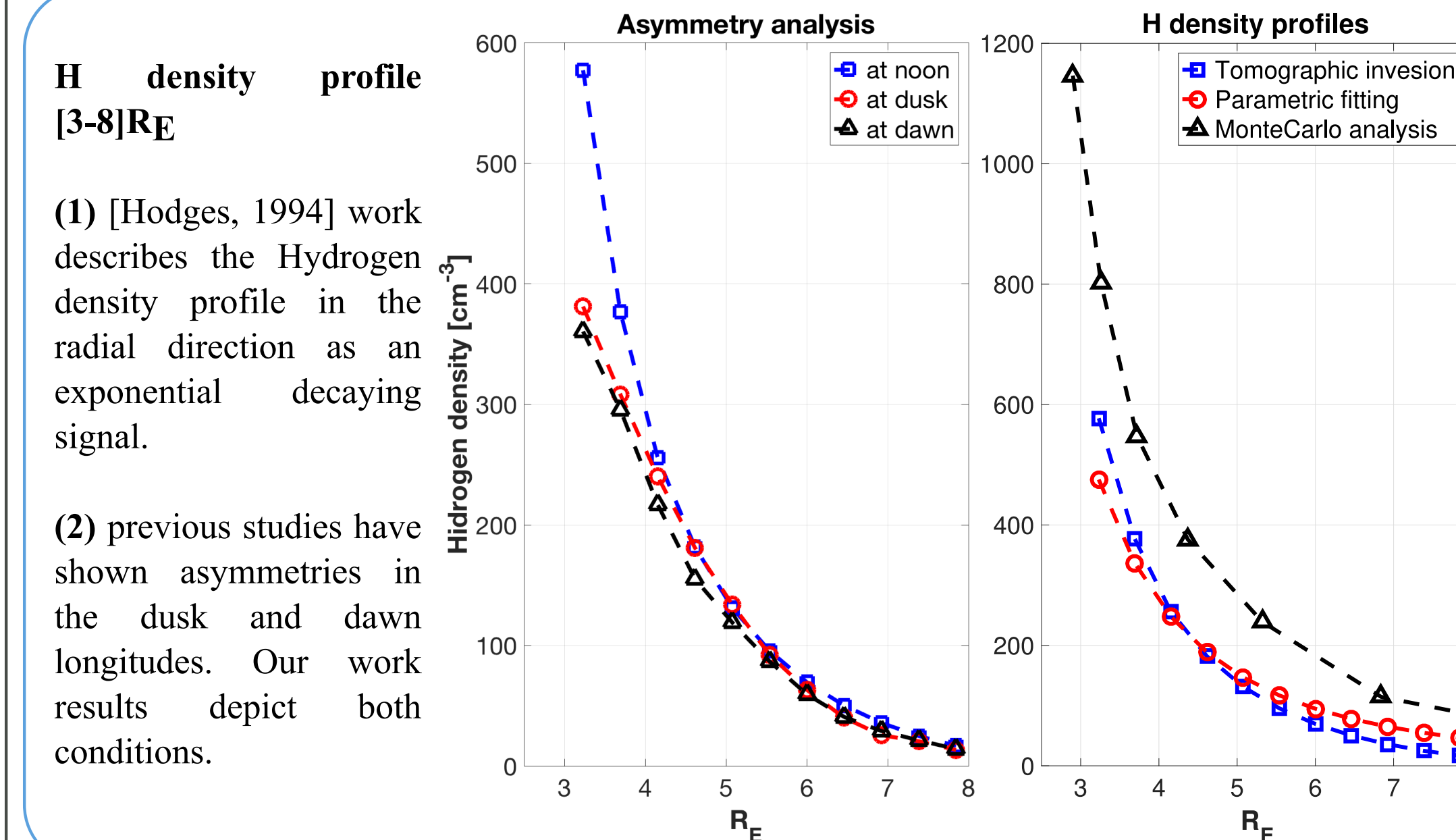


Fig. 6: Comparison between our tomographic inversion work and parametric fitting. Result for three different sectors to identify asymmetries.

Fig. 7: Comparison between our tomographic inversion work and parametric fitting. Result for three different sectors to identify asymmetries.

Fig. 8: Comparison between our tomographic inversion work and parametric fitting. Result for three different sectors to identify asymmetries.

## Discussions



**Quantitative comparison with parametric fitting model.**  
For each shell of constant radius we have identified those voxels with more than 100 LOS (experimental value) through them. The relative difference was calculated between our tomographic inversion and the parametric fitting model.

## Conclusions

We demonstrate that tomographic inversion of optically thin Ly-alpha emission is a promising technique to obtain the 3D structure of the exospheric H.

The azimuthal dependence of our H reconstruction results exhibits superior agreement with Monte Carlo simulations, while the radial dependence agrees best with parametric estimation results.

Future work will focus on applying the new technique systematically to both TWINS and IMAGE Ly-alpha emission data in order to obtain comprehensive spatial and climatological characterization of exospheric structure and variability while avoiding ambiguities of conventional parametric density estimation techniques

## References

Bailey, J. and Gruntman, M. (2011), Experimental study of exospheric hydrogen atom distributions by Lyman-alpha detector on the TWINS mission. *Journal of Geophysical Research: Space Physics*, vol. 116.

Hodges, R. (1994), Monte Carlo simulation of the terrestrial hydrogen exosphere. *Journal of Geophysical Research: Space Physics*, vol. 99.

## Acknowledgments

The authors gratefully thank Dr. J. Bailey and Dr. M. Gruntman for providing the interplanetary background map. We especially thank Dr. Farzard Kamalabadi and Dr. Mark Butala for discussions.