Monte-Carlo simulations of ion velocity distributions and resulting incoherent radar spectra under strong ion frictional heating conditions

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Abstract

In the presence of strong electric fields the ion velocity distribution of the weakly ionized plasma at high latitudes can differ enough from a Maxwellian shape to substantially affect Incoherent Scatter Radar (ISR) spectra, and thus the analysis and interpretation of those spectra. In the present work, an advanced description of the ion velocity distribution and attendant spectra is obtained through improvements made to previous studies that employed Monte-Carlo simulations. These improvements include: 1) a much higher number of collisions to reduce statistical noise, 2) a new velocity distribution fitting technique, 3) the use of Nyquist diagrams to check the plasma stability, 4) a study of more recently published O^+-O resonant charge exchange collision cross-sections, 5) the option to incorporate the effects of ion-ion and ion-electron collisions on the ion velocity distribution, and 6) an improved filtering technique to reduce the statistical noise produced by the Monte Carlo simulations. Through these improvements, it has been found that: 1) ion-ion and ion-electron collisions have a minimal impact on the NO^+ line-of-sight temperature, but a strong impact on the O^+ temperature parallel to the magnetic field; 2) the newer O^+-O resonant charge exchange collision cross-section published by *Pesnell et al.* (1993) produces an electrostatically stable plasma, contrary to what is produced by the older *Knof et al.* (1964) cross-section or the 'scaled' relaxation collision model that has frequently been used in the past; 3) NO^+ spectra can generally be modeled using Maxwellian velocity distributions as far as the impact on ISR spectra is concerned; and 4) O^+ spectra obtained along the magnetic field direction lead to an apparent increase in electron temperature, in contrast with the well-known opposite situation at large angles to the magnetic field.

Monte-Carlo Simulated Ion Velocity Distributions

- From St-Maurice and Schunk [1977], the Lorentz force is written as:

$$\frac{d}{dt}\left(\mathbf{v}-\frac{\mathbf{E}\times\mathbf{B}}{B^2}\right)=\left(\mathbf{v}-\frac{\mathbf{E}\times\mathbf{B}}{B^2}\right)\times\vec{\Omega} \qquad (1)$$

  where $\vec{\Omega}$ is the gyrofrequency along the magnetic field direction, $\mathbf{v}$ is the velocity, $\mathbf{E}$ is the electric field vector, $\mathbf{B}$ is the magnetic field vector, and t is time.
- The motion given by this equation is circular in velocity space and a cycloid in physical space.
- Ion-neutral particle collisions replace the ion velocity with the neutral particle velocity, distorting the ion velocity distribution into a toroid centred on the drift velocity (Figure 1).
- Defining the temperature to be the width of the distribution, these distortions introduce different temperatures parallel and perpendicular to the magnetic field.

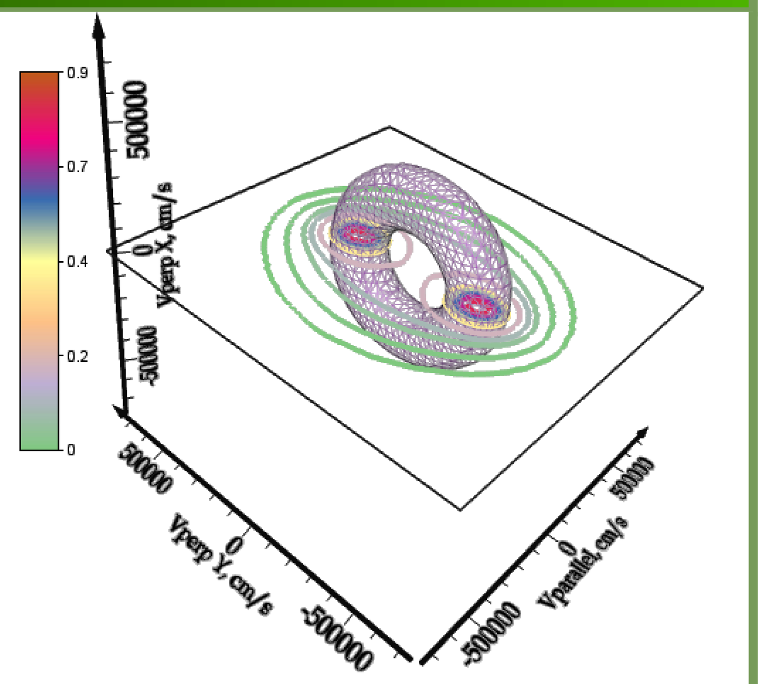

Figure 1: A O^+ velocity distribution subject to O collisions in a 150 mV/m electric field. Simulation developed by *Winkler et al.* [1992].

- A Monte-Carlo (MC) simulation developed by *Winkler et al.* [1992] is used to infer the distribution from a supersonic ion colliding with a population of neutral particles under a given electric field and pre-determined neutral particle parameters.
- By both improving older techniques and introducing better methods, it is possible to return to previous MC simulation studies and explore them in greater detail with less limitations.

Improvements to Previous Studies:

- To provide a complete description of the ion velocity distribution other studies distort a Maxwellian distribution to fit the MC simulated distribution, but this study describes the distribution by taking the exponential of a least squares fit to the logarithm of the distribution.
- To reduce the statistical noise introduced by the MC simulation, ion function values are filtered.
- Previous simulation studies similar to this work have generally ignored if a plasma was unstable because unstable plasma spectra descriptions are not available, but in this study Nyquist diagrams are used to check the stability of a given plasma (Figure 2).
- The Pesnell cross-section is of particular interest because it has been found to be consistently stable up to at least 200 mV/m. Therefore the Pesnell O^+-O RCE cross-section is used in this study.
- Ion-ion and ion-electron collisions are incorporated into the total ion velocity distribution, f_T, using:

$$\nu_T f_T = \nu_{in} f_{in} + \nu_{ie} f_{1i} + \nu_{ii} f_{2i} \qquad (2)$$

  where $\nu_T = \nu_{in} + \nu_{ie} + \nu_{ii}$, ν_{ii} is the ion-ion momentum transfer collision frequency, f_{in} is the ion velocity distribution as determined by ion-neutral particle collisions, f_{1i} is an isotropic, Maxwellian velocity distribution determined by the electron temperature, and f_{2i} is an isotropic, Maxwellian velocity distribution determined by the average ion temperature.
- This work is able to simulate substantially more ion-neutral particle collisions than previous studies, reducing statistical noise.

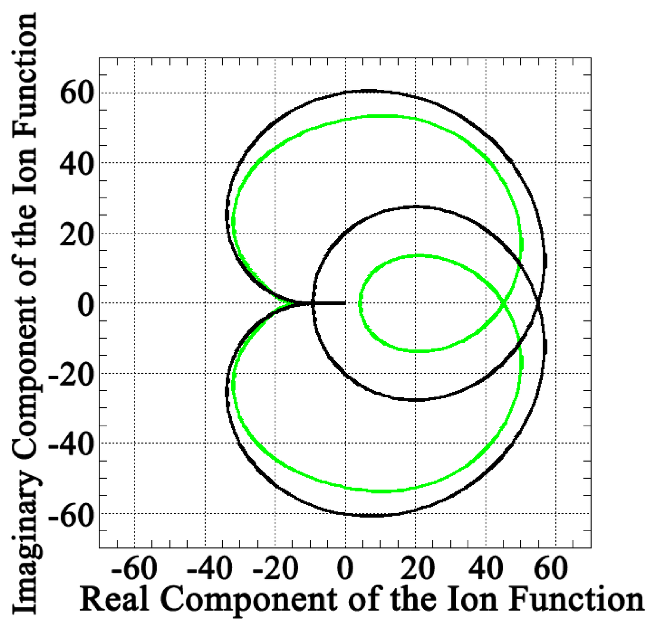

Figure 2: A Nyquist diagram showing a plasma simulated with the "Pesnell" RCE cross-section from *Pesnell et al.* [1993] in green, and a plasma simulated with the "Knof" RCE cross-section from *Knof et al.* [1964] in black, both at 200 mV/m. The Knof cross-section simulates an unstable plasma because the black line circles the origin.

Simulated Ion Temperatures

- The relative ion-neutral temperature difference as a function of the ion-neutral relative drift is explored for O^+-O collisions, and NO^+ with a O and N_2 background (Figure 3).
- As expected the relative ion-neutral temperature difference increases as the relative ion-neutral drift increases and as the aspect angle increases.
- Ion-ion and ion-electron collisions increase the O^+-O relative temperature parallel and near parallel to the magnetic field, but have a negligible impact on the NO^+ with O and N_2 relative temperature
- NO^+ has a strong agreement with the Maxwell molecule formulation at 55°, unlike O^+.

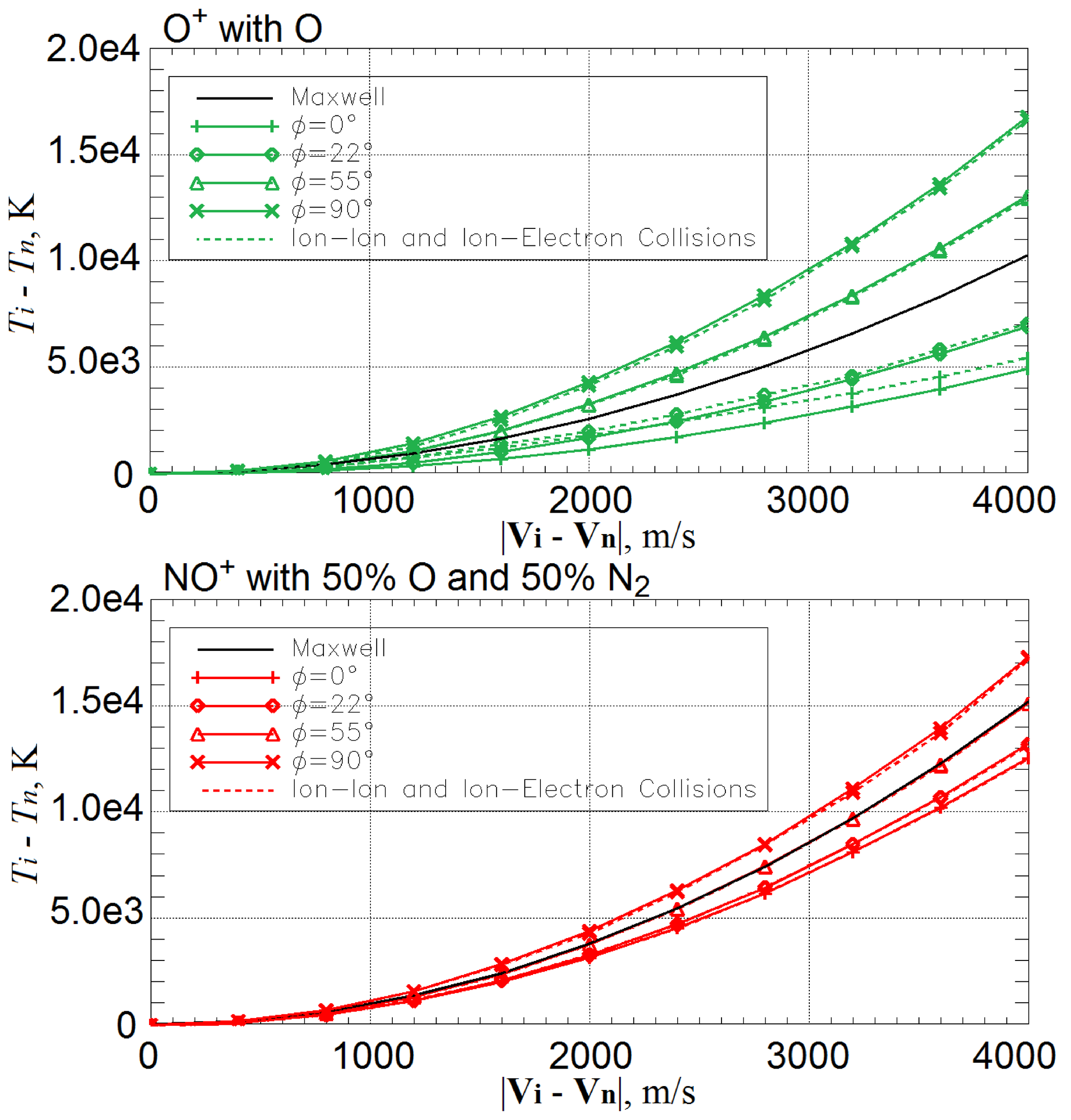

Figure 3: The ion-neutral relative temperature difference, $T_i - T_n$, as a function of the relative drift, $|\mathbf{V_i} - \mathbf{V_n}|$, at a variety of aspect angles. The top panel examines O^+-O collisions using the "Pesnell" RCE cross-section, and the bottom panel examines NO^+ collisions with 50% O and 50% N_2. The black lines gives the average relative ion-neutral temperature difference according to the Maxwell molecule formulation ($T_i - T_n = |\mathbf{V_i} - \mathbf{V_n}|^2 m_n/3k_b$, where m_n is the neutral mass and k_b is the Boltzmann constant). The solid lines reflect the influence of ion-neutral particle collisions, and the dashed lines incorporate self- and ion-electron collisions.

Simulated Spectra

- Spectra created using MC simulated ion velocity distributions are explored (Figure 4).
- Moderate electric fields distort the MC simulated O^+ velocity distributions enough to create spectra substantially different than those from the effective Maxwellian ion velocity distributions, even along the magnetic field.
- Standard Incoherent Scatter Radar (ISR) fitting routines would interpret the elevated "peak-to-trough" ratio in the O^+ spectra along the magnetic field as an increase in the electron to O^+ temperature ratio.
- The MC simulated NO^+ spectra are quite satisfactorily simulated from effective Maxwellian velocity distributions.

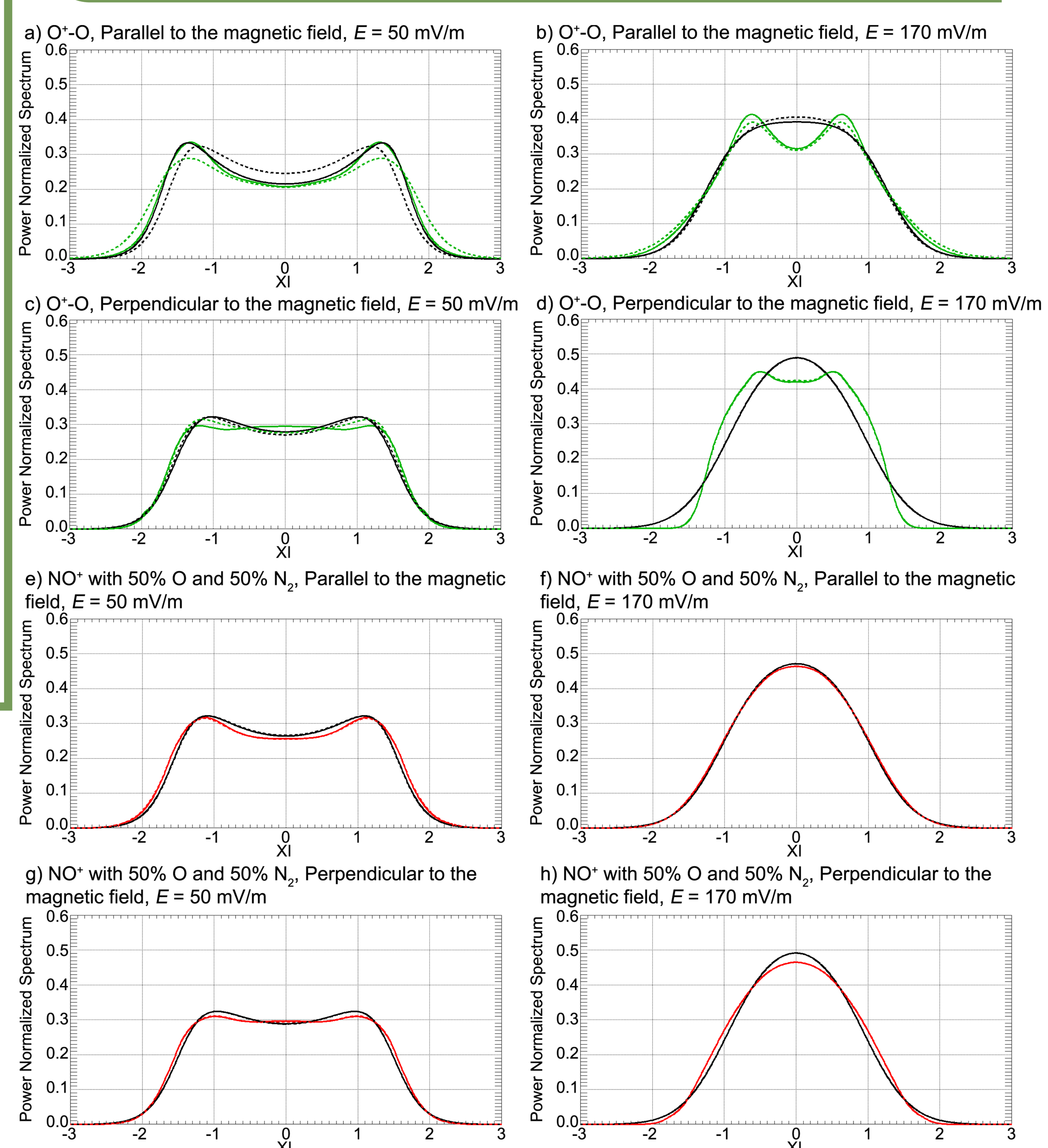

Figure 4: Spectra as a function of XI (XI $= \omega/bk$ where ω is angular frequency, b is the ion thermal speed, and k is the wavenumber) for a variety of cases, all with a 2000 K electron temperature. The green and red lines are produced from MC simulated velocity distributions, the black lines are produced from effective (same line-of-sight ion temperature) Maxwellian distributions, and the dashed lines incorporate self- and ion-electron collisions.

Conclusions and Future Work

- Improvements applied to previous MC simulations have furthered temperature anisotropy, and ISR spectra studies.
- Self- and ion-electron collisions have a minimal impact on the NO^+ velocity distribution, but a substantial impact on O^+ distributions parallel or near parallel to the magnetic field, explaining the high altitude heating in *Goodwin et al.* [2013].
- The *Pesnell et al.* [1993] determination of the RCE O^+-O cross-section stabilizes the plasma against electrostatic fluctuations in spite of a strong toroidal distribution.
- O^+ spectra parallel to the magnetic field show an apparent increase in electron temperature.
- NO^+ spectra can generally be modeled using Maxwellian velocity distributions of the same line-of-sight ion temperature as the MC simulated distribution.
- The results of this work are being compared to ISR observations and satellite images.

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