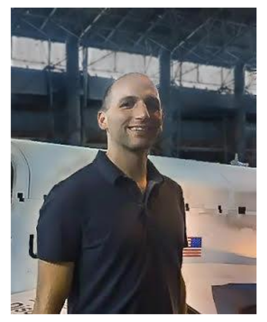

Particle-in-cell simulations of collisional ISR spectra

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Background

Incoherent scatter radars (ISR) reflect radio signals off the ionosphere and measure the returning wave spectra. These spectra are inverted with theoretical models to calculate the density and temperature of the electrons, and ions, and the plasma flow speed. When the radar beam points nearly perpendicular to the Earth's magnetic field, electron-ion Coulomb collisions are the primary mechanism for diffusion across magnetic field lines. This significantly changes the shape of the resulting spectra, and as a result collisionless theories predict incorrect temperature measurements of the low latitude ionosphere [1].

This work is the first to use a fully kinetic, collisional Particle-in-Cell (PIC) code to simulate ISR spectra at small magnetic aspect angles. The PIC approach allows a velocity dependent collision frequency to be used, while allowing the plasma to interact with itself. Previous results are either restricted to single particle simulations with a velocity dependent collision frequency [1,3], or approximate the collision process as Brownian motion with a constant collision frequency [2]. The PIC approach additionally allows for electron-electron and neutral collision effects to be included in the same framework in future work.

Collision Methods

EPPIC Overview: EPPIC is an electrostatic, fully kinetic, massively parallel PIC code [4]. The scatter step uses a linear shape function to aggregate charge density onto the nearest grid cells. The electric field is found by solving Poisson's equation using a spectral technique with a convolution stencil for periodic boundary conditions. A second order Boris mover is used to advance the particle velocities and positions.

Coulomb Collisions: For electron-ion collisions, the ions can be treated as stationary and infinitely massive. Choosing coordinates where the scattering electron is moving in the x-direction, the diffusion tensor from the Fokker-Planck equation is diagonal and with a diffusion coefficient of

$$D(v_e) = \frac{n_i e^4}{4\pi\epsilon_0^2 m_e^2 v_e}\log\Lambda$$

Where $\log\Lambda$ is the Coulomb logarithm and n_i is the density of scattering ions.

Collision Algorithm: The Fokker-Planck Equation is implemented in EPPIC through a Langevin equation. The coordinates are chosen with the initial electron velocity, v_0, in the x-direction. The post collision velocity, v_1, is then

$$v_{1,y} = Q_y$$
$$v_{1,z} = Q_z$$
$$v_{1,x}^2 = v_{0,x}^2 - v_{1,y}^2 - v_{1,z}^2$$

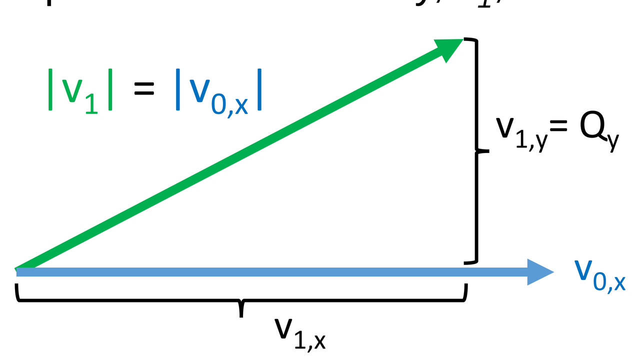


Figure 1: Scattering geometry

Q_y and Q_z are each drawn from a normal distribution with a mean of 0, and a variance of $\sigma^2 = D\Delta t$. This random draw effectively chooses an initial impact parameter for the scattering. This process is repeated for every electron in the simulation. The only assumption in the algorithm is the ions are much heavier than the electrons, so the electron's energy must be conserved through the collision. The energy conservation is enforced by calculating the $v_{1,x}$ component such that $|v_1| = |v_0|$.

Simulation Parameters

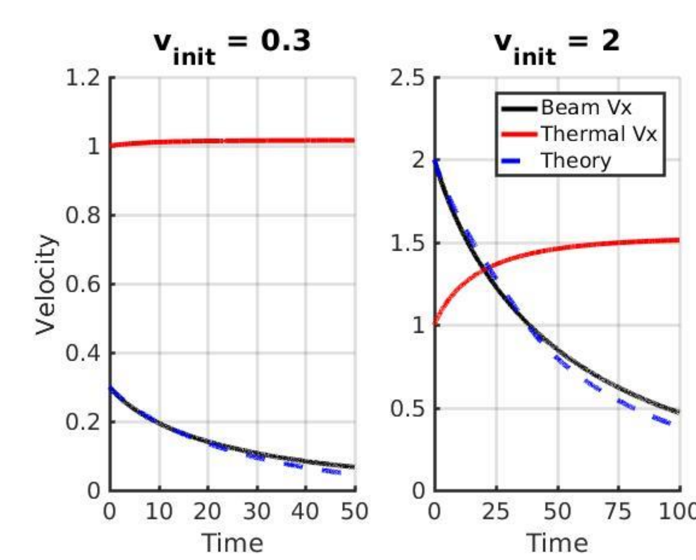
Table 1 provides the parameters used to simulate ISR spectra and are representative of the nighttime F-region. The spectra are obtained by running the PIC code for 100 independent simulations without any initial perturbations, with and without the collision algorithm. The Fourier transformed density of each run is squared, and averaged to obtain $\left\langle \left| n_e(\vec{k},\omega) \right|^2 \right\rangle$, which is proportional to the ISR power spectra. The radar frequency dictates the spatial size of the simulation, so we use the 34 cm Millstone Hill radar wavelength to keep the grid small for our initial results. The methods shown are easily generalized for the 3 m Jicamarca radar wavelength, but the required grid size is computationally demanding.

Grid Size	512 x 512	Electron mass	$m_e = 2.594 \times 10^{-29}$ kg
Grid Step	$\Delta x = \Delta y = 0.8$ cm	Ion mass (O^+)	$m_i = 2.657 \times 10^{-26}$ kg
Total Time Steps	48,000	Electron temperature	$T_e = 2100$ K
Time Step Size	$\Delta t = 50$ ns	Ion temperature	$T_i = 1000$ K
Simulation length	t = 2.4 ms	Average density	$n_i = n_e = 10^5$ cm^{-3}
Magnetic field	$B_x = 0.3$ G	Radar frequency	$f_{rad} = 440$ MHz

Table 1: Parameters used in ISR simulations.

Model Validation

Figure 2: The collision model is validated by simulating a weak beam passing through a background plasma, and comparing its collisional deceleration with theoretical results from the Fokker-Planck equation. The velocities are normalized to the background electron thermal speed, and 2 initial beam speeds are shown.



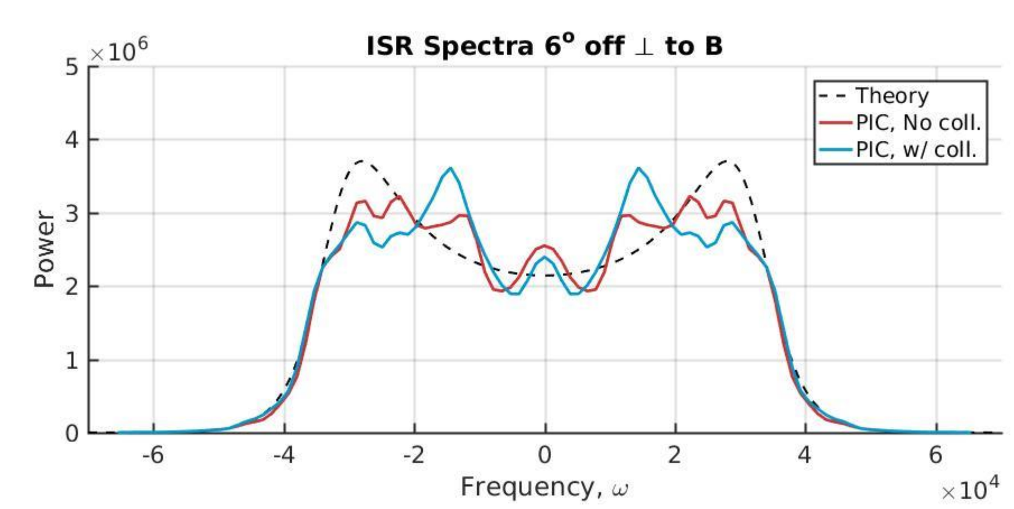


Figure 3: According to [1], the spectra at aspect angles larger than 5° is dominated by the high mobility along the magnetic field lines, and is not affected by Coulomb collisions. At a magnetic aspect angle of 6° our simulations match well with the collisionless theory of [2]. The simulated spectra in Figures 3-7 are scaled up by a factor of 4/3 to account for power lost by the window function.

References

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[2] Kudeki, E. and M. Milla (2011), Incoherent Scatter Spectral Theories – Part 1: A general framework and results for small magnetic aspect ratios, *IEEE Trans. on Geosci. and Remote Sens.*, Vol. 49, Num 1

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Spectra Perpendicular to B

The simulation spectra at small magnetic aspect angles is compared with the theoretical results in [2], which represent electron-ion collisions with a constant collision frequency of 98 Hz. The PIC simulation uses a velocity dependent collision frequency for each electron, so only electrons moving at the thermal speed have a collision frequency of exactly 98 Hz.

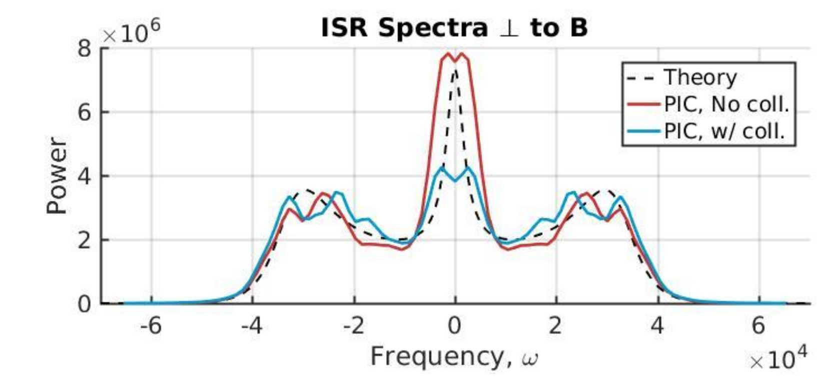

Figure 4: ISR spectra exactly perpendicular to the magnetic field for a 440 MHz radar. The simulated spectra in Figures 3-7 are scaled up by a factor of 4/3 to account for power lost by the window function.

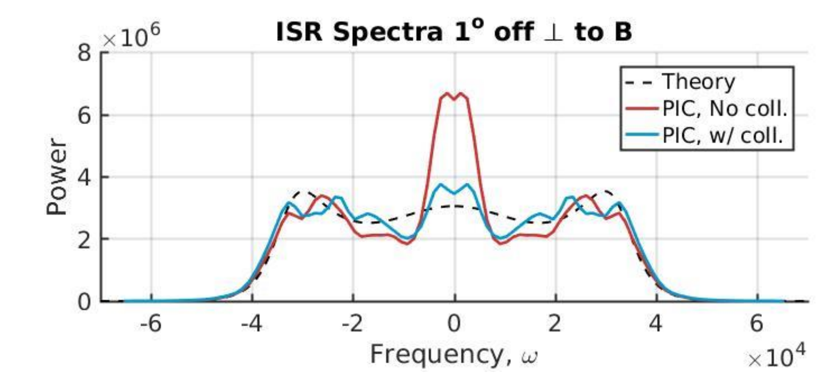

Figure 5: ISR spectra at a 1° magnetic aspect angle. The assumption of a constant collision frequency is tenuous when looking off the perpendicular direction.

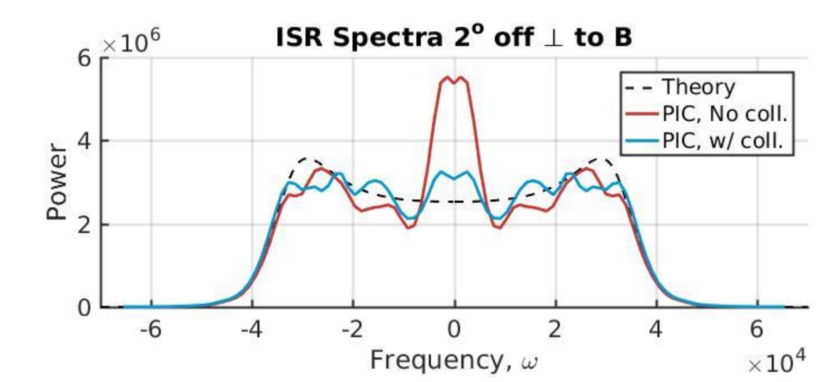

Figure 6: ISR spectra at a 2° magnetic aspect angle.

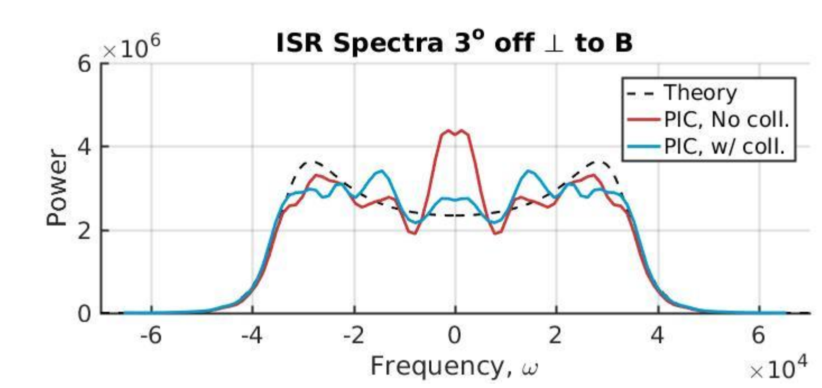

Figure 7: ISR spectra at a 3° magnetic aspect angle, showing the effects of collisions starting to become negligible.

Conclusions

- A fully kinetic, grid-based electron-ion collision algorithm was implemented into a PIC code in order to simulate the effects of collisions on ISR spectra taken at small magnetic aspect angles.
- The collision algorithm validated against kinetic beam theory, and the PIC method was shown to accurately reproduce unmagnetized ISR spectra.
- Fully kinetic collisional simulations of ISR spectra at small magnetic aspect angles significantly deviate from collisionless simulations, and from theoretical results that assume a constant collision frequency.

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