Stochastic modeling of turbulence: applications to the phase speed saturation of Farley–Buneman waves

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Abstract

The **main goal** of this work is to *study the phase speed saturation of Farley– Buneman waves as an interaction with the random turbulent fluctuations in the background and to asses Bourret's averaging method effectiveness to transform the obtained stochastic dynamical system into a deterministic one.*
The results show that we can reproduce a deceleration mechanism and even obtain phase velocities that saturate close to the ion acoustic speed (C_s) as expected.

I. Introduction

Farley–Buneman (FB) waves arise in the E region when the convection speed of the electrons relative to the unmagnetized ions exceeds a threshold speed close to the ion acoustic speed. They are observed in most contexts propagating at phase speeds slower than the convection speed (linear prediction) and closer to the non–isothermal C_s. Sudan attributed phase speed saturation to random, turbulent fluctuations in the electron motion [3].
The present analysis is similar to Sudan's in that it considers the effects of random, turbulent electric fields on the propagation of the waves. However, the analysis is linear, one dimensional, and based on fluid theory. We employ systems theory and the formalism of stochastic differential equations to examine how stochastic fluctuations in the electron flow bias the wave dispersion relation. It shows how phase-speed saturation can be evaluated in terms of the autocorrelation function (ACF) of the flow fluctuations. Figure 1 summarizes our strategy to approach the problem.

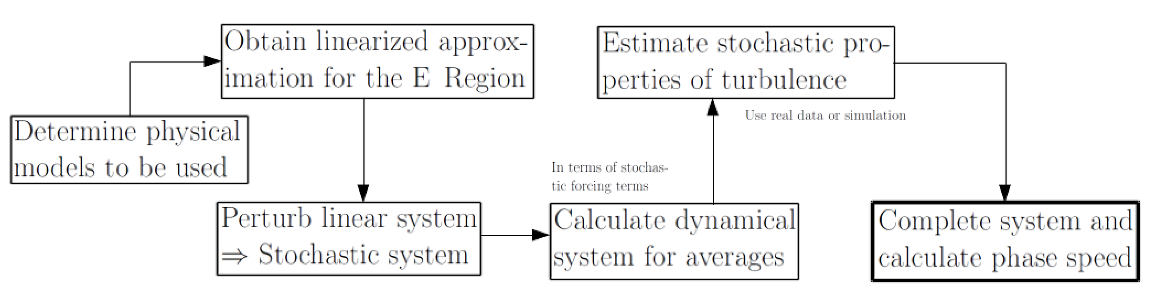


Figure 1: General strategy to calculate the phase speed.

II. Farley–Buneman Linear Model and Bourret's Integral

The fundamental model used consist of the continuity and momentum equations for the e^- and assuming: e^- are inertialess, harmonic spatial variation with wave number k and charge density $n \approx n_i \approx n_e$. Defining:

$$A_\circ = \begin{pmatrix} 0 & 1 \\ -k^2C_s^2 - ikv_\circ\frac{\nu_{in}}{\psi} & -\nu_{in}(1-\psi^{-1}) \end{pmatrix}$$

The linearized system can be written as

$$\dot{u} = A_\circ u \qquad (1)$$

with $u = (\frac{n}{n_\circ}, ikv_i)^T$ and where ψ, $n_\circ$, ν_{in}, $v_\circ$ and v_i are the anisotropy factor, background density, ion–neutral collision frequency, electron convection speed and ion speed respectively. From the dispersion relation of $A_\circ$ we can obtain the condition for instability $v_\circ > C_s(1+|\psi|)$ and the phase speed $v_p^{(lin)} = v_\circ/(1+|\Psi|)^{-1}$.
Extending (1) to model a random perturbation:

$$\dot{u}(t) = (A_\circ + \alpha R(t,\varpi))u(t) \qquad (2)$$

where ϖ is a stochastic variable and α an expansion parameter. System (2) is stochastic, but assuming short correlation times, Bourret's integral

$$A_1 \approx \alpha^2 \int_0^\infty \langle R(t)e^{A_\circ\tau}R(t-\tau)\rangle e^{-A_\circ\tau}d\tau \qquad (3)$$

can transform it into a deterministic system by creating a new operator A_1.

III. Stochastic Modeling of Phase Speed

The interaction with the turbulent background will be through the e^-, so we must replace $v_\circ \to v_\circ(1+\alpha\xi(t))$, where $\xi(t)$ is random. This will define the random matrix

$$R(t) = \begin{pmatrix} 0 & 0 \\ i\alpha\xi(t)kv_\circ\frac{\nu_{in}}{\psi} & 0 \end{pmatrix} \qquad (4)$$

Figure 1 illustrate how this problem simplifies: instead of solving a complex nonlinear system, we are trying to reproduce the nonlinearities by random variations in the e^- convection speed.

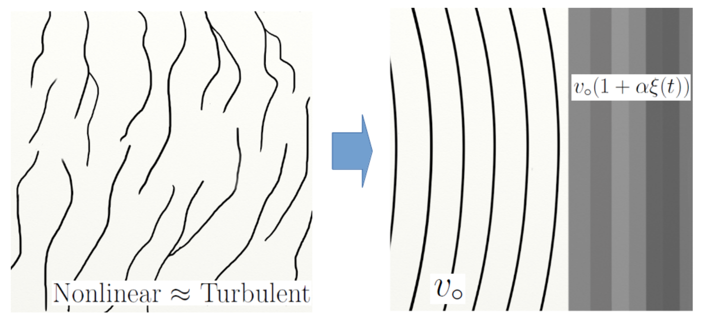


Figure 2: Grayscales represent random medium.

IV. Stochastic Forcing

Using (4) in (3) we will have a deterministic system for the averages $\langle\dot{u}\rangle = (A_\circ + A_1(c_1,c_2))\langle u\rangle$ where:

$$c_1 = \int_0^\infty \langle\xi(t)\xi(t-\tau)\rangle \sinh(\Gamma\tau)\, d\tau \qquad (5)$$

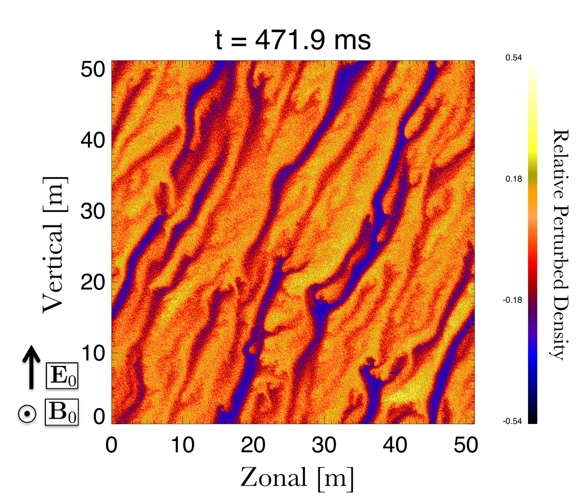
$$c_2 = \int_0^\infty \langle\xi(t)\xi(t-\tau)\rangle(\cosh(\Gamma\tau)-1)\, d\tau \qquad (6)$$

where Γ depends on the system parameters. The structure of $\langle\xi(t)\xi(t-\tau)\rangle$ contains the random behavior of the turbulent e^- speed deviations.

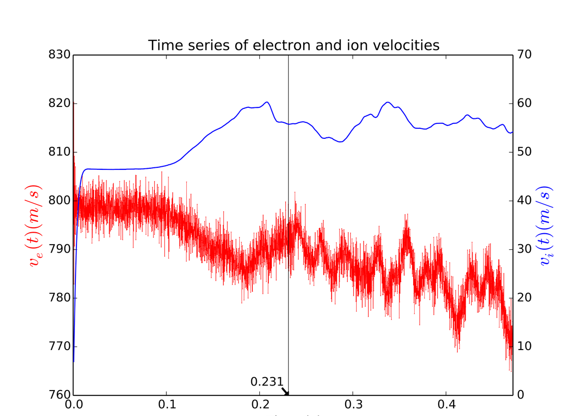
V. Data Processing

$\xi(t)$ was obtained from a full 3D kinetic simulation [2] through the following steps:

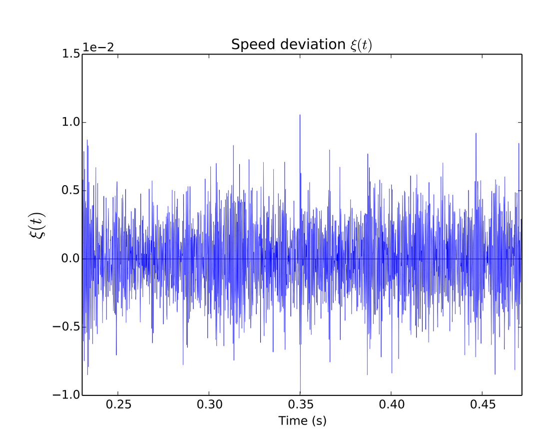
S1. Simulate instability $\to$ get electron fluxes.
S2. Calculate v_o from flux across $\bar{E}\times\bar{B}$ direction.
S3. Compute $\xi(t)$ (remove artifacts and trends).
S4. Construct the function $\langle\xi(t)\xi(t-\tau)\rangle$.



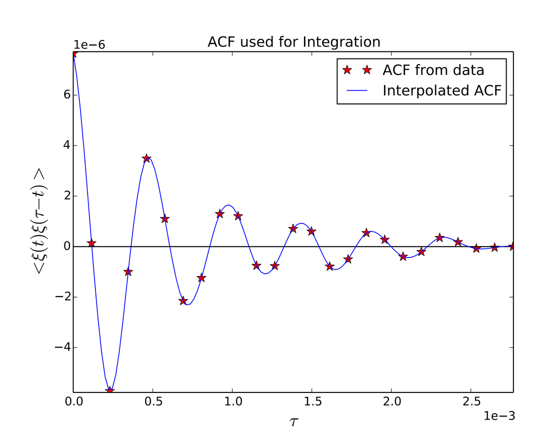
Step 1: Simulation output



Step 2: e^- and ion speed.



Step 3: Speed deviations.



Step 4: $\xi(t)$ ACF

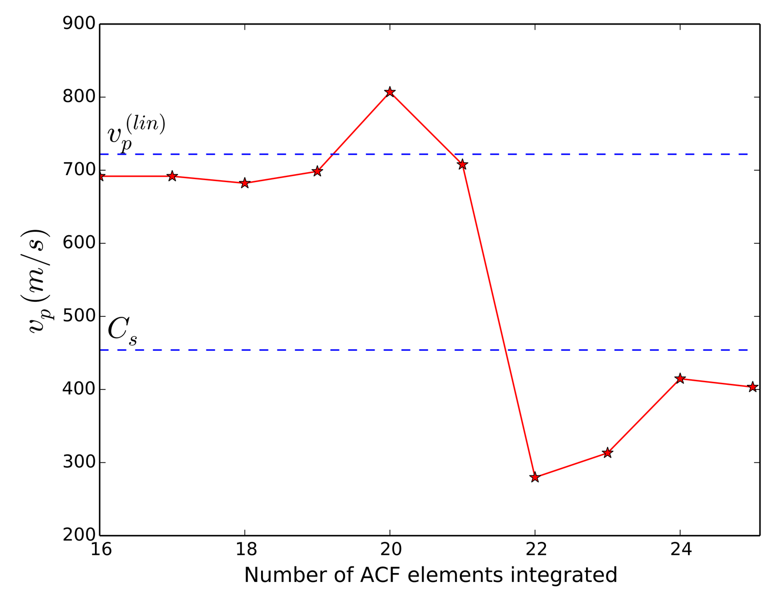


Figure 3: General strategy to calculate the phase speed.

VI. Results

The phase velocity was obtained by the relation $v_p = \Re\{\omega\}/k$, where ω is the positive eigenfrequency of $A_\circ + A_1(c_1,c_2)$. Figure 3 illustrates how v_p approaches the vicinity of the ion acoustic speed as more points are taken from the autocorrelation function. When c_1 and c_2 gets smaller, the estimated v_p gets closer to the phase speed predicted by the linear theory $v_p^{(lin)}$. v_p is going to be in the vicinity of C_s when $\langle\xi(t)\xi(t-\tau)\rangle \approx 0$ for $\tau > 2.1\times10^{-3}$ holds.

Conclusions

- This approach can successfully account for wave deceleration and also for v_p close to C_s
- Differences with C_s may be caused by wave heating, which was not include on our model.
- The fact that no distribution was assumed for $\xi(t)$ should be a motivation to extend this results to similar scenarios.

References

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