Determination of the effective parallel geomagnetic field along a path using Faraday rotation and total electron content from Automatic Dependent Surveillance Broadcast signals

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I: Abstract

A plane polarized electromagnetic (EM) wave that propagates through a plasma, (anti-)parallel to a magnetic field, experiences a gradual rotation of its plane of polarization called Faraday rotation (FR). The FR angle depends on the integrated product of the electron density and the strength of the parallel magnetic field projection to the radio wave propagation direction. The integral is taken along the radio wave propagation direction over the entire path length. Therefore, accurate measurements or a suitable model for both the electron density and the magnetic field as well as the propagation trajectory are required for the interpretation of FR measurements. Many authors use the average value of the parallel magnetic field for estimation of FR from ionospheric total electron content (TEC) measurements. Although it is known that the strength of Earth's geomagnetic field varies slowly at ionospheric altitudes, a reference height characteristic value or characteristic mean value may not always be appropriate. This work considers alternative methods to establish a characteristic value for the average parallel component of the magnetic field, particularly when independent FR and TEC measurements are available.

II: Introduction

Automatic Dependent Surveillance Broadcast (ADS-B) signals are linearly polarized radio waves at 1090 Mhz frequency, which are transmit by most commercial aircraft to track the position and movement of aircraft through intermittent broadcasts containing information about their identity, itinerary, and position state vectors to ground-based receivers and other aircraft within range. ADS-B signals can also be detected by satellites in low Earth orbits and the FR (i.e change of the polarization angle) caused by propagation through the terrestrial ionosphere may be measured.

One of the main benefits of considering ADS-B signals for ionospheric sounding is that ADS-B receivers are able to distinguish and identify signals from different aircraft,which results in potentially hundreds or even up to a couple of thousand independent FR measurements along different propagation paths. The ADS-B signals, currently produced by aircraft, offer a unique opportunity to be used for ionospheric sounding in addition to their main operational purposes. This is similar to how GNSS systems have been used for ionospheric science in addition to their intended purpose of geospatial positioning.

Many authors use an average value for the parallel component of the magnetic field in order to simplify FR estimation. By removing the parallel component of the magnetic field from the integral, the FR can be estimated using the TEC and an average value for the parallel component of the magnetic field. This may be sufficient for some applications such as the estimation of FR from TEC to predict or forecast the amount of FR in satellite systems, but may not be adequate for the inverse problem namely the estimation of TEC from FR measurements and magnetic field models as input for computerized ionospheric tomography (CIT) [Cushley and Noël, 2014], or in applications that require greater precision such as synthetic aperture radar (SAR) imagery correction, and radio astronomy.

In this study, a sensitivity analysis was undertaken comparing different methods (e.g. arithmetic, geometric, mean value theorem for integrals to name a few) of calculating the effective parallel component of the magnetic field in order to find a simple method that works best for a satellite in a circular orbit at 1000 km orbital altitude.

Many methods to determine a characteristic value for the parallel component of the magnetic field that is an improvement over an arithmetic average were investigated by comparing several different methods to one another with respect to a benchmark solution. The work demonstrates the potential of using independent FR and TEC measurements to determine the average parallel component over vertical paths. This method could support the potential exploitation of pre-existing networks and operational data to contribute to current methods of ionospheric sounding.

III: Theory

The FR angle is measured in radians [Budden, 1961; Kraus, 1966] viz.,

$$\Omega = \frac{e^3}{8\pi^2\epsilon_0 m_e^2 c}\frac{1}{f^2}\int n_e(z)B_z(z)dz$$

where $n_e(z)$ is the electron density at altitude z and varies over propagation path of length l, *e* is the charge of an electron, *c* is the speed of light in a vacuum, ϵ_0 is the vacuum permittivity, m_e is the mass of an electron, f is the wave frequency (Hz) of the incident radio wave, and Bz(l) is the component of the magnetic field parallel to the direction of the wave propagation measured in T. Many authors assume an average value for the parallel component of the magnetic field in order to simplify.

$$\Omega \simeq \frac{2.36\times10^4}{f^2}B_{avg}\cos\theta\sec\chi\int_0^l n_e\,\mathrm{d}l$$

where B_{avg} is the average magnetic field intensity in T, Θ is the angle between the wave vector **k** and the magnetic field vector **B**, and *X* is the zenith angle (between the ray wave vector and the vertical).

The total TEC along the raypath of unit cross-section is given by:

$$TEC = \int_0^l n_e(z)dz$$

TEC is usually expressed in TEC units (TECu) equal to 10^{16} electrons m^{-2}, with typical values ranging from 1 to 100 TECu.

By assuming that $B_{||} = B_r$ we restrict our analysis to the case when the satellite is directly overhead [Cushley et al, 2017]. We further assume that the magnitude of the parallel component of the magnetic field can be described by the dipole approximation. Then the component parallel to the propagation of the radio wave can be described using a dipole approximation, written as:

$$B_{||} = B_r = B_{eq}\frac{2R_E^3\sin\lambda}{(z+R_E)^3}$$

where $B_{eq} = 3.1\times10^4$ nT is the equatorial dipole strength at the surface of the Earth and λ is the latitude of the point in consideration.

To properly calculate the average parallel component of the magnetic field <B> in order to extract it from the integral, it should be determined using:

$$\langle B\rangle = \frac{8\pi^2 c\epsilon_o m_e^2 f^2}{e^3}\cdot\frac{\Omega}{TEC} = \frac{1}{TEC}\int_0^l n_e(z)B_z(z)\,\mathrm{d}z$$

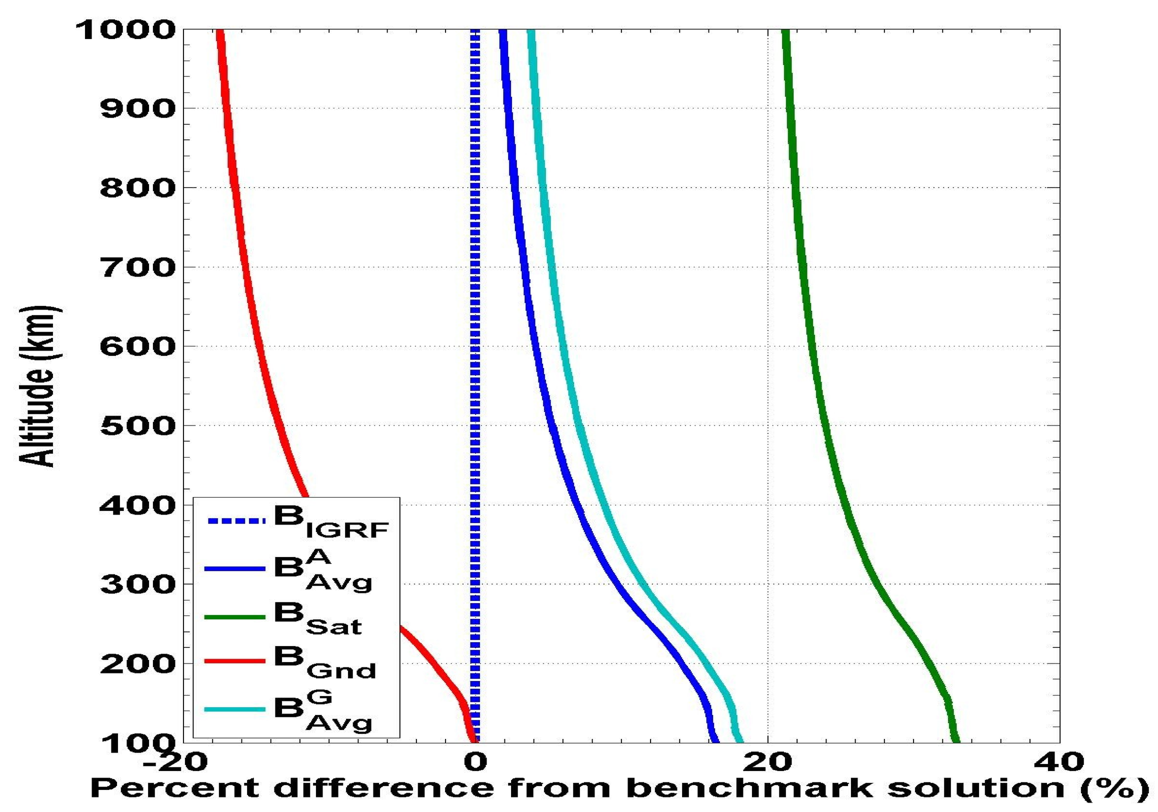
The only way to obtain <B> exactly is to measure both the FR and TEC and then there is no need to compute FR from TEC or vice versa, but both measurements are not usually available from the same satellite. If independent FR and TEC measurements were available from either a single satellite or conjunctions between two satellites they would be useful for mapping the parallel component of the magnetic field, which then could be used for other applications.

In this work, we compare different methods of calculating the effective parallel component of the magnetic field in order to find a simple method that works best for a satellite in a circular orbit at 1000 km orbital altitude. The percent difference *%diff* in the cumulative FR for a given method (Ω_{IGRF}) from the benchmark solution (Ω_{method}) is computed as follows:

$$\%diff = \frac{\Omega_{IGRF}-\Omega_{method}}{\Omega_{IGRF}}\times 100$$

when a particular method overestimates the amount of rotation than the benchmark solution, the percent difference is a negative value.

For this comparison, we arbitrarily selected 1 Jan 2000 at 01h30 UTC at 50°N geographic latitude, 40°E longitude as the location for the comparisons, which is the default location for the International Reference Ionosphere (IRI). The dashed blue curve shows the FR computed for an electron density profile from IRI and the corresponding magnetic field profile from the International Geomagnetic Reference Field (IGRF) evaluated every 10 km, is labelled B_{IGRF} and will be considered as the benchmark solution to which all others will be compared.

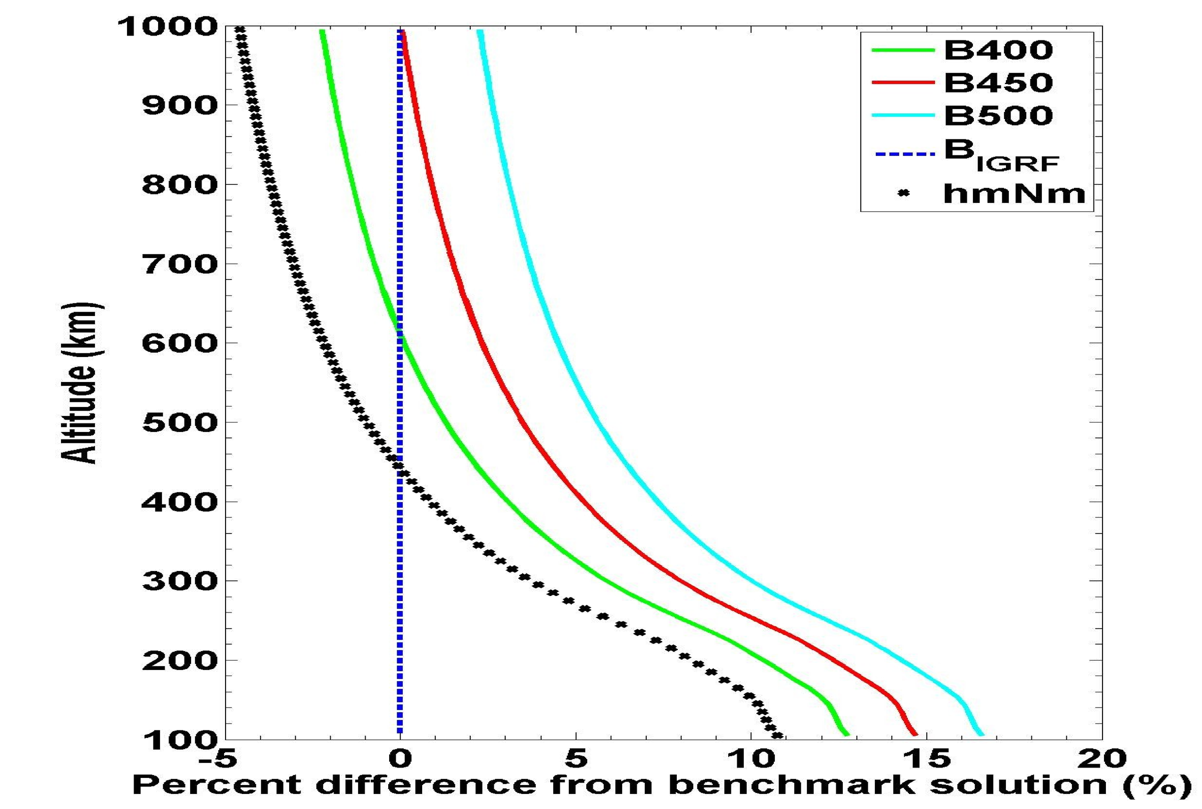


A comparison of the percent difference from benchmark solution using different methods to obtain the characteristic magnetic field from various altitudes. All methods have been normalized as a cumulative percentage as a function of altitude with respect to the total FR (received by a satellite) at 1000 km for the benchmark solution.

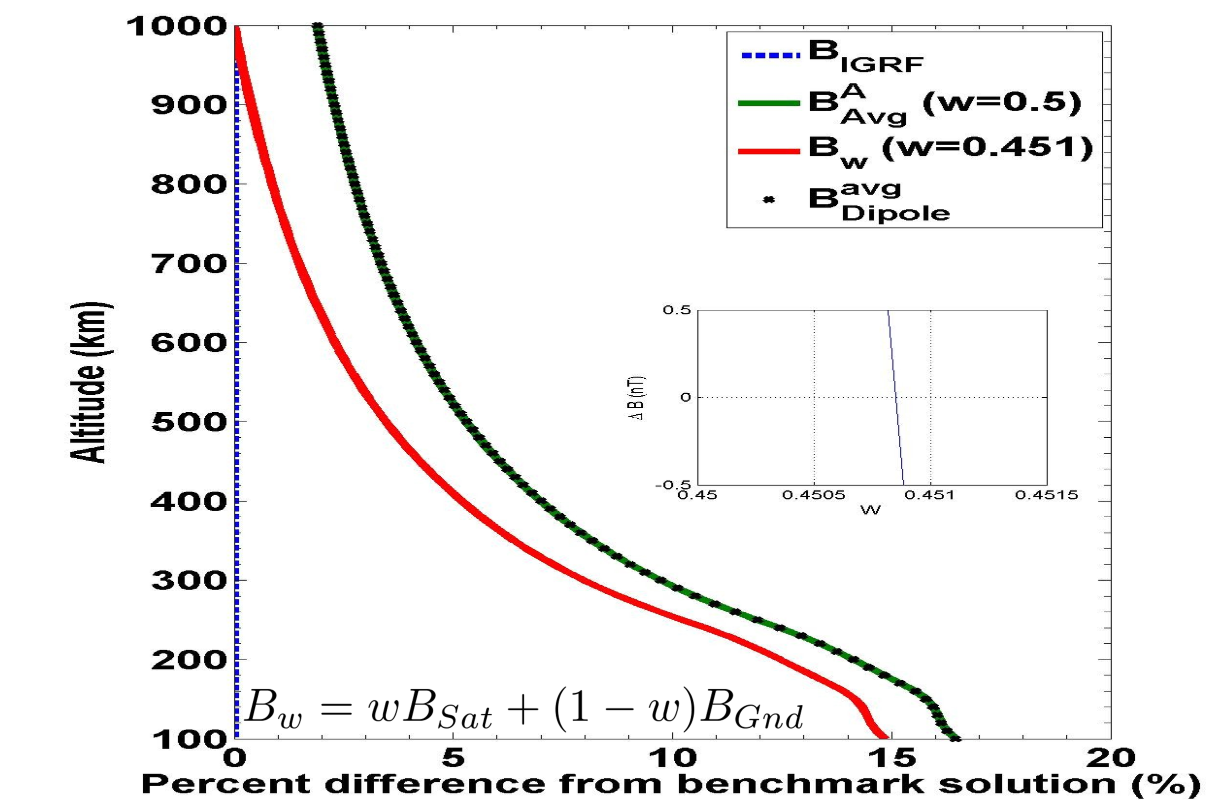
For complete details please refer to:

Cushley, A. C., Noël, J.-M. & Kabin, K., Faraday rotation, total electron content and their sensitivity to the average parallel component of the magnetic field. Submitted (Radio Science).

IV: Results



A comparison of the percent difference from benchmark solution using different methods to obtain the characteristic magnetic field from different altitudes.



A comparison of the integration of the electron density with different methods to obtain a characteristic value for the magnetic field. All methods have been normalized as a cumulative percentage as a function of altitude with respect to the total FR (received by a satellite) at 1000 km for the benchmark solution. The inset shows the difference from the benchmark solution (nT) as a function of weight.

V: Conclusion

FR can affect 30MHz to 300MHz space communications systems that use linear polarization in a significant way. The accuracy of FR estimation from TEC measurements and TEC mapping from FR measurement is limited by the accuracy of magnetic field models. Although it is known that the strength of Earth's geomagnetic field varies slowly at ionospheric altitudes, a reference height characteristic value or mean value may not always be sufficient, although commonly used. Numerical modeling has demonstrated that FR can be calculated more accurately by applying a weighted average in favor of the ground based values. The following are the specific conclusions based on information gathered during this study:

- An average value for the magnetic field tends to underestimate the amount of FR because the magnetic field decreases as an inverse cubic with respect geocentric distance.
- A weighted average (w=0.451) in favor of the magnetic field values from lower altitudes improved results.
- The corresponding altitude (450 km) of the IGRF profile used to create the benchmark solution agreed with the altitude calculated from the weighted method.
- The arithmetic (or some other) average from empirical models is not always reliable and measurements should be used, whenever they are available. This is particularly important during high solar and geomagnetic activity, since magnetic field models may not be capable to account for sporadic space weather events.
- Useful for applications such as computerized ionospheric tomography, SAR correction and radio astronomy.
- The ADS-B signals, currently produced by aircraft, over a unique opportunity to be used for ionospheric sounding in addition to their main operational purposes.
- Aircraft fly in regions (e.g. over oceans) that are not amenable to using other instruments to map the electron density or TEC, specifically oceanic regions and high/polar latitudes.
- The curves diverge at 1000 km and continue to diverge as the altitude increases which shows that the uncertainty also increases when satellites at higher altitudes are considered.
- The FR and TEC can also be obtained for conjunctions using two different satellites to give the exact value for <B>.
- A similar approach can be used to determine the best method for different applications (satellites at different altitudes, latitudes and carrier frequencies).
- More detailed studies are needed to investigate other more general propagation geometries as well as satellites at different altitudes and orbital eccentricities.

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