Incoherent Scatter Radar 101

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Prerequisites:

- Basic electromagnetic theory
- Basic concepts of analog signal processing
- Some plasma physics is helpful
- Logical and geometric thinking (well, pick one; very few of us can do both).

Syllabus:

- Frequency considerations.
- Radar Cross Section
  - Single electron
  - Volume of electrons
- SNR vs. more samples
- ISR spectrum: Nyquist approach
- How the spectrum is measured.
- Some auroral zone examples.

NSF Incoherent Scatter Radars

Jicamarca, Peru

Advanced Modular ISR (AMISR)

Sondrestrom, Greenland

Arecibo, Puerto Rico

Millstone Hill, Massachusetts

What constrains the frequency of an incoherent scatter radar?

ISR operates at frequencies above plasma- and gyro-frequencies,

$$\omega_p = \sqrt{\frac{N_e e^2}{\varepsilon_0 m_e}} \qquad \omega_c = \frac{eB}{m}$$

but not so high that collective behavior of the plasma is lost. This means the wavelength must be greater than the Debye length

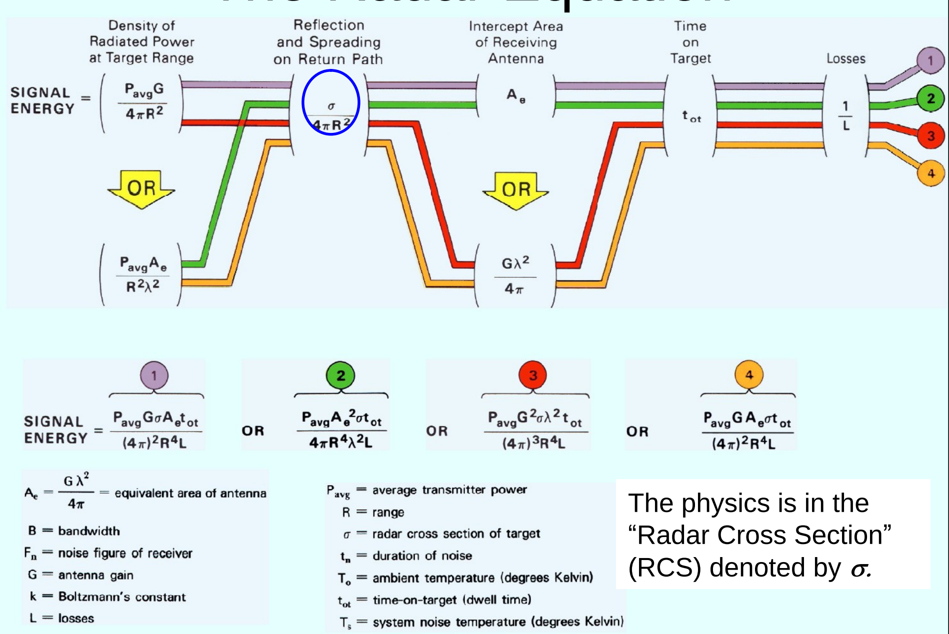
$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_b T_e}{N_e e^2}}$$

Thus the frequency must, therefore, satisfy

$$\omega_p, \omega_c << \omega << \frac{2\pi c}{\lambda_D}$$

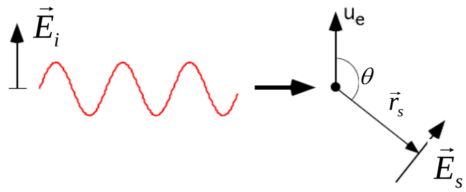
In this frequency range, each electron responds independently to the impinging field. The electrons respond independently (Born approximation), so the scattered signal is the superposition of the Doppler shifted scatter of each electron.

The Radar Equation

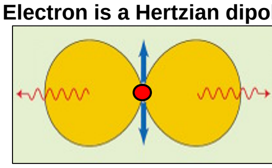


The physics is in the "Radar Cross Section" (RCS) denoted by σ.

Radar cross section: single electron



Electron is a Hertzian dipole



$$\sigma \equiv 4\pi r_s^2 \frac{\text{power per unit area in the scattered wave at receiving antenna}}{\text{power per unit area in wave incident on target}} \quad [\text{m}^2]$$

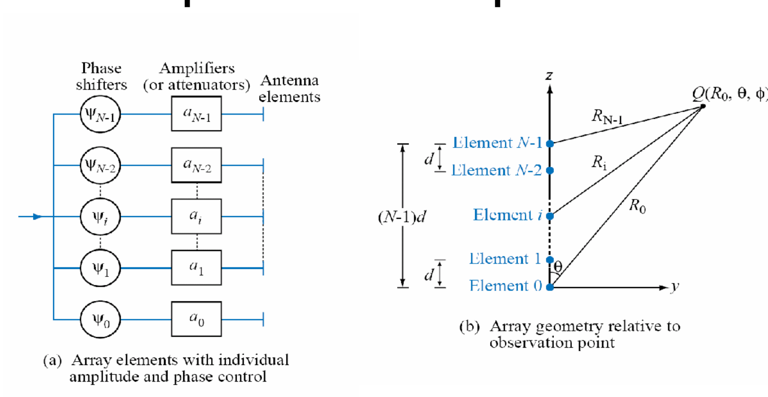
$$= 4\pi r_s^2 \frac{|E_s(r_s)|^2}{|E_i(0)|^2}$$

From classical EM: $\left|\vec{E}_s(\vec{r}_s, t)\right| = \frac{r_e}{r_s}\sin\theta \left|\vec{E}_i(0, t')\right|$

where $r_e = \frac{e^2\mu_0}{4\pi m_e} = 2.82\times10^{-15}$ [m] is the classical electron radius.

Therefore we have $\sigma_e = 4\pi(r_e \sin\theta)^2 \approx 10^{-28}\sin^2\theta \;\; [\text{m}^2]$

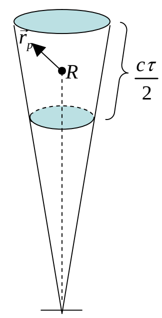
Arrays of radiators and the importance of phase



AMISR relies on constructive interference of individual elemental radiators to focus its beam. ISR relies on constructive interference of electron of dipole radiators in the direction of the receiving antenna.

RCS for volume distribution of electrons

Assume a plane-wave electric field incident on a scattering volume with magnitude E_0 assumed constant within the volume. Let us consider a monostatic radar ($\theta=\pi/2$). The backscattered field at the antenna due to an electron located at position r_p in the volume is given by



$$E_{s,p}(t) = -\frac{r_e}{r_p} E_0 e^{j\left(\omega_0 t - 2\vec{k}_i \cdot \vec{r}_p\right)}$$

where $k_i = 2\pi/\lambda_{radar}$ is the wave number. The total field is the superposition of contributions from individual electrons:

$$E_s(t) = -\frac{r_e}{R} E_0 e^{j\omega_0 t} \sum_{p=1}^{P} e^{-j2\vec{k}_i \cdot \vec{r}_p}$$

Note that, for the purposes of computing amplitude, the entire volume may be considered to be at a single average range *R*.

Of course we cannot make that approximation in the phase term!

RCS for volume distribution of electrons (continued)

We can rewrite the discrete sum as the integral over a differential quantity—the density. In general, the density varies in both space and time, so we have

$$E_s(t) = -\frac{r_e}{R} E_0 e^{j\omega_0 t} \sum_{p=1}^{P} e^{-j2\vec{k}_i \cdot \vec{r}_p} = -\frac{r_e E_0}{R} e^{j\omega_0 t} \int_V N(r,t) e^{-j2\vec{k}_i \cdot \vec{r}_p} d^3 r$$

This expression is a 3D spatial Fourier transform, except that the argument of the exponential is twice the radar *k*-vector. If we let $k=2k_i$ we have

$$E_s(t) = -\frac{r_e}{R} E_0 e^{j\omega_0 t} \int_V N(r,t) e^{-jkr} d^3 r$$

$$= -\frac{r_e}{R} E_0 e^{j\omega_0 t} N(k,t)$$

The magnitude of the scattered signal is proportional to the intensity of spatial fluctuations at ½ the wavelength (twice the wavenumber) of the radar.
Note: the mean value of the density is given by *N(k=0,t),* but this corresponds to an infinite radar wavelength!

Incoherent scatter radars do not measure density directly.
They measure *fluctuations* in density that match the wavelength of the radar.

RCS for volume distribution of electrons (continued)

In the literature, this is often made explicit by substituting *N* with ΔN in the above, and using N_0 for "mean density". With these variables, we can readily obtain

$$E_s(t) = -\frac{r_e E_0}{R} e^{j\omega_0 t} \Delta N(k,t)$$

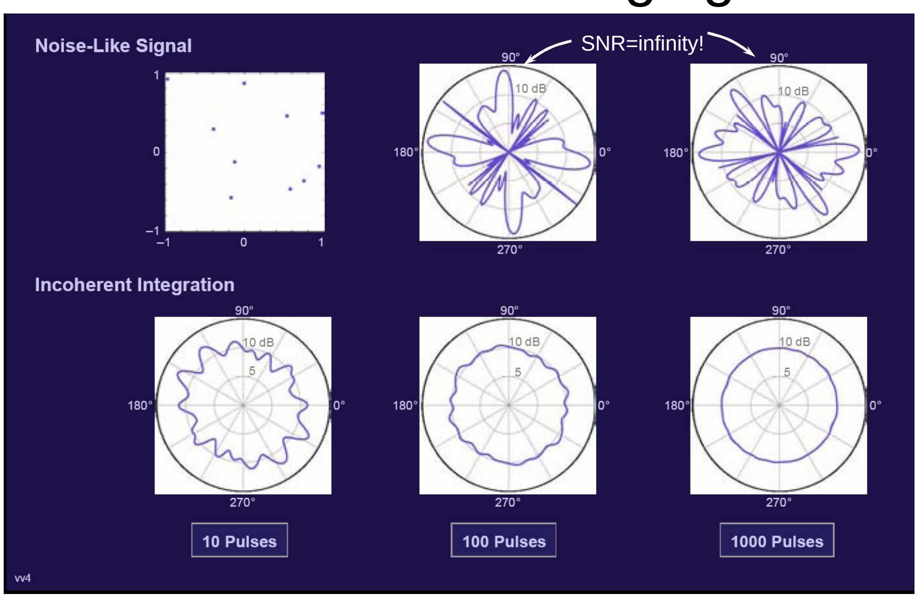
$$\left|E_s(t)\right|^2 = \left(\frac{r_e E_0}{R}\right)^2 \left|\Delta N(k,t)\right|^2$$

$$\sigma = 4\pi R^2 \frac{\left|E_s(t)\right|^2}{\left|E_i(t)\right|^2} = 4\pi r_e^2 \left|\Delta N(k,t)\right|^2$$

Next important observation: $\Delta N(k,t)$ is a random variable, so the instantaneous power (or, instantaneous cross section) is not useful. Rather we are interested in the mean, or expectation, value. Assuming time stationarity we obtain

$$\sigma \propto \left\langle \left|\Delta N(k,t)\right|^2 \right\rangle$$

Incoherent averaging



C. Heinselman, 2004 CEDAR tutorial

SNR versus more samples

The essential result to this effect is from Farley, 1969:

$$\frac{\Delta P_s}{P_s} = \frac{1}{\sqrt{K}}\left(1 + \frac{P_n}{P_s}\right)$$

P_s = signal power
ΔP_s = uncertainty in signal power
K = number of samples
P_n = noise power

Once you have high SNR (P_s/P_n>>1), all that matters is the number of independent measurements

Sixto says:

A big radar helps, but size isn't everything.

Differential cross section

We are not only interested the total power, but how this power is distributed in frequency. Let us now take the Fourier transform in time, noting that the time appears in two places on the right hand side.

$$E_s(t) = -\frac{r_e E_0}{R} e^{j\omega_0 t} \Delta N(k,t) \Longleftrightarrow E_s(\omega_0 + \omega) = -\frac{r_e E_0}{R} \Delta N(k,\omega)$$

The effect of the density fluctuation spectrum appears as an offset to the radar frequency. Using previous developments, we obtain the differential backscatter cross section (I.e., per unit frequency, per unit solid angle) at Doppler frequency $\omega_0+\omega$

$$\sigma(\omega_0 + \omega) d\omega = r_e^2 \left\langle |\Delta N(\mathbf{k},\omega)|^2 \right\rangle d\omega$$

(This is a general result, valid for many scattering problems)

Incoherent scatter radars measure fluctuations in density, not density itself.
Are these two things related? Yes….

What should $\langle |\Delta N(\mathbf{k},\omega)|^2 \rangle$ to look like?

- Two essential theoretical approaches: microscopic and macroscopic.
- For once in plasma physics, particle approach and macroscopic approach agree **exactly**!
- Microscopic: Dressed test particle
- Macroscopic: Ion-acoustic wave spectrum.
  - Based on Nyquist dissipation theory
  - Radar picks one component of the thermal spectrum
  - Spectrum represents Landau-damped, Doppler-shifted scatter from this component.

ISR spectrum: Nyquist approach

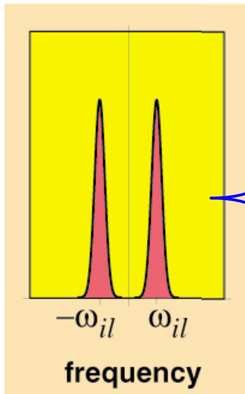
- Send out a wave at ω_o which corresponds to $k=\omega_o/c$.
- The back-scattered signal is Doppler shifted by the forward and backward ion acoustic wave matching that *k*-vector (or wavelength).
- The backscattered signal has peaks at $\omega_o + kC_s$ and $\omega_o - kC_s$ where

$$|C_s| = \omega / |\mathbf{k}| = \sqrt{\frac{k_b(T_e + T_i)}{m_e + m_i}}$$

- But not really: the wave modes are Landau damped!

ISR spectrum: Nyquist approach

The natural density fluctuations are ion acoustic modes.
The collisionless ion acoustic wave has velocity:



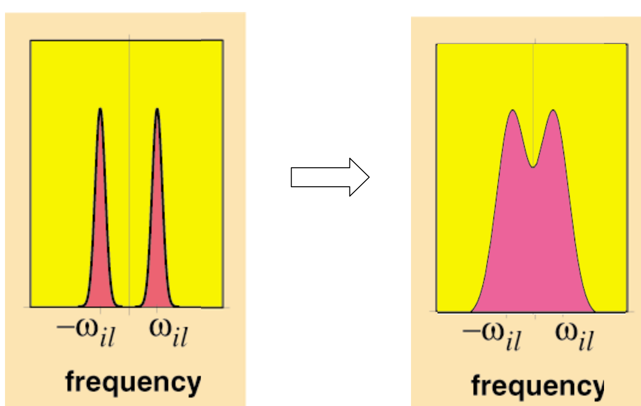
$$|C_s| = \omega / |\mathbf{k}| = \sqrt{\frac{k_b(T_e + T_i)}{m_e + m_i}}$$

In the absence of wave damping, the backscattered power spectrum would look like this.

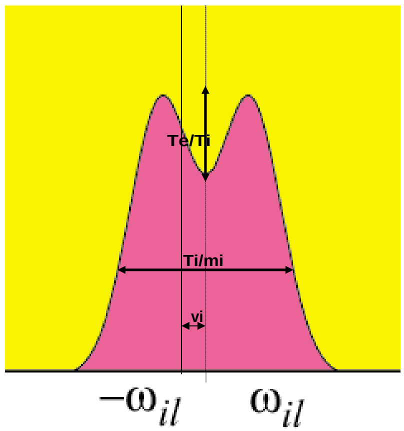
$$|\omega_{il}| = |kC_s| = \frac{4\pi |C_s|}{\lambda_{\text{radar}}}$$

ISR Power Spectrum

Landau damping affects the ion line because the ion thermal velocities are similar to the ion acoustic speed. The degree of damping is strongly affected by the ratio of electron and ion temperatures.



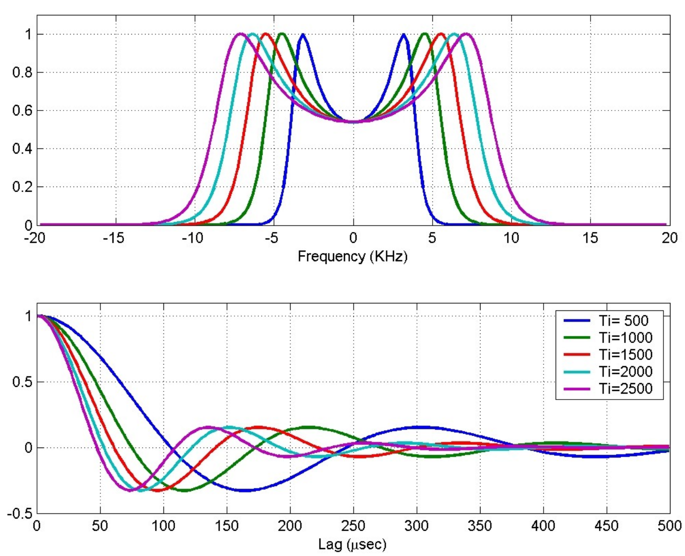
Primary information in the spectrum



- •Ion temperature (Ti) to ion mass (mi) ratio from the width of the spectra
- •Electron to ion temperature ratio (Te/Ti) from "peak_to_valley" ratio
- •Electron (= ion) density from total area (corrected for temperatures)
- •Ion velocity (vi) from the Doppler shift

Courtesy C. Heinselman

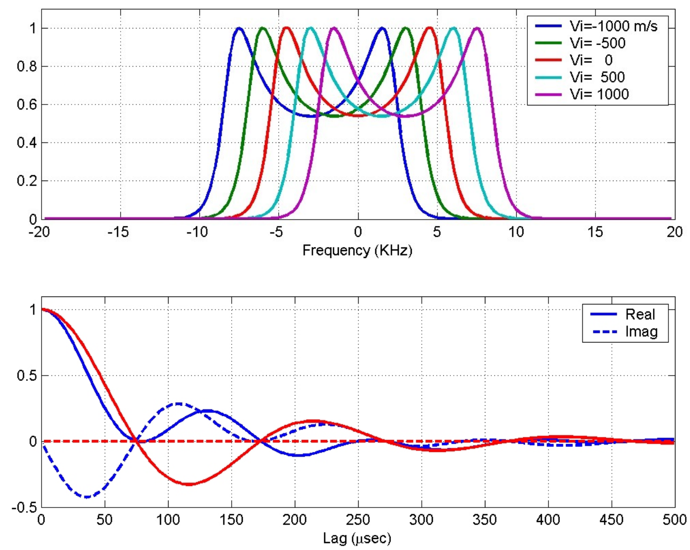
Ion Temperature



Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Te: 2*Ti
Comp: 100% O^+
n_{in}: 10^{-6} KHz

Courtesy C. Heinselman

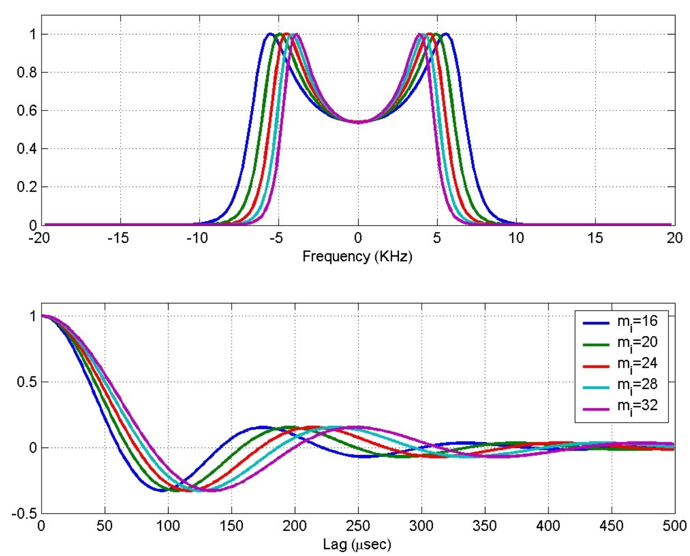
Ion Velocity



Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1000 K
Te: 2000 K
Comp: 100% O^+
n_{in}: 10^{-6} KHz

Courtesy C. Heinselman

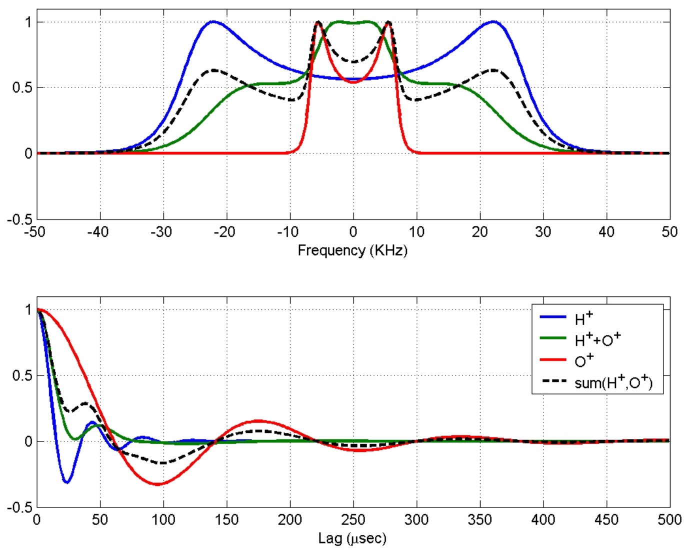
Ion Mass



Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1500 K
Te: 3000 K
ν_{in}: 10^{-6} KHz

Courtesy C. Heinselman

Ion Composition (O^+ vs. H^+)

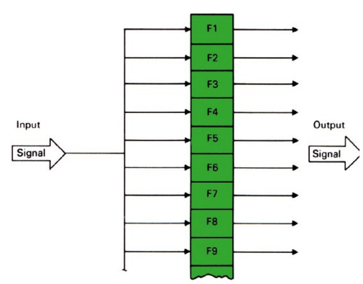


Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1500 K
Te: 3000 K
n_{in}: 10^{-6} KHz

Courtesy C. Heinselman

How do we measure this spectrum?

Could use bank of filters,
but not easy to make high-Q filters



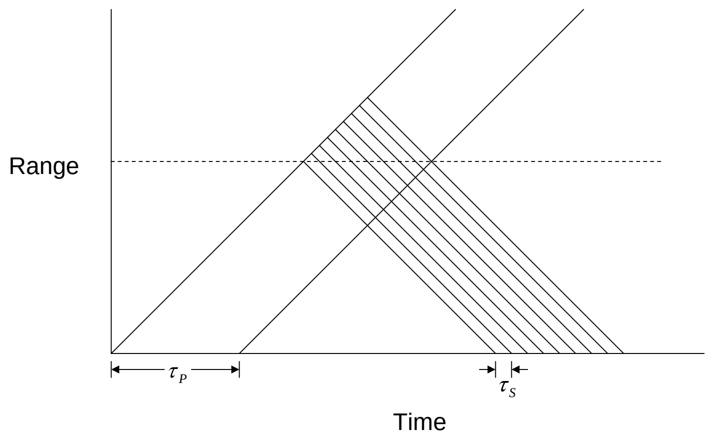
Power spectrum and autocorrelation function are Fourier transform pairs and, so compute autocorrelation

$$R(\tau) = \int v(t) v^*(t-\tau) dt$$

$$R[j] = \sum_n x_n x_{n-j}^*$$

In other words, just complex multiplications of the return voltage samples

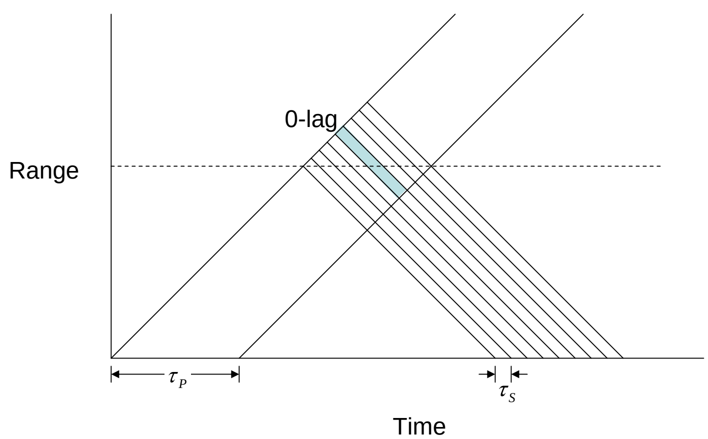
Range-time diagram



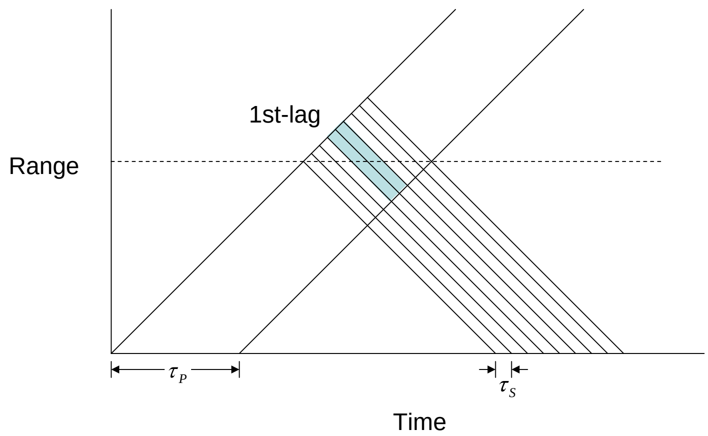
τ_P = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

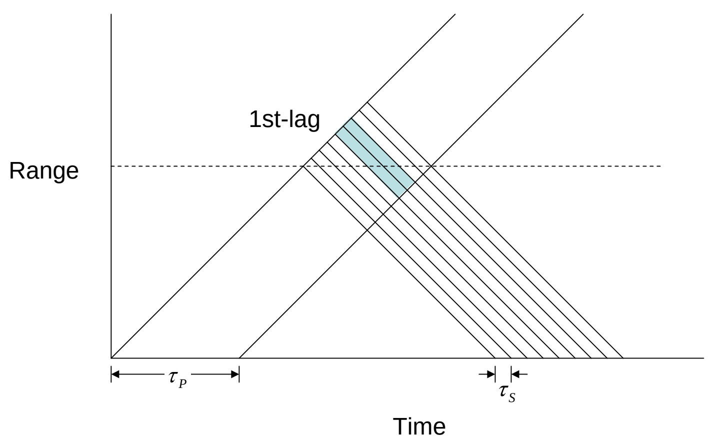
Range-time diagram



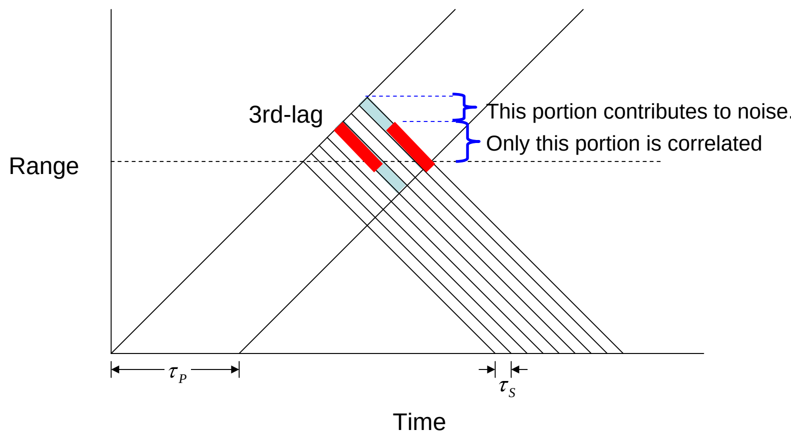
Range-time diagram



Range-time diagram



Range-time diagram



Deriving ionospheric state parameters from spectrum

Ne, Te, Ti, Vi are derived by nonlinear fitting to the theoretical model

$$\sigma(\omega) = \frac{\left| 1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_1 \left(\frac{1}{D_i}\right)^2 F_i(\omega) \right|^2 \overline{\left|N_e^0(\omega)\right|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 \left|F_e(\omega)\right|^2 \sum_i \overline{\left|N_i^0(\omega)\right|^2}}{\left| 1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{ \left(\frac{1}{D_e}\right)^2 \cdot F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega) \right\} \right|^2}$$

where:

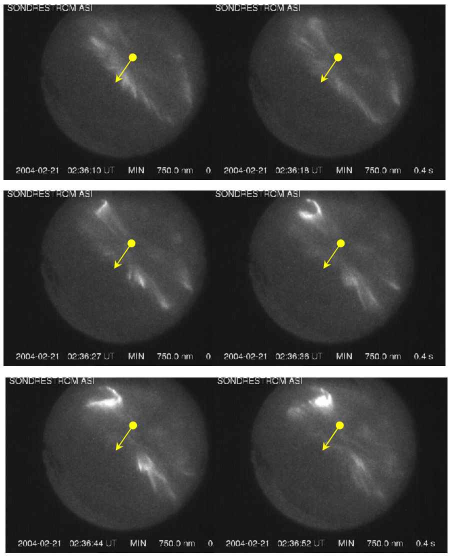
$$F_e(\omega) = 1 - \omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)\,d\tau$$

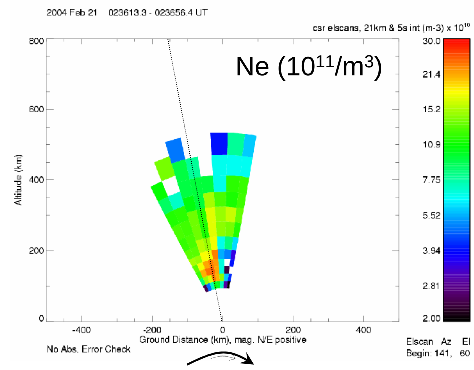
$$F_i(\omega) = 1 - \omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\cos(\omega\tau)\,d\tau$$

Evans, 1969 IEEE review

Plasma parameters at auroral arc boundaries

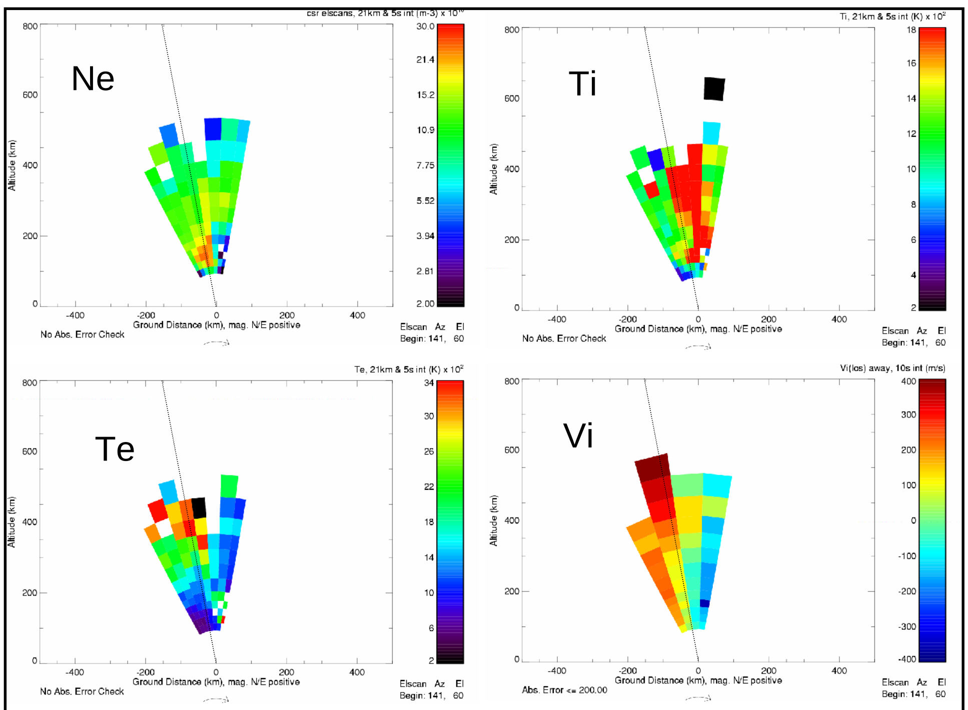
Allsky images, 750 nm





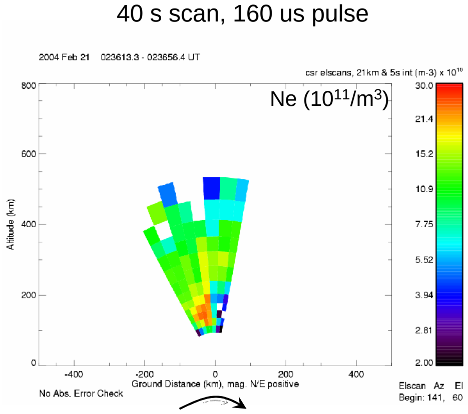
40 s scan, 48 km pulse

{ Allsky camera images, 750 nm (N2 1PG), taken during the radar scan.

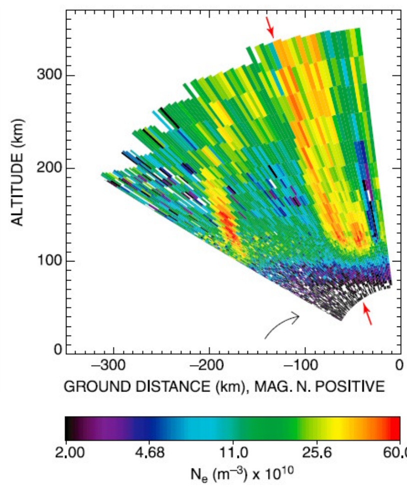


Single Pulse vs. Barker Code

40 s scan, 160 us pulse

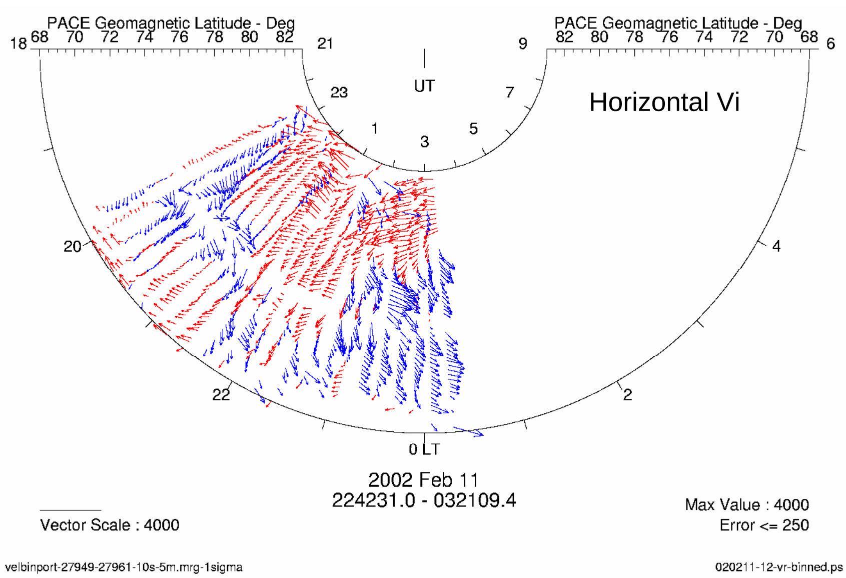


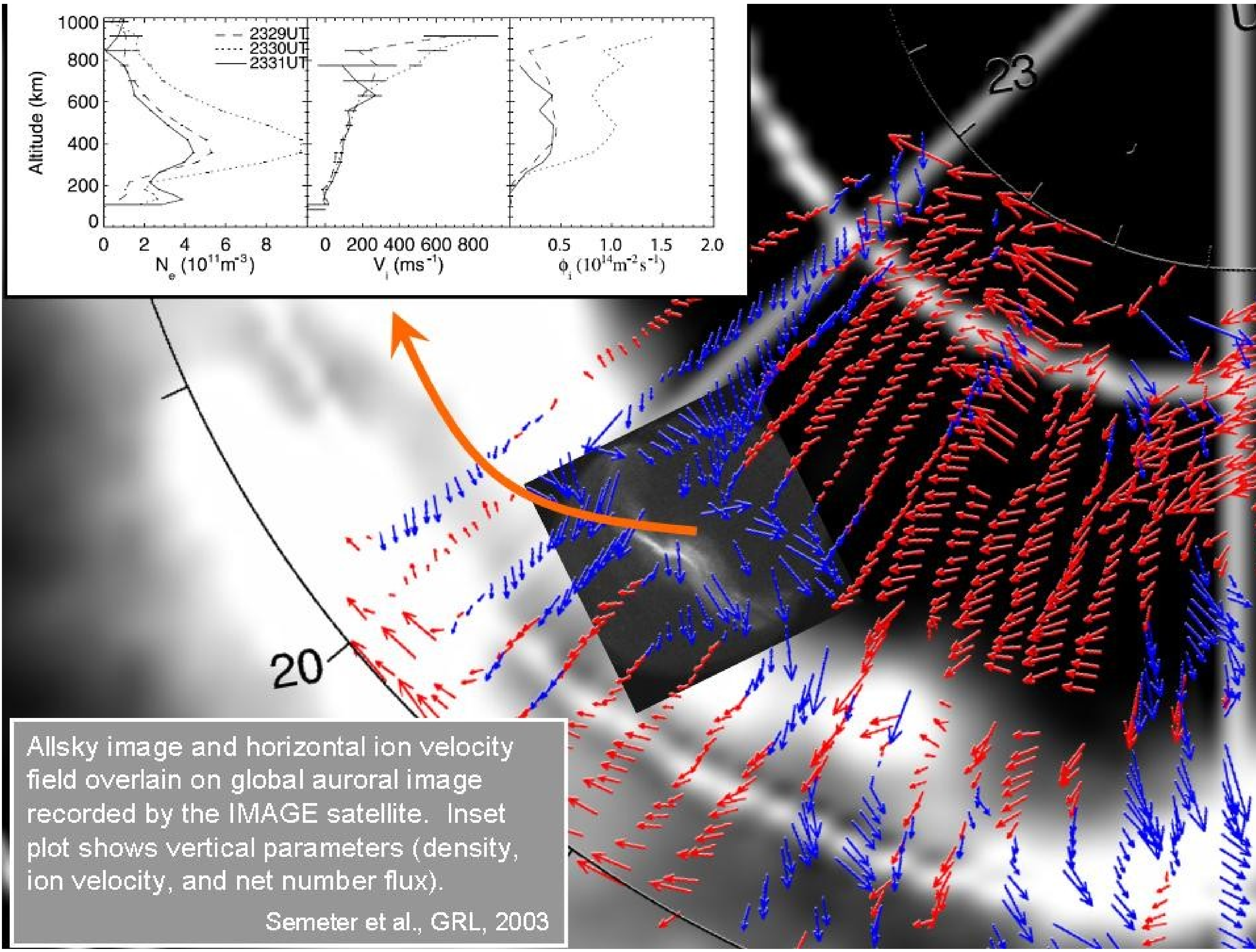
2-min scan, 5-baud Barker code



Semeter et al., JGR 2005

3-dimensional ion motion





1000
800
600
400
200
0
Altitude (km)
2329UT
2330UT
2331UT
0 2 4 6 8
N_e ($10^{11}m^{-3}$)
0 200 400 600 800
V_i (ms^{-1})
0.5 1.0 1.5 2.0
ϕ_i ($10^{14}m^{-2}s^{-1}$)
23
20
Allsky image and horizontal ion velocity field overlain on global auroral image recorded by the IMAGE satellite. Inset plot shows vertical parameters (density, ion velocity, and net number flux).
Semeter et al., GRL, 2003