The Ionosphere in Motion: Winds, Waves, and Electrodynamics

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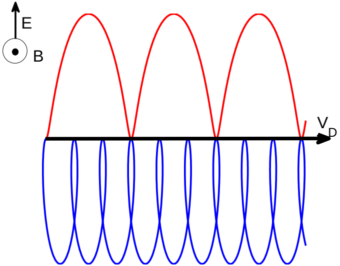
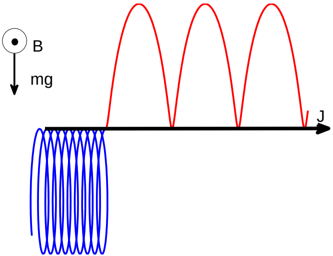
Plasma Drifts





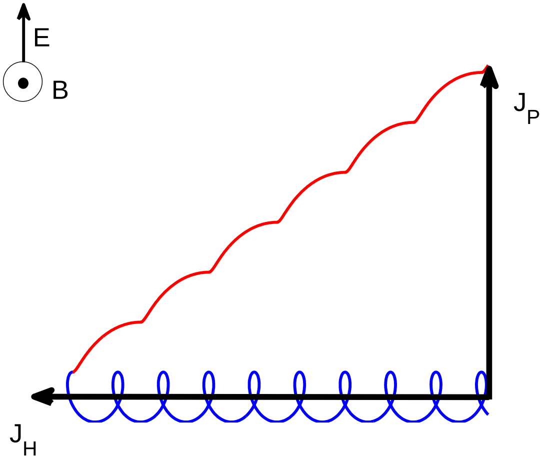
$$\mathbf{V}_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

$$\mathbf{V}_{Di} = \frac{m_i \mathbf{g} \times \mathbf{B}}{q_i B^2}$$

$$\mathbf{J} = N_i \frac{m_i \mathbf{g} \times \mathbf{B}}{B^2}$$

The Effects of Collisions

Suppose $\nu_e \ll \Omega_e$, $\nu_i \sim \Omega_i$



The Conductivity Tensor

$$\mathbf{J} = \begin{pmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix} \cdot \mathbf{E}$$

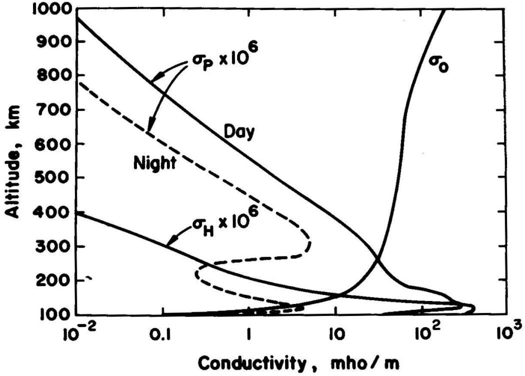
$$\mu_j = \frac{q_j}{m_j \nu_{jn}}$$

$$\kappa_j = \frac{\Omega_j}{\nu_{jn}} = \frac{q_j B}{m_j \nu_{jn}}$$

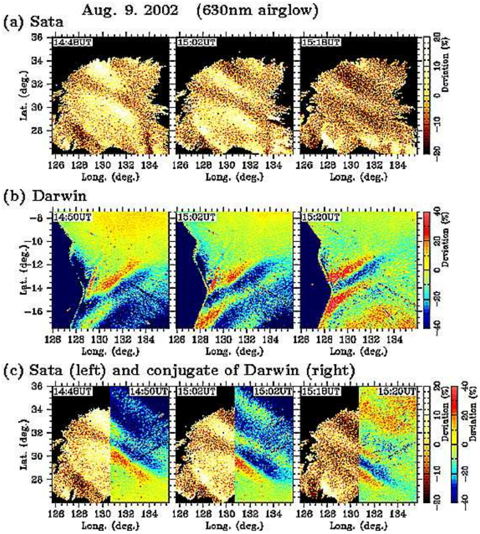
$$\sigma_0 = \sum_j q_j N_j \mu_j$$

$$\sigma_P = \sum_j q_j N_j \frac{\mu_j}{1+\kappa_j^2}$$

$$\sigma_H = -\frac{1}{B} \sum_j q_j N_j \frac{\kappa_j^2}{1+\kappa_j^2}$$



Electric Field Mapping



[Otsuka et al. (2004). *Geophys. Res. Lett.* **31**, L15803]

Neutral Winds and Frame Transformations

Ionospheric electrodynamics problems are usually done in a reference frame moving with the neutral wind, **U**.
Using special relativity:

$$\mathbf{E}' = \frac{\mathbf{E} + \mathbf{U} \times \mathbf{B}}{\sqrt{1 + U^2/c^2}} \approx \mathbf{E} + \mathbf{U} \times \mathbf{B}$$

$$\mathbf{B}' = \frac{\mathbf{B} - \frac{1}{c^2}(\mathbf{U} \times \mathbf{E})}{\sqrt{1 + U^2/c^2}} \approx \mathbf{B}$$

Fundamental Equations of the Neutral Atmosphere

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0$$ Conservation of Mass

$$\rho \frac{\partial \mathbf{U}}{\partial t} = \rho \mathbf{g} - \nabla p$$ Conservation of Momentum

$$\frac{\partial p}{\partial t} + \mathbf{U} \cdot \nabla p_0 = c_s^2 \left[\frac{\partial \rho}{\partial t} + \mathbf{U} \cdot \nabla \rho_0 \right]$$ Energy Balance

$$c_s^2 = \frac{\gamma p_0}{\rho_0}$$ Speed of Sound Squared

Equilibrium Solutions

$$0 = \rho_0 \mathbf{g} - \nabla p_0$$

$$p_0 \propto \rho_0 \propto \exp\left(-\frac{z}{H}\right)$$

$$H = \frac{c_s^2}{\gamma g} = \frac{k_B T}{mg}$$ Scale Height

Linear Perturbations

Assume

$$\mathbf{U} = 0 + u\hat{\mathbf{x}} + w\hat{\mathbf{z}}$$
$$u \propto w \propto \frac{\delta p}{p_0} \propto \frac{\delta\rho}{\rho_0} \propto \exp\left[i(K_x x + K_z z - \omega t)\right]$$

$$\begin{pmatrix} -i\omega & 0 & iK_x & -\frac{1}{H} + iK_z \\ 0 & iK_x\frac{c_s^2}{\gamma} & -i\omega & 0 \\ g & -\frac{c_s^2}{\gamma}\left(\frac{1}{H} - iK_z\right) & 0 & -i\omega \\ i\omega c_s^2 & -i\omega\frac{c_s^2}{\gamma} & 0 & (\gamma-1)g \end{pmatrix} \begin{pmatrix} \delta\rho/\rho_0 \\ \delta p/p_0 \\ u \\ w \end{pmatrix} = 0$$

$$\omega^4 - \omega^2 c_s^2(K_x^2 + K_z^2) + (\gamma - 1)g^2 K_x^2 - i\gamma g\omega^2 K_z = 0$$

The Gravity Wave Dispersion Relation

Define $K_x = k_x$, $K_z = k_z - \frac{i}{2H}$

$$u \propto \exp\left(+\frac{z}{2H}\right) \exp\left[i(k_x x + k_z z - \omega t)\right]$$

$$\frac{(\omega^2 - \omega_a^2)\omega^2}{c_s^2} - \omega^2(k_x^2 + k_z^2) + N^2 k_x^2 = 0$$

$$\omega_a \equiv \frac{\gamma g}{2c_s}$$

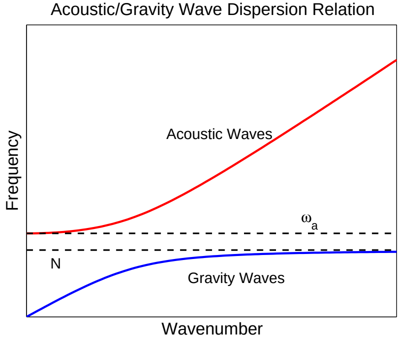
$$N \equiv \frac{\sqrt{\gamma - 1}g}{c_s}$$

$\omega \gg \omega_a$

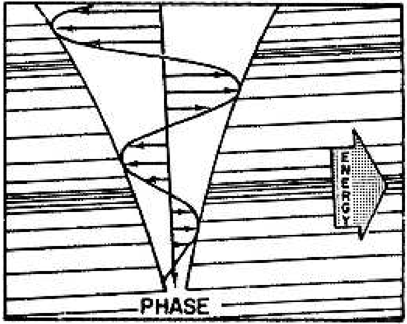
$$\omega^2 = c_s^2(k_x^2 + k_z^2)$$

$\omega \ll N$

$$\omega^2 = \frac{N^2 k_x^2}{k_x^2 + k_z^2 + (1/2H)^2}$$



Group and Phase Velocities



The vertical phase and group velocities have opposite signs!

$$\frac{\omega}{k_z} = \frac{Nk_x}{k_z\sqrt{k_x^2 + k_z^2 + (1/2H)^2}}$$

$$\frac{d\omega}{dk_z} = -\frac{Nk_x k_z}{\left(k_x^2 + k_z^2 + (1/2H)^2\right)^{3/2}}$$

Tides

Global scale waves called tides are driven by the 24 h heating cycle and the gravity of the moon.

Analysis of extremely large scale waves must account for

- Additional gravitational perturbations (e.g. the motion of the Moon)
- The rotation of the Earth (Coriolis force)

The new linearized momentum equation becomes

$$\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} + \nabla p_1 - \rho_1 \mathbf{g} + \rho_0 \nabla \Psi_1 + 2\rho_0 \mathbf{\Omega}_r \times \mathbf{u}_1 = 0$$

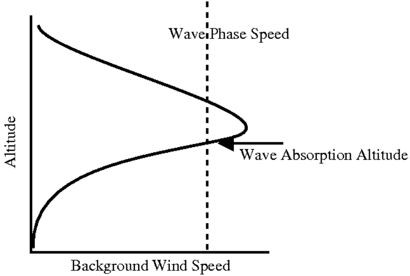
Furthermore, for a spherical Earth

- The Earth's circumference is an integer number of wavelengths
- Periods of 24 h, 12 h, 8 h, etc.

Gravity Wave Filtering

If the background winds and temperatures (N) vary slowly with altitude, then so will k_x and k_z.

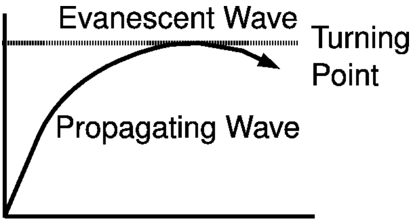
Wave Absorption



$$\omega_I = \omega - \mathbf{k} \cdot \mathbf{U}_0$$

$$\text{If } \frac{\omega}{k} = \mathbf{U}_0 \Rightarrow \omega_I \to 0 \text{ and } k_z \to \infty$$

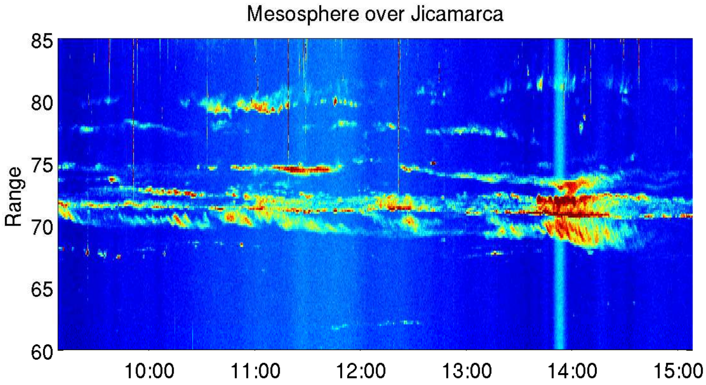
Total Internal Reflection



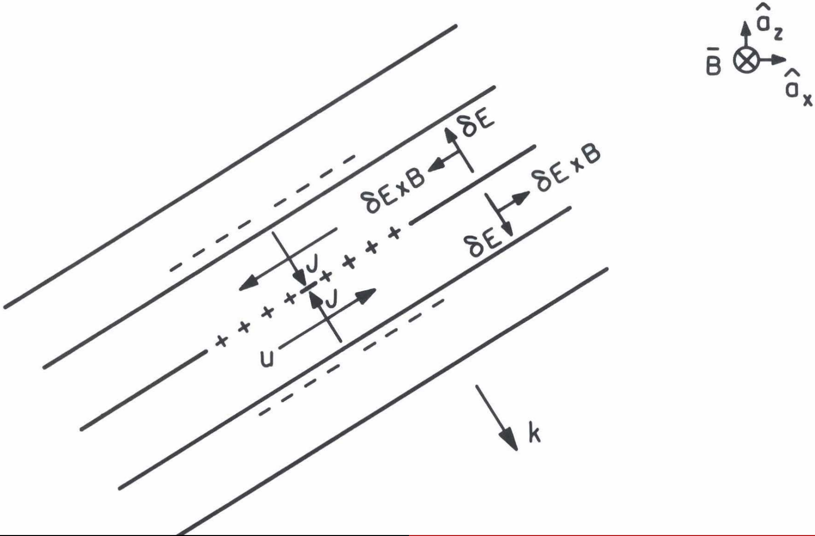
$$k_z \to 0$$

Gravity Wave Breaking and Mesospheric Turbulence

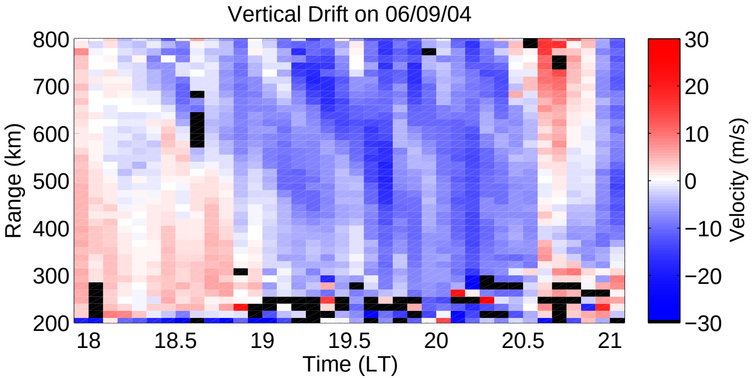
Waves are unstable when $u > \omega/k$.



Electrified Gravity Waves at High Altitudes

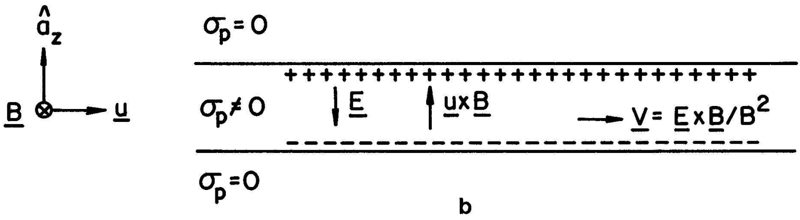


Electrified Gravity Waves at High Altitudes



[Varney et al. (2009) *J. Geophys. Res.* **114**, A02304.]

Slab Model of the F-region Dynamo



In a frame moving with the wind:

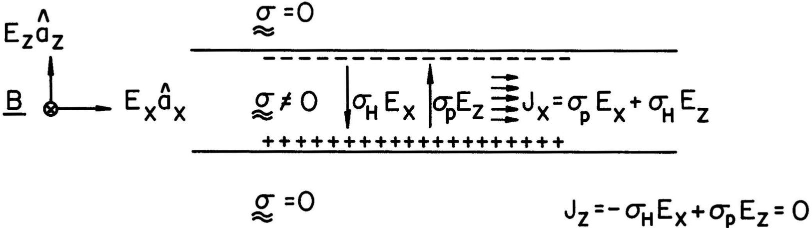
$$\mathbf{E}' = \mathbf{E} + \mathbf{U} \times \mathbf{B} \Longrightarrow \mathbf{J} = \sigma_P \left(\mathbf{E} + \mathbf{U} \times \mathbf{B} \right)$$

If there are no parallel currents $\mathbf{J} = 0$, thus $\mathbf{E} = -\mathbf{U} \times \mathbf{B}$
In the Earth frame:

$$\mathbf{V}_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2} = \frac{-\mathbf{U} \times \mathbf{B} \times \mathbf{B}}{B^2} = \mathbf{U}$$

Slab Model of the E-region Dynamo

Consider applying an eastward electric field, E_x, to the E-region.



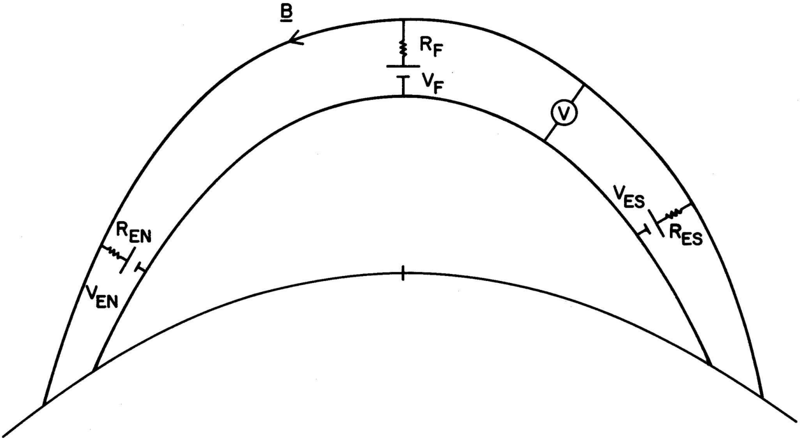
A vertical electric field forms to oppose the vertical Hall current.

$$\sigma_H E_x = \sigma_P E_z \Longrightarrow E_z = \frac{\sigma_H}{\sigma_P} E_x$$

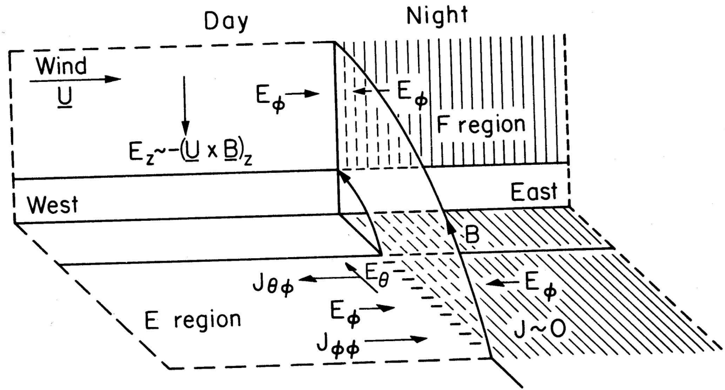
The Hall current from this new E_z adds to the existing Pedersen current from E_x

$$\mathbf{J}_x = \sigma_H E_z + \sigma_P E_x = \left[(\sigma_H/\sigma_P)^2 + 1 \right] \sigma_P E_x \equiv \sigma_C E_x$$

Coupling of the Dynamos

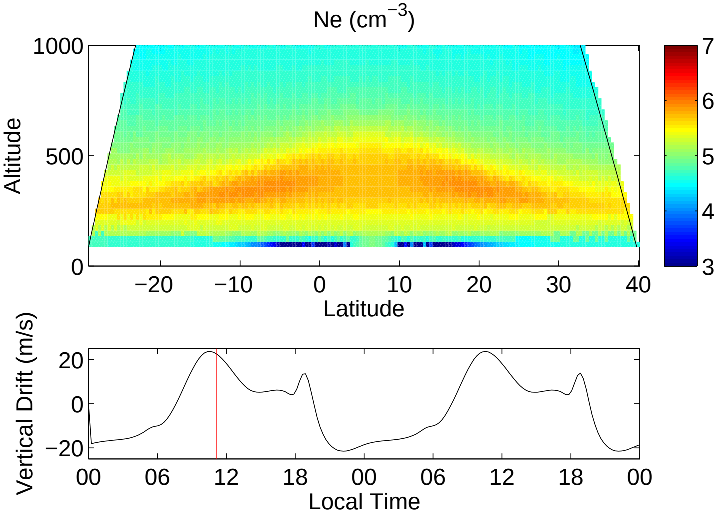


The Prereversal Enhancement



[Farley et al. (1986). *J. Geophys. Res.* **91**, 3,723.]

Effects of Electric Fields on Ionospheric Morphology

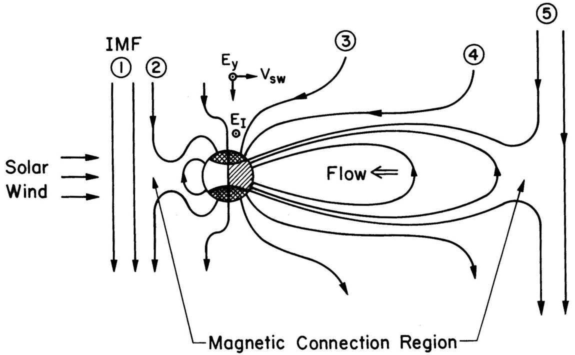


Electric Fields Produced by the Solar Wind

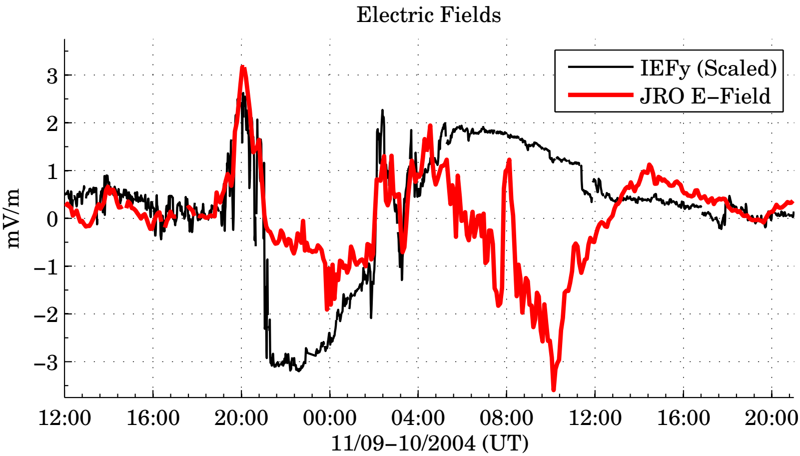
The highly conductive solar wind has no Electric Field.
In the reference frame of the stationary Earth:

$$\mathbf{E} = -\mathbf{V}_{sw} \times \mathbf{B}$$

$\sim$ 10 % of this field is not shielded out by the magnetosphere.

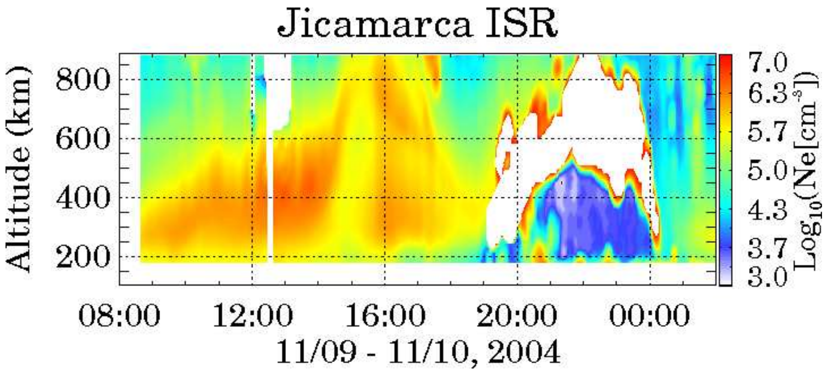


The November 2004 Superstorm



[Kelley et al. (2010). *J. Atmos. Solar-Terr. Phys.* **72**, 285-291.]

A Double Ionosphere



[Retterer et al. (2010). *J. Atmos. Solar-Terr. Phys.* **72**, 350-357.]

Acknowledgements

Thanks to Michael Kelley and Ronald Ilma for several plots.

The Sami2 ionosphere model was written and developed by J. D. Huba, G. Joyce, and M. Swisdak of the Naval Research Laboratory.

Further Reading

Plasma drifts and the conductivity tensor:

- Chapter 2 of Kelley, M. C., 2009. *The Earth's Ionosphere: Plasma Physics and Electrodynamics*. 2$^{\mathrm{nd}}$ ed. Academic Press.

Gravity Waves:
The classic original paper:

- Hines, C. O., 1960. Internal atmospheric gravity waves at ionospheric heights. Can. J. Phys. 38, 1441–1481.

A modern review paper:

- Fritts, D. C., Alexander, M. J., 2003. Gravity wave dynamics and effects in the middle atmosphere. Rev. Geophys. 41, 1003–1066.

Further Reading

Gravity Waves Cont.:
Brief textbook descriptions:

- Section 10.5 of Schunk, R. W., and A. F. Nagy, 2009. *Ionospheres: Physics, Plasma Physics, and Chemistry*. 2^{nd} ed. Cambridge University Press.
- Sections 6.2, 7.2 and 7.3 of Kelley (2009).

A more extensive text:

- Gossard, E., and W. Hooke, 1975. *Waves in the Atmosphere*, Elsevier Scientific Publishing Company.

Further Reading

Tidal Theory:
A brief description:

- Section 10.6 of Schunk and Nagy (2009).

The classic book on the subject:

- Chapman, S. C., and R. S. Lindzen, 1970. *Atmospheric Tides*, D. Reidel.

Dynamo Theory and Low Latitude Electrodynamics:

- Chapter 3 of Kelley (2009).