Incoherent Scattering
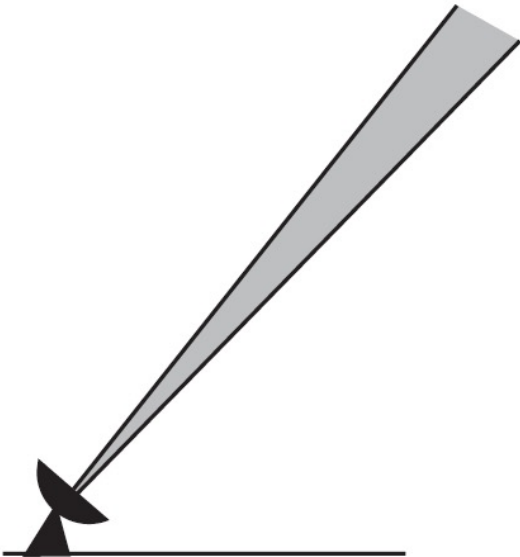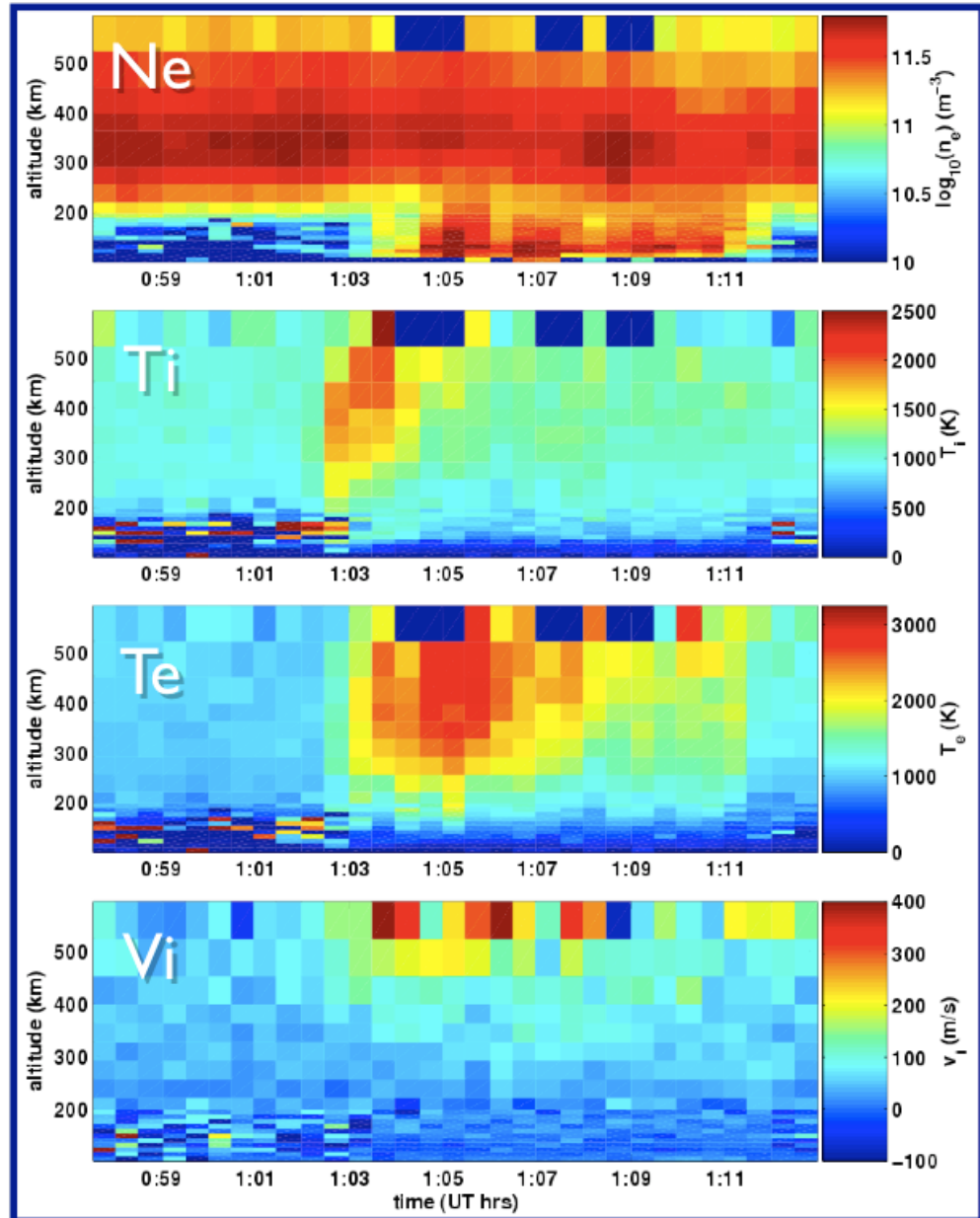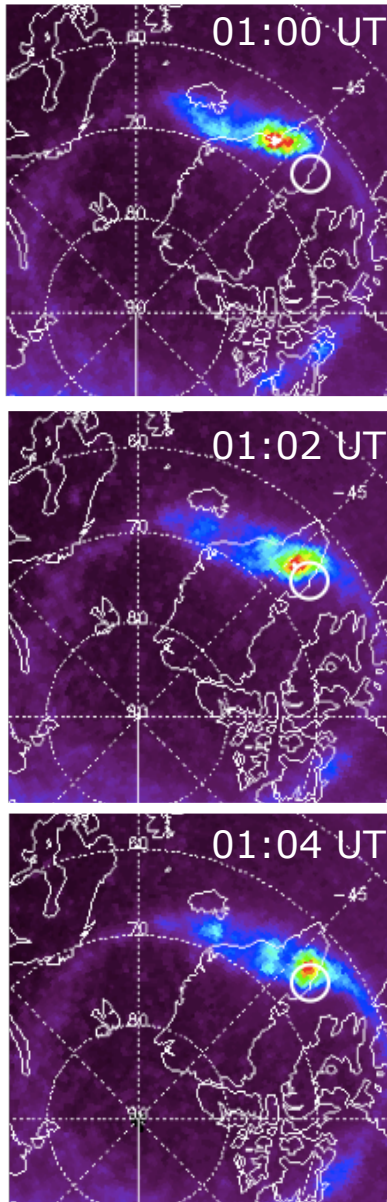
Hassan Akbari

Boston University

Outline

- Thomson Scattering.

- Scattering from a collection of electrons.
  - Coherent Scattering.
  - Incoherent Scattering.

- Plasma waves approach to study the received signal.
  - Shape of the ion line.

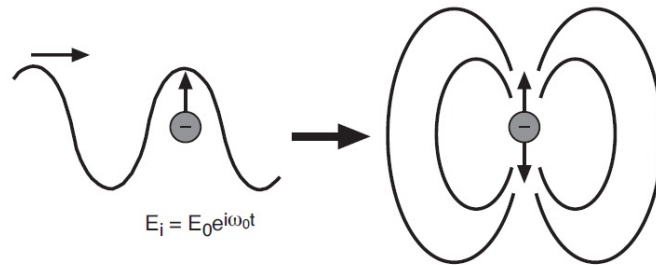- How to measure the spectrum/autocorrelation function with radars.

- Fitting process.

Ionospheric response to the aurora

# Electronically steerable ISR

Thomson Scattering.

$E_i = E_0 e^{i\omega_0 t}$
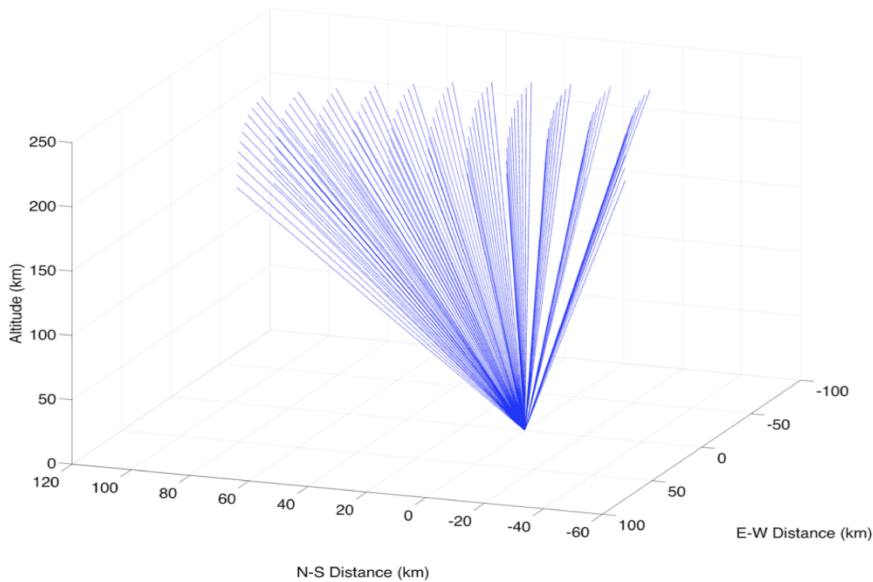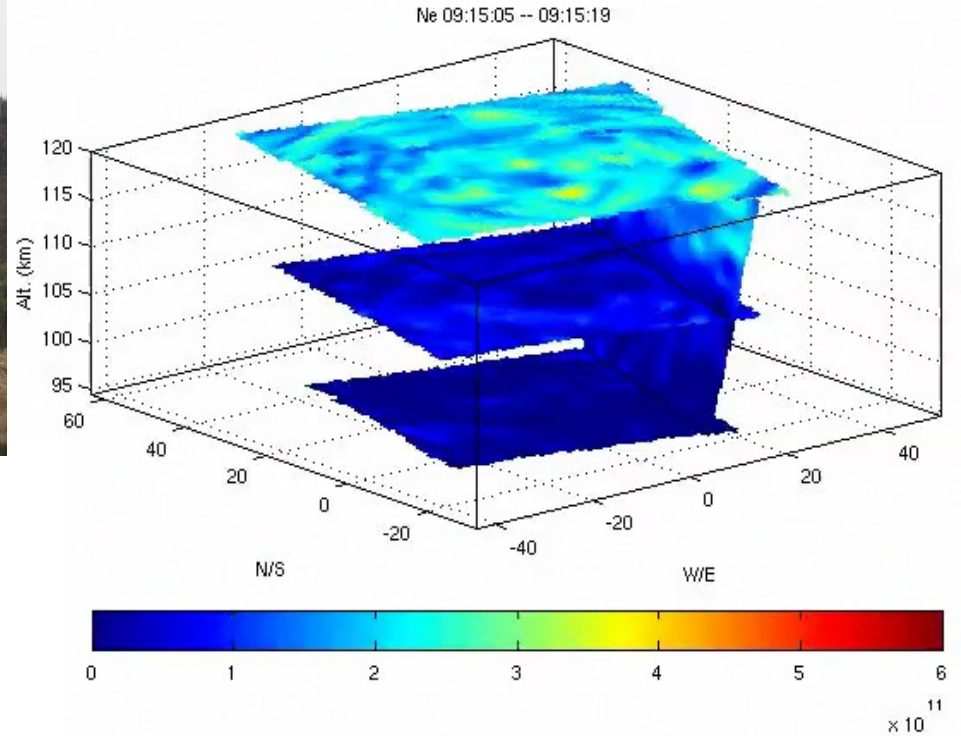
- Thomson scattering is a process by which the energy of an electromagnetic wave is partly scattered in various directions by a free electron.

- Incident EM wave accelerates each particle it encounters, particles then re-radiate an EM wave.

- Because of the large mass of the ions their scattering is negligible.

Thomson Scattering.


Doppler Shift

Unseen planet
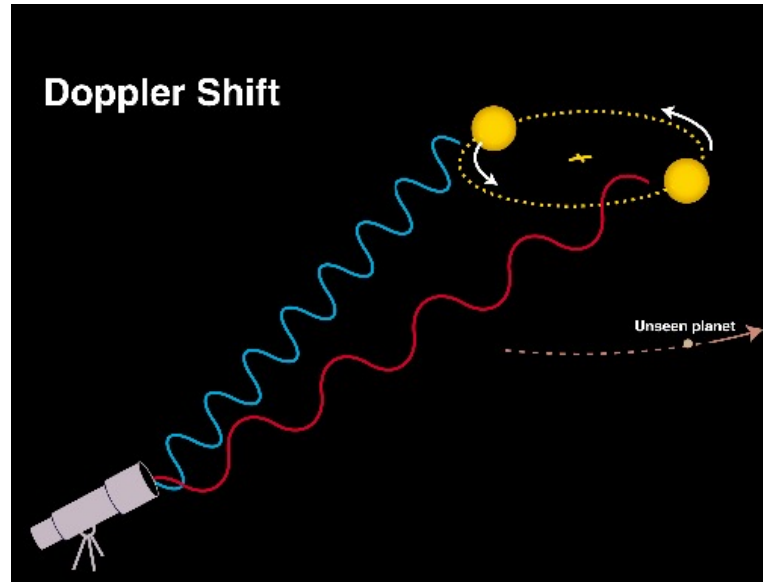
- If the electron is moving then we have Doppler effect.
  - Frequency of the scattered wave is determined by the LOS velocity of the electron.

- In the ionosphere we have lots of electrons.
  - The scattered signal is not monochromatic.
  - Spectrum of the received signal looks like the velocity distribution of the electrons.

Scattering from a collection of electrons.
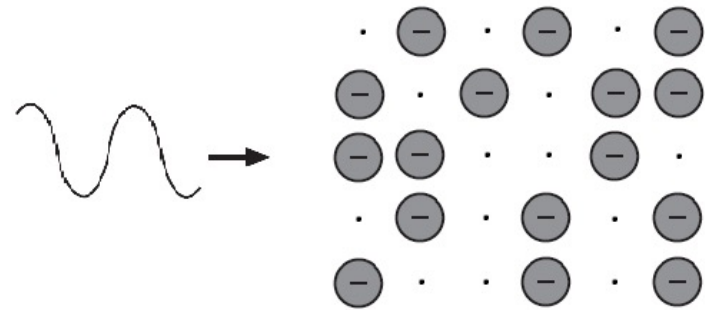
Imagine three cases:

1) Homogeneous medium.

No irregularities, no scattering.
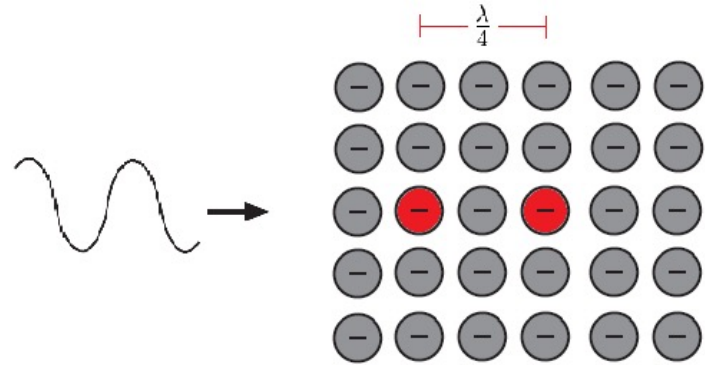
2) Randomly distributed electrons.
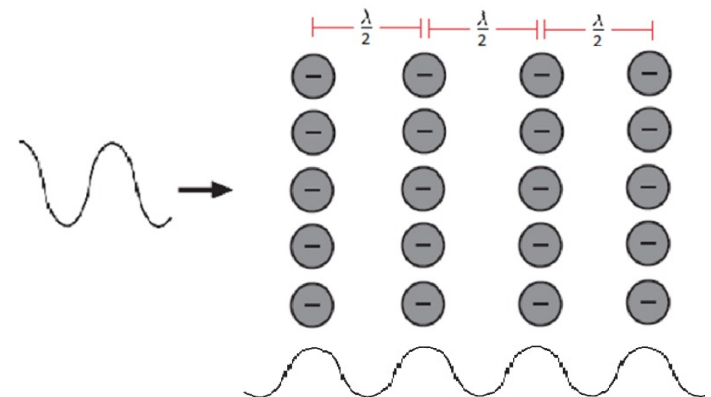
Destructive and constructive scattering.
Incoherent scattering.

3) Non randomly distributed electrons.

Bragg scattering.
Coherent scattering.

Scattering from a collection of electrons.

- Scatter from some particular electron at $\mathbf{r}_p$

$$E_{s,p}(t) = \frac{r_e}{r_s} E_0 \sin \chi e^{-i(\omega_0 t + \mathbf{k} \cdot \mathbf{r}_p)}$$

Where $k = \dfrac{4\pi}{\lambda}$ in backscatter case.

- By knowing the scattering from a single electron, now all we have to do is add up the scattered fields from many electrons within the scattering volume, keeping careful track of the phase term.
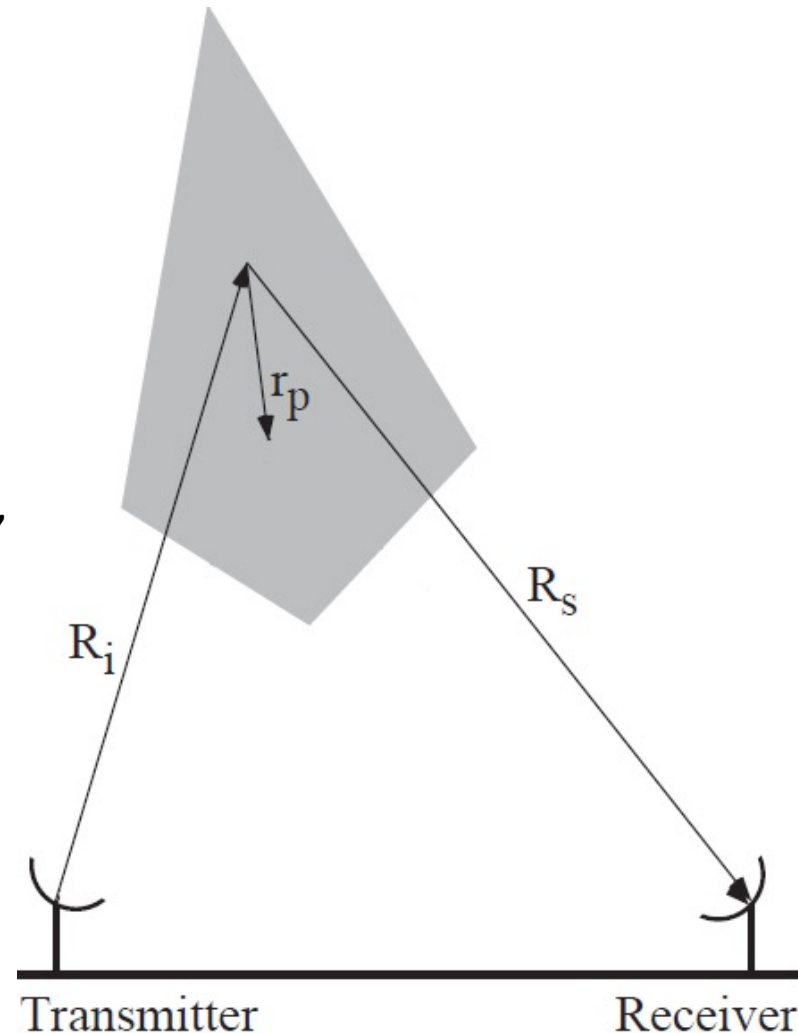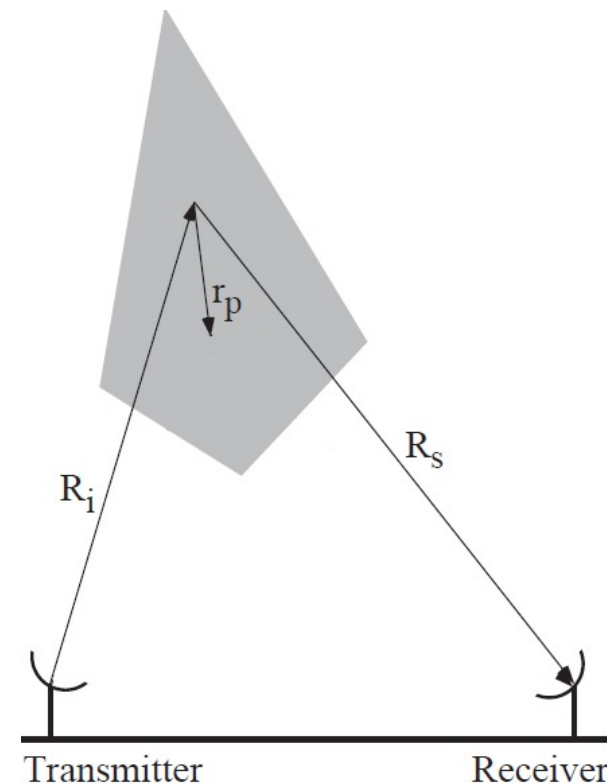
Scattering from a collection of electrons.

Now sum over the N electrons to get the total scattered signal.

$$E_{s,p}(t) = \frac{r_e}{r_s} E_0 \sin \chi e^{-i(\omega_0 t + \mathbf{k} \cdot \mathbf{r}_p)}$$

$$E_s(t) = \frac{r_e E_0}{R_s} e^{-i\omega_0 t} \sin \chi \sum_{p=1}^{N_s} e^{-i\mathbf{k} \cdot \mathbf{r}_p(t')}$$

$$= \frac{r_e E_0}{R_s} e^{-i\omega_0 t} \sin \chi \int_V N(\mathbf{r}, t') e^{-i\mathbf{k} \cdot \mathbf{r}} \, d^3\mathbf{r}$$

E is the sum of many small random contributions,

Central Limit Theorem ➡ E is Gaussian random variable.

All the information is in Autocorrelation function.
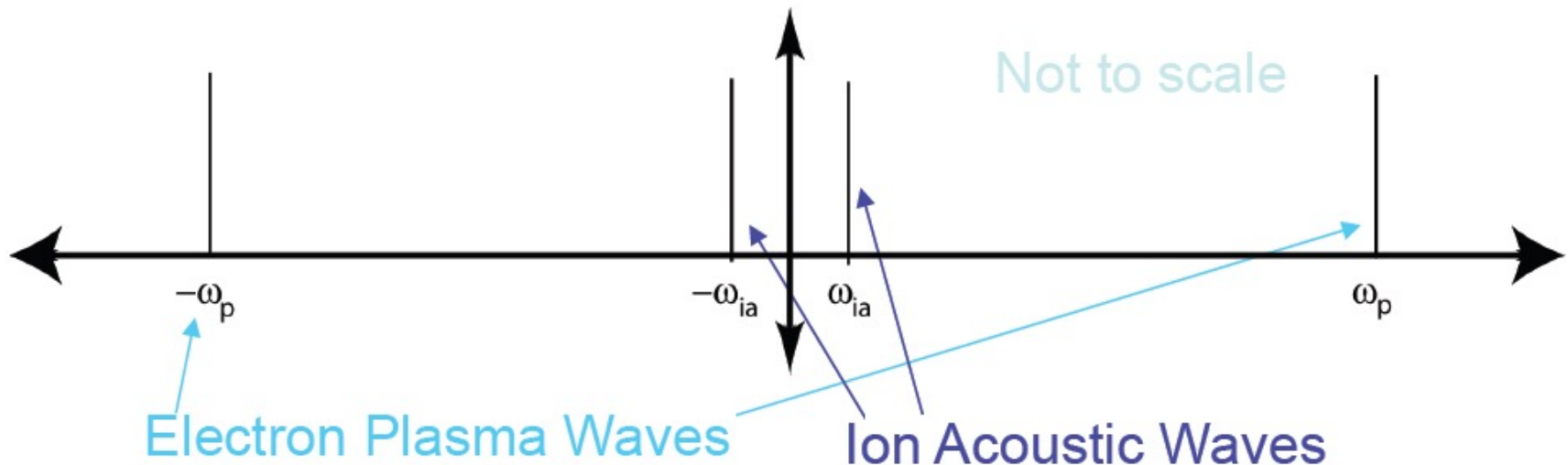
Plasma waves approach.

- We calculated the scattered electric field before:

$$E_s(t) \simeq \frac{r_e E_0}{R_s} e^{-i\omega_0 t} \sin \chi \int_V N(\mathbf{r}, t') e^{-i\mathbf{k}\cdot\mathbf{r}} \, d^3\mathbf{r}$$

- Do you recognize the integration?

- By this integration we rewrite the density fluctuations as the sum of infinite number of waves in every direction.

- The time dependence of the density fluctuations reflects the fact that the electrons are in perpetual thermal motion and waves are moving.

- Each component of the scattered signal corresponds essentially to a single component (known as the Bragg component) of the three-dimensional spatial Fourier transform of the electron density fluctuations.

Plasma waves approach.

- The two main wave modes which contribute to density fluctuations are the ion acoustic waves and the Langmuir waves.

- Thus the scattering spectrum should contain four lines, two of them caused by ion acoustic waves and two by Langmuir waves.

Not to scale

$-\omega_p$ $-\omega_{ia}$ ω_{ia} ω_p

Electron Plasma Waves   Ion Acoustic Waves

Plasma waves approach.

- If the plasma waves were undamped, we might observe only discrete frequencies in the radar probing experiment, but we usually see a continuous band.

- Two lines due to ion acoustic waves are actually broadened to merge into a single ion line.
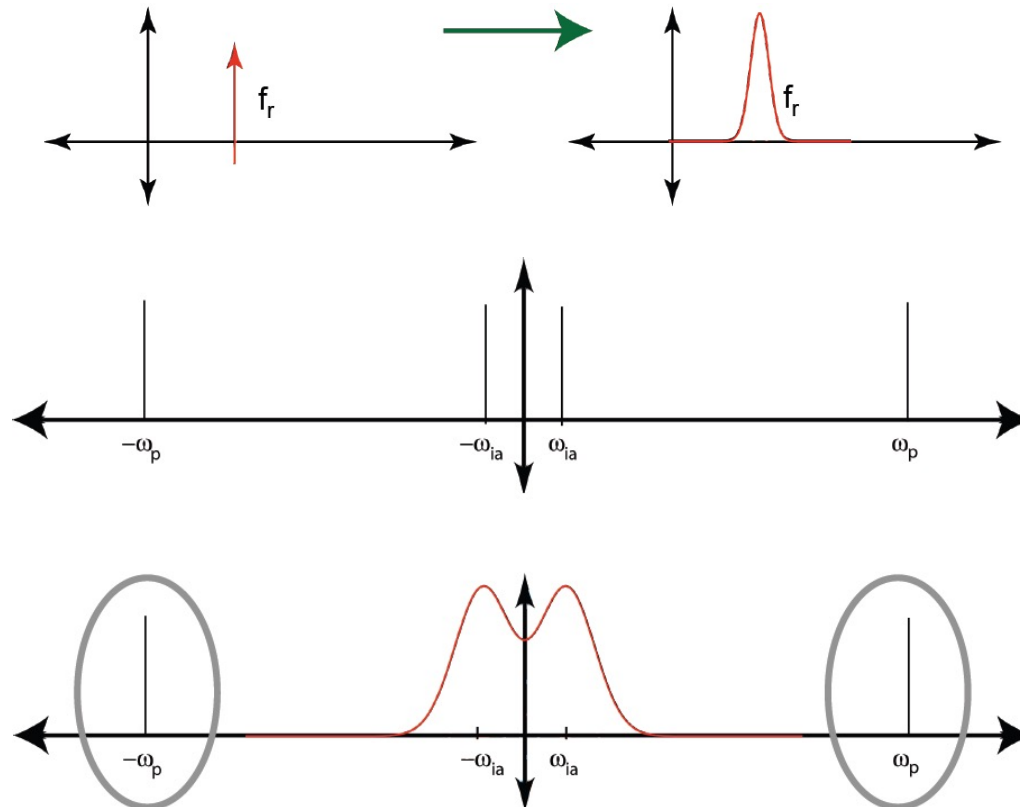
Spectral characteristics.

The Spectrum can be seen to contain two terms:

Electronic component.

Ionic component.

At small wavelengths the scattered energy is entirely due to the electronic component and has a Gaussian spectrum. As wavelength increases the amount of power in the electronic component decreases and appears in a single line at a Doppler shift approximately equal to the plasma frequency.
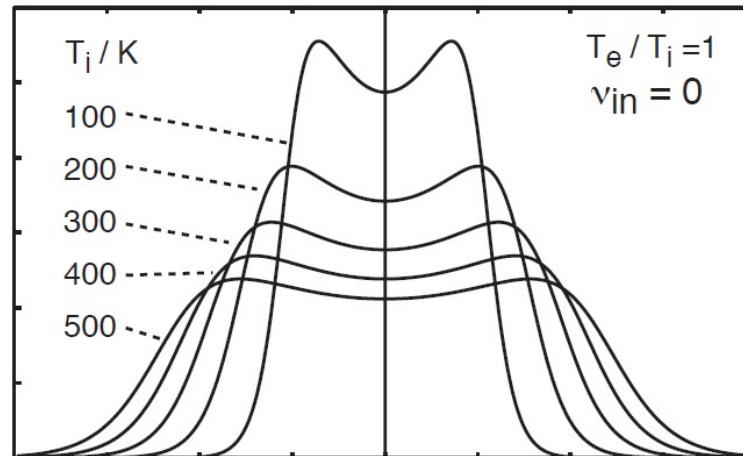

COMPLETE SPECTRA FOR VARIOUS VALUES OF α

$\varkappa (= 4\pi D \lambda)$

POWER $W(\Delta I_o)/N r_e^2$

DOPPLER SHIFT (ΔI_o)
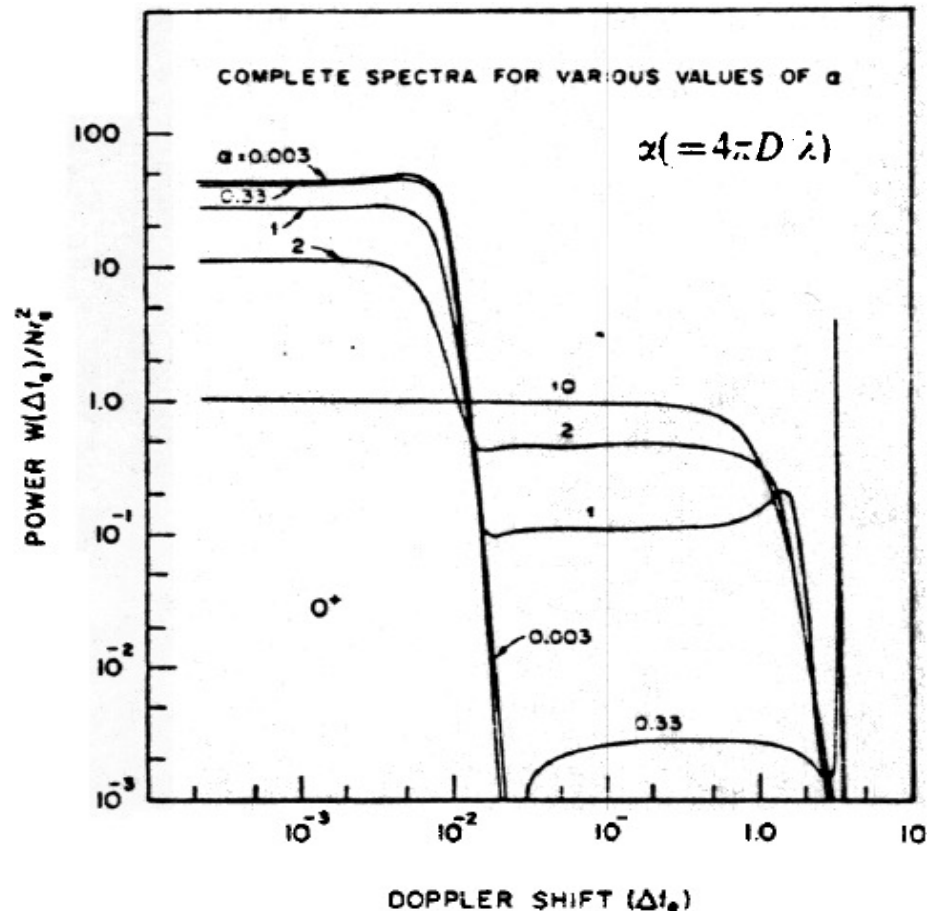
Shape of the ion line.

Let's study the shape of the ion line and how it behaves when we change the plasma parameters.
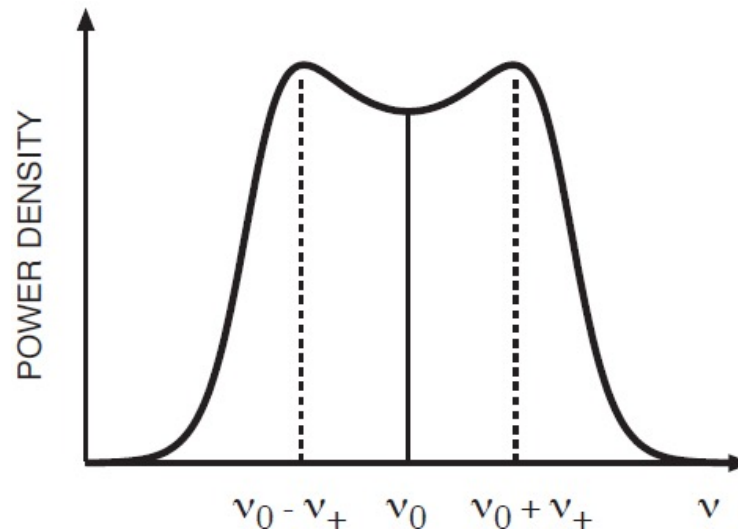
Shape of the ion line, increasing the ion temperature

- Radar frequency: 931.5MHz.
- Ion mass: 30.5 u (mixture of O2+ and NO+)
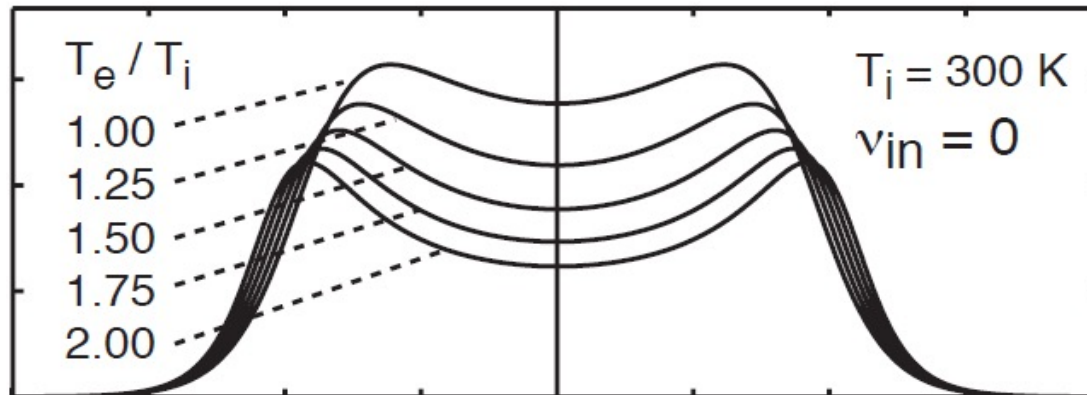- Constant electron to ion temperature ratio.

By increasing the ion temperature:
- Shoulders move outward. (Ion line broadening.)
- Amplitude decreases. (Area under the curve should be constant.)

Shape of the ion line, increasing the electron to ion temperature ratio

- Radar frequency: 931.5MHz
- Ion mass: 30.5 u (mixture of O2+ and NO+)
- Constant ion temperature.

By increasing the temperature ratio:

- Shoulders move outward, edges become steeper, and the minimum becomes deeper due to less damping of ion acoustic waves.

- Amplitude decreases. (Area under the curve should be constant.)

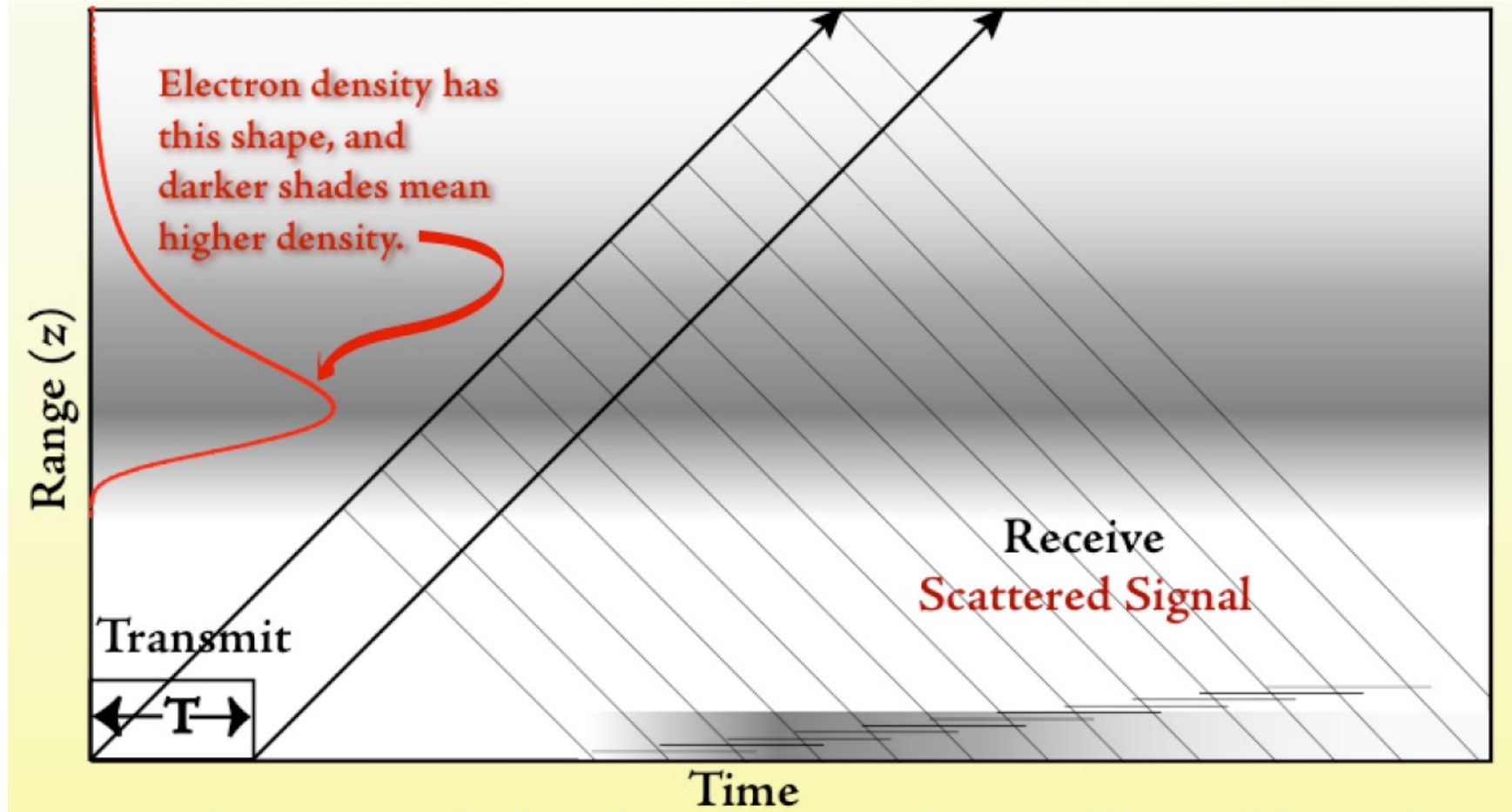# Shape of the ion line, increasing the collision frequency

- Radar frequency: 931.5MHz
- Ion mass: 30.5 u (mixture of O2+ and NO+)
- Constant temperatures.
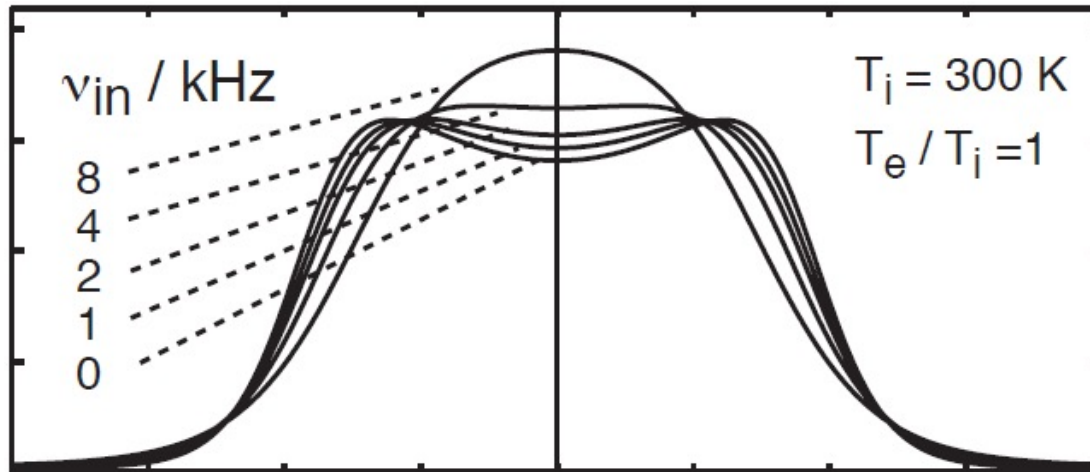
By increasing the collision frequency:
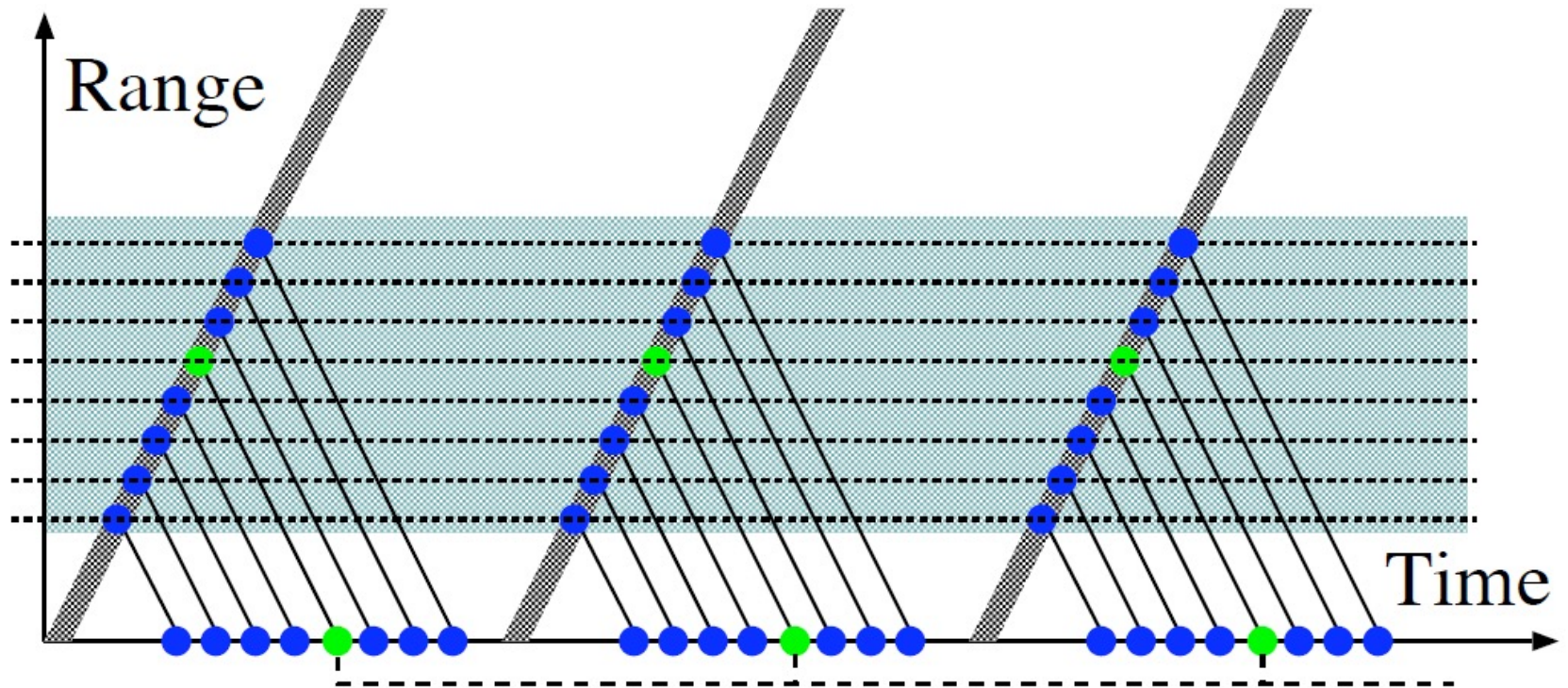- Minimum disappears due to damping of ion acoustic waves via ion-neural collisions.
- Amplitude decreases. (Area under the curve should be constant.)

How can we measure the spectrum?

By Michael Sulzer, Arecibo Observatory/NAIC
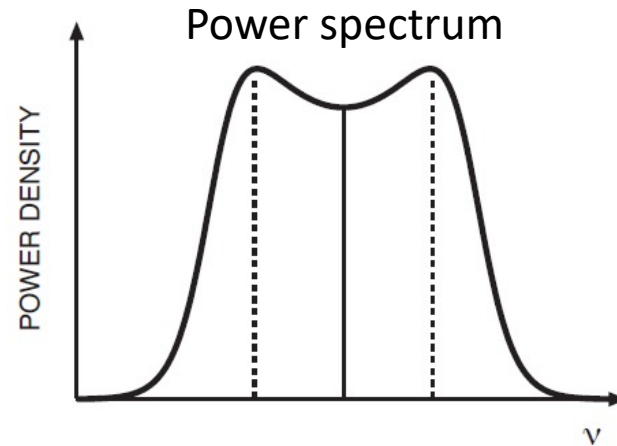
How can we measure the spectrum?

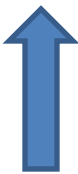By John Sahr, University of Washington.

Fitting process.

Plasma Parameters:
Electron Density
Ion Temperature
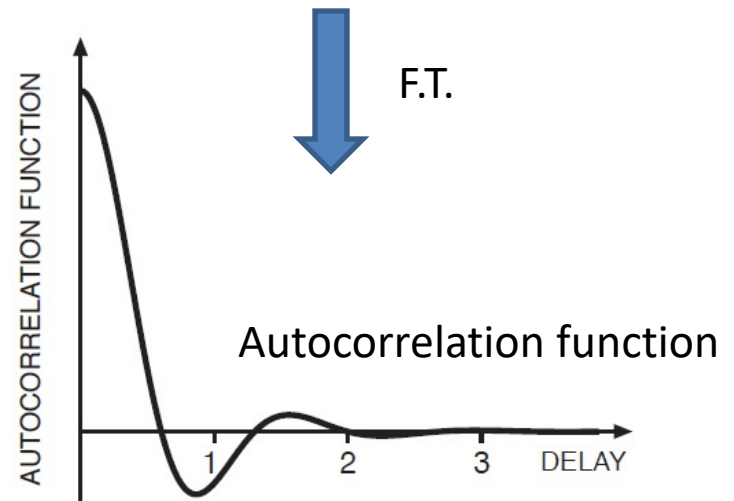Electron Temperature
Ion composition
Collision frequency

ISR theory

Power spectrum

POWER DENSITY

ν

Inverse problem

F.T.

AUTOCORRELATION FUNCTION

1   2   3   DELAY

Autocorrelation function

Noise
Pulse Coding effects

Add

We measure this with the Radar

Acknowledgements

- Tor Hagfors. Max-Planck-Institute.
- Philip Erickson. MIT Haystack Observatory.
- Michael Sulzer. Arecibo Observatory/NAIC.
- John Sahr. University of Washington.
- Joshua Semeter. Boston University.

Thank you!