Introduction to Waves and Tides

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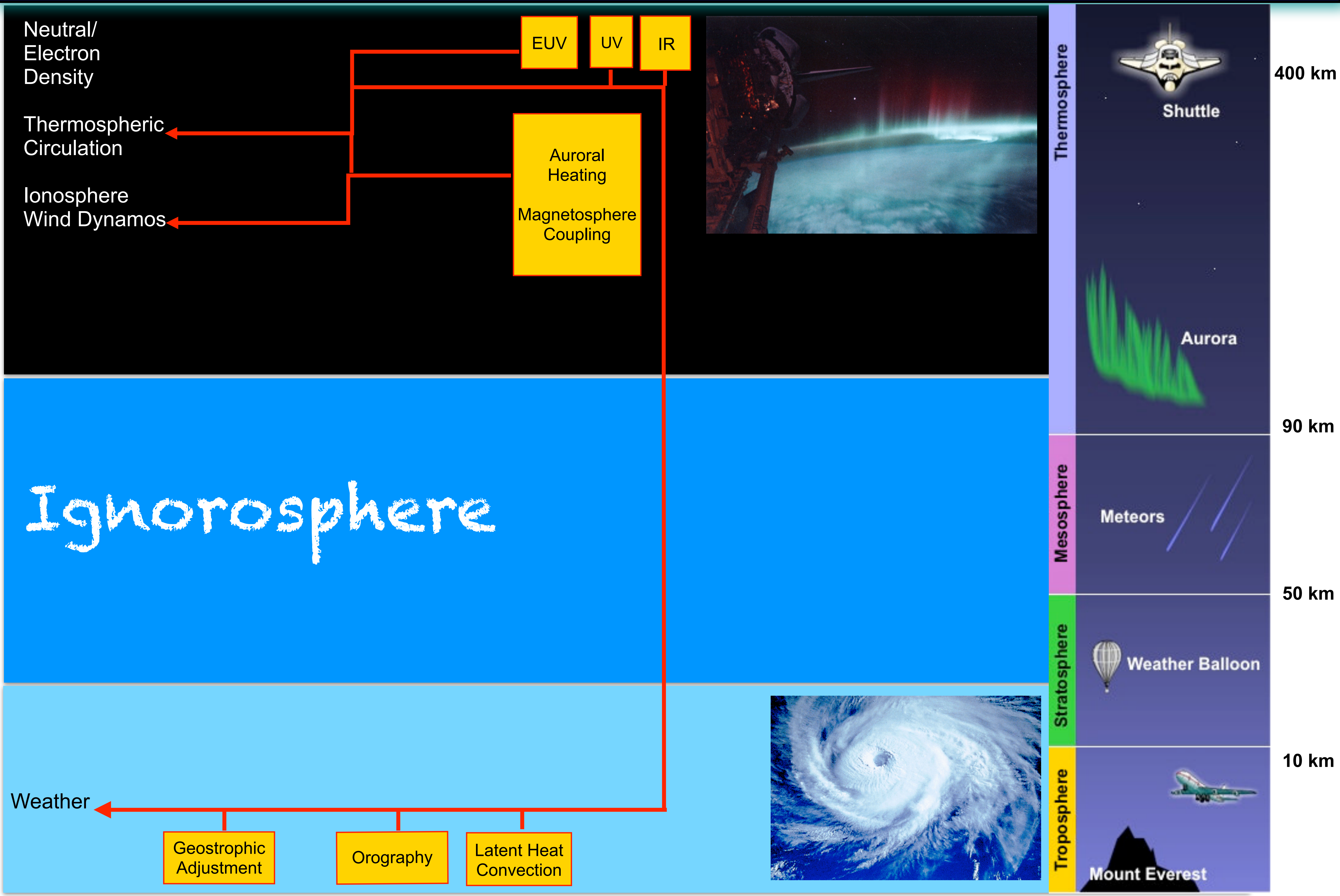
CEDAR Student Workshop

The Atmosphere as I knew it...



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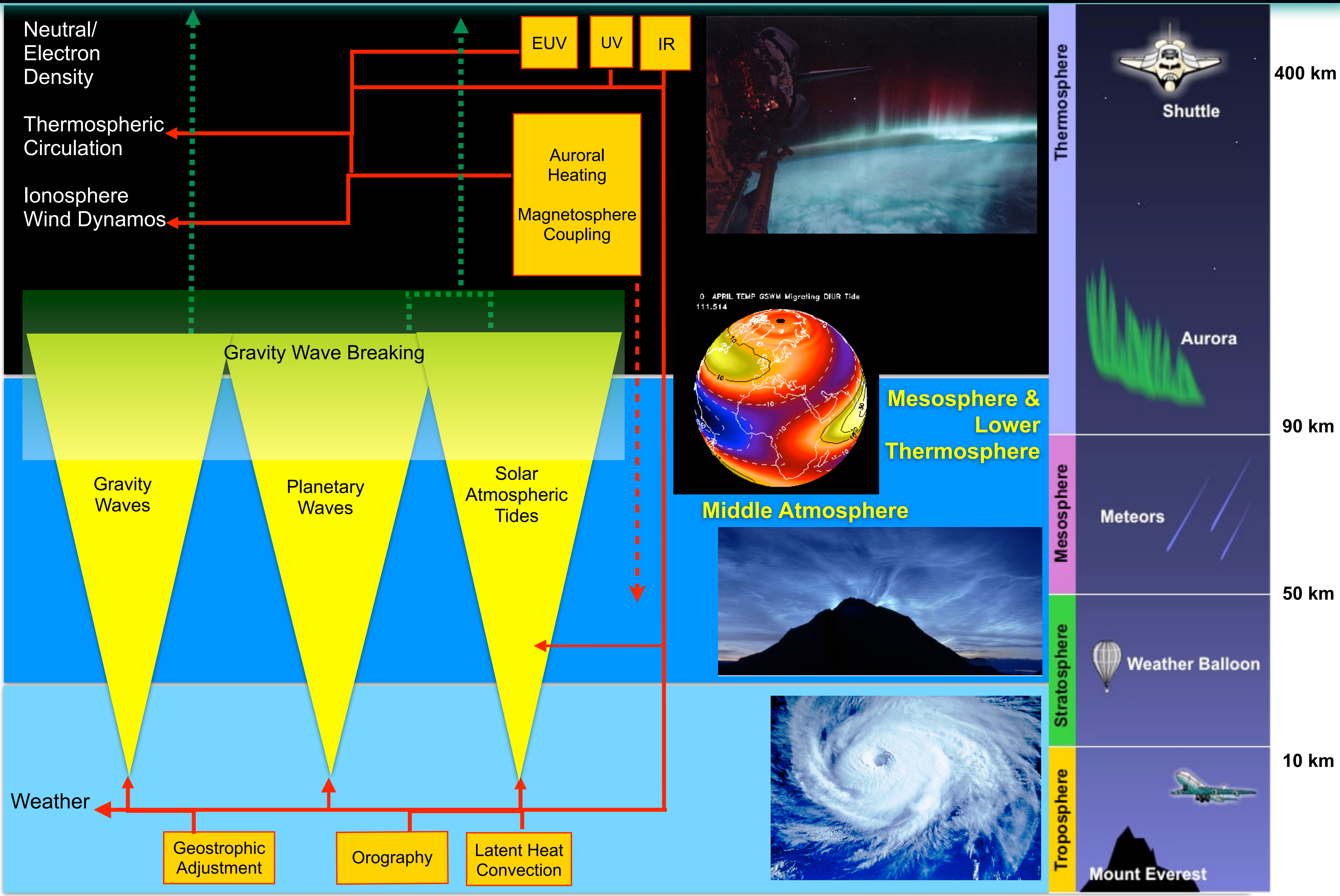


Governing Equations

Momentum Conservation

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + f_c v + F_x$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - f_c u + F_y$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + F_z$$

Energy Conservation

$$\frac{\partial \theta}{\partial t} = Q$$

Continuity

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Reynolds Decomposition

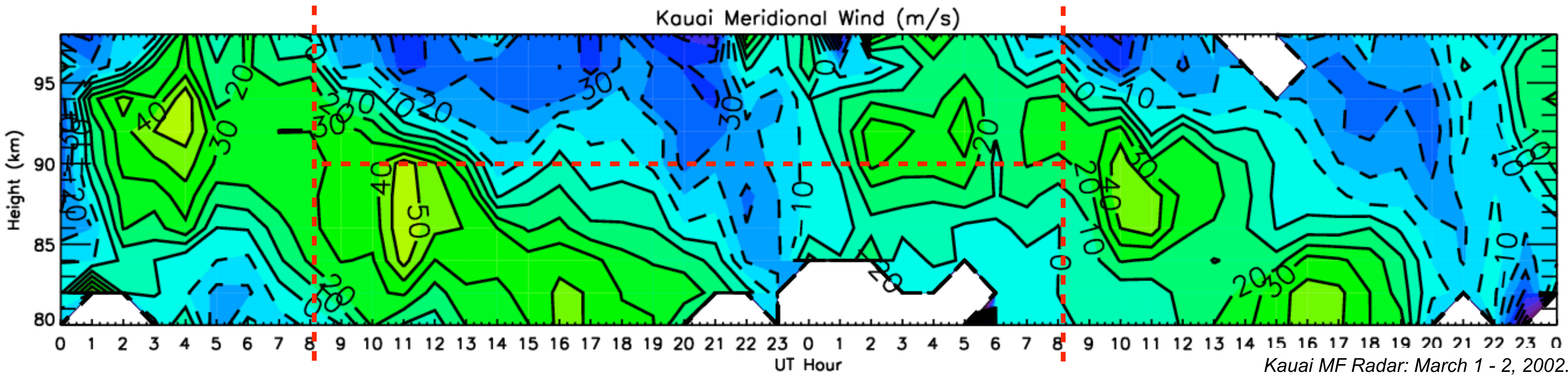
$$f\left(x, y, z, t\right) = \bar{f} + f'\left(x, y, z, t\right)$$

Time Variation

All waves and tides are characterized as **sinusoidal in time...**

$$f'(x, y, z, t) = \hat{f} \exp\left[i\left(\omega t + \ldots\right)\right]$$

$\omega = \Omega$ = 2π/(24 hours)



Kauai MF Radar: March 1 - 2, 2002.
Data courtesy D. Riggin.

Tides are **persistent global scale oscillations forced by solar heating** with periods that are **harmonics of one solar day.**

(24 hours = **D**iurnal, 12 hours = **S**emidiurnal, 8 hours = **T**erdiurnal)

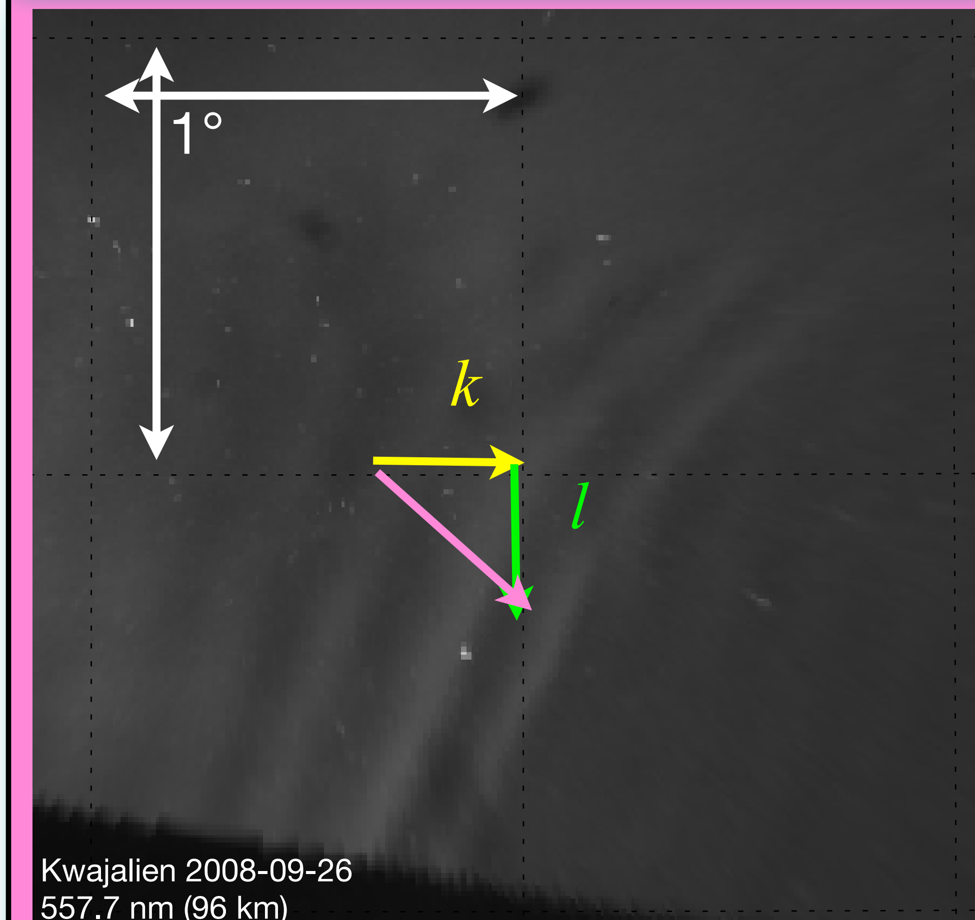
Non-solar harmonics:

- **Planetary Waves:** Periods ~ >1-20 days
- **Gravity Waves:** Periods ~ Minutes to < 1 day

Spatial Structure

$$f'(x,y,z,t) = \hat{f}\exp\left[i\left(kx + ly + mz - \omega t\right) + \frac{z}{2H}\right]$$

$$f'(\lambda,\phi,z,t) = \hat{f}(\phi,z)\exp\left[i\left(\omega t - s\lambda\right)\right]$$



Data courtesy T. Pederson, AFRL

GWs

Spatial scale small compared to fluid volume.

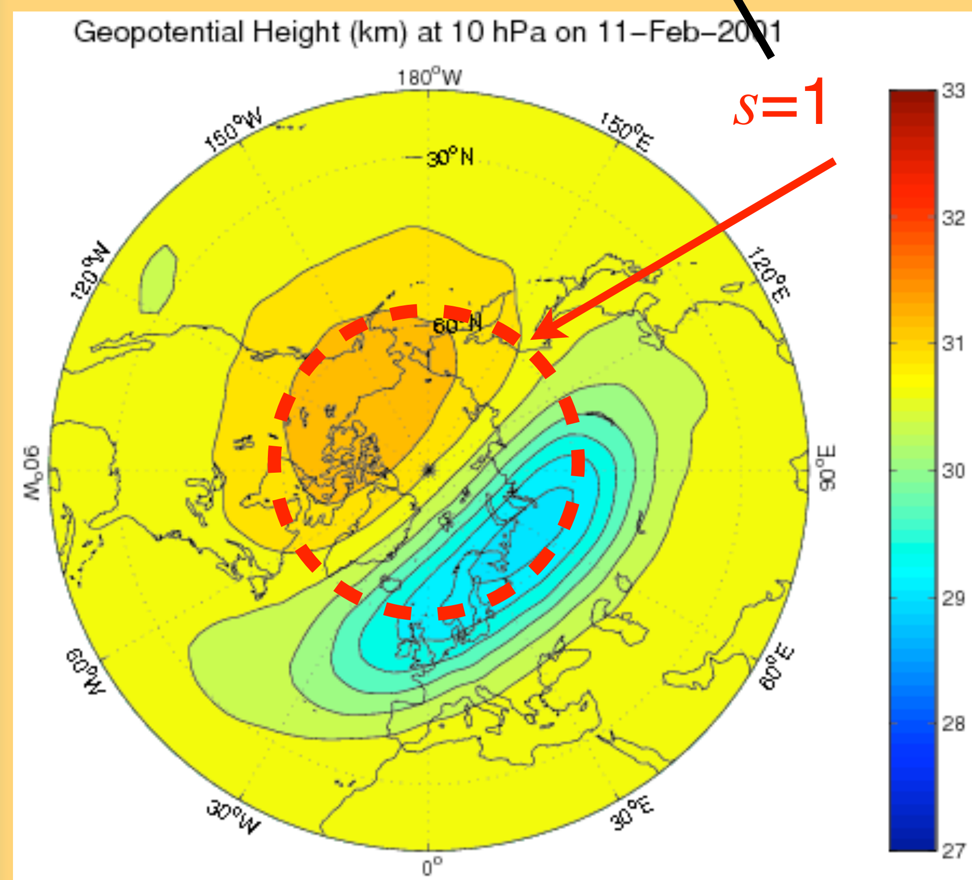


Image: http://www.appmath.columbia.edu/ssws

PWs and Tides

Zonal wavelength is harmonic of Earth's circumference.

Phase velocity ($c = \frac{\omega}{s}$) can be used to subdivide tidal components:

Migrating tides:

$$c = -\Omega$$
$$= -\frac{2\pi}{24hours}$$

Nonmigrating tides:

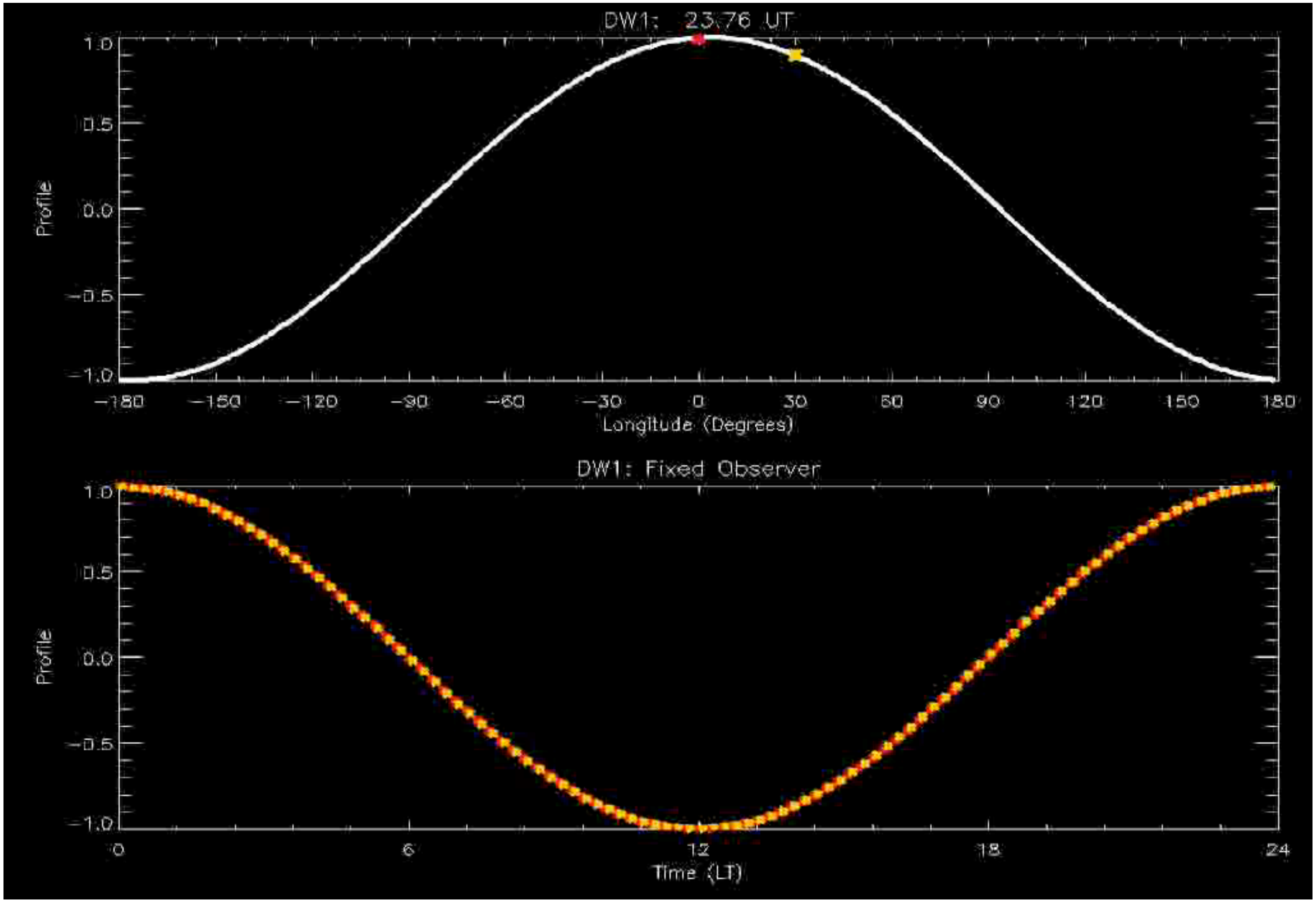
$$c \neq -\Omega$$

Refer to individual tidal components based on period and zonal wavenumber:

DE3 for **D**iurnal (24 hour period), **E**astward zonal wavenumber **3** (*s*=3)

SW2 for **S**emidiurnal (12 hour period), **W**estward zonal wavenumber **2** (*s*=-2)

Migrating Diurnal Tide (DW1)



Frequency

$$\omega = \Omega$$

Zonal wavenumber

$$s = -1$$

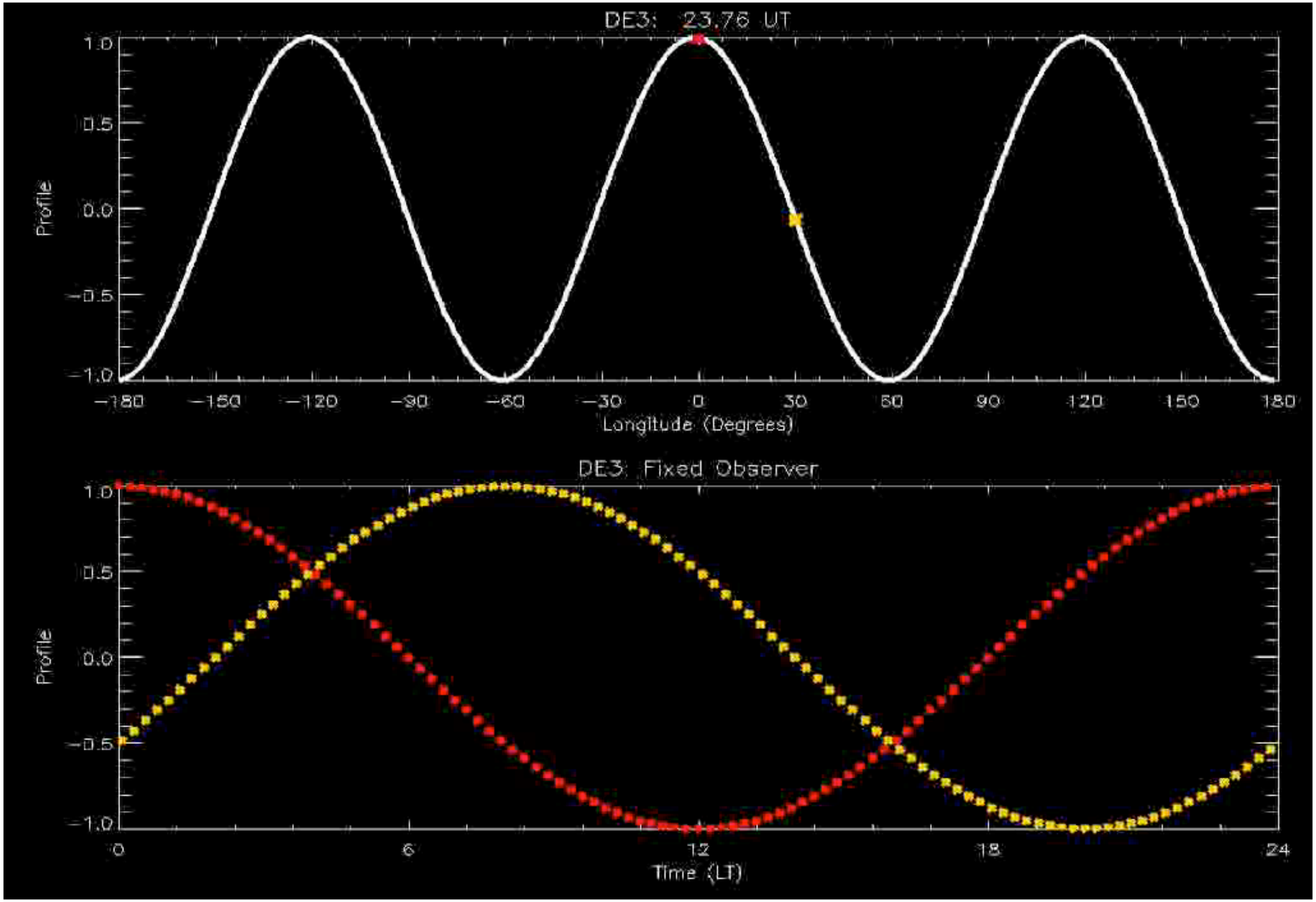
Phase velocity

$$c = -\Omega$$

Migrating tide

Migrating tides show the same local time variation at all longitudes!

Nonmigrating Diurnal Tide (DE3)



Frequency

$$\omega = \Omega$$

Zonal wavenumber

$$s = 3$$

Phase velocity

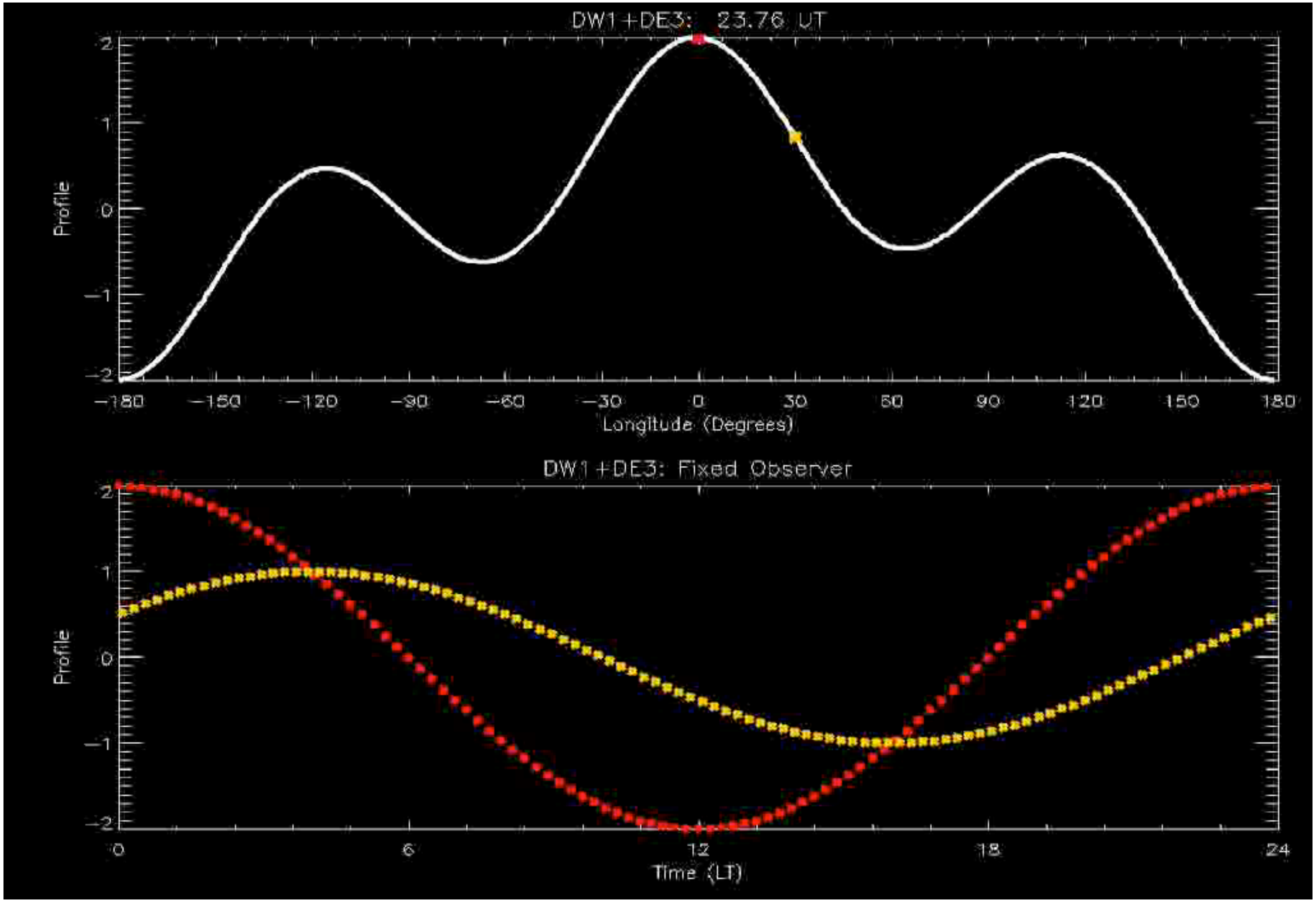
$$c = \frac{\Omega}{3} \neq -\Omega$$

Nonmigrating tide

Observed phase for a nonmigrating tide is longitude dependent!

Superposition

DW1 + DE3: Identical period, different zonal wavenumber.



Superposition results in amplitude variation when observed at different longitudes

Separation of variables

$$u'(\lambda,\phi,z,t) = \sum_s \sum_n U_n^s(\phi)\, G_n(z)\, e^{i(\omega t - s\lambda)}$$

$$v'(\lambda,\phi,z,t) = \sum_s \sum_n V_n^s(\phi)\, G_n(z)\, e^{i(\omega t - s\lambda)}$$

$$\Phi'(\lambda,\phi,z,t) = \sum_s \sum_n \Theta_n^s(\phi)\, G_n(z)\, e^{i(\omega t - s\lambda)}$$

$$J'(\lambda,\phi,z,t) = \sum_s \sum_n \Theta_n^s(\phi)\, J_n(z)\, e^{i(\omega t - s\lambda)}$$

Laplace's Tidal Equation

$$\mathcal{L}\Theta_n = \frac{4\Omega^2 a^2}{g h_n}\Theta_n$$

Vertical Structure Equation

$$\frac{\partial^2 G_n'}{\partial z'^2} + \left(\frac{\kappa H}{h_n} - \frac{1}{4}\right) G_n' = F_n$$

Solve as wave equation:

$$\frac{\partial^2 G_n'}{\partial z'^2} + \alpha^2 G_n' = F_n$$

For $F_n \neq 0$

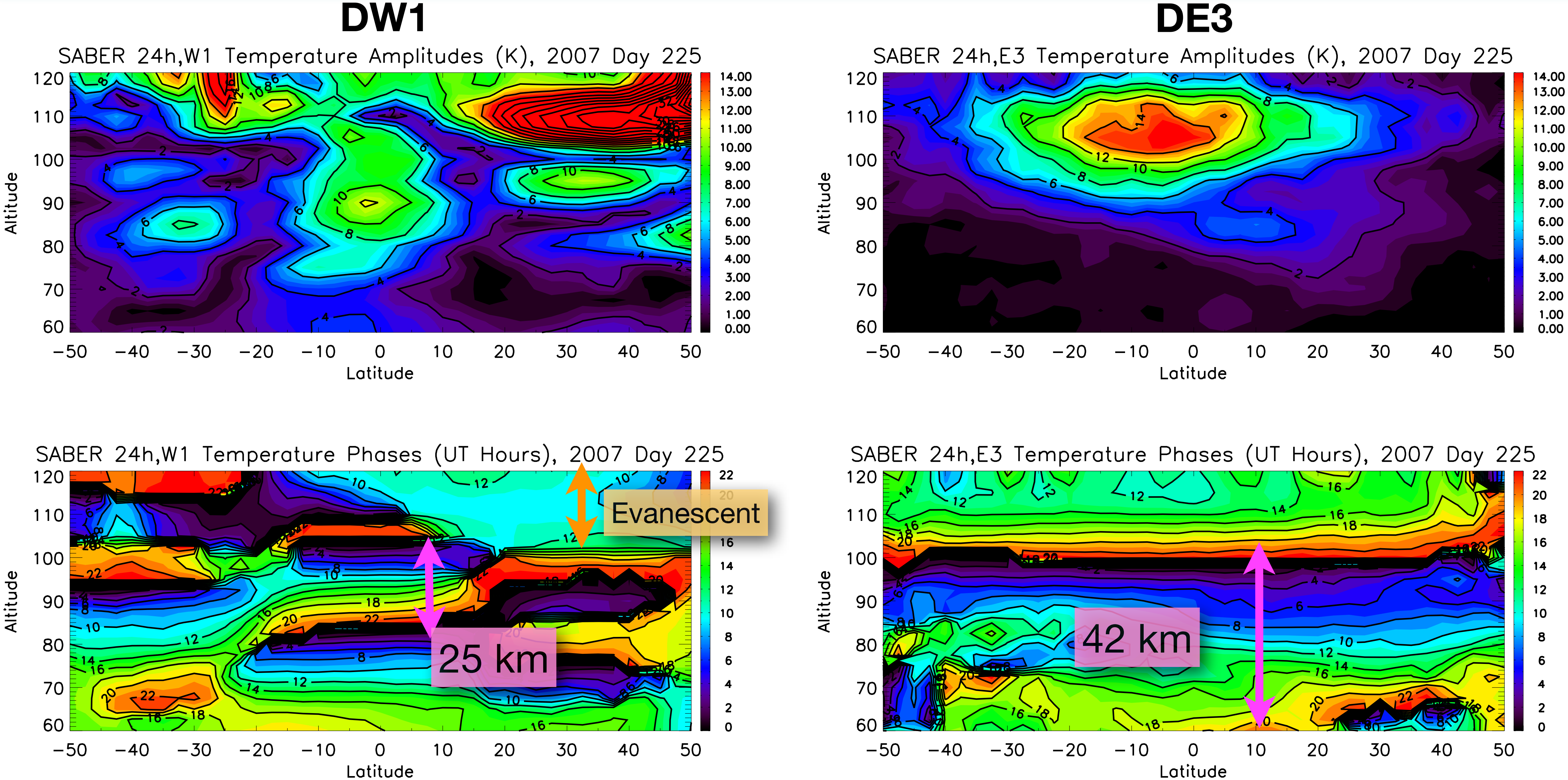
Vertically propagating:

$$G_n' \sim e^{i\alpha z'}$$

Trapped (Evanescent):

$$G_n' \sim e^{-|\alpha| z'}$$

What goes up?



- Long vertical wavelengths allow for penetration to higher altitudes.
- Trapped waves are confined to areas surrounding their region of excitation.

Conclusions

- Waves and tides can be identified by ***period*** and ***wavenumber***:
    - Small scale, short period oscillation: **Gravity Waves**
    - Zonal wavelength harmonic of Earth's circumference:
        - 24 hour harmonic period: **Tides**
            - Sun-synchronous: **Migrating tides**
            - Non-Sun-synchronous: **Nonmigrating tides**
        - Other periods: **Planetary Waves**

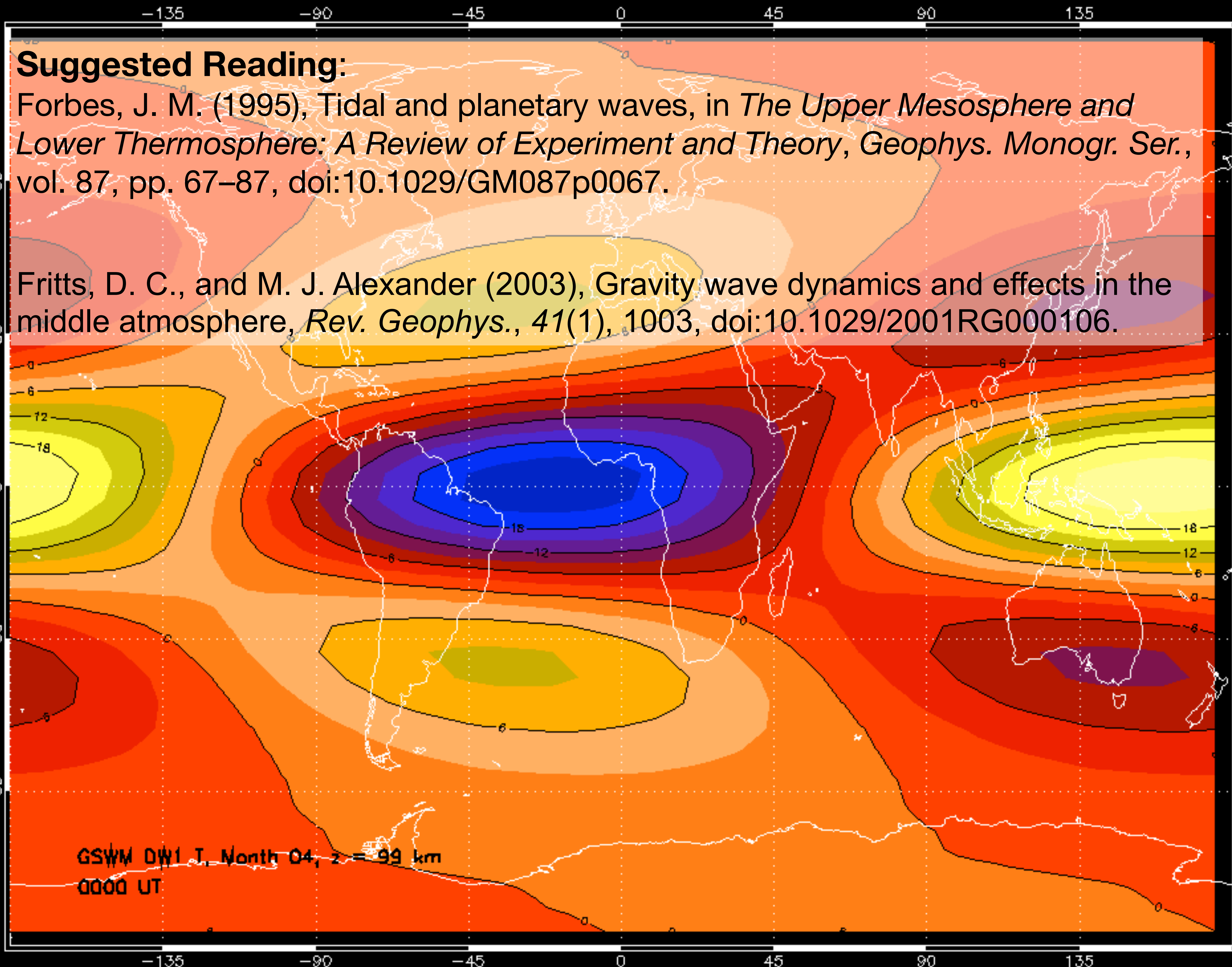
- Tides and PWs of identical period but different zonal wavenumbers are cannot be differentiated from a single location.

- Tidal and PW modes have distinct latitudinal and vertical structure determining their regions of occurrence.

Suggested Reading:

Forbes, J. M. (1995), Tidal and planetary waves, in *The Upper Mesosphere and Lower Thermosphere: A Review of Experiment and Theory*, *Geophys. Monogr. Ser.*, vol. 87, pp. 67–87, doi:10.1029/GM087p0067.

Fritts, D. C., and M. J. Alexander (2003), Gravity wave dynamics and effects in the middle atmosphere, *Rev. Geophys.*, *41*(1), 1003, doi:10.1029/2001RG000106.



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Vertical Structure Equation

$$\frac{\partial^2 G_n'}{\partial z'^2}+\left(\frac{\kappa H}{h_n}-\frac{1}{4}\right)G_n'=F_n$$

SABER DW1 Temperature Amplitudes