# Rossby Waves

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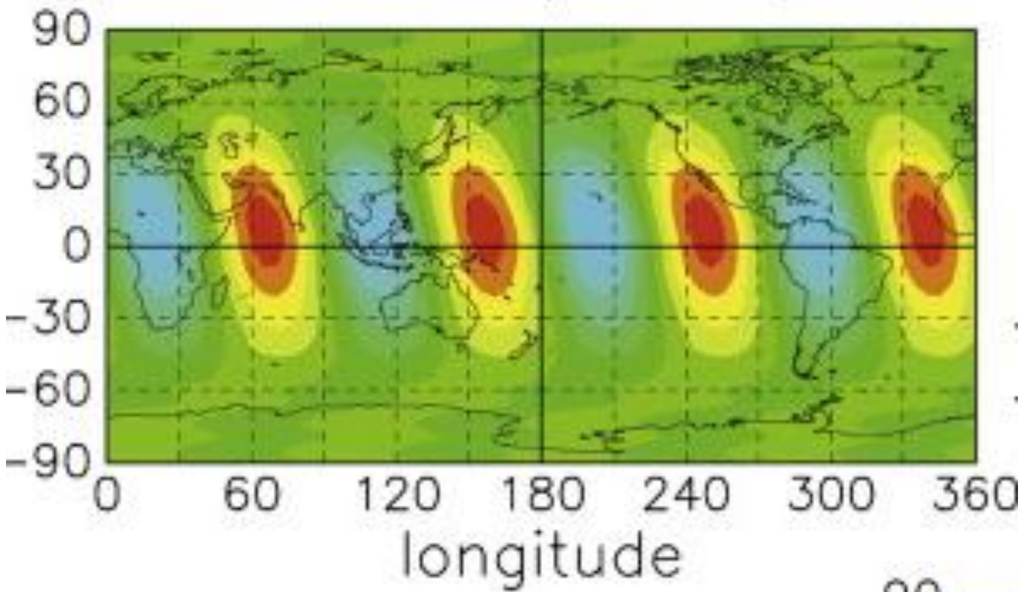
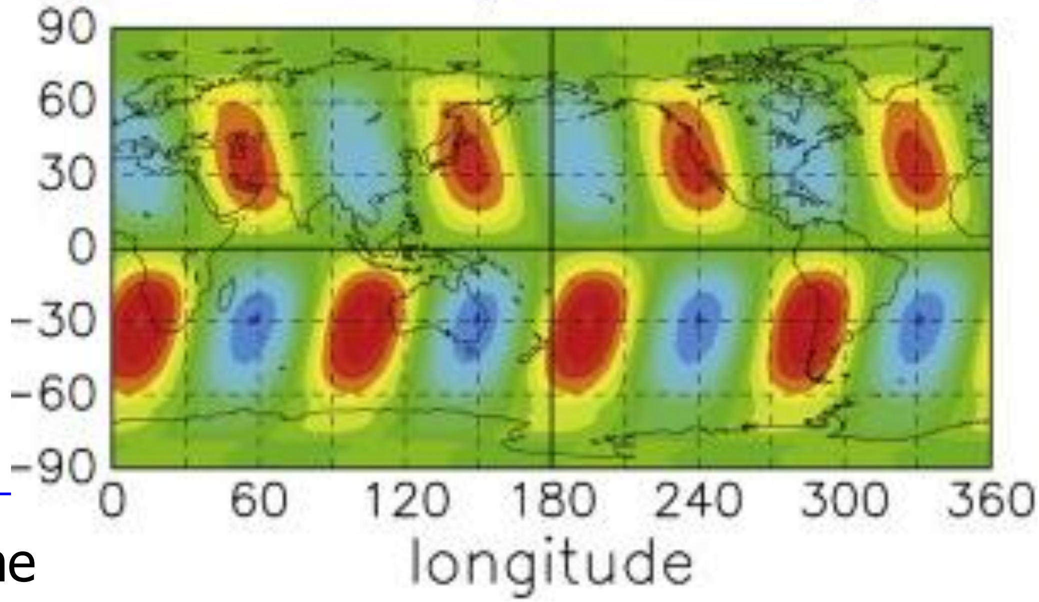
# What is a Rossby Wave?

- Rossby waves are global scale atmospheric waves
- Low zonal wavenumber s=1-4
- Large zonal wavelength $\lambda_x = \dfrac{2\pi a}{s}$ km
  - a=6370*cos($\phi$) km
  - s=1, $\phi$=0 implies $\lambda_x$= 40023 km
- Can be stationary or propagate westward (Don't propagate eastward)
- Have periods longer than 1 day
  - 2, 5, 10 and 16 days
- Have low phase speeds $C_{ph} = \dfrac{484\cos(\phi)}{Ts}$ m/s
  - 10 day wave at 60$^o$N has phase speed of 24 m/s

# Zonal Wavenumber 4




Equatorially symmetric




Equatorially asymmetric

10day wave movie

These patterns are a snapshot in time

# What is a Rossby Wave?

- Rossby waves are generally barotropic (no vertical motion)
- Rossby waves are dispersive (group and phase velocities are different)
- Rossby wave are transient
- Generally important in the mid and high latitudes
- Can propagate from the troposphere to the MLT
- Rossby waves owe their existence to the gradient of vorticity - this is the wave restoring force

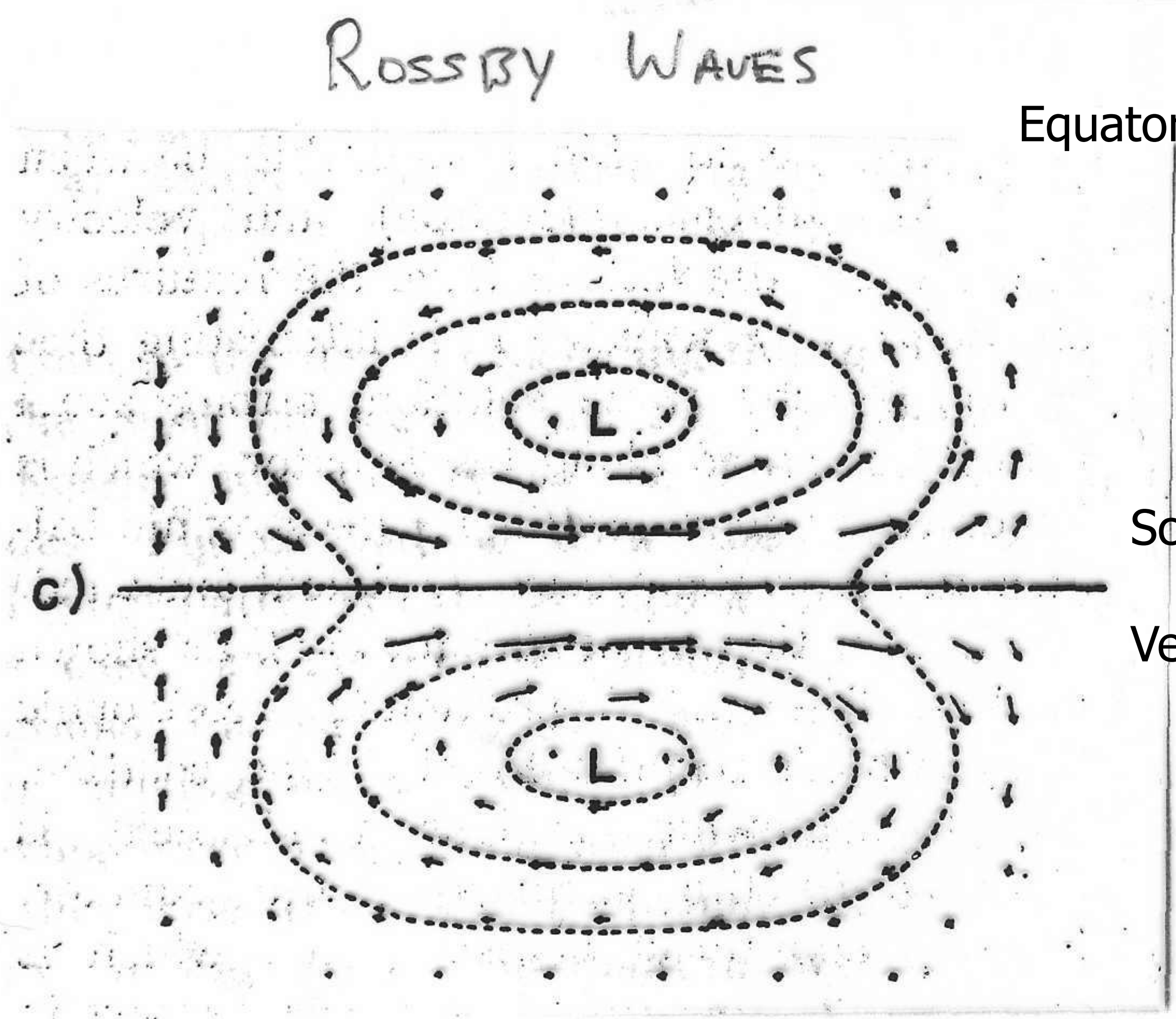
$$f = 2\Omega \sin(\phi)$$ Coriolis parameter

$$\beta = \frac{\partial f}{\partial y}$$ Latitudinal gradient of f

$$\beta = \frac{2\Omega cos(\phi)}{a}$$ $\beta > 0$ for Rossby wave to exist

# Symmetric Rossby Wave

Equatorially symmetric




Northward

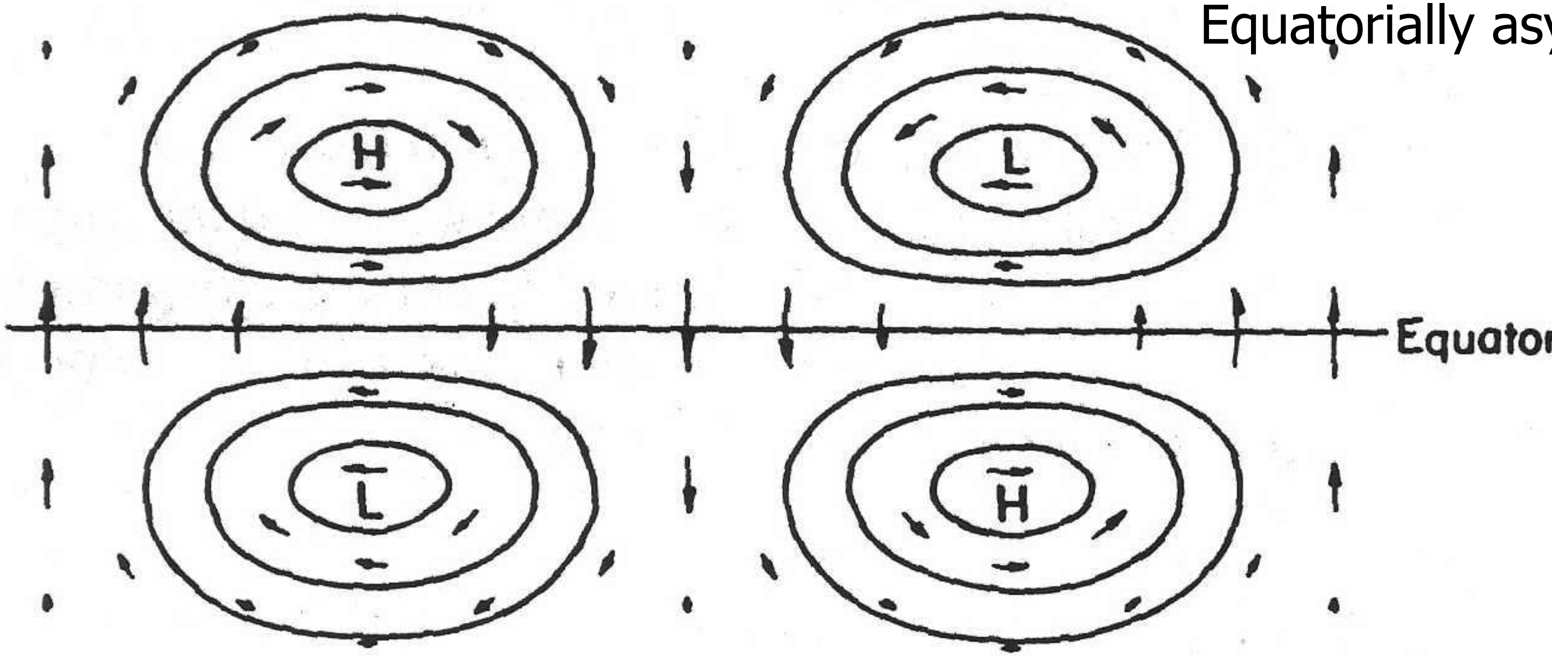
Eastward

Scalar: Temperature

Vector: Winds

# Mixed Rossby Gravity Mode

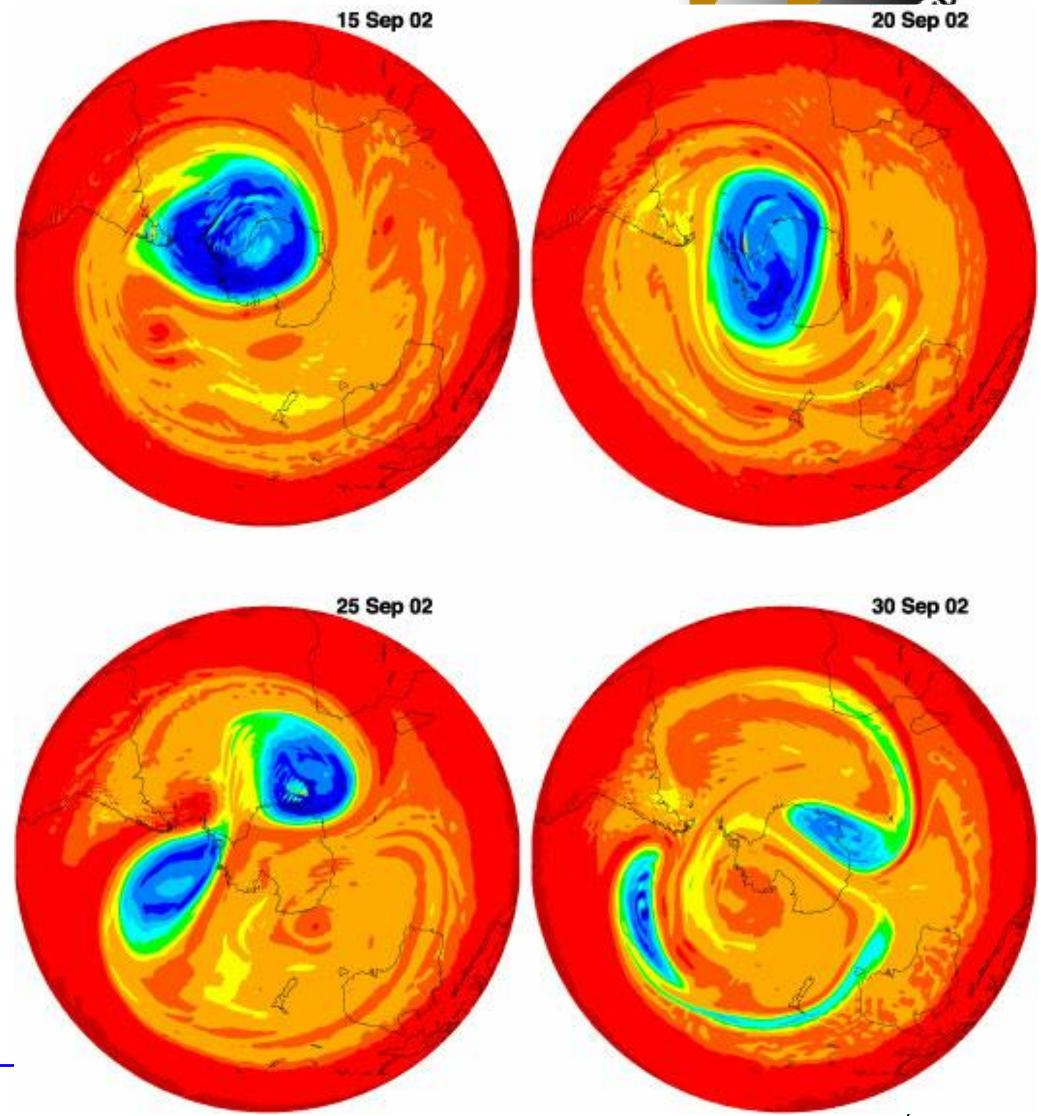
Equatorially asymmetric




Plan view of horizontal velocity and height perturbations associated with an equatorial Rossby–gravity wave. (Adapted from Matsuno, 1966.)

# Stationary Planetary Waves

- Wave 1 & 2 stationary planetary waves (SPW1, SPW2)
- Play a key role in sudden stratospheric warmings (SSWs)



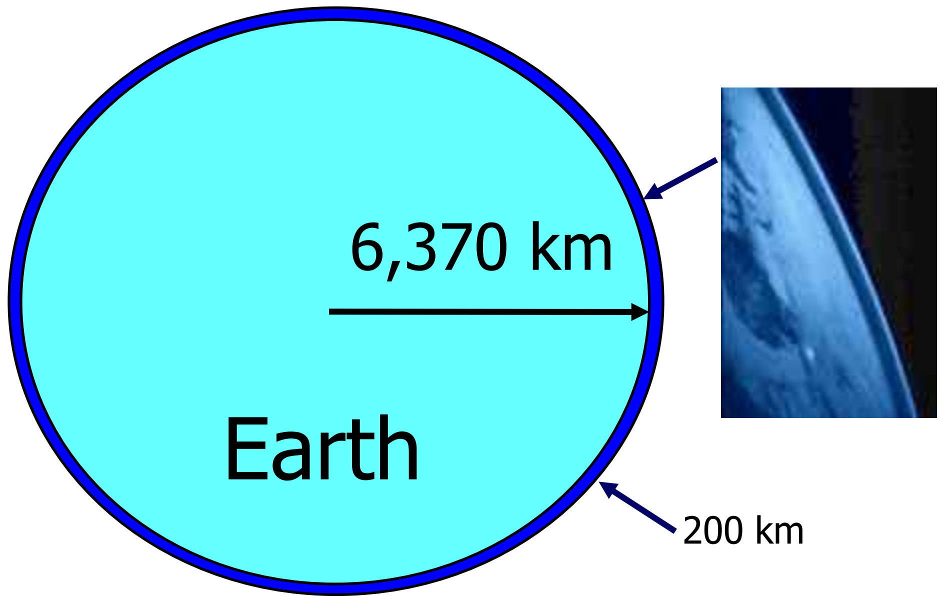

SPW Antarctic movie

SPW global movie

Baldwin et al.
http://www.atmosp.physics.utoronto.ca/SPARC/News20/20_Baldwin.html

# What is the theory?

Navier-Stokes equation on
a thin fluid surrounding
a rotating sphere.



# The Primitive Equations on geometric height coordinates

- Horizontal Momentum $$\frac{D\mathbf{U}}{Dt} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{U} + \mathbf{F}_{visc}$$

- Vertical Momentum (hydrostatic balance) $$\frac{dp}{dz} = -\rho g$$

- Thermodynamic (conservation of energy) $$Q = c_p \frac{DT}{Dt} + \frac{1}{\rho}\frac{Dp}{Dt}$$

- Continuity (conservation of mass) $$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{U} = 0$$

- Ideal gas law for a dry atmosphere $$p = \rho R T$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \qquad u \equiv \frac{Dx}{Dt} \qquad v \equiv \frac{Dy}{Dt} \qquad w \equiv \frac{Dz}{Dt}$$

# How are planetary waves defined?

**Primitive Equations**

$$f(t,\lambda,\phi,z)$$

**Zonal Mean Equations**

$$\overline{f}(t,\phi,z) = \int_{\lambda} f(t,\xi,\phi,z)d\xi$$

**Nonlinear Perturbation Equations**

$$f'(t,\lambda,\phi,z) = f(t,\lambda,\phi,z) - \overline{f}(t,\phi,z)$$

**Solutions are Periodic in Time ($\sigma$-frequency) and Longitude (s-wavenumber)**

$$f'(t,\lambda,\phi,z) = \sum_{\sigma}\sum_{s} A^{\sigma,s}(\phi,z)\cos(\Omega\sigma t - s\lambda + \Psi^{\sigma,s}(\phi,z))$$

$\Omega=2\pi/24$ radians/hour, $\sigma$ = normalized freq (cycles/day), $T=24/\sigma$ period (hours)

# Analytical Solutions

Assume:

No zonal mean winds $\overline{U} = 0$

Isothermal atmosphere $T = T_0$

No dissipation

Solve LaPlace's Tidal Equation

$$f'(t,\lambda,\phi,z) = \sum_{\sigma}\sum_{s}\sum_{n} F_n^{\sigma,s}(\phi) G_n^{\sigma,s}(z) \cos(\Omega\sigma t - s\lambda + (\lambda_z)_n^{\sigma,s} z + \Psi_n^{\sigma,s}(\phi))$$

$F_n^{\sigma,s}(\phi) = \Phi_n^{\sigma,s}(\phi)$ – related to Hough Functions

$\Phi_n^{\sigma,s}(\phi) = \Theta_n^{\sigma,s}(\phi)$

$$F_n^{\sigma,s}(\phi) = V_n^{\sigma,s}(\phi) = \frac{-j}{2a\Omega(\sigma^2 - \sin^2\phi)}\left(s\tan\theta + \sigma\frac{\partial}{\partial\phi}\right)\Phi_n^{\sigma,s}(\phi)$$

$G_n^{\sigma,s}(\phi)$– solution to the vertical structure equation

# Solutions for a Linearized, Isothermal Atmosphere

$$f'(t,\lambda,\phi,z) = \sum_{\sigma}\sum_{s}\sum_{n} F_n^{\sigma,s}(\phi) G_n^{\sigma,s}(z) \cos(\Omega\sigma t - s\lambda + (\lambda_z)_n^{\sigma,s} z + \Psi_n^{\sigma,s}(\phi))$$

Solutions are periodic in
- Time (t) – $\Omega\sigma$ is frequency
- Longitude ($\lambda$) – s is wavenumber
- Altitude (z) – $\lambda_z$ is vertical wavelength

Solar tides are harmonics of $\Omega$ ($\sigma >= 1$)
- $\sigma$=1 Diurnal Tide (T=24 hours)
- $\sigma$=2 Semidiurnal Tide (T=12 hours)
- $\sigma$=3 Terdiurnal Tide (T=8 hours)

Rossby Waves are subharmonics of $\Omega$ (($\sigma >= 1$)
- $\sigma$=0.5 2 day wave
- $\sigma$=0.2 5 day wave
- $\sigma$=.01 10 day wave

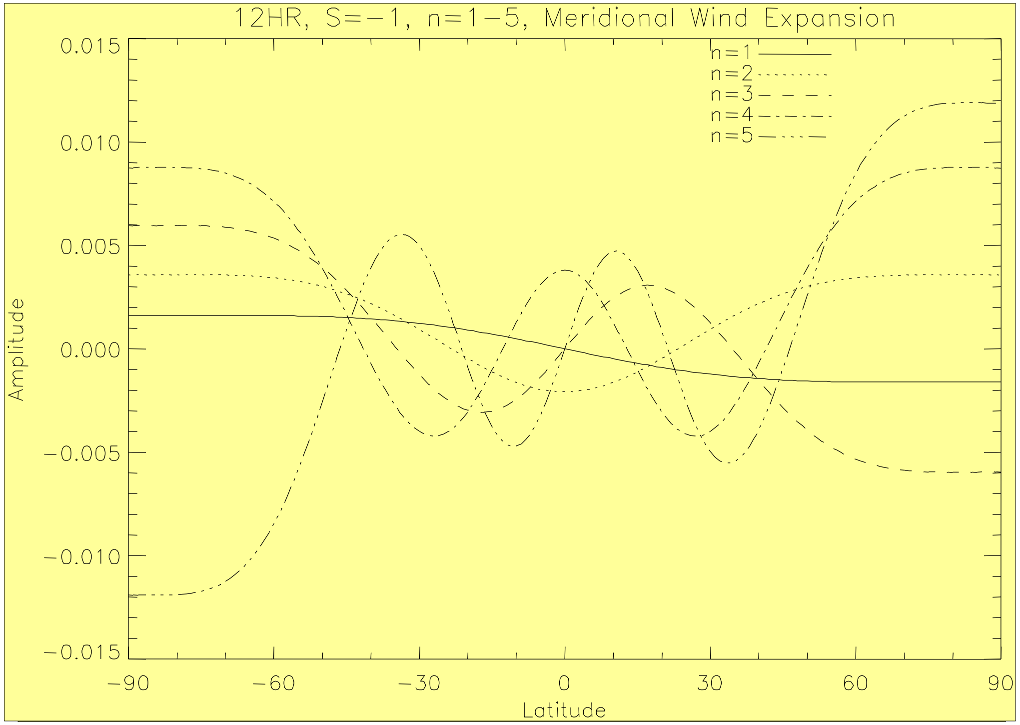
# Latitudinal Structure

$$f'(t,\lambda,\phi,z) = \sum_{\sigma}\sum_{s}\sum_{n} F_n^{\sigma,s}(\phi) G_n^{\sigma,s}(z)\cos(\Omega\sigma t - s\lambda + (\lambda_z)_n^{\sigma,s} z + \Psi_n^{\sigma,s}(\phi))$$

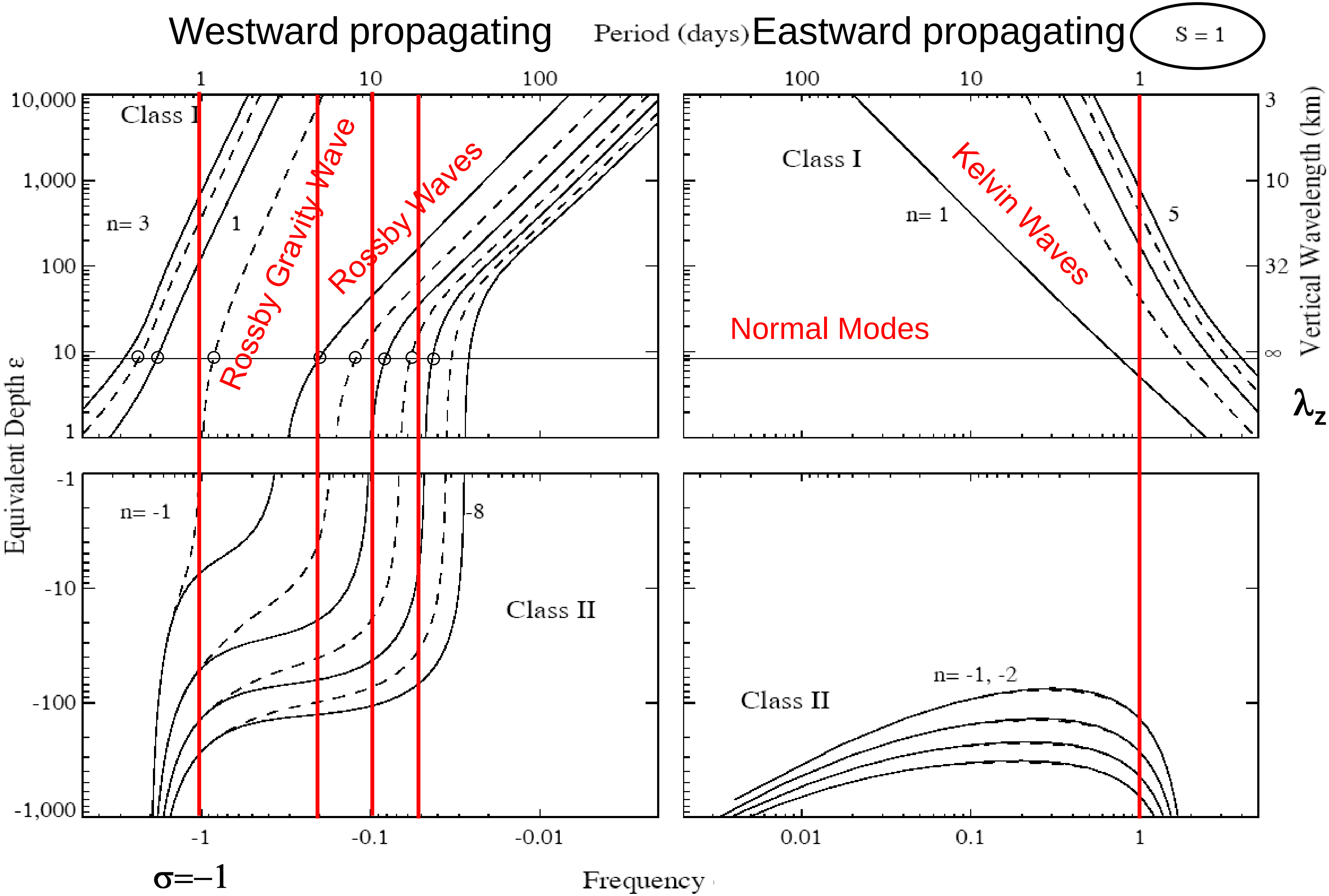
Latitudinal amplitude structure

Depends upon:
- Frequency ($\sigma$)
- Wavenumber (s)
- Meridional index (n)

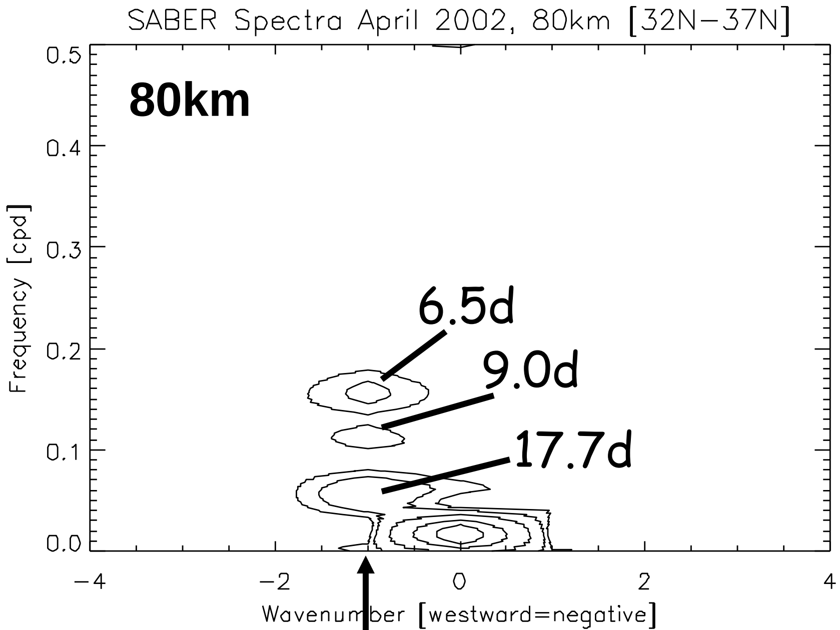
Non Migrating Semidiurnal Tide ($\sigma$=2, s=-1, n=1-5)



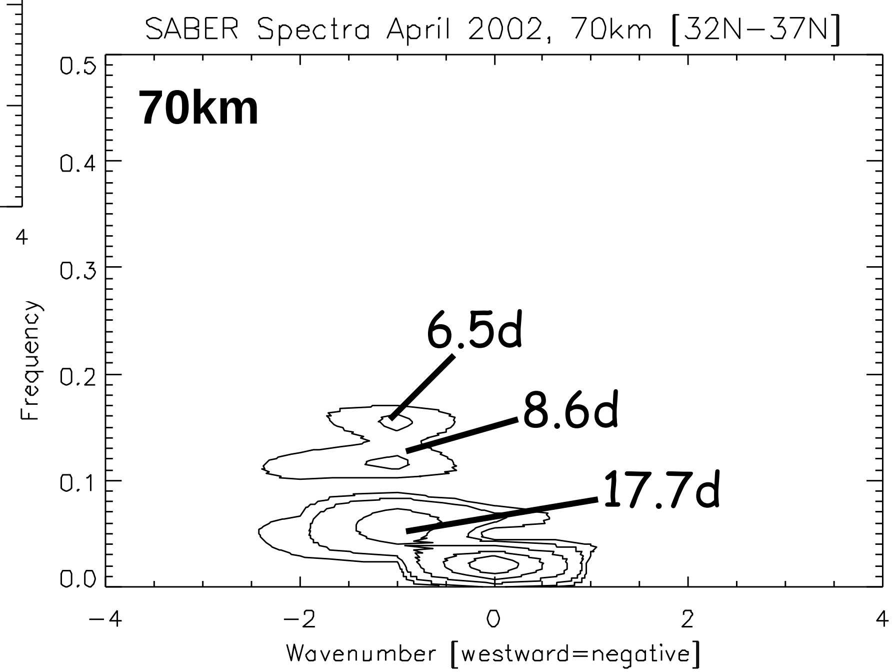
# Dispersion relation for s=1



# SABER Power Spectra, 32N-37N




**April 2002**




Zonal wavenumber 1
Westward propagating

# 2-day wave

Mueller and Nelson, 1978

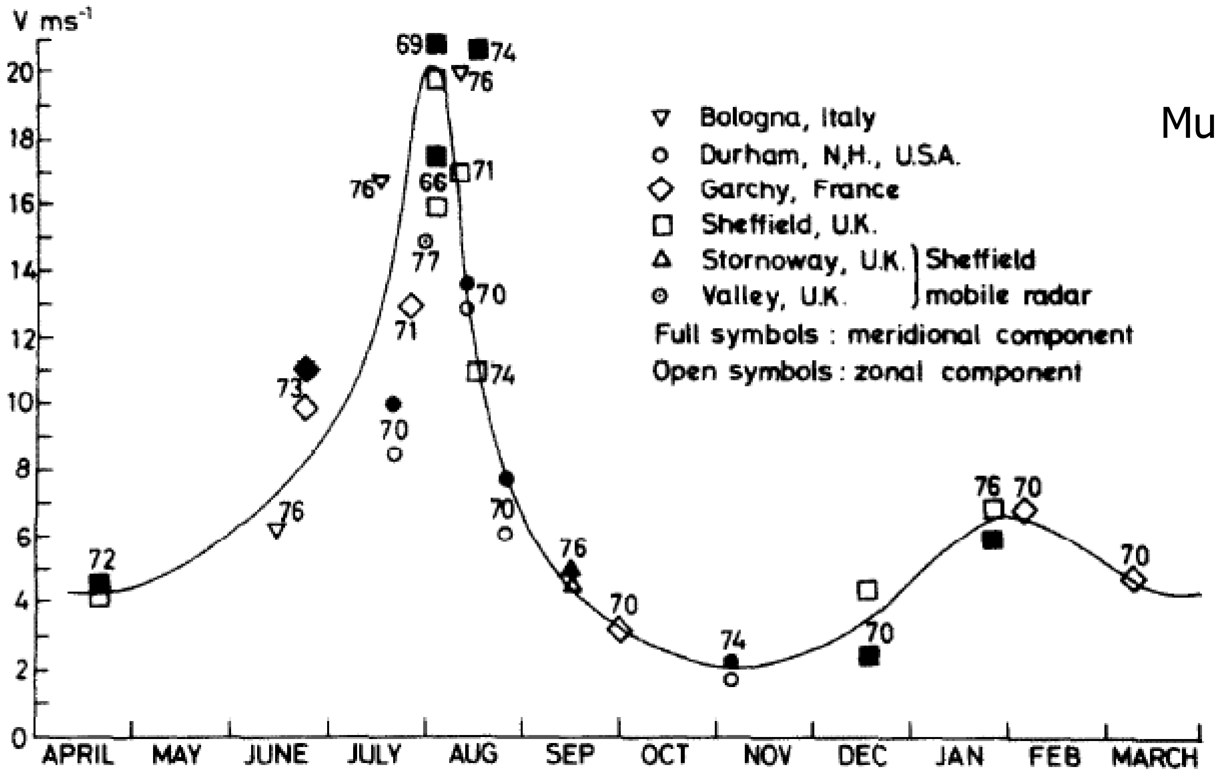
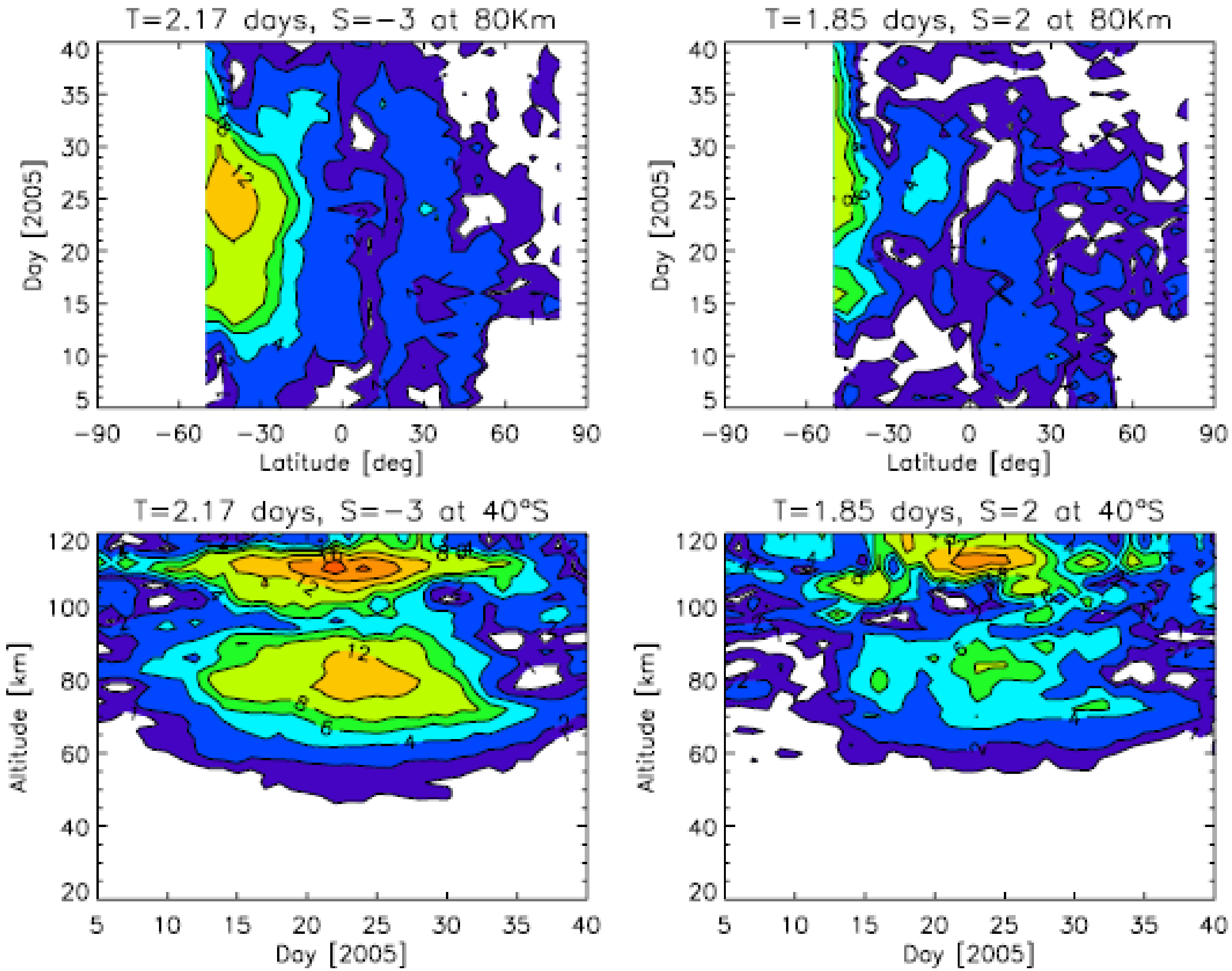



Figure 2. The amplitude of the 51 h meteor wind oscillation at various times of the year based on mid latitude records for the northern hemisphere during the period 1966 – 1977. Values relate to an average altitude of 95 km.

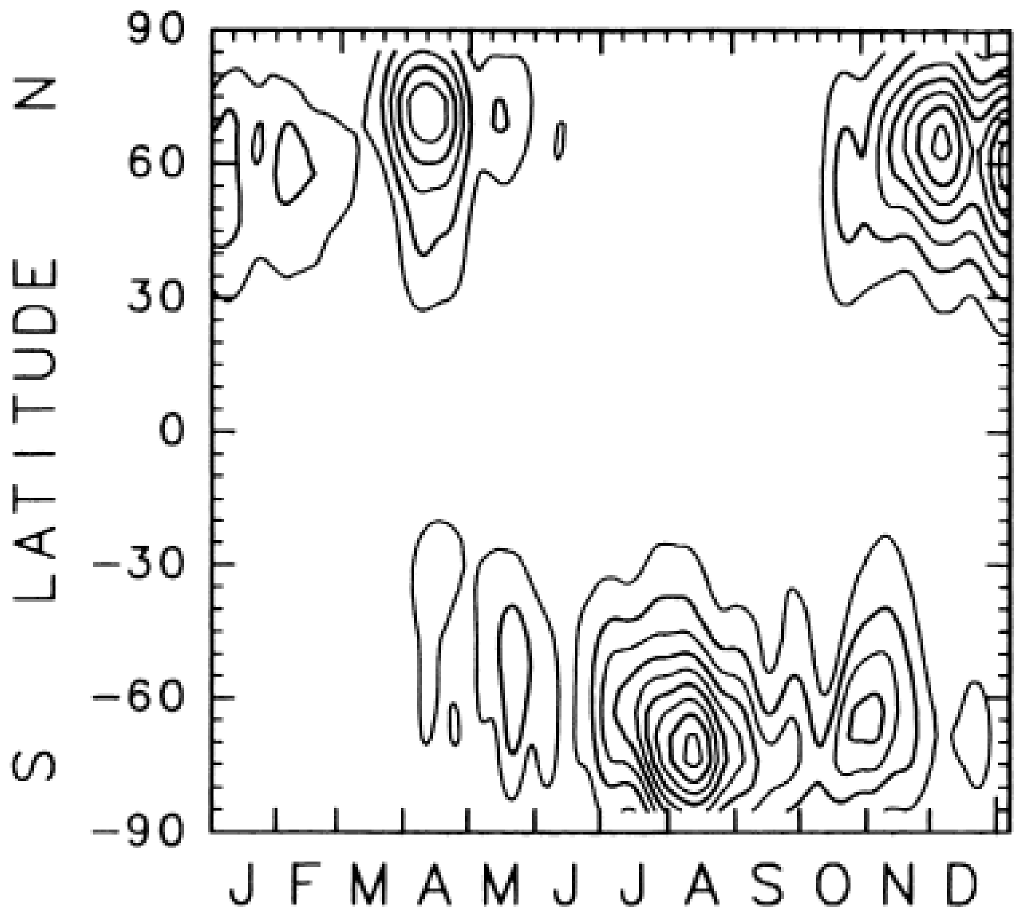
# 2 Day Wave in SABER




2day wave movie

Palo et al, 2007

# 16-day wave




(c)

Miyoshi, 1999

# Rossby Normal Modes

| Wave | (s,n) | (s,\|n\|-s) | $h_n$(km) | Cph@eq m/s | |
|---|---|---|---|---|---|
| 5-day | (1,-2) | (1,1) | 10.5 | 97 | First symmetric |
| 10-day | (1,-3) | (1,2) | 10.5 | 48 | First asymmetric |
| 16-day | (1,-4) | (1,3) | 10.5 | 30 | Second symmetric |
| 4-day | (2,-3) | (2,1) | 10.5 | 60 | First symmetric |
| 2-day | (3,-3) | (3,0) | 10.5 | 81 | Mixed Rossby-Gravity, asymmetric |

Table adapted from Forbes, 1995

# Summary

- Rossby waves are global scale disturbances (s=1-4)
- $\beta$ is the restoring force for Rossby waves
- Stationary planetary waves (SPW1, SPW2) are Rossby waves and are important in northern hemsphere winter dynamics and play a major role in sudden stratospheric warmings.
- Many of the large scale disturbances observed in the MLT are Rossby Waves. These waves propagate westward and are related to atmospheric normal modes
  - Quasi 2-day wave
  - Quasi 4-day wave
  - Quasi 10-day wave
  - Quasi 16-day wave

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Note that dissertations often provide seminal references on a subject and tutorial information

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