Global Model and Data Assimilation

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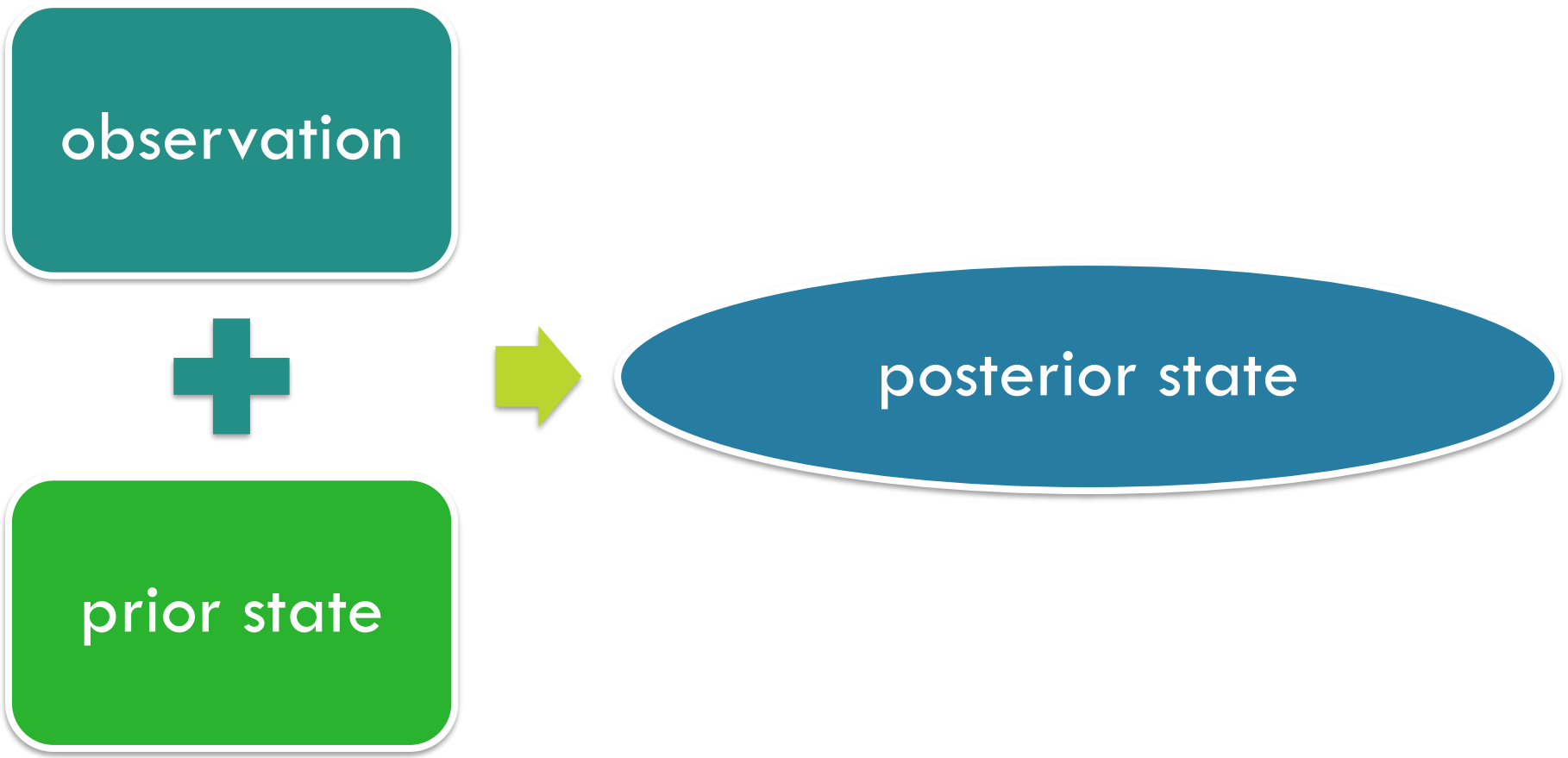
This material is based upon work supported by the National Center for Atmospheric Research, which is a major facility sponsored by the National Science Foundation under Cooperative Agreement No. 1852977.

What is Data Assimilation (DA)?



If we only have single point …

a. Variance minimum methods

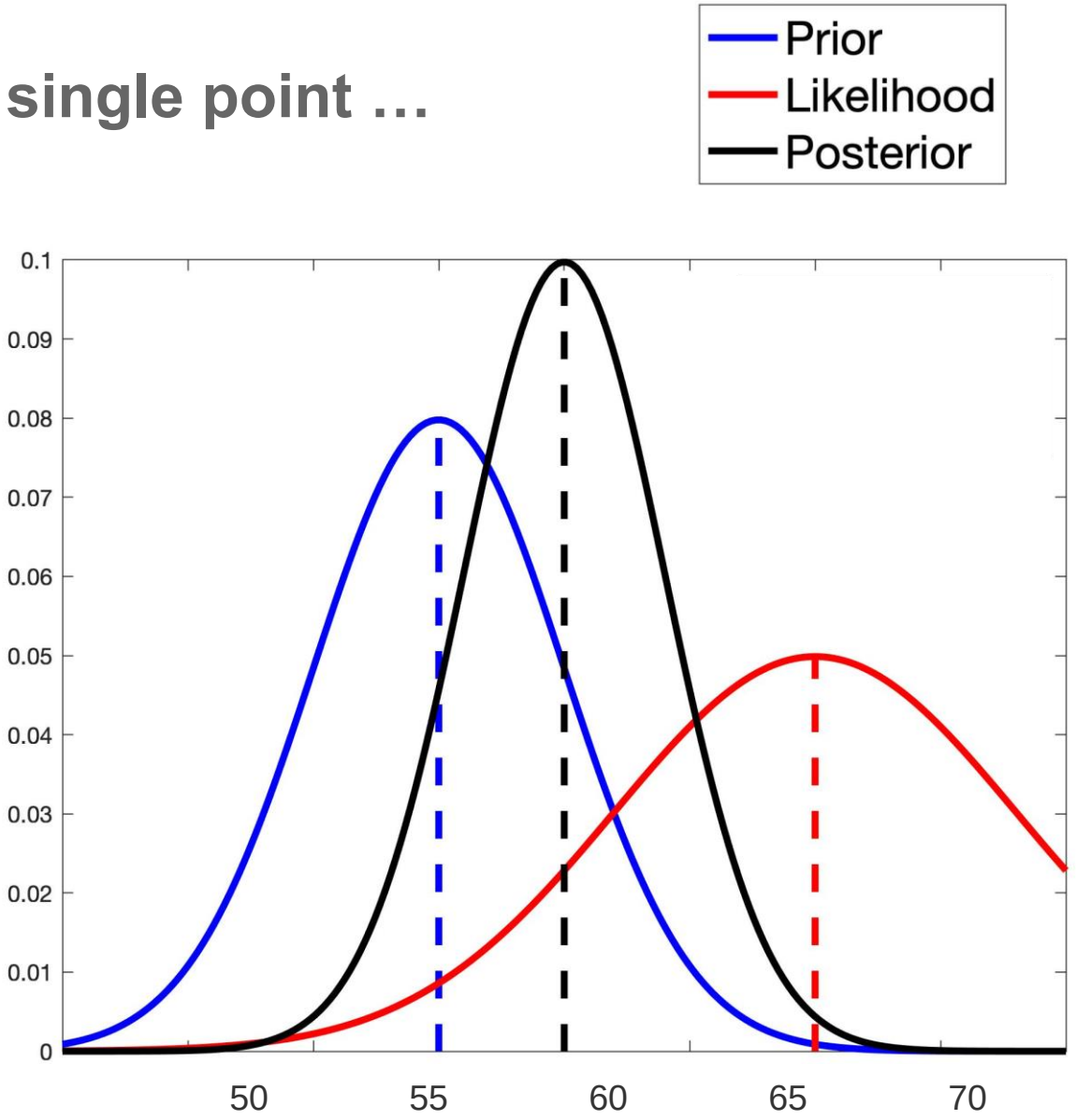
- Find the state with minimum variance

➔ e.g. Kalman Filter

b. Maximum likelihood methods

- Find the most possible state

➔ e.g. variational method



Kalman Filter

Update equation

$$\mathbf{X}^a = \mathbf{X}^b + \mathbf{K}\left(\mathbf{y}^o - \mathbf{H}(\mathbf{X}^b)\right)$$

$$\mathbf{P}^a = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}^b$$

$$\mathbf{K} = [\mathbf{P}^b\mathbf{H}^T][\mathbf{H}\mathbf{P}^b\mathbf{H}^T + \mathbf{R}]^{-1}$$

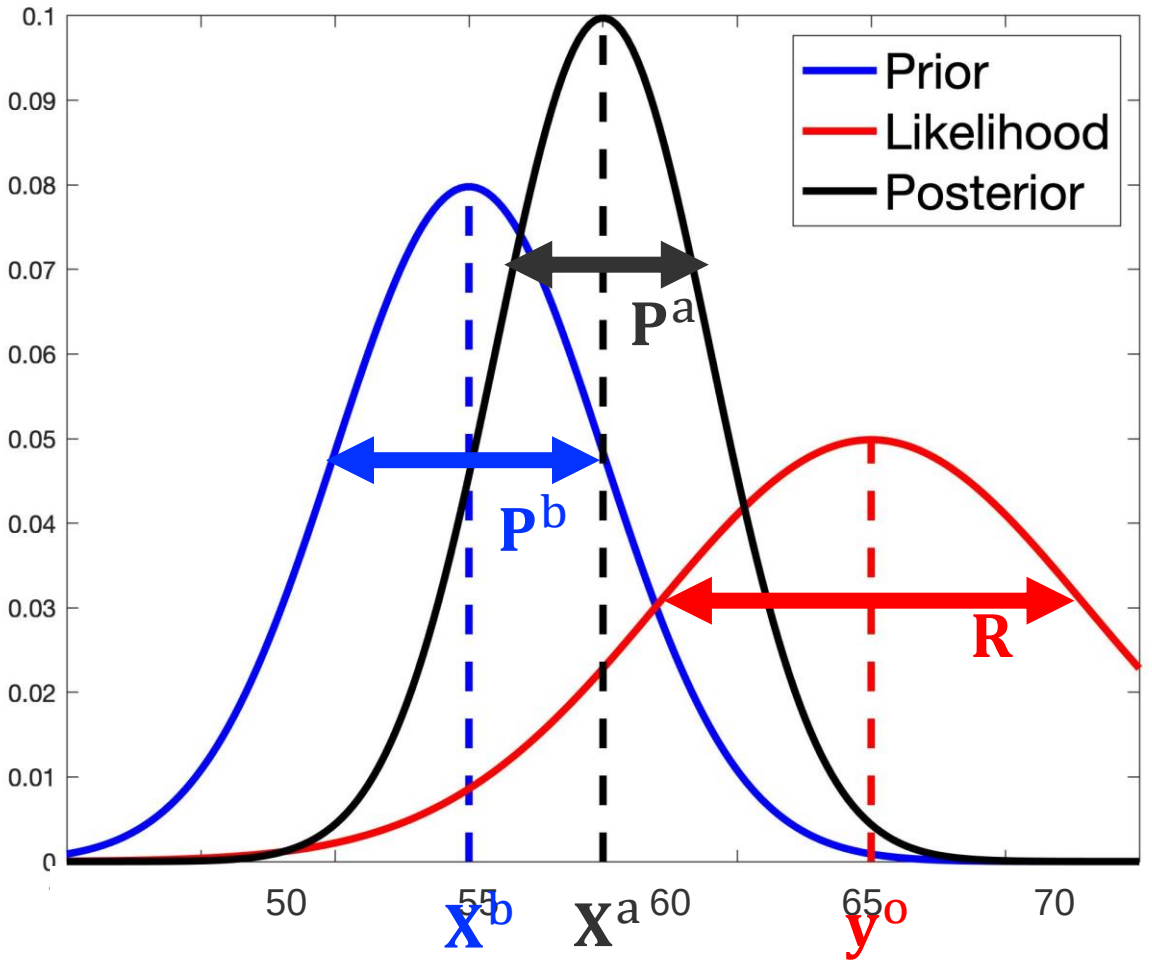
$\mathbf{K}$ Kalman gain

$\mathbf{H}$ forward operator

$\mathbf{P}^a$ analysis error covariance

$\mathbf{P}^b$ background error covariance

$\mathbf{R}$ observation error covariance



Data Assimilation Cycle

Update equation

$$\mathbf{X}_t^{\,a} = \mathbf{X}_t^{\,b} + \mathbf{K}_t(\mathbf{y}_t^{\,o} - \mathbf{H}(\mathbf{X}_t^{\,b}))$$

$$\mathbf{P}_t^{\,a} = (\mathbf{I} - \mathbf{K}_t\mathbf{H}_t)\mathbf{P}_t^{\,b}$$

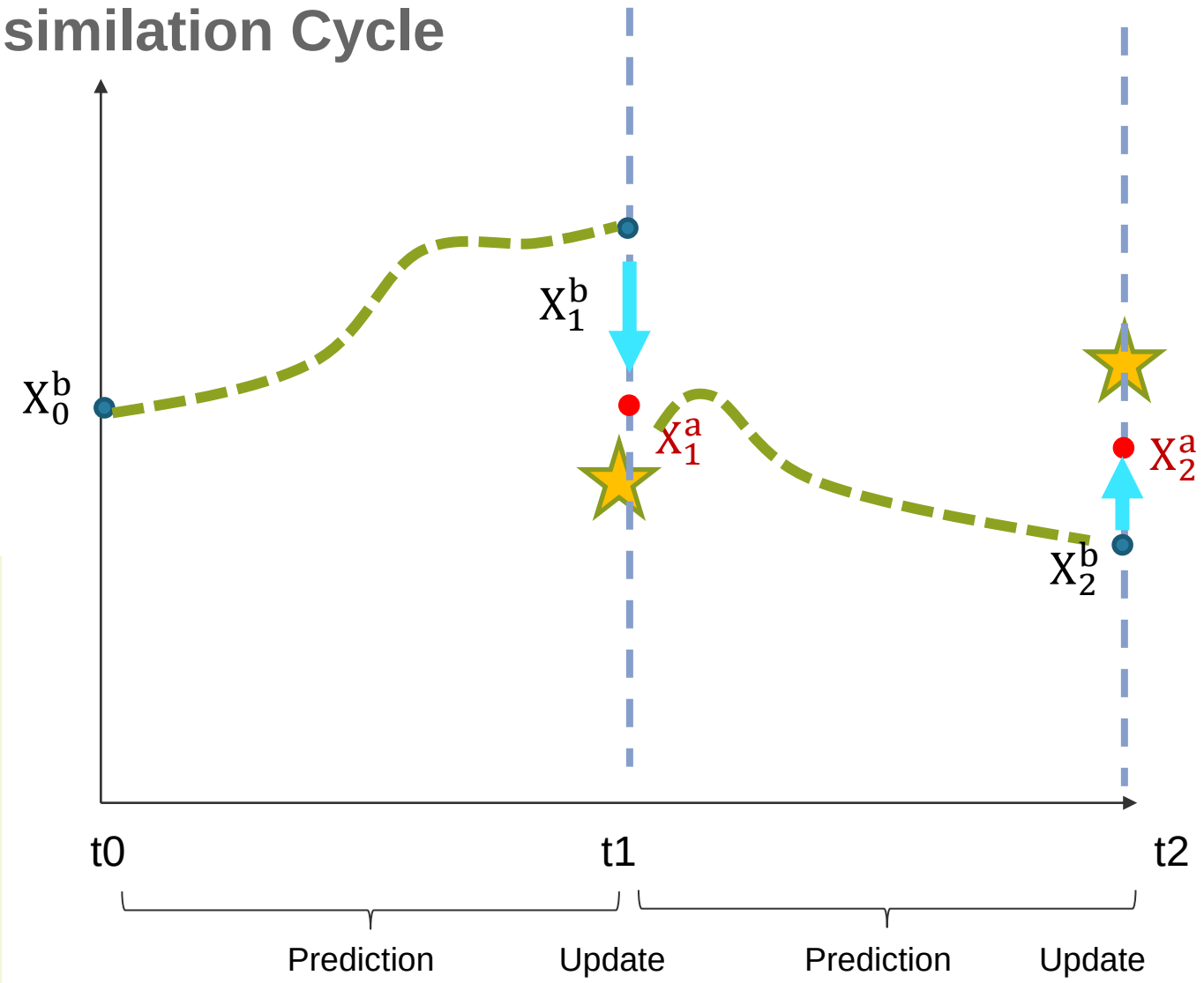
$$\mathbf{K}_t = [\mathbf{P}_t^{\,b}\mathbf{H}_t^{\,T}][\mathbf{H}_t\mathbf{P}_t^{\,b}\mathbf{H}_t^{\,T} + \mathbf{R}_t]^{-1}$$

Predict equation

$$\mathbf{X}_t^{\,b} = \mathbf{M}_t\,\mathbf{X}_{t-1}^{\,a}$$

$$\mathbf{P}_t^{\,b} = \mathbf{M}_t\,\mathbf{P}_{t-1}^{\,a}\mathbf{M}_t^{\,T} + \mathbf{Q}$$

$\mathbf{M}_t$ forecast operator



Application of Data Assimilation on the global I-T system

There are many applications, including, but not limited to,

A. Generate model initial condition and time-dependent forcing for upper atmospheric weather forecasting

B. Extend the potential usage of observing system

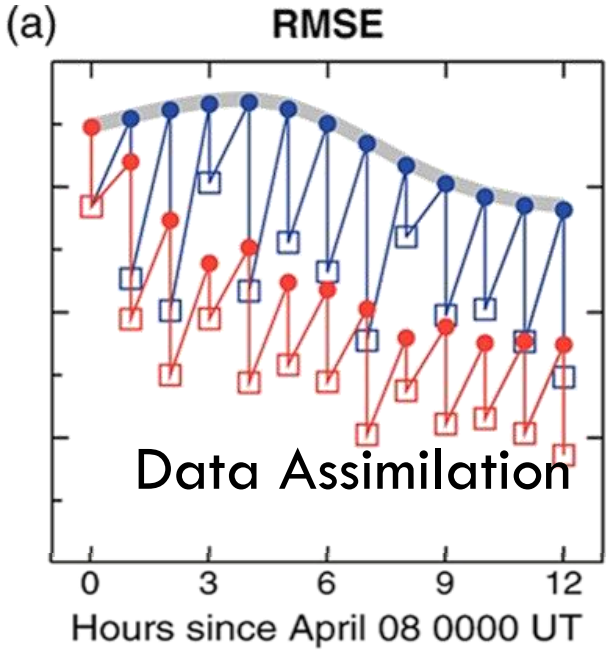
C. Produce better upper atmospheric specification for science investigation purpose

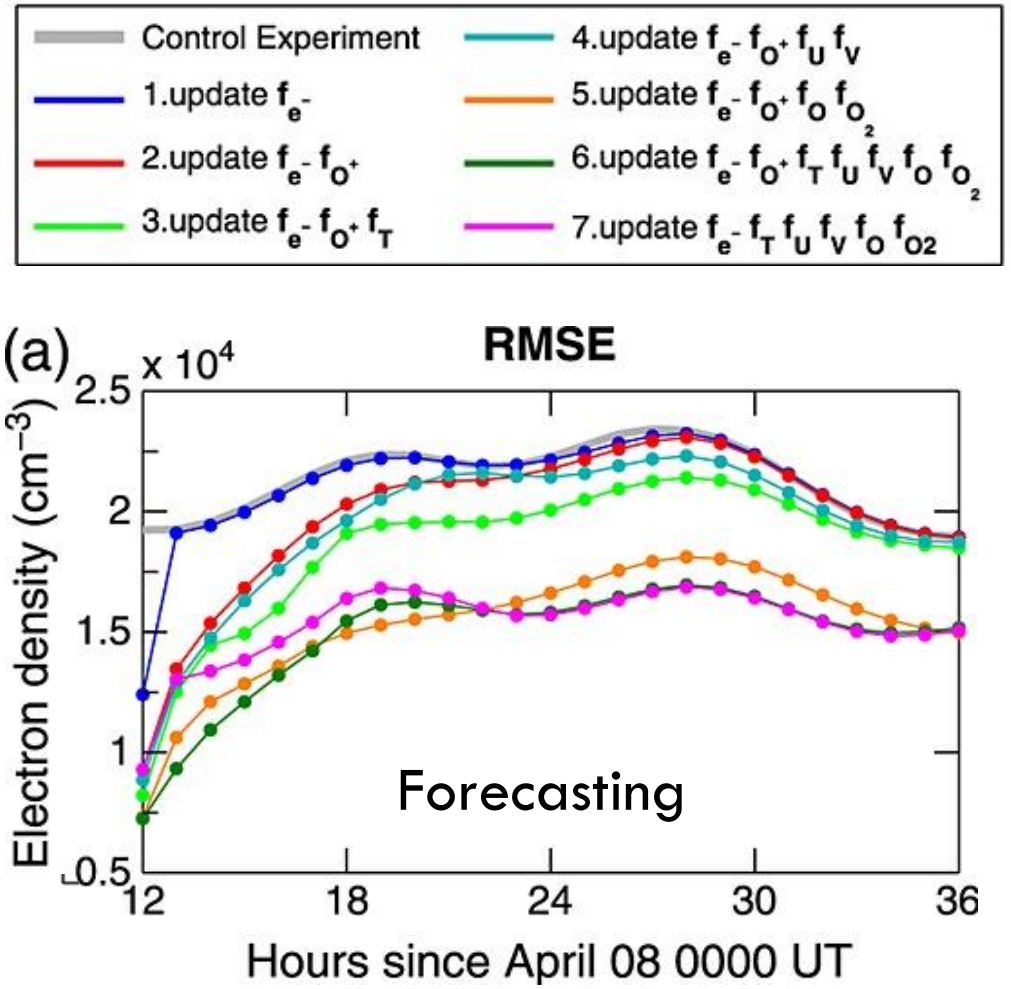
A. Upper Atmospheric Weather Forecasting – Hsu et al . [2014]

To predict the thermosphere and ionosphere, we need the knowledge of current states from observation.

- Model: TIEGCM
- Observation: RO electron density
- DA Method: EAKF (Ensemble Adjustment Kalman Filter)

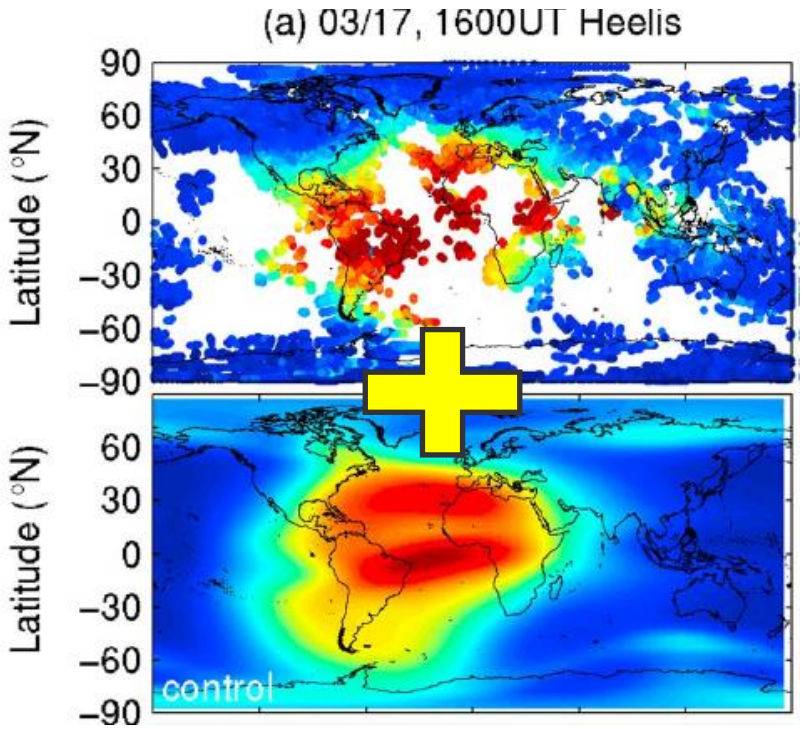
*RMSE: Root-mean-square error





B. Extend the potential usage of observing system Chen, et al. [2017]

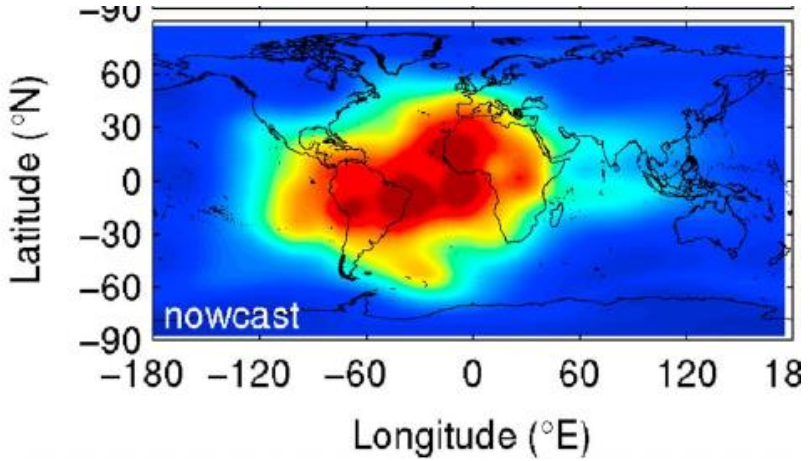
Ground-based GPS TEC



TIE-GCM

- Model: TIEGCM
- Observation: GPS TEC
- DA Method: EAKF

Data Assimilation Result



C. Produce better upper atmospheric specification for science investigation purpose
Hsu et al., [2021]

Observation

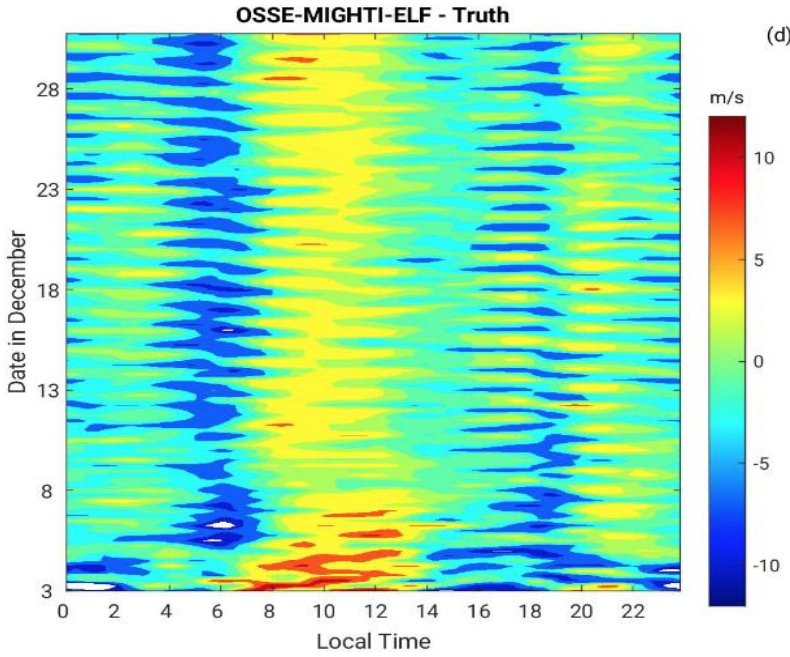
Data Assimilation

Updated state

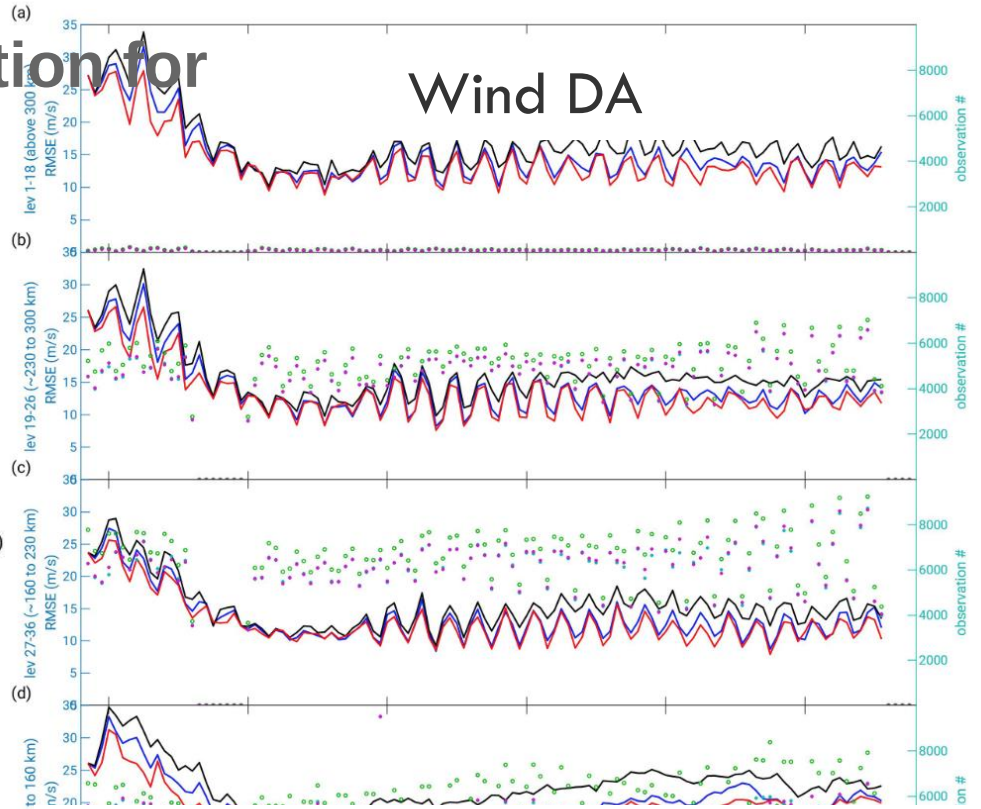
Model Physics

Hidden state

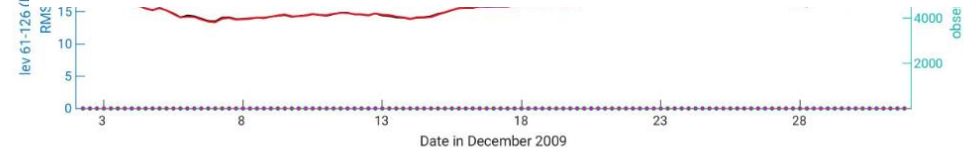
Equatorial Plasma Drift



Wind DA

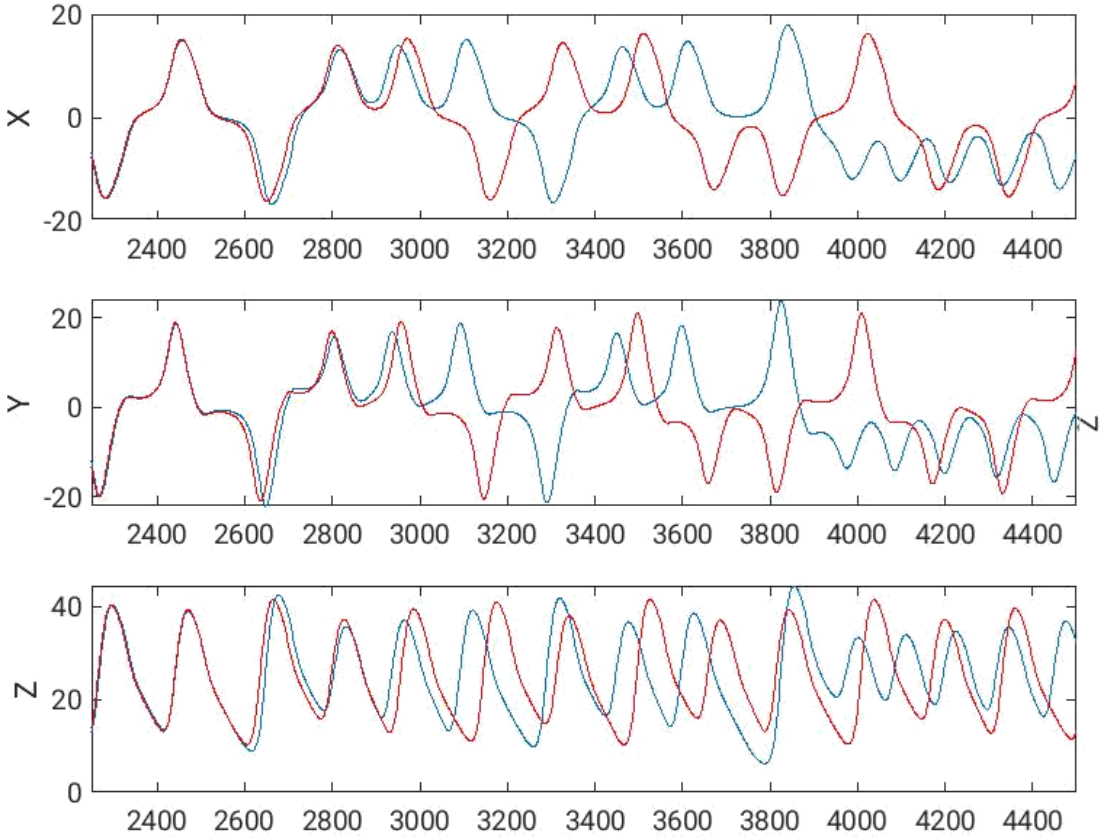


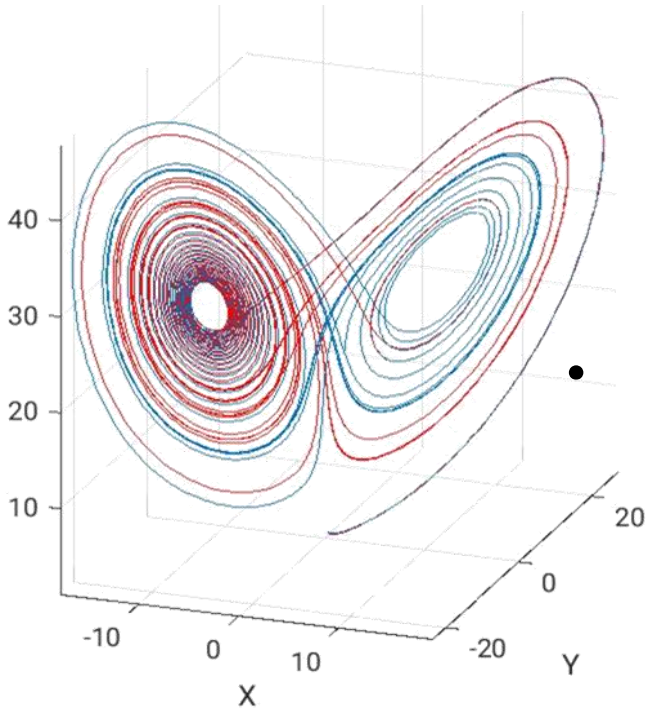
- Model: WACCM-X
- Observation: ICON MIGHTI
- DA Method: EAKF



Overview of this small tool

[xi, yi, zi]=[0 1 1.03]
[xi, yi, zi]=[0 1 1.02]





- Model : Lorenz Model
- DA method: Kalman Filter (without updating background error covariance)
- Purpose: Help student to understand how the whole data assimilation cycle works.

Overview of this small tool

Update equation

$$\mathbf{X}_t^{\,a} = \mathbf{X}_t^{\,b} + \mathbf{K}_t(\mathbf{y}_t^{\,o} - \mathbf{H}(\mathbf{X}_t^{\,b}))$$

$$\mathbf{P}_t^{\,a} = (\mathbf{I} - \mathbf{K}_t\mathbf{H}_t)\mathbf{P}_t^{\,b}$$

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Predict equation

Lorenz Model

$$\mathbf{X}_t^{\,b} = \mathbf{M}_t\,\mathbf{X}_{t-1}^{\,a}$$

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- Model : Lorenz Model
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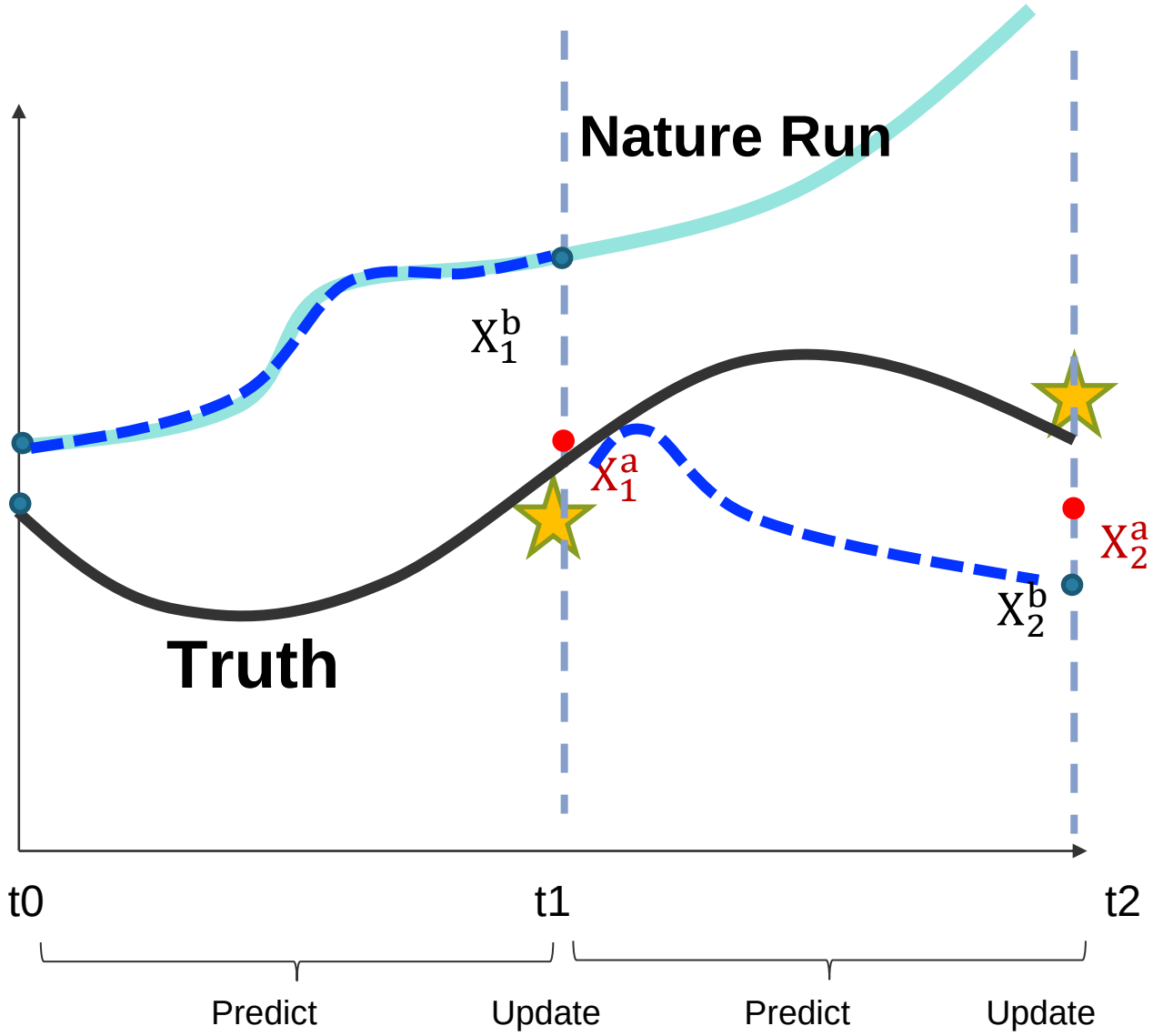

- Set up two initial conditions
- Set up observation error covariance and sampling frequency
- Set up background error covariance

References

Chen, C.-H., C. Lin, W.-H. Chen, and T. Matsuo (2017), Modeling the ionospheric prereversal enhancement by using coupled thermosphere-ionosphere data assimilation, Geophys. Res. Lett., 44, 1652–1659, doi:10.1002/2016GL071812.

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