U.S. NAVAL RESEARCH LABORATORY

IRI in Python for Rapid Global Runs
Global-IRI

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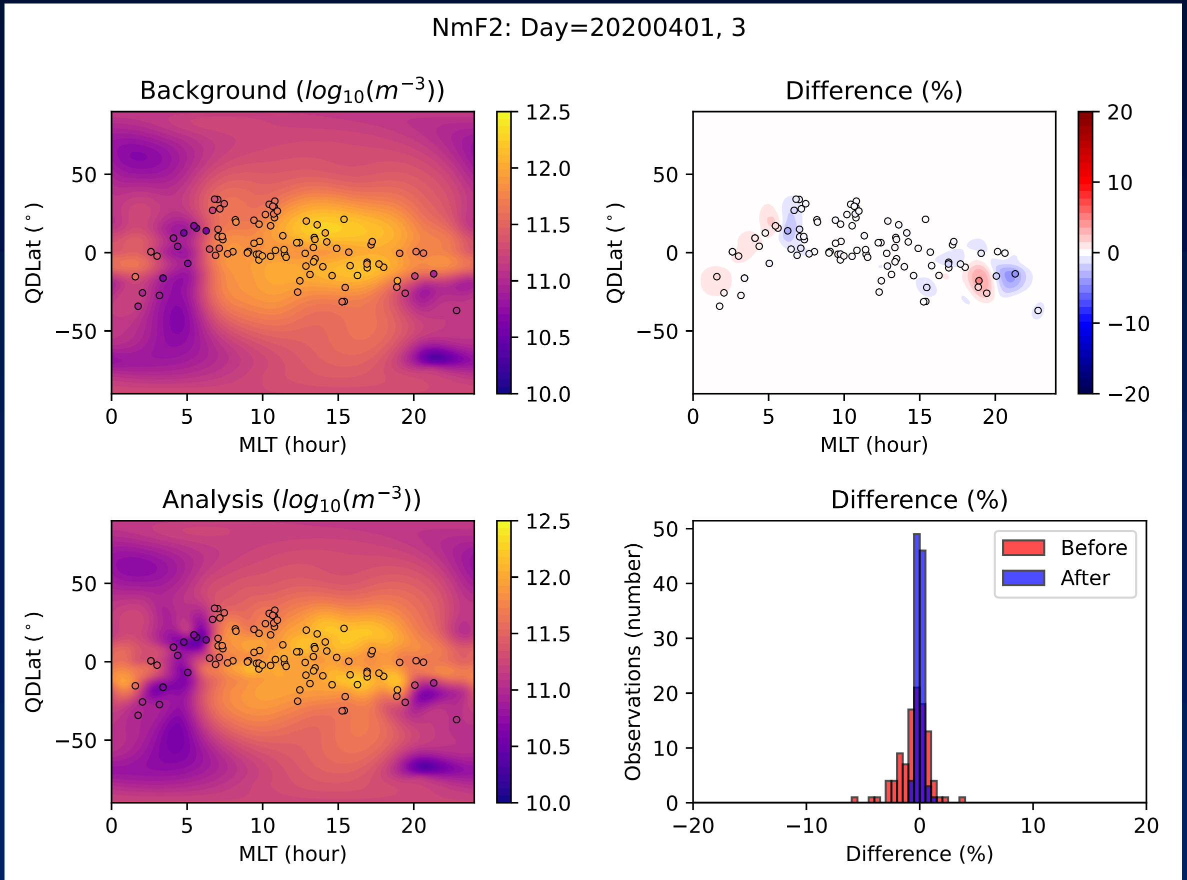
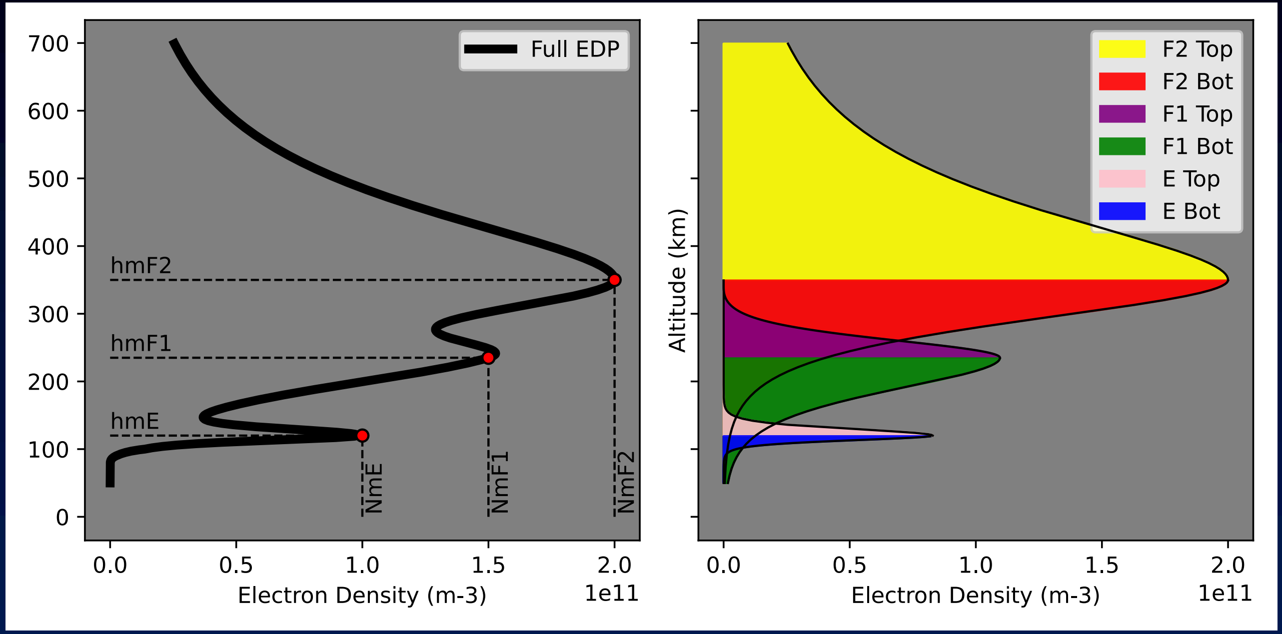
Motivation: ANCHOR

- A new DA model is being developed at NRL.
- It extracts anchor (hence the name) points from data and assimilates them in 2D maps using Kalman Filter.





- In order to try different assimilative techniques, including Ensemble Kalman Filter, we needed an ionospheric model that would produce rapid global outputs of the main parameters.
- Unfortunately, there is no such option to run IRI-2020 globally
- Usually the irregular grid is used for the DA, requiring point-by-point model run

Introduction: NmF2, hmF2

- The F2-peak is the most important parameter for any ionospheric model, because it is the highest value reached in the ionosphere [Bilitza et al., 2022]
- IRI uses CCIR coefficients (or URSI coefficients (same format)) to obtain the NmF2 and hmF2.

JOURNAL OF RESEARCH of the National Bureau of Standards—D. Radio Propagation
Vol. 66D, No. 4, July–August 1962

Representation of Diurnal and Geographic Variations of Ionospheric Data by Numerical Methods*

William B. Jones and Roger M. Gallet

NBS

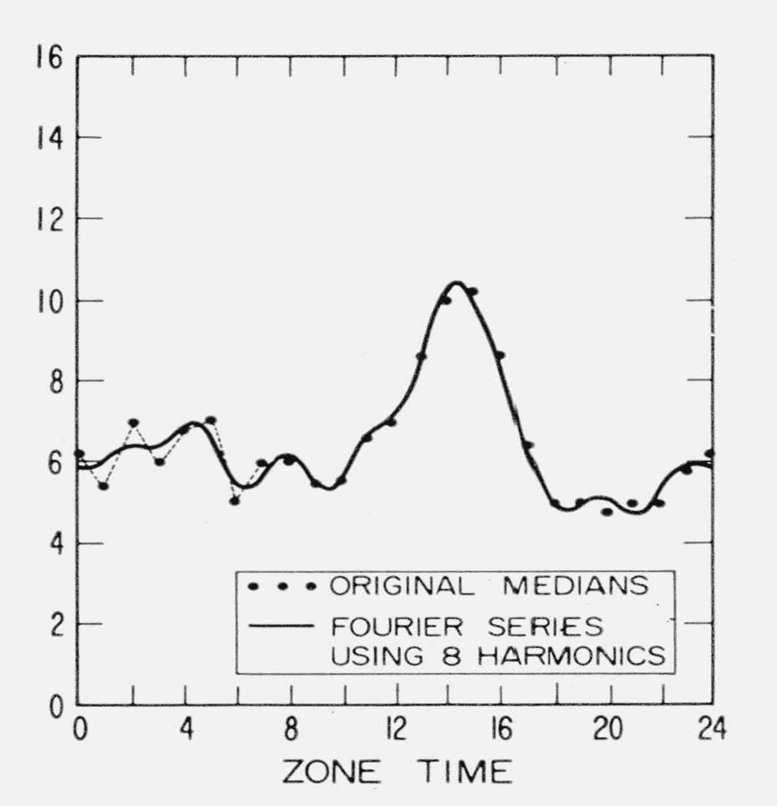
Technical Note No. 337

ADVANCES IN IONOSPHERIC MAPPING BY NUMERICAL METHODS

WILLIAM B. JONES, RONALD P. GRAHAM AND MARGO LEFTIN

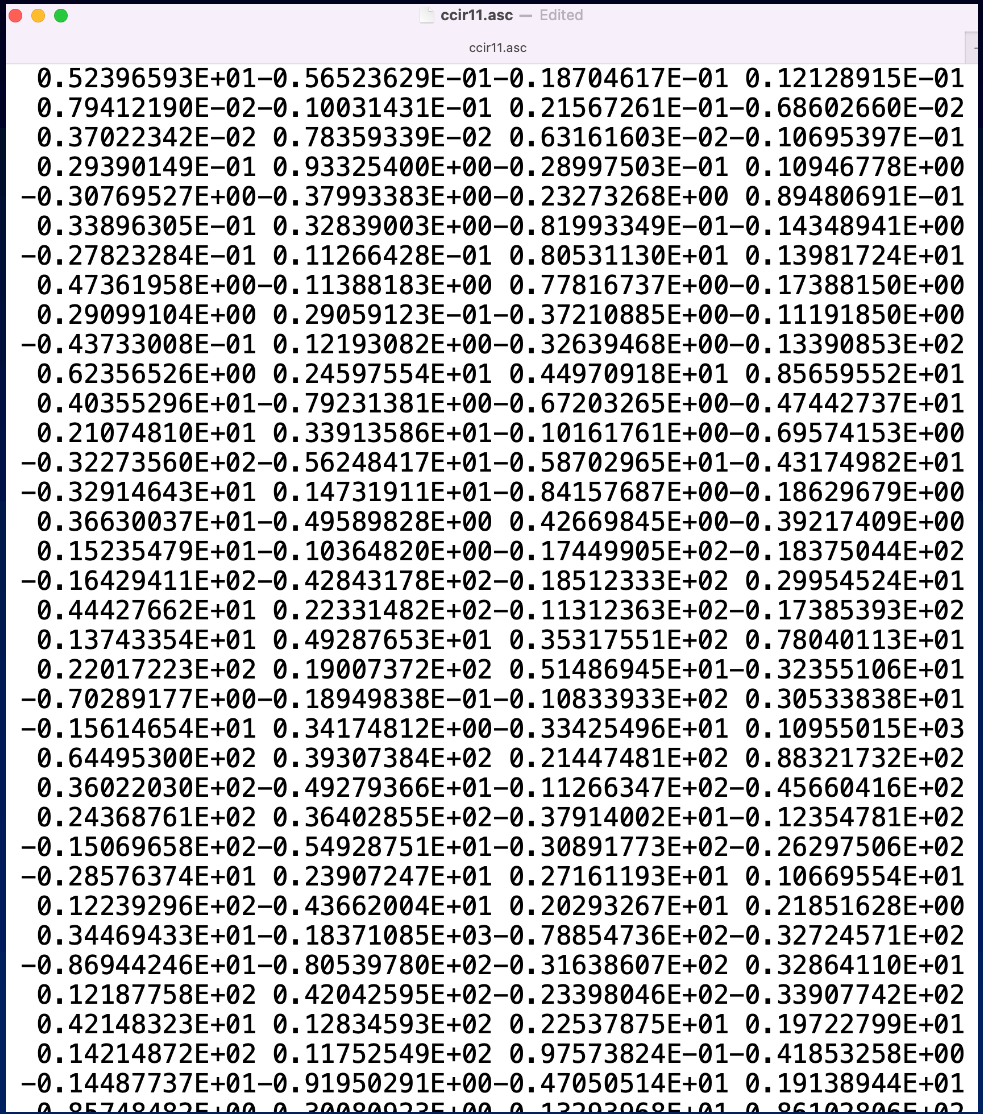
Coefficients

- Jones & Gallet, 1962 applied Least Squares method to find coefficients for global and diurnal functions that best describe the variations in the mean values of the ionospheric parameters.
- The diurnal variation of the parameters is represented by Fourier analysis of the 24-hour data from each station.
- Then the worldwide geographic variation of each Fourier coefficients is expanded in a series of functions similar to surface spherical harmonics.
- The noise is reduced by the truncations of the series.
- Result: table of coefficients that describe diurnal and geographical variation of the monthly mean ionospheric parameters.



CCIR files

- Inside one can find 4 columns of coefficients
- Very inconvenient to read (no space delimiters) and hard to understand what is what
- Inside coefficients for 2 levels of solar activity for foF2 and M3000
- Both have different sizes due to the truncation of series
- 2858 coefficients


ccir11.asc — Edited

ccir11.asc

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 0.37022342E-02 0.78359339E-02 0.63161603E-02-0.10695397E-01
 0.29390149E-01 0.93325400E+00-0.28997503E-01 0.10946778E+00
-0.30769527E+00-0.37993383E+00-0.23273268E+00 0.89480691E-01
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 0.62356526E+00 0.24597554E+01 0.44970918E+01 0.85659552E+01
 0.40355296E+01-0.79231381E+00-0.67203265E+00-0.47442737E+01
 0.21074810E+01 0.33913586E+01-0.10161761E+00-0.69574153E+00
-0.32273560E+02-0.56248417E+01-0.58702965E+01-0.43174982E+01
-0.32914643E+01 0.14731911E+01-0.84157687E+00-0.18629679E+00
 0.36630037E+01-0.49589828E+00 0.42669845E+00-0.39217409E+00
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-0.15614654E+01 0.34174812E+00-0.33425496E+01 0.10955015E+03
 0.64495300E+02 0.39307384E+02 0.21447481E+02 0.88321732E+02
 0.36022030E+02-0.49279366E+01-0.11266347E+02-0.45660416E+02
 0.24368761E+02 0.36402855E+02-0.37914002E+01-0.12354781E+02
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 0.12239296E+02-0.43662004E+01 0.20293267E+01 0.21851628E+00
 0.34469433E+01-0.18371085E+03-0.78854736E+02-0.32724571E+02
-0.86944246E+01-0.80539780E+02-0.31638607E+02 0.32864110E+01
 0.12187758E+02 0.42042595E+02-0.23398046E+02-0.33907742E+02
 0.42148323E+01 0.12834593E+02 0.22537875E+01 0.19722799E+01
 0.14214872E+02 0.11752549E+02 0.97573824E-01-0.41853258E+00
-0.14487737E+01-0.91950291E+00-0.47050514E+01 0.19138944E+01
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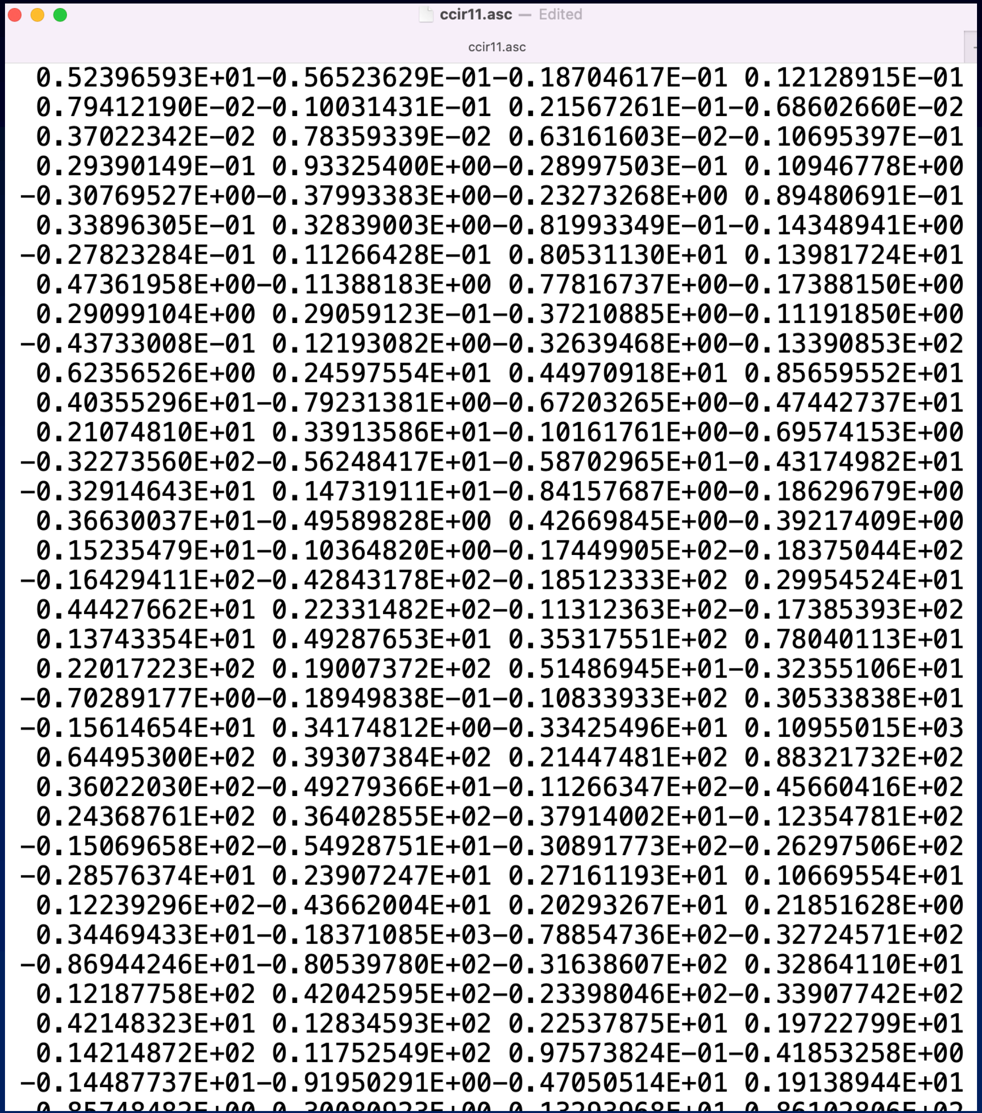
CCIR files

2858 numbers represent 2 arrays:

1976 → [13, 76, 2]

- the first array that contains coefficients for foF2



ccir11.asc — Edited

ccir11.asc

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-0.14487737E+01-0.91950291E+00-0.47050514E+01 0.19138944E+01
```

foF2 CCIR Coefficients

$$U = [13, 76, 2]$$

Diurnal functions

Global functions

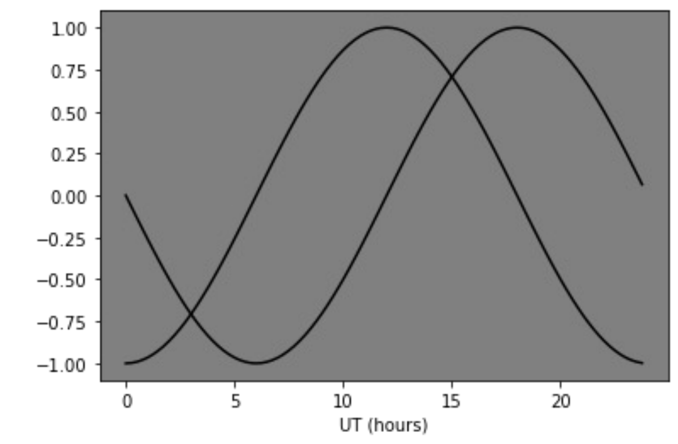
Min and Max levels of solar activity

- foF2 uses:
- 13 functions to describe diurnal variation
- 76 functions to describe global variation

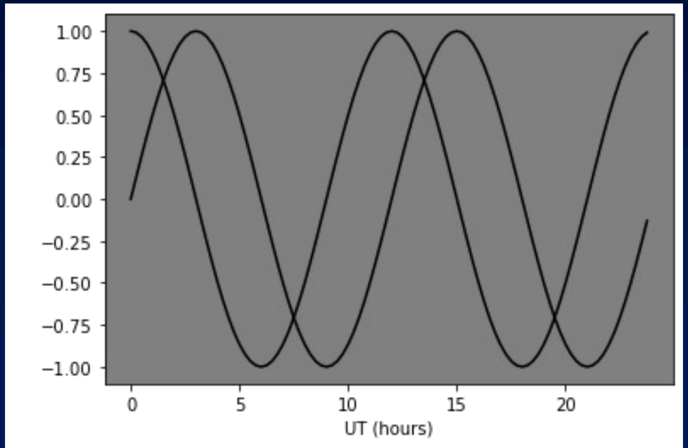
Diurnal Functions F_D (Fourier Series)

$U = [13, 76, 2]$

$$D_k(t) = C_{0,k} + \sum_{j=1}^{H} \left[C_{2j-1,k} \sin(jT_0) + C_{2j,k} \cos(jT_0) \right]$$



$\sin(T_0)$
$\cos(T_0)$



$\sin(2T_0)$
$\cos(2T_0)$

Global Functions F_G

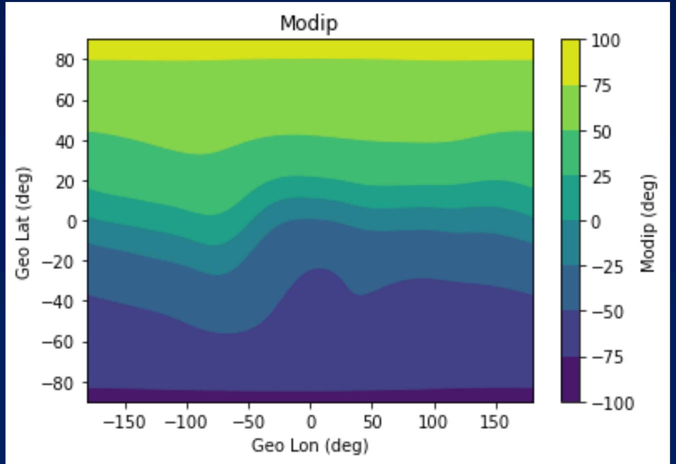
$U = [13, 76, 2]$

$$\sin^Q(x)\ \cos^L(\lambda)\ \begin{matrix}\cos(L\theta)\\ \sin(L\theta)\end{matrix}$$

8-degree harmonic expansion, L=0-8

Truncations:

$Q = [12,12,9,5,2,1,1,1,1]$



- Grid is defined as 1D array of Lat and 1D array of Lon
- Suitable for irregular grid
- 1°× 1° resolution gives $N_G = 65341$

k	$G_k(\lambda,\theta)$
0	1
1	$\sin x$
2	$\sin^2 x$
3	$\sin^3 x$
4	$\sin^4 x$
5	$\sin^5 x$
6	$\sin^6 x$
7	$\sin^7 x$
8	$\sin^8 x$
9	$\sin^9 x$
10	$\sin^{10} x$
11	$\sin^{11} x$
12	$\cos\lambda\ \cos\theta$
13	$\cos\lambda\ \sin\theta$
14	$\sin x\ \cos\lambda\ \cos\theta$
15	$\sin x\ \cos\lambda\ \sin\theta$
16	$\sin^2 x\ \cos\lambda\ \cos\theta$
17	$\sin^2 x\ \cos\lambda\ \sin\theta$
18	$\sin^3 x\ \cos\lambda\ \cos\theta$
19	$\sin^3 x\ \cos\lambda\ \sin\theta$
20	$\sin^4 x\ \cos\lambda\ \cos\theta$
21	$\sin^4 x\ \cos\lambda\ \sin\theta$
22	$\sin^5 x\ \cos\lambda\ \cos\theta$
23	$\sin^5 x\ \cos\lambda\ \sin\theta$
24	$\sin^6 x\ \cos\lambda\ \cos\theta$
25	$\sin^6 x\ \cos\lambda\ \sin\theta$
26	$\sin^7 x\ \cos\lambda\ \cos\theta$
27	$\sin^7 x\ \cos\lambda\ \sin\theta$
28	$\sin^8 x\ \cos\lambda\ \cos\theta$
29	$\sin^8 x\ \cos\lambda\ \sin\theta$
30	$\sin^9 x\ \cos\lambda\ \cos\theta$
31	$\sin^9 x\ \cos\lambda\ \sin\theta$
32	$\sin^{10} x\ \cos\lambda\ \cos\theta$
33	$\sin^{10} x\ \cos\lambda\ \sin\theta$
34	$\sin^{11} x\ \cos\lambda\ \cos\theta$
35	$\sin^{11} x\ \cos\lambda\ \sin\theta$
36	$\cos^2\lambda\ \cos 2\theta$
37	$\cos^2\lambda\ \sin 2\theta$
38	$\sin x\ \cos^2\lambda\ \cos 2\theta$
39	$\sin x\ \cos^2\lambda\ \sin 2\theta$

k	$G_k(\lambda,\theta)$
40	$\sin^2 x\ \cos^2\lambda\ \cos 2\theta$
41	$\sin^2 x\ \cos^2\lambda\ \sin 2\theta$
42	$\sin^3 x\ \cos^2\lambda\ \cos 2\theta$
43	$\sin^3 x\ \cos^2\lambda\ \sin 2\theta$
44	$\sin^4 x\ \cos^2\lambda\ \cos 2\theta$
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49	$\sin^6 x\ \cos^2\lambda\ \sin 2\theta$
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52	$\sin^8 x\ \cos^2\lambda\ \cos 2\theta$
53	$\sin^8 x\ \cos^2\lambda\ \sin 2\theta$
54	$\cos^3\lambda\ \cos 3\theta$
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58	$\sin^2 x\ \cos^3\lambda\ \cos 3\theta$
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66	$\sin x\ \cos^4\lambda\ \cos 4\theta$
67	$\sin x\ \cos^4\lambda\ \sin 4\theta$
68	$\cos^5\lambda\ \cos 5\theta$
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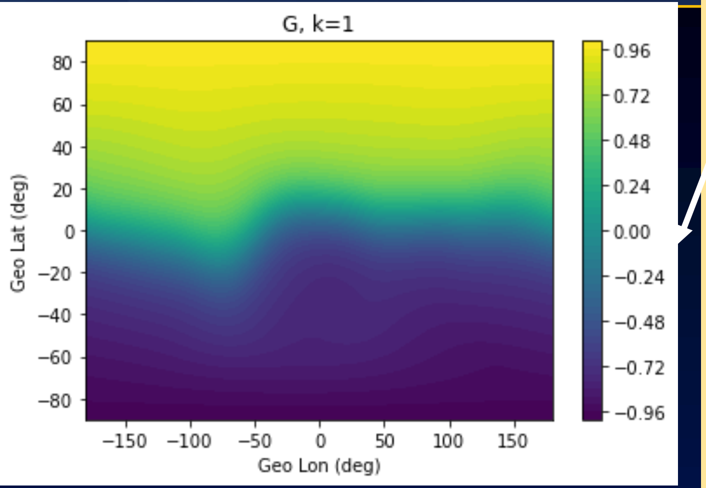
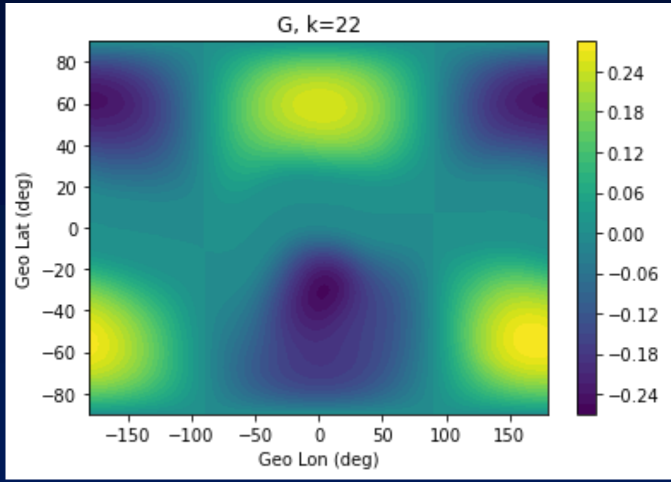
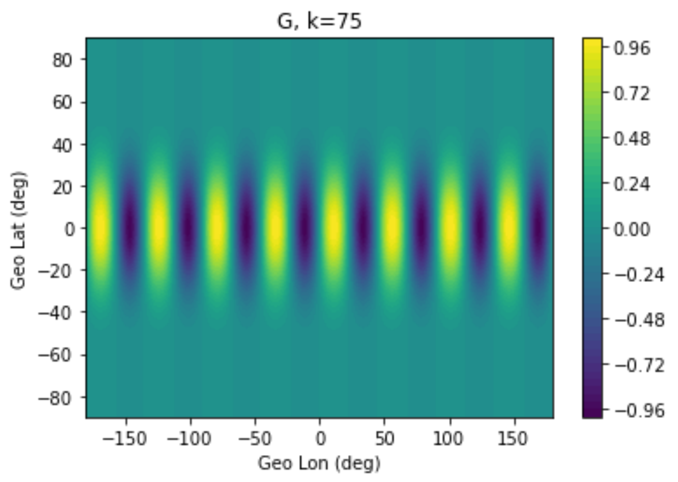
70	$\cos^6\lambda\ \cos 6\theta$
71	$\cos^6\lambda\ \sin 6\theta$
72	$\cos^7\lambda\ \cos 7\theta$
73	$\cos^7\lambda\ \sin 7\theta$
74	$\cos^8\lambda\ \cos 8\theta$
75	$\cos^8\lambda\ \sin 8\theta$

Global Functions F_G

$$U = [13, 76, 2]$$







$$\sin^Q(x)\ \cos^L(\lambda)\ \begin{matrix}\cos(L\theta)\\ \sin(L\theta)\end{matrix}$$

k	$G_k(\lambda,\theta)$
0	1
1	$\sin x$
2	$\sin^2 x$
3	$\sin^3 x$
4	$\sin^4 x$
5	$\sin^5 x$
6	$\sin^6 x$
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55	$\cos^3\lambda \sin 3\theta$
56	$\sin x \cos^3\lambda \cos 3\theta$
57	$\sin x \cos^3\lambda \sin 3\theta$
58	$\sin^2 x \cos^3\lambda \cos 3\theta$
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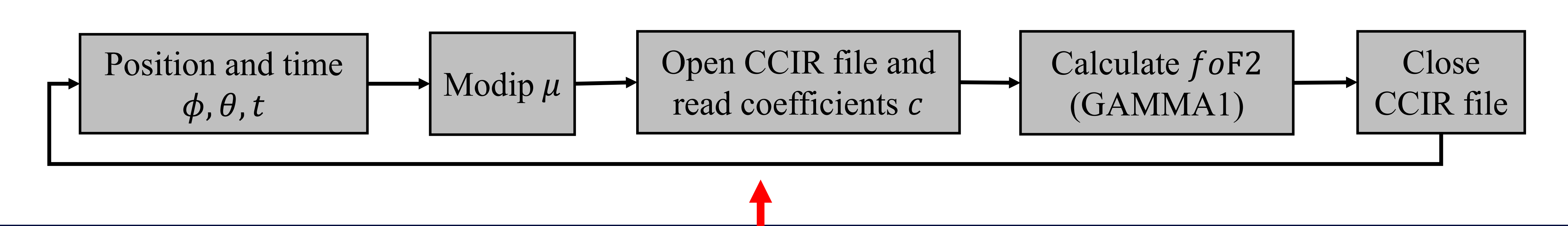
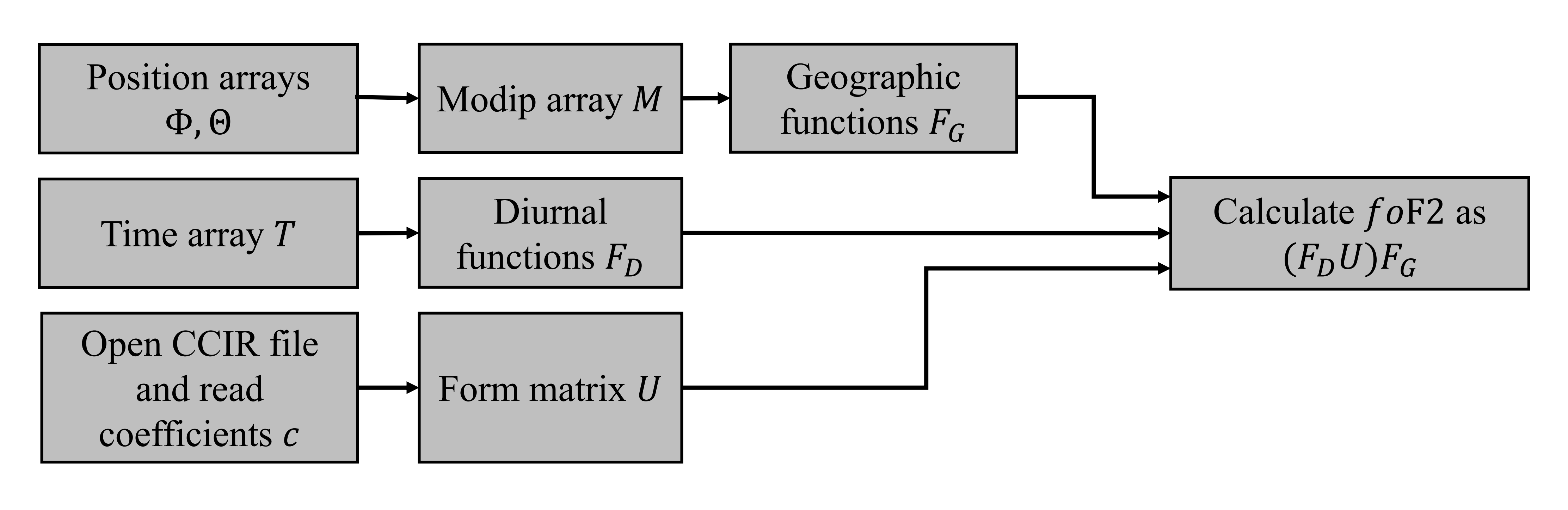
How IRI uses GAMMA1 function:



This file contains information for ALL locations, and ALL time frames

How Global-IRI does it



- Evaluation involves matrix multiplication instead of additions in the closed loops
- Suitable for high spatial and temporal resolution grids
- The output is 24-hour global distribution

Diurnal Functions to Matrix Form

$$D_k(t) = C_{0,k} + \sum_{j=1}^{H} \left[C_{2j,k} \cos(jT_0) + C_{2j-1,k} \sin(jT_0)\right]$$

index	j	function
0	0	1
1	1	$\sin(T_0)$
2	1	$\cos(T_0)$
3	2	$\sin(2T_0)$
4	2	$\cos(2T_0)$
5	3	$\sin(3T_0)$
6	3	$\cos(3T_0)$
…	…	…
13	6	$\cos(6T_0)$

$$U = [13, 76]$$

Calculate F_D for the array of UT with size $[N_T]$

$$F_D = [N_T, 13]$$

Global Functions to Matrix Form

k	$G_k(\lambda,\theta)$
0	1
1	$\sin x$
2	$\sin^2 x$
3	$\sin^3 x$
4	$\sin^4 x$
5	$\sin^5 x$
6	$\sin^6 x$
7	$\sin^7 x$
8	$\sin^8 x$
9	$\sin^9 x$
10	$\sin^{10} x$
11	$\sin^{11} x$
12	$\cos\lambda \cos\theta$
13	$\cos\lambda \sin\theta$
14	$\sin x \cos\lambda \cos\theta$
15	$\sin x \cos\lambda \sin\theta$
16	$\sin^2 x \cos\lambda \cos\theta$
17	$\sin^2 x \cos\lambda \sin\theta$
18	$\sin^3 x \cos\lambda \cos\theta$
19	$\sin^3 x \cos\lambda \sin\theta$
20	$\sin^4 x \cos\lambda \cos\theta$
21	$\sin^4 x \cos\lambda \sin\theta$
22	$\sin^5 x \cos\lambda \cos\theta$
23	$\sin^5 x \cos\lambda \sin\theta$
24	$\sin^6 x \cos\lambda \cos\theta$
25	$\sin^6 x \cos\lambda \sin\theta$
26	$\sin^7 x \cos\lambda \cos\theta$
27	$\sin^7 x \cos\lambda \sin\theta$
28	$\sin^8 x \cos\lambda \cos\theta$
29	$\sin^8 x \cos\lambda \sin\theta$
30	$\sin^9 x \cos\lambda \cos\theta$
31	$\sin^9 x \cos\lambda \sin\theta$
32	$\sin^{10} x \cos\lambda \cos\theta$
33	$\sin^{10} x \cos\lambda \sin\theta$
34	$\sin^{11} x \cos\lambda \cos\theta$
35	$\sin^{11} x \cos\lambda \sin\theta$
36	$\cos^2\lambda \cos 2\theta$
37	$\cos^2\lambda \sin 2\theta$
38	$\sin x \cos^2\lambda \cos 2\theta$
39	$\sin x \cos^2\lambda \sin 2\theta$
40	$\sin^2 x \cos^2\lambda \cos 2\theta$
41	$\sin^2 x \cos^2\lambda \sin 2\theta$
42	$\sin^3 x \cos^2\lambda \cos 2\theta$
43	$\sin^3 x \cos^2\lambda \sin 2\theta$
44	$\sin^4 x \cos^2\lambda \cos 2\theta$
45	$\sin^4 x \cos^2\lambda \sin 2\theta$
46	$\sin^5 x \cos^2\lambda \cos 2\theta$
47	$\sin^5 x \cos^2\lambda \sin 2\theta$
48	$\sin^6 x \cos^2\lambda \cos 2\theta$
49	$\sin^6 x \cos^2\lambda \sin 2\theta$
50	$\sin^7 x \cos^2\lambda \cos 2\theta$
51	$\sin^7 x \cos^2\lambda \sin 2\theta$
52	$\sin^8 x \cos^2\lambda \cos 2\theta$
53	$\sin^8 x \cos^2\lambda \sin 2\theta$
54	$\cos^3\lambda \cos 3\theta$
55	$\cos^3\lambda \sin 3\theta$
56	$\sin x \cos^3\lambda \cos 3\theta$
57	$\sin x \cos^3\lambda \sin 3\theta$
58	$\sin^2 x \cos^3\lambda \cos 3\theta$
59	$\sin^2 x \cos^3\lambda \sin 3\theta$
60	$\sin^3 x \cos^3\lambda \cos 3\theta$
61	$\sin^3 x \cos^3\lambda \sin 3\theta$
62	$\sin^4 x \cos^3\lambda \cos 3\theta$
63	$\sin^4 x \cos^3\lambda \sin 3\theta$
64	$\cos^4\lambda \cos 4\theta$
65	$\cos^4\lambda \sin 4\theta$
66	$\sin x \cos^4\lambda \cos 4\theta$
67	$\sin x \cos^4\lambda \sin 4\theta$
68	$\cos^5\lambda \cos 5\theta$
69	$\cos^5\lambda \sin 5\theta$
70	$\cos^6\lambda \cos 6\theta$
71	$\cos^6\lambda \sin 6\theta$
72	$\cos^7\lambda \cos 7\theta$
73	$\cos^7\lambda \sin 7\theta$
74	$\cos^8\lambda \cos 8\theta$
75	$\cos^8\lambda \sin 8\theta$

$$U = [13, 76]$$

Calculate F_G for the array of grid with size $[N_G]$

$$F_G = [76, N_G]$$

Multiplication with Coefficients

$$U = [13, 76]$$

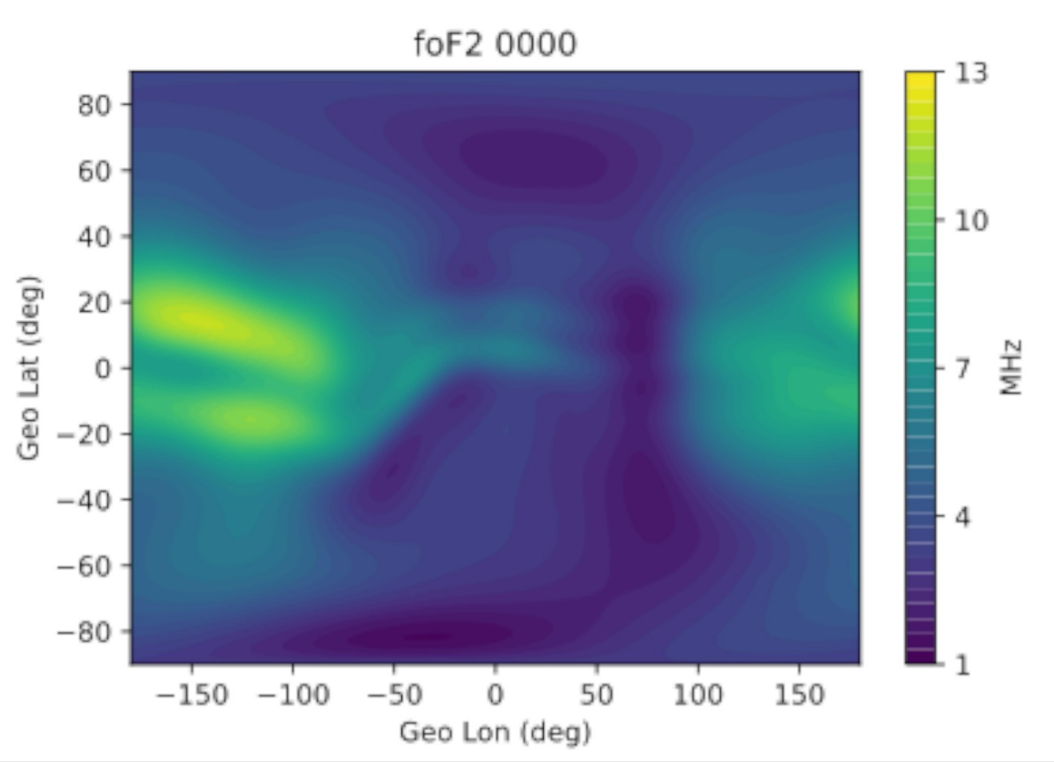
$$F_G = [76, N_G]$$

$$F_D = [N_T, 13]$$

$$GAMMA = (F_D\ U)\ F_G = [N_T, N_G]$$

- To obtain the same result, it would require to execute IRI for $N_T * N_G$ times
- 6,272,736 times
- Same can be done in 3 seconds

- Suitable for high spatial and temporal resolution grids



Future Directions

- This work will soon be published and the code will be made available to the community as a Python package.
- Stay tuned for the first release.