Time-of-Flight Positioning Enables Quantified Uncertainties and Easier Deployments for Meteor Radar

Avoiding Interferometry's Headaches with Few System Changes

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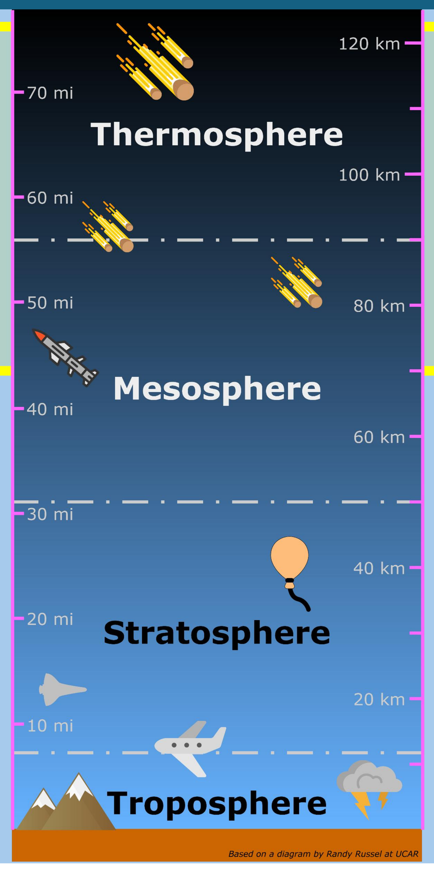
BACKGROUND



Specular meteor radar (SMR) remains one of the few tools able to densely sample the **mesosphere/lower thermosphere** (MLT, 70–120 km).[1]

- SMR systems **reflect radar waves off the plasma trails** left by ablating meteors.
- **Environmental characteristics** at the trail's location are estimated by comparing the signal sent to the echo received.[2,3]
- A single **transmit–receive** link can record 1,000+ meteor echoes every hour.[4]
- Helps reveal the structure and dynamics of the MLT.

SMRs currently use **interferometry** to determine the locations of meteor echoes. Utilizing a precisely placed **array of transmitting (TX)** antennas, a **receiver (RX) measures the relative phase** of the reflected signals **to infer the echo's direction**.[3,5]

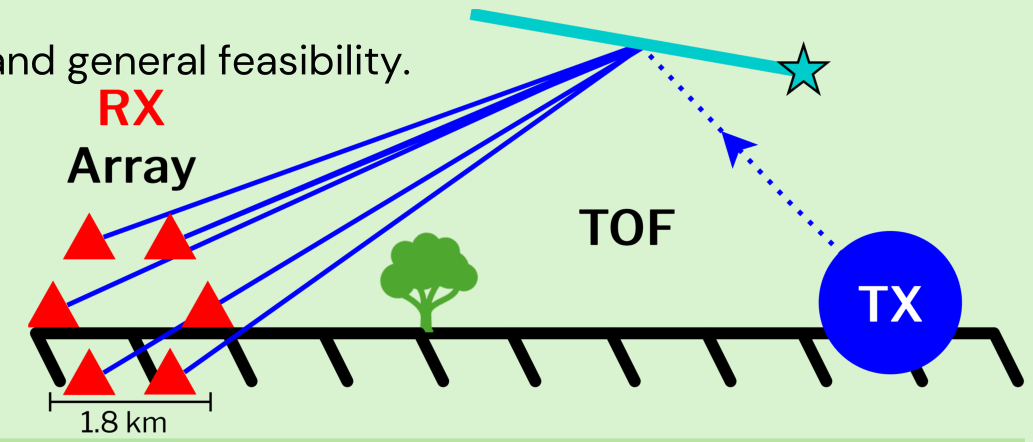
SUMMARY

Interferometry has drawbacks that **makes deployment, operation, and analysis difficult**, including tedious set-up and difficult to derive uncertainties.[6,7]

This work explores an **alternate time-of-flight** (TOF) **position solution** that avoids interferometry's problems while requiring few system changes. Using only 4+ range measurements, TOF positioning **enables easier SMR instrument deployments and scientific analysis**; this makes large-scale meteor radar networks more viable, which provide detailed mesoscale observations of the MLT and its dynamics.

This poster details the technique's development, focusing its precision and general feasibility. Key considerations include:

1. *Precision*: Measurement errors are estimated and propagated through to the final position solution.
2. *Required system changes*: increased spacing between antennas reduce positional uncertainties.
3. *Consistent spatial sampling*: Each TX–RX measurement should measure the same region of atmosphere.



Simulations show that **TOF positioning provides a calibration-free alternative to interferometry, achieving < 1 km vertical positioning accuracy** for the vast majority of observations and meeting the precision required for scientific analysis.

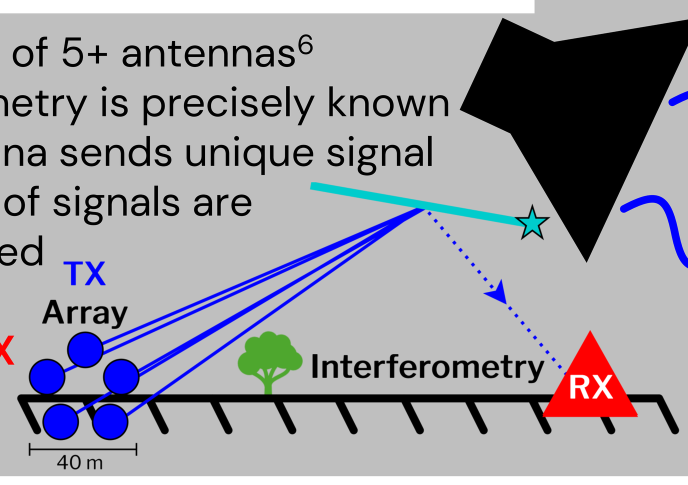
CONVENTIONAL POSITION SOLUTION

Interferometry

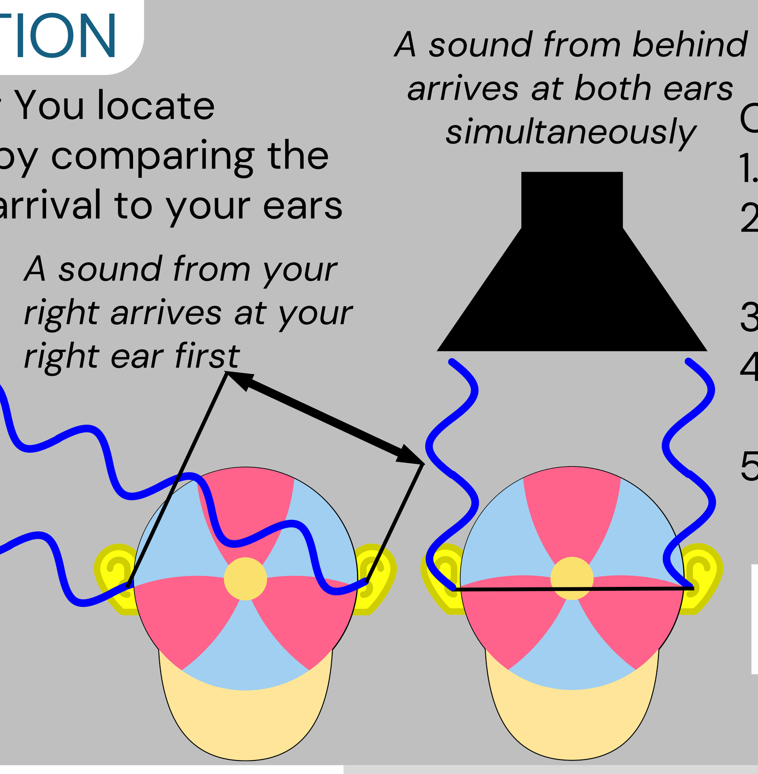
Using an **array of antennas**, differences in signals' phase are used to **determine the direction to the meteor**

Uses an array of 5+ antennas[6]

- Array geometry is precisely known
- Each antenna sends unique signal
- The **phase** of signals are synchronized
- Array can be TX or RX



Analogy: You locate sounds by comparing the time of arrival to your ears



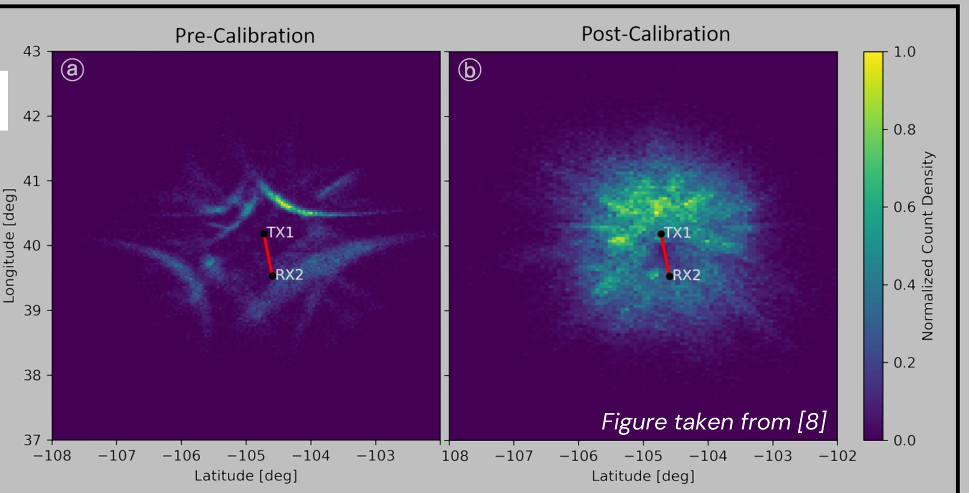
Overview of an interferometric position solution:

1. Each TX antenna transmits a unique signal.
2. Each signal arrives at the meteor trail using slightly different pathlengths.
3. Signals bounce off the meteor trail towards RX
4. The RX measures each signal's relative delay by measuring their phase.
5. Math is used to find the direction of arrival that best fits the signals' delays.

Each point in the sky will have a unique set of phase delays

Interferometry has drawbacks

These drawbacks make deployment, operation, and analysis difficult.

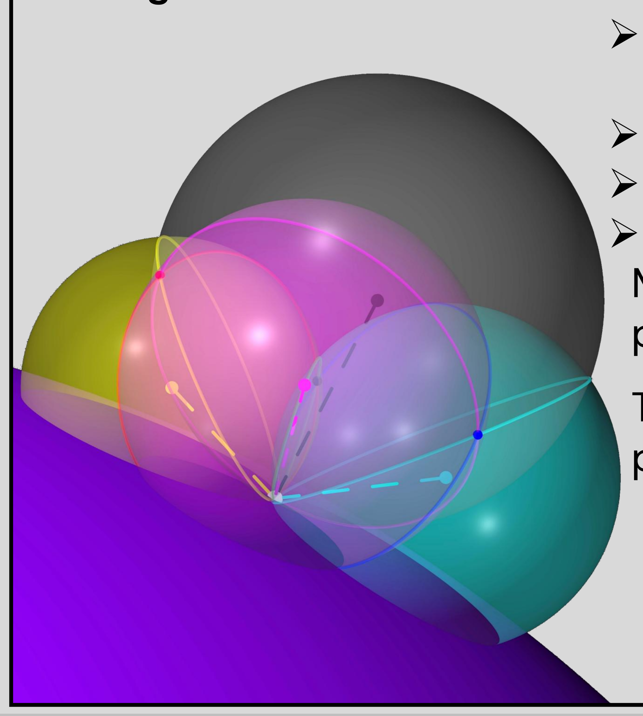


- Arrays must be **precisely placed**
- Requires **tedious calibration** to ensure accurate comparisons of phase[6]
- **Positional uncertainties are difficult to derive**[7]
- Interferometry fails for low SNR echoes, and these **observations are discarded**

TIME-OF-FLIGHT (*TOF*) POSITION SOLUTION

TOF Positioning in GNSS

Global Navigation Satellite Systems (GNSS) **estimate position using 4+ range measurements** from satellites with known locations[9]



- 1 range measurement constrains position to a *sphere*
- 2 measurements → *circle*
- 3 measurements → *2 points*
- **4 range measurements** → ***1 point!***

More measurements give more precision

TOF positioning solves many problems

- Provides **easily derived uncertainties**
- Robust at low SNRs
- **No user calibration required**

Adapting TOF Positioning for Meteor Radar

SMR measures **bistatic range:**
Distance from TX, to meteor trail, to RX

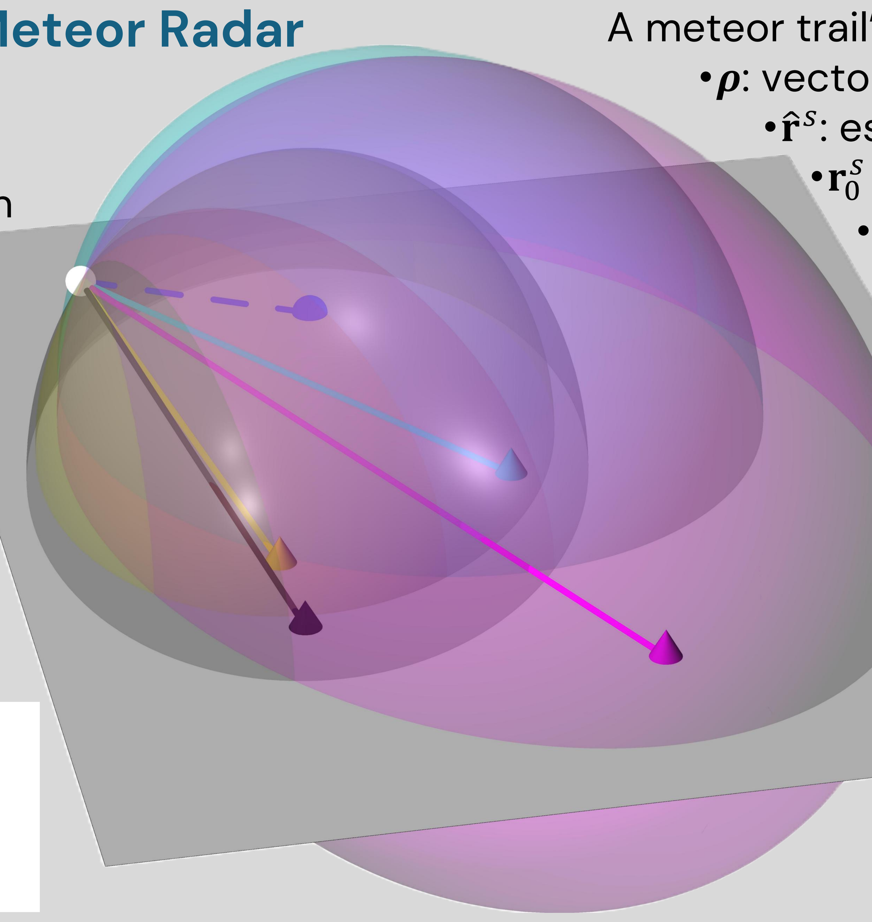
Each measurement constrains reflection point to an ellipsoid[10]

- Foci at TX, RX
- Major axis = bistatic range

Using RX_1, RX_2, RX_3, and RX_4:

- 1 bistatic range measurement constrains position to an *ellipsoid*
- 2 measurements → *ellipse*
- 3 measurements → *2 points*
- **4 range measurements** → ***1 point!***

A larger array footprint provides more geometric diversity in measurements for **a more precise TOF position solution**



A meteor trail's reflection point is estimated via iterative linearized least squares:

- $\boldsymbol{\rho}$: vector of 4+ bistatic range measurements
- $\hat{\mathbf{r}}^s$: estimated reflection point
- $\mathbf{r}_0^s = [x_0^s, y_0^s, z_0^s]$: linearization point
- $\mathbf{r}_{R_j} = [x^{R_j}, y^{R_j}, z^{R_j}]$: RX locations
- $\mathbf{r}_R = [x^T, y^T, z^T]$: TX locations
- $\mathbf{G}$: geometry matrix, relating TX/RX/$\mathbf{r}^s$ locations to bistatic range

$$\hat{\mathbf{r}}^s = \left(\mathbf{G}^\top \mathbf{G}\right)^{-1} \mathbf{G}^\top \left(\boldsymbol{\rho} - \boldsymbol{\rho}(\mathbf{r}_0^s)\right) + \mathbf{r}_0^s$$

$$\mathbf{G} = \begin{bmatrix} \frac{x_0^s - x^T}{|\mathbf{r}_0^s - \mathbf{r}^T|} + \frac{x_0^s - x^{R_1}}{|\mathbf{r}_0^s - \mathbf{r}^{R_1}|} & \frac{y_0^s - y^T}{|\mathbf{r}_0^s - \mathbf{r}^T|} + \frac{y_0^s - y^{R_1}}{|\mathbf{r}_0^s - \mathbf{r}^{R_1}|} & \frac{z_0^s - z^T}{|\mathbf{r}_0^s - \mathbf{r}^T|} + \frac{z_0^s - z^{R_1}}{|\mathbf{r}_0^s - \mathbf{r}^{R_1}|} \\ \vdots & \vdots & \vdots \\ \frac{x_0^s - x^T}{|\mathbf{r}_0^s - \mathbf{r}^T|} + \frac{x_0^s - x^{R_J}}{|\mathbf{r}_0^s - \mathbf{r}^{R_J}|} & \frac{y_0^s - y^T}{|\mathbf{r}_0^s - \mathbf{r}^T|} + \frac{y_0^s - y^{R_J}}{|\mathbf{r}_0^s - \mathbf{r}^{R_J}|} & \frac{z_0^s - z^T}{|\mathbf{r}_0^s - \mathbf{r}^T|} + \frac{z_0^s - z^{R_J}}{|\mathbf{r}_0^s - \mathbf{r}^{R_J}|} \end{bmatrix}$$

Each range measurement has uncertainty:

$$\sigma_\rho = \sqrt{2\sigma_{clk}^2 + 2\sigma_{pos}^2 + \sigma_{\Delta\tau}^2} = 8.99\ \mathrm{m}\ (\textit{typical})$$

- $\sigma_{clk} = 3$ m: TX/RX clock instability[11]
- $\sigma_{pos} = 1$ m: TX/RX positional uncertainty
- $\sigma_{\Delta\tau} \approx 7.8$ m: median correlation error[9,12]

Uncertainties are easily propagated:

$$\mathbf{H} = \left(\mathbf{G}^\top \mathbf{G}\right)^{-1}$$

$$\sigma_{\text{tot}} = \sigma_\rho \cdot \sqrt{H_{1,1} + H_{2,2} + H_{3,3}} \qquad \sigma_E = \sigma_\rho \cdot \sqrt{H_{1,1}}$$

$$\sigma_{\text{horiz}} = \sigma_\rho \cdot \sqrt{H_{1,1} + H_{2,2}} \qquad \sigma_N = \sigma_\rho \cdot \sqrt{H_{2,2}} \qquad \sigma_U = \sigma_\rho \cdot \sqrt{H_{3,3}}$$

A smaller array footprint ensures consistent spatial sampling

Problem:
Each RX has slightly different reflection point along trail

- Breaks TOF assumptions
- Samples different parts of wind field

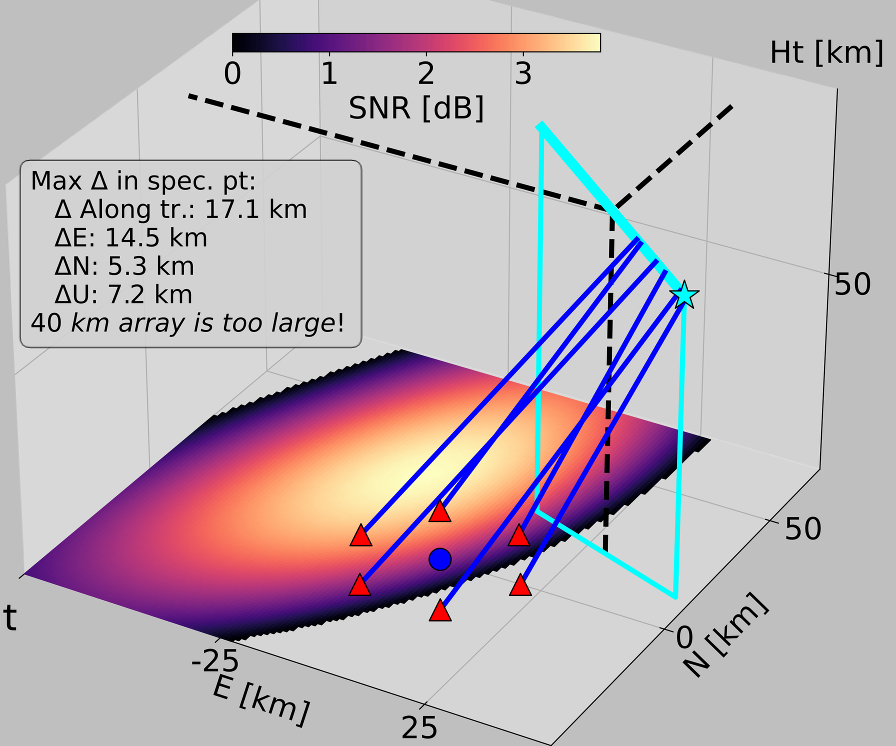
Solution:
Ensure change in reflection point is small

- Max change = $\frac{\text{Max RX Separation}}{2}$

All RX should overlap **Fresnel zones**[1]

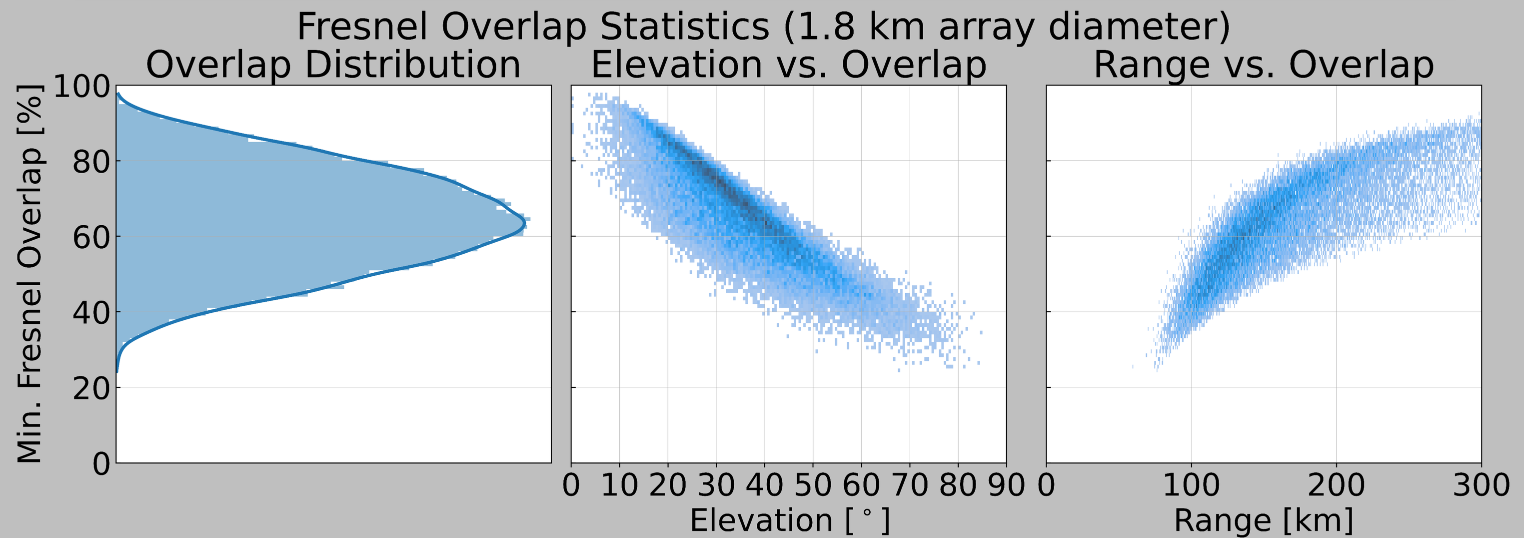
- Region that reflects >90% power
- ~0.7 km on either side of reflection point
- Overlapping Fresnel zones mean similar spatial sampling

Investigating the Change in Specular Point
40 km array diameter



Having all RX within 1.8 km ensures significant Fresnel zone overlap

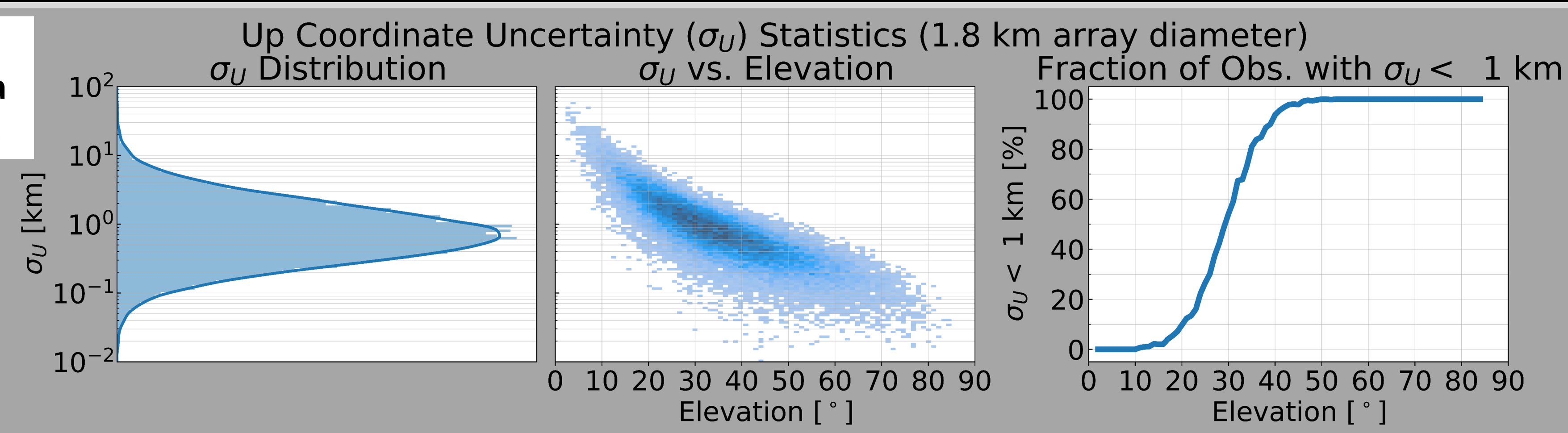
Fresnel Overlap Statistics (1.8 km array diameter)



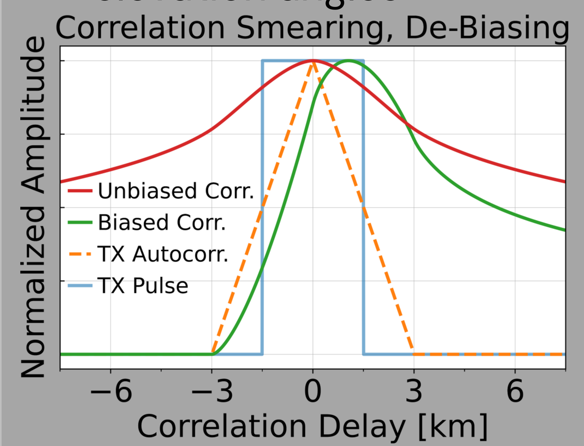
A Monte Carlo simulation is used to estimate positional uncertainties for a representative population of meteors

- Scientific analyses require < 1 km of uncertainty in the "up" direction[4]
- Most observations >30° elevation meet this requirement
- Uncertainties get larger at low elevation angles

Up Coordinate Uncertainty (σ_U) Statistics (1.8 km array diameter)



Correlation Smearing, De-Biasing



Range uncertainty due to correlation ($\sigma_{\Delta\tau}$) is estimated for long meteor trails

- Range is estimated via correlation-derived time delay
- A point-like object provides a sharp correlation function for small correlation uncertainty (orange)
- A long meteor trail smears and biases the correlation function towards positive ranges (green)
- The smeared correlation function is de-biased by accounting for the coherent reflection along the entire trail (red)

$$\sigma_{\Delta\tau} \approx T_c \sqrt{\frac{1}{2\,T_{coh} C/N_0}}$$

$\sigma_{\Delta\tau}$ depends on chip duration (T_c), carrier-to-noise ratio (C/N_0), and coherent integration time (T_{coh}).[9,12]

CONCLUSIONS

A time-of-flight position solution locates a meteor trail's specular point of reflection using multiple range measurements. This shows promise as **a viable alternative** to the current standard of interferometry.

- TOF solution requires physically larger arrays, but no other system changes
- No calibration required, making **instrument deployment easier**
- **Easily derived uncertainties** help scientific analyses weight each measurement
- For most observations, **TOF precision meets science requirements**
- TOF's robustness at low SNR mean **fewer observations are discarded**

TOF allows larger networks to be more easily deployed and analyzed, enabling dense mesoscale observations of the MLT. **Alternatively, TOF could be used in conjunction with interferometry** to aid calibration and reduce ambiguities of low-SNR observations.

FUTURE WORK

Next steps to show TOF's viability:

- Verify de-biasing by performing correlations with real data.
- Examine effects of error in assumed TX/RX positions.
- Validate in an experimental campaign to directly compare interferometric and TOF solutions.

Contact & References