Extended MHD Equatorial Ionosphere Model

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Introduction

We present recent advances in our Extended Magnetohydrodynamics (XMHD) model, derived from the PERSEUS solver, optimized for simulating **Equatorial Spread F (ESF)** irregularities. These interchange instabilities manifest in the ionospheric F-region as large-scale, turbulent plasma bubbles, which severely disrupt global navigation (GPS), satellite communications, and radar systems.

Traditional models often simplify these phenomena by treating plasma under isothermal (or single-temperature fluid) and electrostatic assumptions. Our framework uniquely includes independent multi-species **thermodynamic equations** and implements a **fully electromagnetic** formulation alongside the **fluid dynamics**. In this work, we demonstrate the solver's capability to capture the non-linear, structural evolution of the underlying **Rayleigh–Taylor instability**, providing a comprehensive platform to study how rising plasma bubbles directly excite and radiate **Alfvén waves** into the background ionosphere.

Numerical Framework

Finite Volume Method ensures conservation at the discrete level by:

- Solving XMHD as conservation law for each cell:

$$\frac{\partial \vec{q}}{\partial t} + \mathrm{div}\overleftrightarrow{F}(\vec{q}) = \vec{S}(\vec{q}) \longrightarrow \vec{q}^{\,*} = \vec{q}^{\,n} - \frac{\Delta t}{V}\Phi + \Delta t \vec{S}^{n}$$

- Tracking volume averages and computing net fluxes across cell's faces with 5th order Rusanov Flux, TVD, Flux limiters.
- Predictor-corrector Hybrid Implicit/Explicit time advance scheme.
- Relaxation method for Generalized Ohm's law.
- Continuous BC (0th order extrapolation).
- Built for orthogonal curvilinear coordinates.
- Parallelized architecture utilizing MPI.

Governing Physics and Instability mechanism

The ionospheric plasma is modeled as an electrically conducting fluid governed by a **combination of Navier-Stokes and Maxwell's equations:**

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\bar{u} = 0$$

$$\frac{\partial \bar{u}}{\partial t} + \nabla\cdot(\bar{u}\bar{v}) = \bar{J}\times\bar{B} - \nabla p + \rho\bar{g} - \left(\frac{m_e}{m_i}\nu_{en} + \nu_{in}\right)[\bar{u} - \rho\bar{w}] + \nu_{en}\frac{m_e}{e}\bar{J}$$

$$\frac{\partial \varepsilon_e}{\partial t} + \nabla\cdot[\bar{v}(\varepsilon_e + p_e)] = -n_e e \bar{v}_e\cdot\bar{E} - 3\nu_{ei}n_i(T_e - T_i) - 3\nu_{en}n_e(T_e - T_n) + \frac{m_e}{e}\nu_{en}\bar{J}\left(\bar{w} - \frac{1}{ne}\bar{J}\right)$$

$$\frac{\partial \varepsilon_i}{\partial t} + \nabla\cdot[\bar{v}(\varepsilon_i + p_i)] = \bar{v}\cdot(\bar{J}\times\bar{B}) - 3\nu_{ei}n_i(T_i - T_e) - 3\nu_{in}n_i(T_i - T_n) - \nu_{in}\bar{v}[\bar{u} - \rho\bar{w}]$$

$$\frac{\partial \bar{B}}{\partial t} + \nabla\times\bar{E} = 0 \qquad \frac{\partial \bar{E}}{\partial t} - c^2\nabla\times\bar{B} = -\mu_0 c^2\bar{J}$$

$$\frac{\partial \bar{J}}{\partial t} + \nabla\cdot\left(\bar{v}\bar{J} + \bar{J}\bar{v} - \frac{\bar{J}\bar{J}}{n_e e}\right) - \frac{e}{m_e}\nabla p_e = \frac{n_e e^2}{m_e}\left(\bar{E} + \bar{v}\times\bar{B} - \frac{1}{ne}\bar{J}\times\bar{B} - \eta\bar{J}\right) + \nu_{en}\frac{e}{m_i}[\bar{u} - \rho\bar{w}]$$

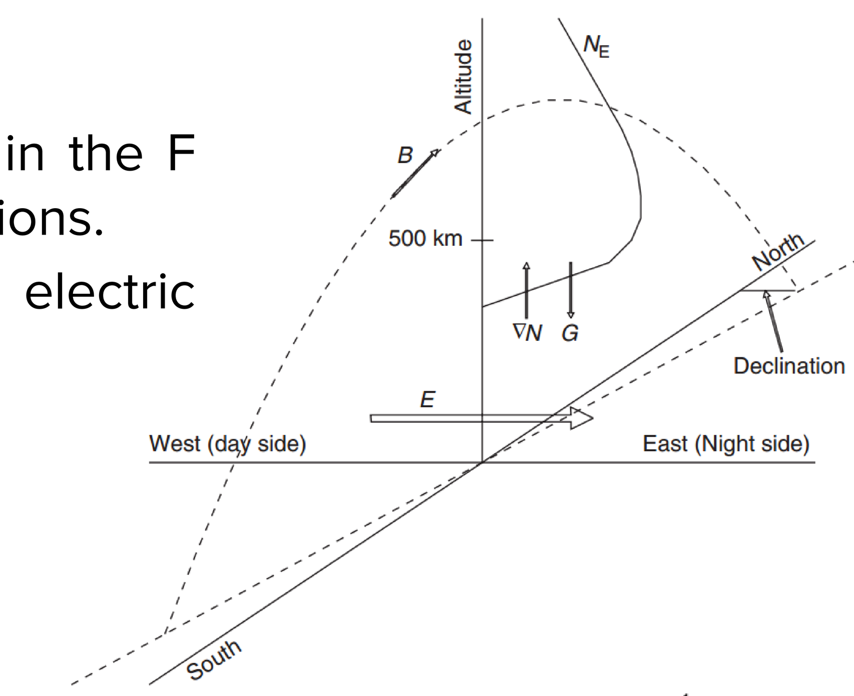
$$\varepsilon_s = \rho_s v_s^2 + \frac{p_s}{\gamma - 1} \qquad p_s = n_s k_B T_s \qquad \eta = \frac{m_e\nu_{ei}}{n_e e^2}$$

ESF Instability Mechanism:

- Triggered by steep density gradients in the F region produced by post-sunset conditions.
- Pressure gradients, gravitational and electric fields drive $\vec{F}\times\vec{B}$ drifts.

$$\vec{v}_F = \frac{\vec{F}\times\vec{B}}{qB^2}$$

  - Gravitational force: $\vec{F} = m\vec{g}$
  - Pressure gradient: $\vec{F} = -\nabla p$
  - Electric field: $\vec{F} = q\vec{E}$ ➔ leads to $\vec{E}\times\vec{B}$ **drift**
- Any initial boundary perturbation wiggles the interface, creating local charge separation and secondary polarization electric fields, The resulting upward drift forces the low density to rise forming plasma plumes.



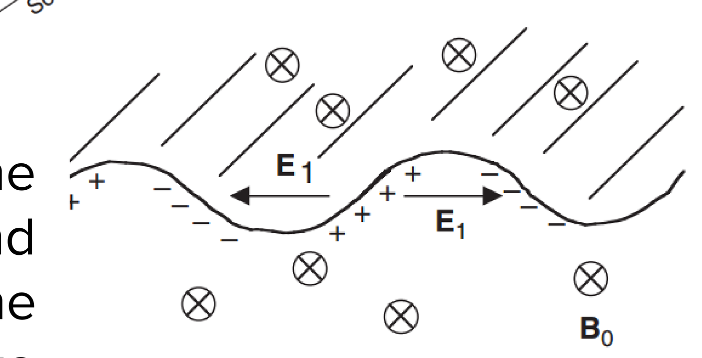


Fig 1: Starting stage for Rayleigh–Taylor instability.

Simulation Results

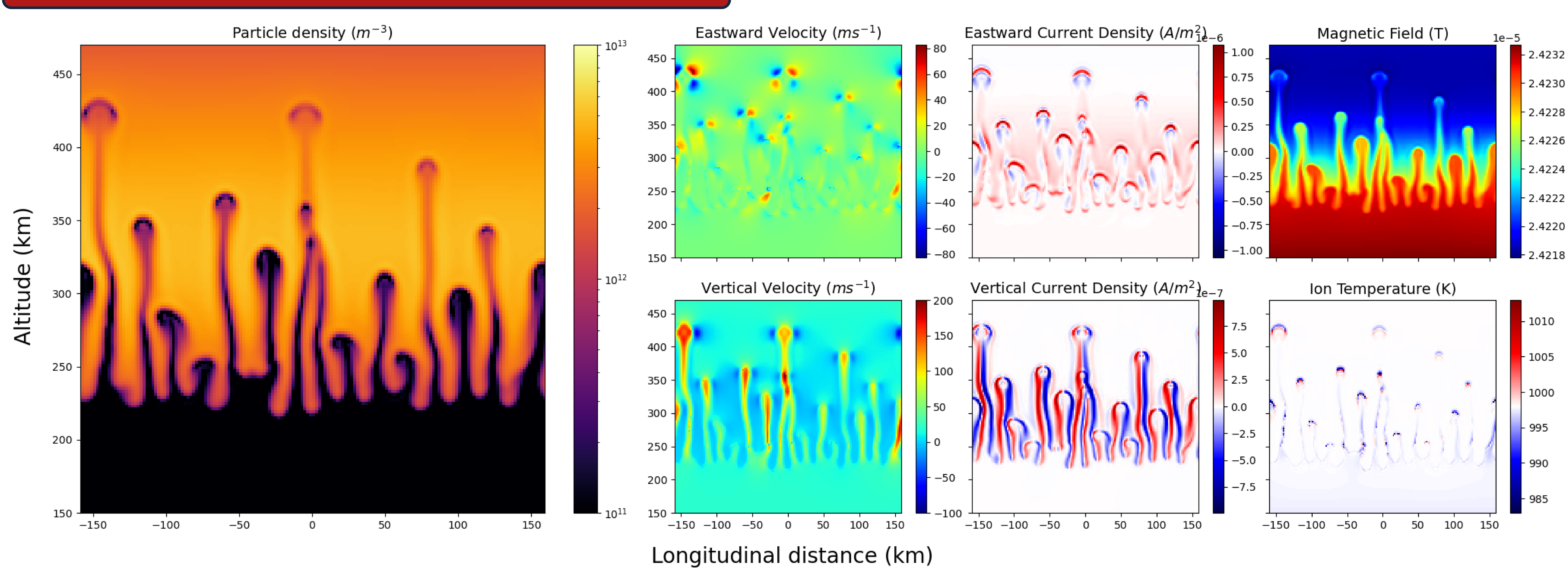


Fig 2: Fully developed ESF instability recovered by our XMHD solver.

- Plasma density: Our XMHD solver recovers the classic "mushroom-head" plumes that drifts upward into the upper F-region.
- Velocity and Current Fields: High-speed plasma is injected into the bubble cores, while current sheets are formed along the bubble walls.
- Temperature fluctuations of plasma bubbles are captured as a consequence of adiabatic expansion and collisional Joule heating.
- B field lines mirrors the plasma density, since in a macroscopic view, "frozen-in" flux lines are dragged upward by the rising bubbles. At small spatial scales, however, the Generalized Ohm's law allows the magnetic field lines to sliver, diffuse, and slip slightly past the bulk fluid and thus driving the excitation of Alfvén waves.

Conclusions

The 3D XMHD solver serves as a highly robust tool for modeling ionospheric phenomena. By successfully coupling multi-species energy transport and the complete Maxwell equations, the solver natively resolves sub-fluid timescales while remaining fully electromagnetic and non-isothermal.

The FVM cleanly allows us to use orthogonal curvilinear coordinates.

The solver also incorporates different background parameters as neutral wind coupling, and dynamic neutral and Coulomb collisions, which could also be integrated with data-driven circulation models.

The next step is to study the spectral energy distribution of Alfvén Waves excited by plasma bubbles.

[1] **Seyler, C. E.**, & Martin, M. R. (2011). Relaxation model for extended magnetohydrodynamics: Comparison to magnetohydrodynamics for dense Z-pinches. *Physics of Plasmas*, *18*(1).