

ABSTRACT

Romashets and Vandas (2020, 2022) derived Euler potentials for field-aligned currents (FACs) above the ionosphere, introducing a local parameter g controlling current density. The case of constant g was modeled in Romashets and Vandas (2025) to analyze vertical shell current distribution. Modeling with variable g is more complex. Here, we present results for shell current density, magnetic field, and Euler potentials inside the ionosphere, using a realistic, observation-based g derived from Korth et al. (2010) FAC data for quiet and disturbed periods.

INTRODUCTION

- In previous research, we constructed Euler potentials to describe Earth's magnetosphere—combining Earth's dipole field, field-aligned currents, the ring current, and surface currents modeled by Dungey's term.
- Field-aligned currents, or Birkeland currents, connect the magnetosphere to the ionosphere and close through horizontal currents known as shell currents. These are key to regulating Earth's magnetic environment.
- While FACs can be traced from satellite data, their ionospheric closure paths—especially Pedersen and Hall currents—are harder to define due to the complex conductivity structure of the ionosphere.
- To overcome these challenges, we divide the system into three regions and incorporate both the dipole field and FAC-generated fields. This lets us calculate current densities in a realistic, finite-thickness ionosphere.

ACKNOWLEDGMENT

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METHOD

Euler Potentials for Inner & Outer Regions:

$$\alpha = B_0 r_0^3 \frac{\sin^2 \theta}{r}$$

Below 100 km: $\beta = -\phi$

Above 400 km: $\beta = -\phi + g_0 r^2 \frac{\cos \theta}{r_0^2 \sin^4 \theta}$

Using Tanh Function for Smooth ϕ Field

$$B_\phi = \frac{B_0 g_0 r_0}{2r \sin \theta} \left\{ 1 + \tanh \left[\frac{6(r - r_c)}{\Delta r} \right] \right\}$$

The smooth transition of the magnetic field is modeled using the function:

$$st(r) = \frac{1}{2} \left[1 + \tanh \left(\frac{6(r - r_c)}{\Delta r} \right) \right]$$

This function is approximated by a power series:

$$sm(r) = \sum_{i=1}^I c_i \left(\frac{r}{r_0} \right)^{n_i}$$

Once the coefficients are known, β is constructed as:

$$\beta = -\varphi + \sum_{i=1}^I c_i R_i(r) \Theta_i(\theta)$$

Divide 100–400 km into 4 subintervals; fit each separately and apply $f_k(\alpha)$ to keep β smooth and continuous.

Current density is calculated after obtaining smooth β , we get $J_r = 0$, $J_\phi = 0$, and $J_\theta = -\frac{3B_0 g_0 r_0}{\mu_0 \Delta r r \sin \theta} \cdot \text{sech}^2 \left(\frac{6(r - r_c)}{\Delta r} \right)$.

CONCLUSION

This study develops continuous Euler potentials to model charged-particle motion with smooth magnetic field transitions. A tanh function ensures continuity, with adjustments to across four regions. The approach applies to ionospheric currents, solar wind interactions, and magnetospheric boundaries, providing a useful tool for space weather and plasma studies.

RESULTS

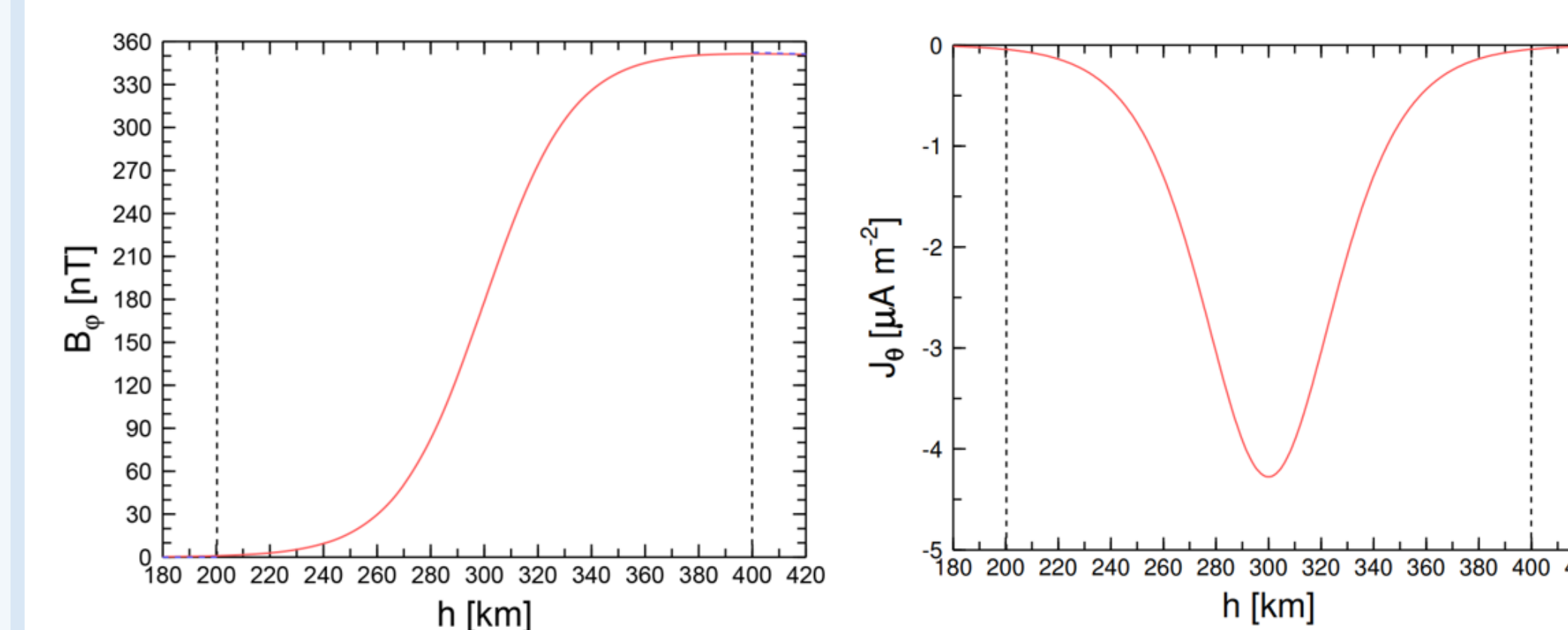


Fig 1: Profiles of (a) ϕ component of the magnetic field and (b) θ component of the electric current density.

- The ϕ component of the magnetic field (for $\theta = \frac{\pi}{6}$) shows a smooth transition across the ionosphere, using $g_0 = 0.006$ to represent strong geomagnetic activity.
- The electric current density (in the ϕ direction) peaks around 300 km altitude, confirming the expected behavior of shell currents in the ionosphere.
- The tanh (step-up) function used to smooth the magnetic field is validated, showing excellent fit (Figure 2a).

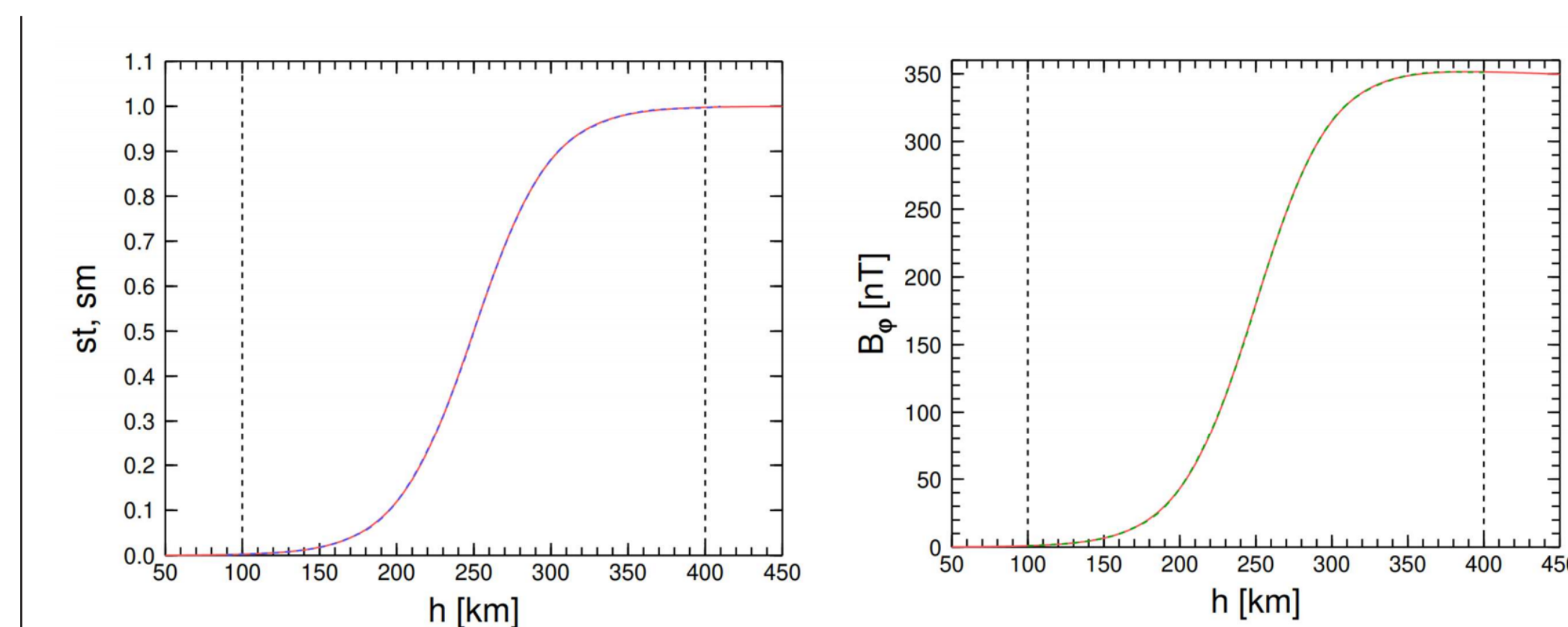


Fig 2: (a) Profiles of Fit of the step-up function, red line, by a sum of power functions, dashed blue line, in the ionosphere. (b) The given (red line) and model (dashed green line) ϕ component of the magnetic field.

- The total magnetic field, recalculated using:

$$\mathbf{B} = \nabla \alpha \times \nabla \beta$$

matches the original field very well (Figure 2b), validating the model's accuracy.

- The model successfully reproduces smooth, realistic variations in magnetic field and current density, consistent with observations during geomagnetic storms.

DISCUSSIONS

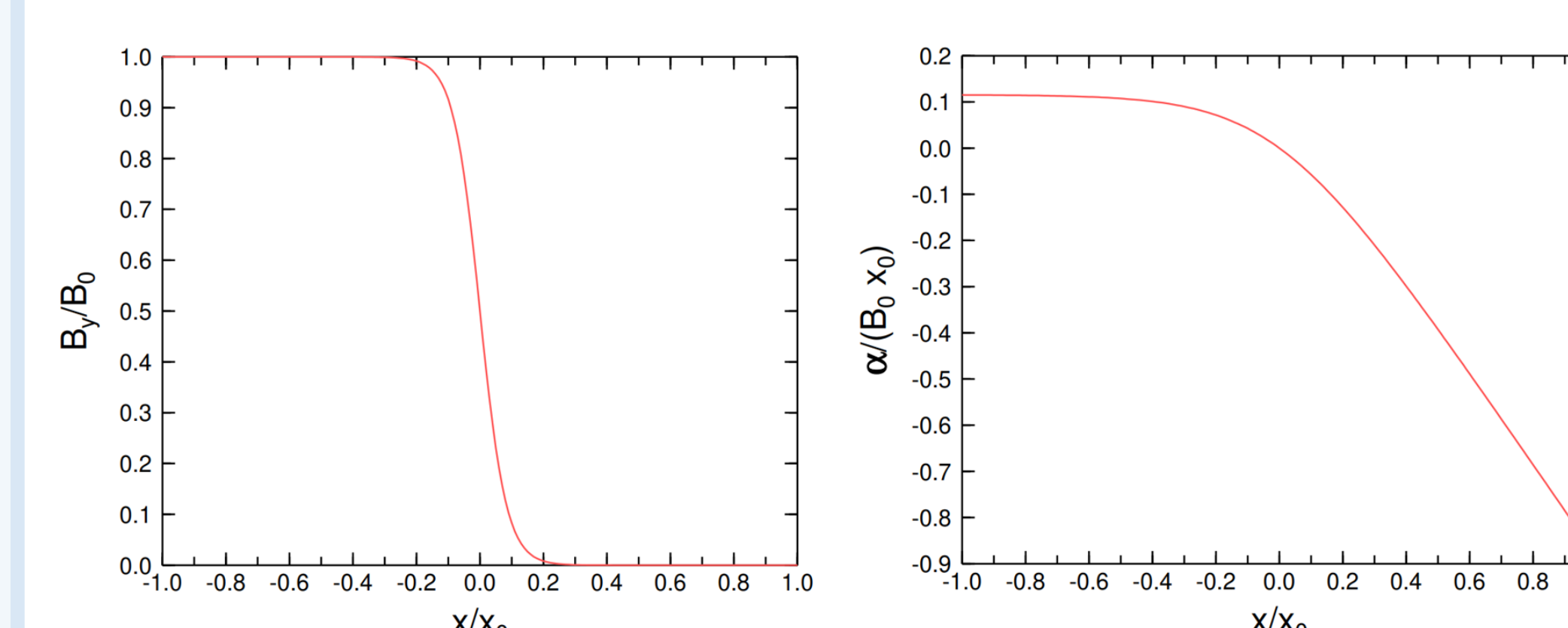


Fig 3 (a) B_y in the layer. (b) Euler potential α in the layer for $y = 0$.

In the planar model ($-x_0 < x < x_0$), the y -component of the magnetic field is:

$$B_y = \frac{1}{2} B_0 \left(1 + \tanh \left(\frac{3x}{x_0} \right) \right)$$

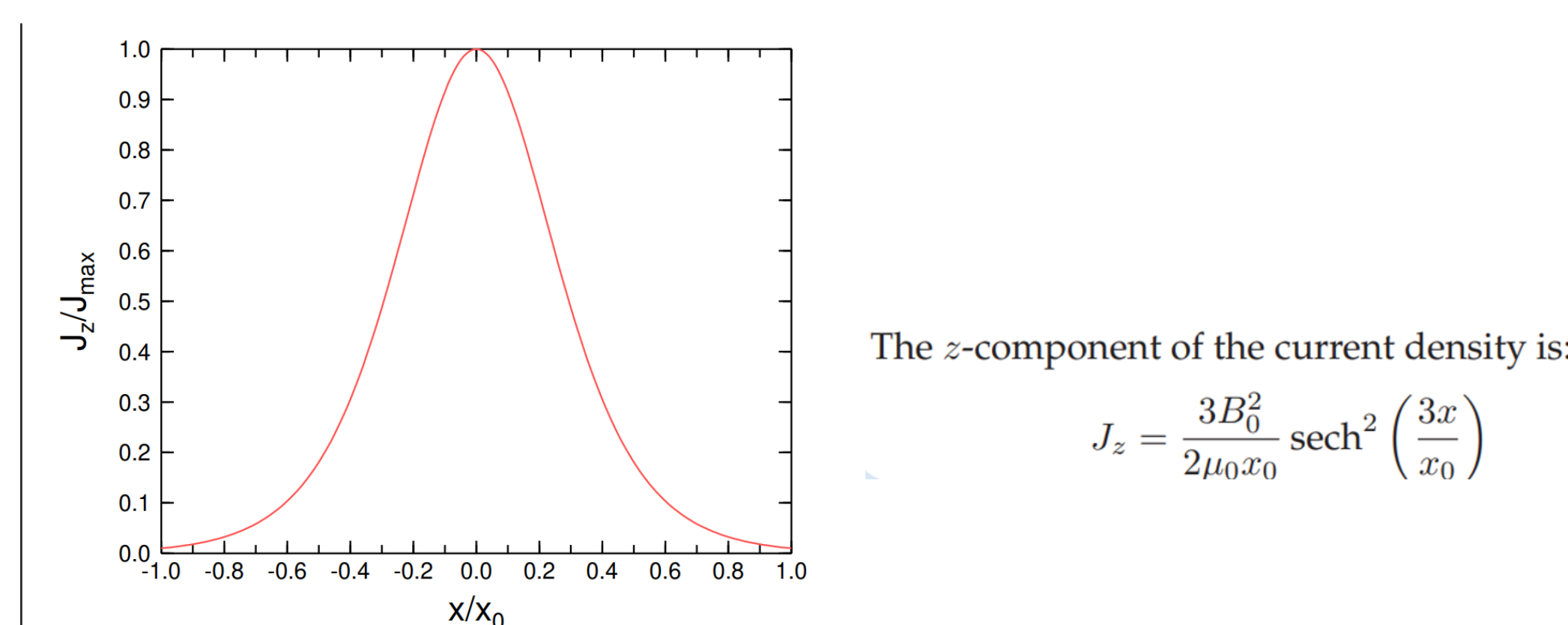


Fig 4: Current density in the layer.

The spherical ionospheric model describes a tangential magnetic field discontinuity above the ionosphere and accurately captures the balance between Pedersen and Hall currents by combining toroidal and poloidal components.

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