High-Latitude Ionosphere TEC Mapping using Ground-based and Spire Nanosatellite GNSS Observations

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Abstract

This study presents a mapping of high-latitude ionospheric total electron content (TEC) on March 23, 2023. High latitudes, particularly polar regions, are inadequately covered by ground-based GNSS receivers for ionospheric observation. To address this, the study investigates the integration of space-based GNSS measurements from Spire Nanosatellites to improve the 2D mapping of vertical TEC. The data used include line-of-sight (LOS) TEC measurements from the Madrigal database, as well as radio occultation (RO) and grazing-angle GNSS reflectometry (GNSS-R) measurements from Spire Nanosatellites. During the selected period from 09:30-10:00 on March 23, there were 120 sets of RO and 137 sets of grazing-angle GNSS-R data available. Using all these measurements, we formulated a 2D TEC mapping problem for high latitudes (e.g., northern hemisphere latitudes > 45 degrees) with a spatial resolution of 1-degree latitude by 1-degree longitude. The electron density profiles simulated by the Whole Atmosphere Model are employed to support the use of space-based GNSS. The detailed methodology, mapping results, and some comparative analysis will be presented.

2. TEC Mapping Formulation

Assuming vTEC is constant within the grid, forming a 1-deg by 1-deg global vTEC map, the relationship between the vTEC and sTEC measurements can be expressed as,

$$=\sum_{j=1}^{N}A_{ij}x_j + B_ie_i \tag{4}$$

where y_i is the *i*-th sTEC measurement, x_j is the unknown vTEC at the *j*-th grid, A_{ij} is the functions that relates y_i and x_j and comprises of two elements: the fraction of electron content (α_{ij}) and the vertical-to-slant TEC mapping function (β_{ij}), B_i is the mapping factor that assigns each observation to its profile's bias, and e_i is the error for the *i*-th measurement which is the combination of the measurement noise, receiver DCB, unavailable leveling factor Z, etc. Note that each RO profile shares a single common bias. *N* is the number grids passed through the LOS (line-of-sight) ray path. Only the nearest grid point was considered along the LOS path. Eq. (4) can also be written in the matrix form,

$$Y = [A B] \begin{bmatrix} X \\ \varepsilon \end{bmatrix}$$
(5)

(6)

where
$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{bmatrix}$$
, $A = \begin{bmatrix} \alpha_{1,1}\beta_{1,1} & \alpha_{1,2}\beta_{1,2} & \cdots & \alpha_{1,m}\beta_{1,m} \\ \alpha_{2,1}\beta_{2,1} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_{n,1}\beta_{n,1} & \cdots & \cdots & \alpha_{n,m}\beta_{n,m} \end{bmatrix}$, $B = \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,p} \\ b_{2,1} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ b_{n,1} & \cdots & \cdots & b_{n,p} \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_m \end{bmatrix}$, n is the number of measurements, p is the number of RO profiles, and m is the number grids.

Therefore,

 $X = (A^T A + \lambda I)^{-1} A^T Y$

where λ is the regularization parameter which serves to control the balance between fitting the data and suppressing large values coming from overfitting or numerical instability caused by ill-conditioned matrices. It typically occurs in the grid where data is sparse.



Spire Global Nanosatellite Constellation

Spire Global operates up to a few tens of nanosatellites for GNSS radio occultation and grazing-angle reflectometry. These nanosatellites are equipped with multi-use radio frequency (RF) sensors that collect radio occultation and reflectometry data.

- Orbit altitudes range from 400-600 km with various inclinations and local times.
- Over 10,000 RO profiles and 2,000 grazing angle reflection arcs per day.
- Products:
- 1. POD (precise orbit determination) GNSS navigation data
- 2. GNSS-RO (radio occultation)
- 3. GNSS-R (Grazing angle GNSS reflectometry): one set of direct line-of-sight (DLOS) and reflected signals at a time.
- For more details, please visit: https://www.earthdata.nasa.gov/about/csda/vendor-spire



2023/03/23 09:30-10:00 180[°] E

Madrigal TEC database

Madrigal database at Massachusetts Institute of Technology Haystack Observatory (Rideout & Coster, 2006) provides sTEC measurements from groundbased GNSS receivers

Madrigal supports sophisticated searching and enables the real-time delivery of data in a variety of standardized formats including that used by the NCAR CEDAR data system, (Sica et al. 1988)

The sTEC data were converted to vertical TEC (vTEC) by assuming a slab ionosphere at 350 km. This means that only elevation needs to be considered in mapping sTEC to vTEC.

The 1-deg by 1-deg vTEC maps were routinely published by CEDAR Madrigal database every 5 min.



WAM-IPE (Whole Atmosphere Model-Ionosphere Plasmasphere Electrodynamics)

A coupled forecast system of ionosphere and thermosphere specification

- WAM is an extension of Global Forecast System (GFS) with a spectral hydrostatic dynamical core utilizing an enthalpy thermodynamic variable to 150 vertical levels on a hybrid pressure-sigma grid, with a model top of approximately 3 x 10-7 Pa (typically 400-600km)
- The IPE model provides the plasma component of the atmosphere. It is a time-dependent, global 3D model of the ionosphere and plasmasphere from 90 km to approximately 10,000 km
- Ion species for IPE: O⁺, H⁺, He⁺, N⁺, NO⁺, O₂⁺, N₂⁺

400

• Resolution: $2^{\circ} \times 4^{\circ}$ in latitude-longitude





Density (#/m³)

Mapping function $\beta = \frac{1}{\cos(\theta_z)}$ where θ_z is the zenith angle



3. TEC Mapping Using Madrigal and Spire LOS sTEC (a) Madrigal vTEC map (1-deg product) 2023/03/23 09:30-10:00 (b) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and Spire tangent points and reflectometry ipp 2023/03/23 09:30-10:00 (c) Madrigal vTEC and spire tangent points

1. Madrigal Database & Spire sTEC



- (a) GNSS receiver network from the Madrigal database over high latitudes (northern hemisphere latitudes > 45 degrees)
 (b) Madrigal LOS vTEC and Spire RO tangent points (black) and reflectometry SP (specular point) positions (grey) during the second second
- (b) Madrigal LOS vTEC and Spire RO tangent points (black) and reflectometry SP (specular point) positions (grey) during 09:30-10:00UT, March 23, 2023.
- (c) Altitudinal profiles of Spire RO sTEC.
- (d) Altitudinal profiles of DLOS (direct LOS) sTEC.



(d) Reconstructed vTEC map by integrating Madrigal and Spire data during 09:30-10:00UT.

Compare the reconstructed map with SWARM electron density



TEC retrieval methodology

TEC is a line integral of electron density on a path between the transmitter (Tx) and receiver (Rx). The path separation between the two signals is usually small, and the assumption is often made that they both travel along the same straight line. The TEC can be estimated by forming the geometry-free combination (Teunissen & Montenbruck, 2017; Angling et al., 2021). The RO antennas onboard the Spire STRATOS payload measured the atmospheric excess phases on L1 and L2 channels. Given L1 and L2 excess phases and pseudorange, sTEC estimated from phase $(sTEC_L)$ and pseudorange $(sTEC_P)$ measurements can be expressed as,

$$sTEC_L = \frac{f_2^2 \cdot f_1^2}{40.3(f_2^2 - f_1^2)} (\varphi_2 - \varphi_1 - Z)$$
(1)

$$sTEC_P = \frac{f_2^2 \cdot f_1^2}{40.3(f_2^2 - f_1^2)} (P_1 - P_2 - DCB_{rx} - DCB_{tx})$$
(2)

where f_1 and f_2 are L1 and L2 frequency, φ_1 and φ_2 are L1 and L2 excess phases in cycles, P_1 and P_2 are L1 and L2 pseudorange in meters, and DCB_{rx} and DCB_{tx} are the differential code bias of the receiver and the transmitter.

 $sTEC_P$ is an absolute measurement, while it is noisy due to multipath and to limitations of the receiver front end bandwidth. $sTEC_L$ has less noise but includes ambiguity (Z), which is the bias between pseudoraange and carrier phase measurements. One approach to obtain the bias is to level the phase TEC to the pseudorange TEC by simply modifying the mean value,

$$Z = \frac{P_1 - P_2 + \varphi_2 - \varphi_1}{N}$$

where N is the number of measurements.

Acknowledgements This work has been supported by NASA grant #80NSSC23K1056

- ⁽³⁾ With Spire LOS sTEC measurements, the vTEC map has been effectively improved, particularly over the region around 120-180°E, where the Madrigal data were previously limited.
 - The mapping results presented revealed various fine structures that were previously absent in the Madrigal 1-deg map.