Assimilating Grape Doppler Receiver Data into Bayesian Frameworks

Rachel Boedicker [1], Nathaniel Frissell [2], John Gibbons [1], Kristina Collins [3], and David Kazdan [1]

[1]Case Western Reserve University, [2]University of Scranton, [3]The Space Science Institute

Introduction and Data

The Ham Radio Science Citizen Investigation (HamSCI) joins scientists and volunteers to collect data as part of a personal space weather station (PSWS) project. As part of the PSWS network, the distributed array of high frequency Grape receivers record Doppler shifted signals from national time standard stations such as WWV in Fort Collins, Colorado. The recorded Doppler shift is a result of the signal passing through the earth's atmosphere and therefore provides insight into the state and variability of the ionosphere. The Grape network includes Grape 1 Doppler receivers, as well as multi-channel Grape 2 and WSPR (Weak Signal Propagation Reporter) Grapes which generate spectral data.

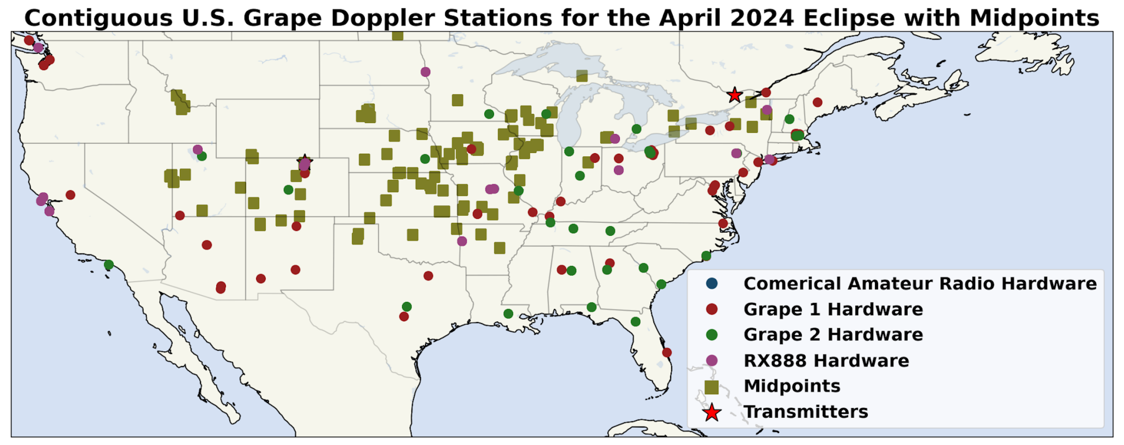


Figure 1. Map of stations active during the 2025 eclipse.

Ionospheric measurements are sparse by nature and can't give a complete picture of the ionosphere. Similarly, although there are extensive models of the ionosphere, they don't replicate the variability and real-time conditions that data indicates. Therefore reconstruction of an ionospheric event, such as the activity during the 2023 and 2024 solar eclipses, can benefit from incorporating measured data into assumed prior models to estimate the value of a measurement. This poster examines the construction of the Grape inverse problem and mathematical algorithms and techniques to produce an approximation of the ionospheric state during an event.

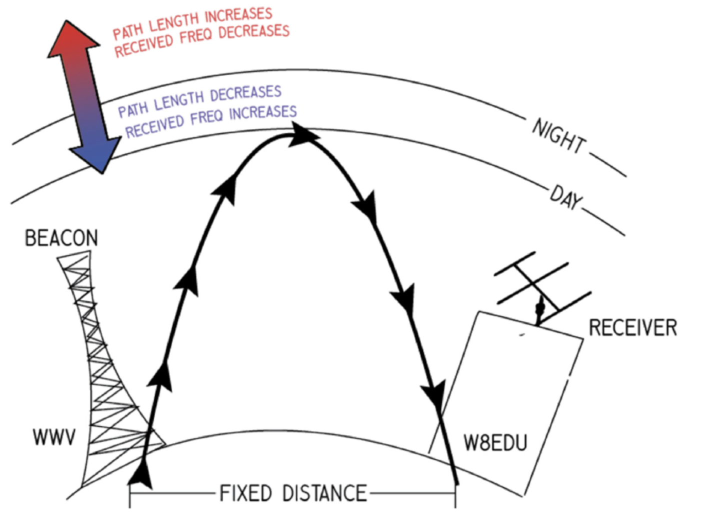


Figure 2. (Left) Basic ionospheric signal propagation model for a Grape receiver. Image from Kristina Collins, KD8OXT (2021).

To begin, we note that the Doppler shift, or frequency change from transmission frequency, measured by a Grape receiver assumed to be caused by either movement in the ionospheric layers or change in the width of the layers. Therefore, we have a large amount of uncertainty which is represented mathematically in terms of probability distributions.

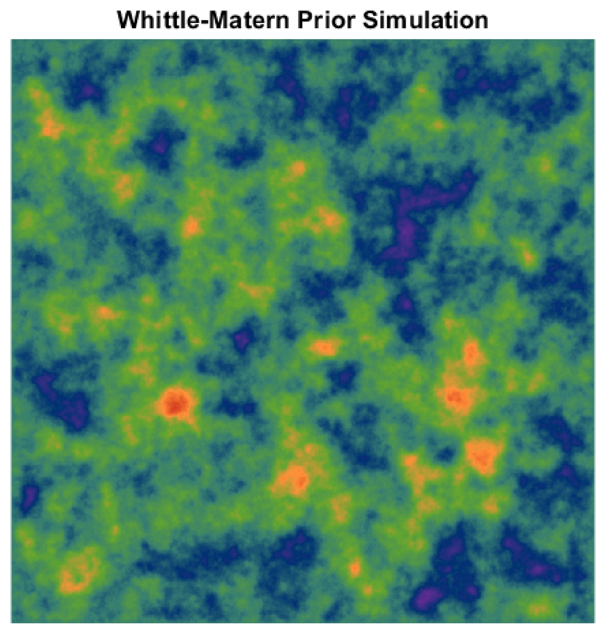
Bayesian Methods of Assimilating Data

In the case of a Grape receiver, we know that there is an originating continuous-time signal, $s(t)$, that passes through an unknown system (the ionosphere) and is then received as a stored discrete vector, b. Therefore we have the structure of an inverse problem,

$$Ax + e = b$$

where $A \in \mathbb{R}^{m\times n}$ is an unknown transform, $e \in \mathbb{R}^m$ is an additive noise vector, and $x \in \mathbb{R}^n$ is the discrete version of the original signal. We can begin to fill in information about the system using data and assumptions.

Bayesian methods are sensitive to prior assumptions, or beliefs held about the solution to a problem. for example, if we believe that the solution to a problem will be smooth up to a certain order, then we enforce these beliefs on the solution to fill in unknown information.



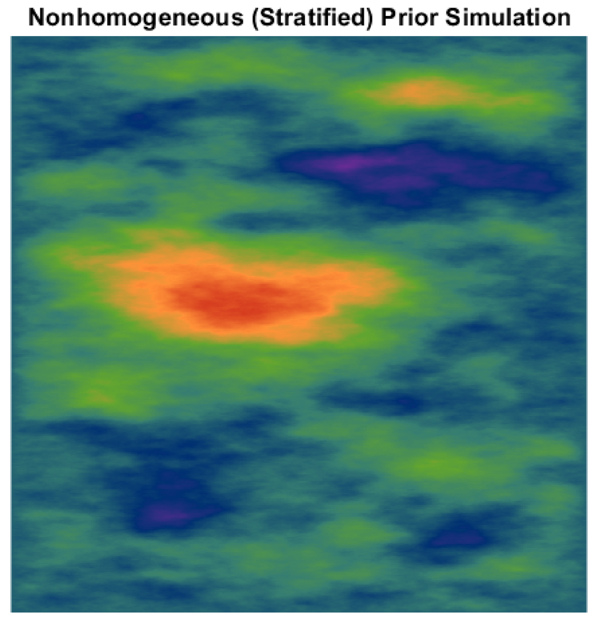


Figure 3. (Left) Example a Whittle-Matérn prior generated with a correlation length around 30. The prior assumes uniform scaling in both directions and is derived using a discrete Laplacian approximation [1].

Figure 4. (Right) An example of a stratified prior with boundary conditions. This is similar to a two dimensional Gaussian prior except that the horizontal and vertical components can be separately scaled.

Since our signal is received in a particular order, we may characterize this as a stochastic process where we can observe only secondary measurement (Doppler frequency) which is related to a more difficult measurement. As data becomes available, we can use it to perform Bayesian filtering [1], by applying the observation to refine a prior distribution.

The Kalman Filter is an iterative process that allows us to update our estimates given a sequence of observations. Consider a stochastic process $H_0, H_1, \dots$ in $\mathbb{R}^n$ and an observation process $B_1, B_2, \dots$ taking on values in $\mathbb{R}^m$ such that

$$H_t = FH_{t-1} + V_t \qquad\qquad B_t = AH_t + W_t$$

where t is an iteration, $F \in \mathbb{R}^{n\times n}$ and $A \in \mathbb{R}^{m\times n}$, and we assume the following are normally distributed random variables [1]

$$H_0 \sim \mathcal{N}(\overline{h}_0, D_0) \qquad V_t \sim \mathcal{N}(0, \Gamma_t) \qquad W_t \sim \mathcal{N}(0, \Sigma_t)$$

which are mutually independent. We can formulate a prediction step based on the last mean, $\mathbb{E}(H_{t-1}) = \overline{h}_{t-1}$, and the covariance of H_t given the prior observations, $B_1, \dots, B_{t-1}$

$$\hat{h}_t = F\bar{h}_{t-q} \qquad\qquad \hat{D}_t = FD_{t-1}F^T + \Gamma_t$$

we note that

$$K_t = \hat{D}_t A^T (A\hat{D}_t A^T + \Sigma_t)^{-1}$$

is called the Kalman Gain Matrix. We can use the gain to update our prediction for the next mean and covariance according to the next observation, b_t

$$\overline{h}_t = \hat{h}_t + K_t(b_t - A\hat{h}_t) \qquad\qquad D_t = \hat{D}_t - K_t A\hat{D}_t$$

The measurement residual, $\Delta_t = b_t - A\hat{h}_t$ gives us am idea of how well the current state predicted the next measured state.

The ensemble Kalman filter combines the evolution-observation of a Kalman filter with the sample-evolution method of a particle filter to apply the evolution to each of a set of distribution samples.

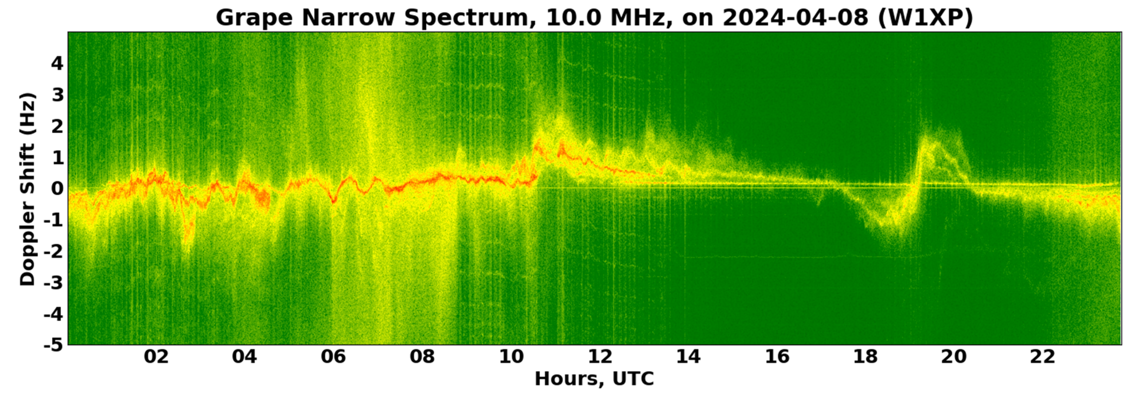


Figure 5. Example of a spectrogram for Grape 1 data in the Digital Radio Frequency (DRF) format. Data was recorded during the 2024 eclipse by Bob Reif in Massachusetts.

Additional work needs to be done to incorporate spatial correlations between the Grape network stations. This is to fit the assumption that nearby stations also measure similar sections of the ionosphere. Future work should also include Grape 2 data, which is still being retrieved and uploaded to a central location.

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rachel.boedicker@case.edu

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