## Elves as the Optical Signature of Lightning Precursor Discharges in the Lower lonosphere

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## 1. Introduction

Narrow bipolar events (NBEs) are compact intracloud discharg Narrow bipolar events (NBEs) are compact intracloud discharg es, with spatial extent of 100s of meters, that usually precede
lightning flashes. There is growing evidence that these discharglightning flashes. There is growing evidence that these discharges are associated with elves [Neubert et al, Science, 367, 183 2020, Nature, 589, 371, 2021; Østgaard et al., JGR, 126, e2020 D033921, 2021]; a type of transient luminous event observed a $\sim 80-95 \mathrm{~km}$ altitude as laterally expanding rings of light. Elves are produced when electromagnetic pulses from high peak curren lightning discharges interact with the lower ionosphere [Fukunisł et al., GRL, 23, 2157,1996; Inan et al., GRL, 24, 583, 1997].


Figure 1. Schematic illustration of the known Transient Luminous Events by citizen scientists Frankie Lucena and Hank Schyma [Lyons, Weatherwis $75,14,2022$ ]. Emphasis to NBEs and elve added.
In this work we study the association of NBEs and elves by 1. Using electric field measurements of an observed NBE to re cover its source current using the method given in [Pervez e al., GRL, e2023GL107789, 2024].
2. Using the recovered source current to estimate optical emissions at elve altitudes.
2. Model Geometry

Cylindrical domain of radius 200 km at elve altitudes

Figure 2. Schematic illustration of model geometry, depicting the verti-
cal conducting source channel of the NBE discharge of length $l$, at height $h$ above a perfectly conducting ground. The electric field measurements are made at a point of observation on the ground at source. A cylindrical domain of radius 200 km located at elve altitudes is used to calculate optical emissions from the NBE source.

Perfectly conducting ground

## 3. Source Current Extraction

We use an NBE observed by Eack [GRL, 31, L20102, 2004], for which electric fields were measured both in the near field $(d=2.8 \mathrm{~km})$ and the far field $(d=$ 200 km ). The waveforms used here (shown in Figure 3) come from the model of Watson and Marshall [GRL, 34, L04816, 2007].


Figure 3. NBE electric field waveforms observed in (a) the near field ( $d=2.8 \mathrm{~km}$ ), and (b) the far field ( $d=200 \mathrm{~km}$ ).
The electric field spectrum observed at a point on the ground from a linear vertical conducting source is given by

$$
E_{z}(\omega)=-\frac{j \eta}{2 \pi \beta} \int_{h-l / 2}^{h+l / 2} I\left(z^{\prime}, \omega\right)\left[G_{1}+z^{\prime 2} G_{2}\right] e^{-j \beta R} d z^{\prime}
$$

where $\eta$ is the characteristic impedance of free space, $\beta=\omega / c$ is the phase constant, $I(z, \omega)$ is the spectrum of source current $I(z, t)$ in the frequency domain, that is, $I(z, \omega)=\mathcal{F}[I(z, t)]$, where $\mathcal{F}$ denotes Fourier transform, and

$$
G_{1}=\frac{-1-j \beta R+\beta^{2} R^{2}}{R^{3}}, \quad \text { and } \quad G_{2}=\frac{3+j 3 \beta R-\beta^{2} R^{2}}{R^{5}} .
$$

The current is modeled in the following manner

$$
I(z, t)=g(z) i(t), \quad g(z)=\cos (\pi(z-h) / l), \quad z \in[h-l / 2, h+l / 2]
$$

This current polarizes the channel similar to a dipole, with one half of the channel being positively charged, and the other half negatively charged. The nel being positively charged, and the other haff negatively charged. The the current $i(t)$ can be recovered using the field impulse response $E_{z}^{\text {IR }}$ as
$i(t)=\mathcal{F}^{-1}\left[\frac{\mathcal{F}\left[E_{z}(t)\right]}{E_{z}^{\mathrm{IR}}(\omega)}\right]$


Figure 4. The current $i(t)$ recovered from the NBE waveforms of figure 3 using the methodology described, with parameters $h=7 \mathrm{~km}$, and $l=600 \mathrm{~m}$.

## 4. Optical Emissions at Elve Altitudes

Using the NBE source current from Figure 4, we can calculate the radiated electric field in the cylindrical domain at elve altitudes, and consequently the optical emissions. The chemical reactions considered here are given in Table 1. The intensity of each optical emission in Rayleighs is given by:

$$
I_{m}=10^{-6} \int_{\Gamma} A_{m} n_{m} d \Gamma
$$

where $n_{m}\left[1 / \mathrm{cm}^{3}\right]$ is the number density of excited particles in state $m, A_{m}[1 / \mathrm{s}]$ is the radiative transition frequency, and the integral is taken along the horizontal line of sight $\Gamma$ [cm]. We calculate optical emissions from the first ( $B^{3} \Pi_{g} \rightarrow A^{3} \Sigma_{u}^{+}$) and second $\left(C^{3} \Pi_{u} \rightarrow B^{3} \Pi_{g}\right)$ positive band systems of $\mathrm{N}_{2}$. Assuming steady-state, i.e., $\partial n_{m} / \partial t=0$, the densities of the $\mathrm{N}_{2}(B)$ and $\mathrm{N}_{2}(C)$ states, $n_{1}$ and $n_{2}$, respectively, are given by

$$
n_{2}=\tau_{2} \nu_{2} n_{e}, \quad \text { and } \quad n_{1}=\tau_{1}\left[n_{2} A_{2}+\left(k_{N_{2}}^{2} / 2\right) N_{\mathrm{N}_{2}} n_{2}+\nu_{1} n_{e}\right],
$$

where $\tau_{m}=\left(A_{m}+k_{\mathrm{N}_{2}}^{m} N_{\mathrm{N}_{2}}+k_{\mathrm{O}_{2}}^{m} N_{\mathrm{O}_{2}}\right)^{-1}$ is the total lifetime of state $m$

We use the following vertical profile for the electron number density $n_{e}$ [Marshall et al., GRL, 42, 6112, 2015]:

$$
n_{e}=n_{e 0} e^{-0.15 h^{\prime}} e^{(\alpha-0.15)\left(z-h^{\prime}\right)}
$$

$$
\begin{gathered}
n_{e 0}=1.43 \times 10^{7} \mathrm{~cm}^{-3} \\
\alpha=0.8 \mathrm{~km}^{-1} \\
h^{\prime}=87 \mathrm{~km}
\end{gathered}
$$

Table 1. List of chemical reactions considered
where,

| Reactions | Frequency (1/s) or Rate Constant (cms ${ }^{\text {s }}$ ) |
| :---: | :---: |
| $\mathrm{N}_{2}+e \rightarrow \mathrm{~N}_{2}(B)+e$ | $v_{1}(1 / s)\left(\right.$ from Bolsig ${ }^{+}$) |
| $\mathrm{N}_{2}+e \rightarrow \mathrm{~N}_{2}(\mathrm{C})+e$ | $v_{2}(1 / s)$ (from BoLSIG + ) |
| $\mathrm{N}_{2}(B) \rightarrow \mathrm{N}_{2}(A)+h v\left(1 \mathrm{PN}_{2}\right)$ | $A_{1}=1.7 \times 10^{5} 1 / \mathrm{s}$ |
| $\mathrm{N}_{2}(C) \rightarrow \mathrm{N}_{2}(B)+h v\left(2 \mathrm{PN}_{2}\right)$ | $A_{2}=2.0 \times 10^{7} 1 / \mathrm{s}$ |
| $\mathrm{N}_{2}(B)+\mathrm{O}_{2} \rightarrow \mathrm{~N}_{2}+\mathrm{O}+\mathrm{O}$ | $k_{\mathrm{o}_{2}}^{1}=3 \times 10^{-10} \mathrm{~cm}^{3} / \mathrm{s}$ |
| $\mathrm{N}_{2}(B)+\mathrm{N}_{2} \rightarrow \mathrm{~N}_{2}(A)+\mathrm{N}_{2}$ | $k_{\mathrm{N}_{2}}^{1}=10^{-11} \mathrm{~cm}^{3} / \mathrm{s}$ |
| $\mathrm{N}_{2}(\mathrm{C})+\mathrm{O}_{2} \rightarrow \mathrm{~N}_{2}+\mathrm{O}+\mathrm{O}(\mathrm{D})$ | $k_{\mathrm{O}_{2}}^{2}=2.5 \times 10^{-10} \mathrm{~cm}^{3} / \mathrm{s}$ |
| $\mathrm{N}_{2}(C)+\mathrm{N}_{2} \rightarrow \mathrm{~N}_{2}(B)+\mathrm{N}_{2}$ | $k_{\mathrm{N}_{2}}^{2} / 2=10.11 \mathrm{~cm}^{3} / \mathrm{s}$ |
| $\mathrm{N}_{2}(C)+\mathrm{N}_{2} \rightarrow \mathrm{~N}_{2}(a)+\mathrm{N}_{2}$ | $k_{\mathrm{N}, 2}^{2} / 2=10 \cdot 1 \mathrm{~cm}^{3} / \mathrm{s}$ |

(a) Optical emissions for 1 st positive band system of $\mathrm{N}_{2}$
(b) Optical emissions for 2nd positive band system of $\mathrm{N}_{2}$

 nd positive band systems of $N_{2}$ in the cylindrical domain at elve altitudes

## 5. Conclusion

The source current recovered for the NBE results in observable first-order estimates of optical emissions at elve altitudes, providing further support to the association between NBEs and elves.

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