# A synthetic study on Kalman smoother coupling to a data assimilation algorithm

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## Motivation

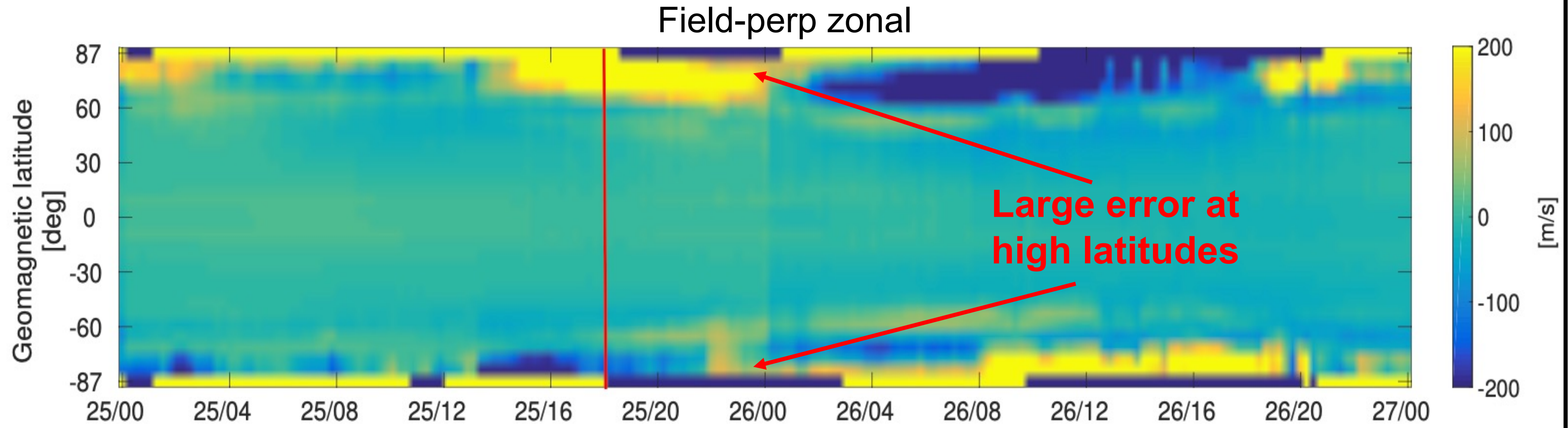

Fig. 1 EMPIRE performance analysis from primary setup[1]

- EMPIRE (Estimating Model Parameters from Ionospheric Reverse Engineering) is a data assimilation method that estimates ion drifts and neutral winds based on plasma density rate observations
- Recent estimation error analysis[1] shows **Large** uncertainties for ion drift driver estimation at high latitudes observed for the data assimilation algorithm of EMPIRE

## Objective

- Develop an algorithm setup for reducing the high-latitude error
  - Adopt the **Kalman smoother**[2] to refine the state calculations

## Background – EMPIRE [3,4]

- Based on **Ion continuity equation**

$$\frac{\partial N}{\partial t} = \underbrace{s_{prod}}_{a_{prod,0}} + \underbrace{s_{loss}}_{a_{loss,0}} \underbrace{-\vec{\nabla}\cdot(N\vec{v}_{\perp})}_{a_{\perp}} \underbrace{-\vec{\nabla}\cdot(N\vec{v}_{\parallel})}_{a_{\parallel}}$$

$a_0 + \delta a_{\perp} + \delta a_u$

$\delta a_{\perp}$: ExB transport correction
$\delta a_u$: Neutral wind transport correction

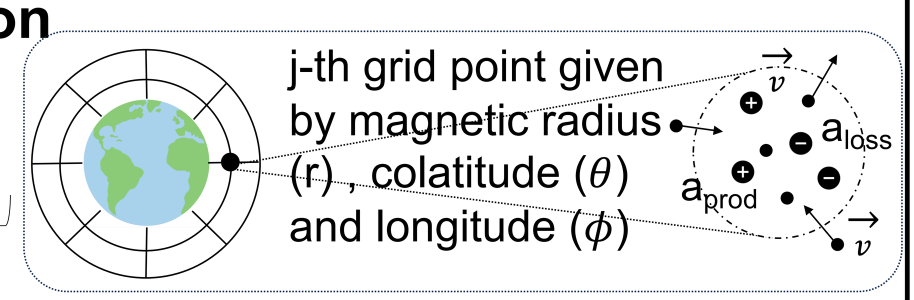

Fig. 2 EMPIRE global grid map with ion transports
Image credit: A. L. Rubio

- Kalman filter setup

$$\underbrace{\frac{\partial \mathbf{N}}{\partial t} - \mathbf{a_0}}_{\mathbf{z}} = \delta \mathbf{a}_{\perp} + \delta \mathbf{a_u} = \underbrace{\begin{bmatrix} \mathbf{H_V} & \mathbf{H_u} \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} \delta \mathbf{x_V} \\ \delta \mathbf{x_u} \end{bmatrix}}_{\mathbf{x}} + \epsilon$$

$H_V$: ExB transport mapping matrix
$H_u$: Neutral wind mapping matrix
$\delta x_V$: Electric potential state vector
$\delta x_u$: Neutral wind state vector
ϵ : observation update error

- Algorithm flowchart

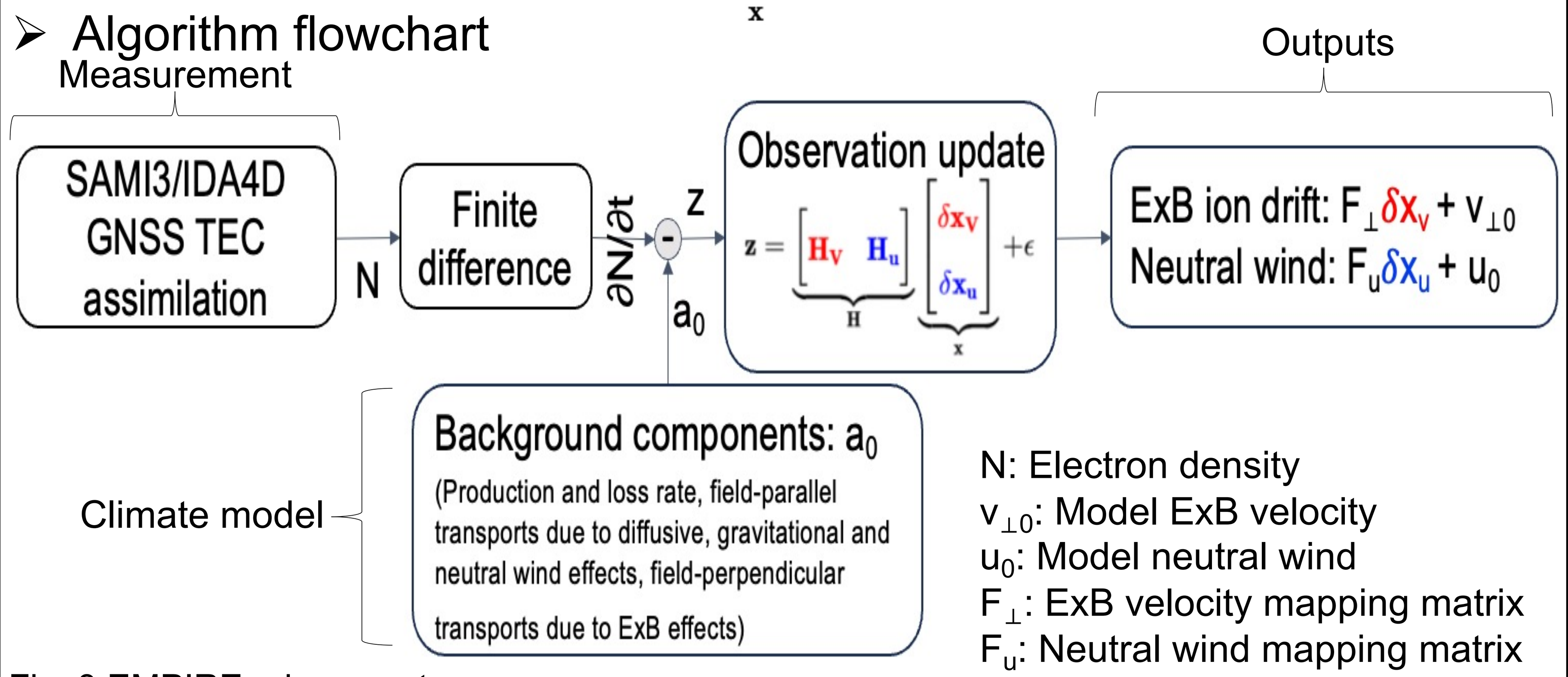

Fig. 3 EMPIRE primary setup

## New method – Couple the Kalman smoother

- Kalman filter + smoother

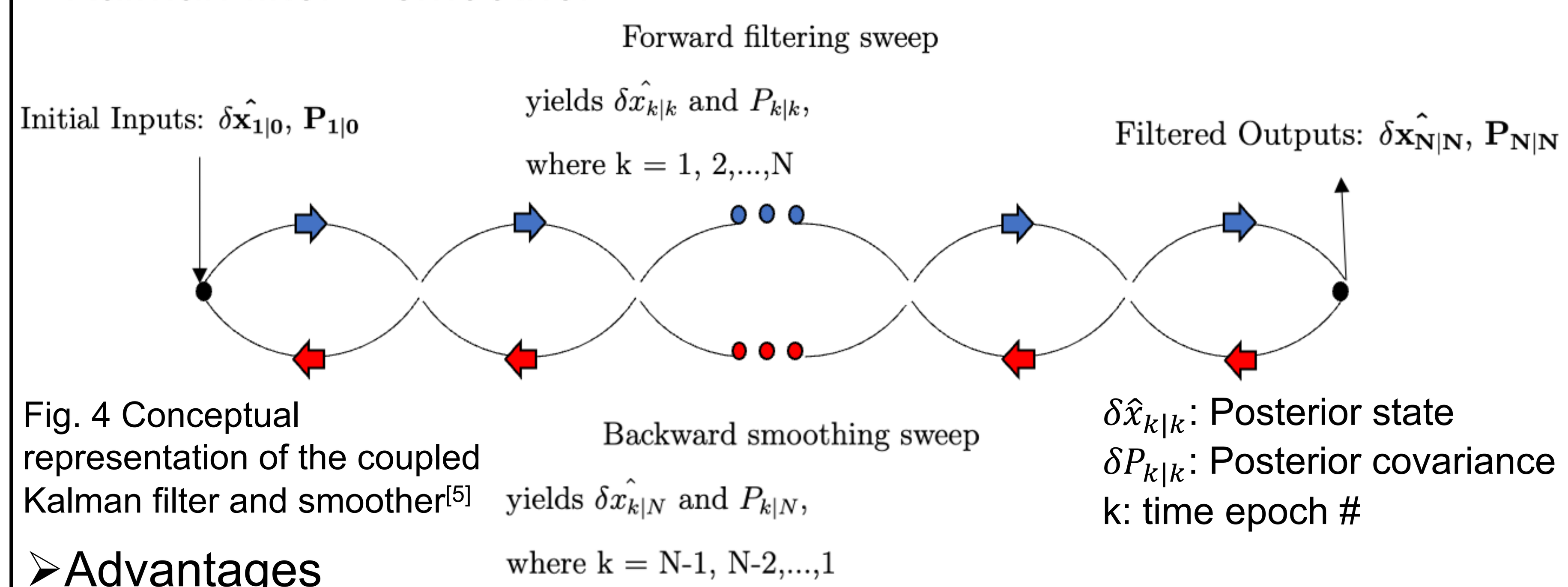

Fig. 4 Conceptual representation of the coupled Kalman filter and smoother[5]

- Advantages
  - Improves the error statistics to the algorithm for historical event analysis
  - Refines the state estimation within a time frame

## Experimental setup

- Conduct a synthetic study on August 25th – 26th 2018
  - Treat SAMI3 data as the synthetic truth
  - SAMI3 is self-consistently driven by Weimer, MSIS and HWM14
  - Compare EMPIRE filtered and smoothed estimates of ion drift to the EMPIRE configuration of filter only as a function of longitude and time

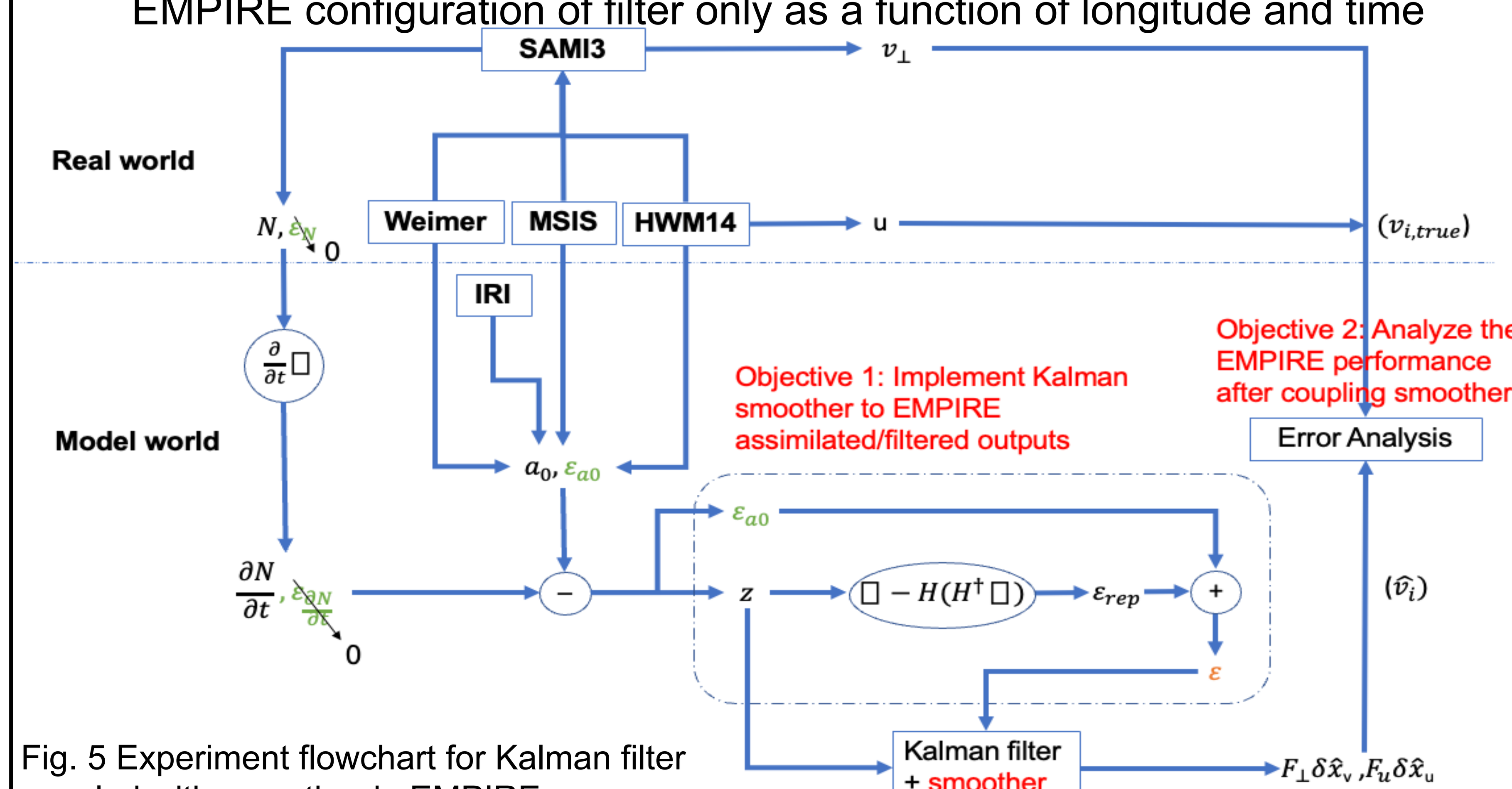

Fig. 5 Experiment flowchart for Kalman filter coupled with smoother in EMPIRE

## Conclusion & Future work

- Compared EMPIRE configurations w/ and w/o smoother
- The Kalman smoother has the potential to reduce errors and sigma for EMPIRE state estimations at high latitudes, based on a simulated quiet and storm period analysis
- Future work: Implement the coupled Kalman smoother and filter for actual storm observation data

## Acknowledgement

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## Results

- The Kalman smoother **reduces** zonally averaged errors and residual sigma in both perpendicular zonal and meridional directions for ExB drift estimations at high-latitudes

* Previous results for comparison with the new setup is published[1]

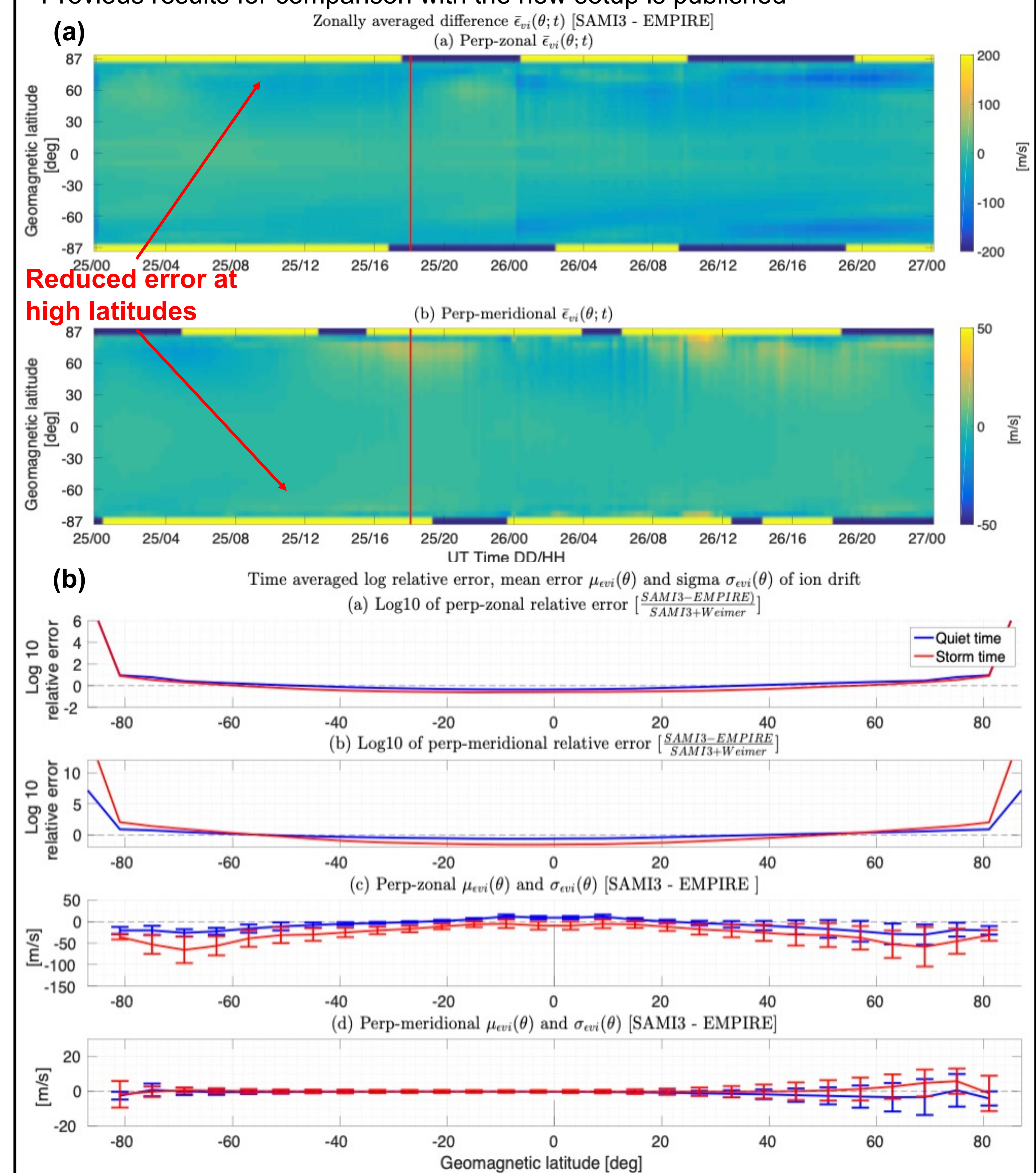

Fig. 6 The (a) zonally averaged errors vs. time (b) time averaged ion drift relative errors on a log base-10 scale vs. geomagnetic latitude

## References

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