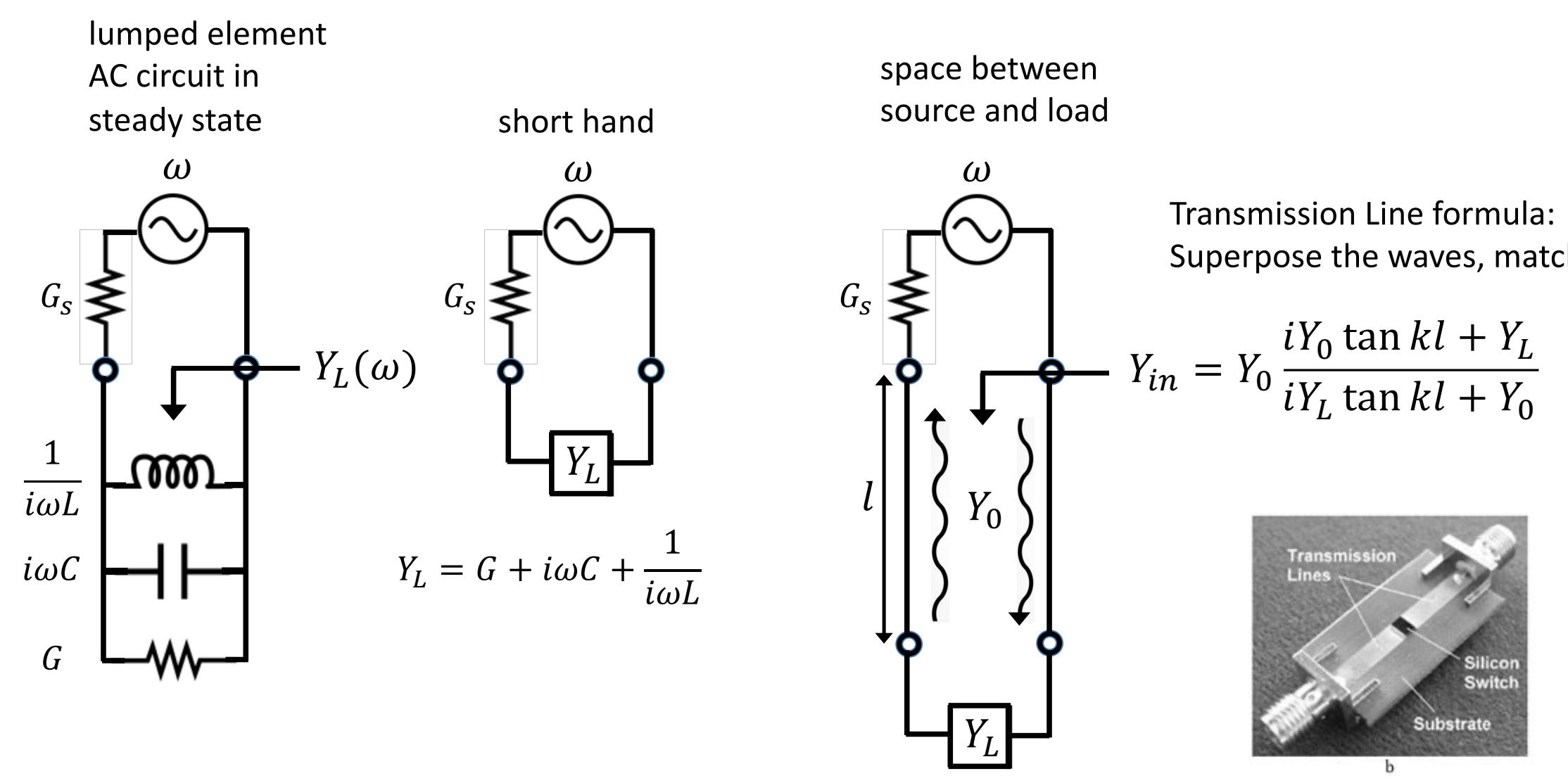
The new electromagnetic ionosphere and how it differs from the old electrostatic one

Russell B. Cosgrove

1 Abstract

Using a rigorous solution for the electromagnetic fluid equations, it is found that they do not predict the electric-field-mapping that is usually expected, and that even if they did, the ionospheric conductance would have a significantly smaller value. In fact, these equations predict wavelike effects on all transverse scales investigated, which are partially associated with short parallel wavelengths, and partially associated with the interaction of multiple modes. It is also found that the electrostatic-wave theory that is used, for example, to derive the spectrum of incoherent scatter, will likely produce unphysical results if extended to transverse scales longer than about one hundred meters. By way of comparison, the new solution is a linearized, causal, driven steady-state solution for the ionospheric conductance and electric field mapping, but it does not include the nonlinear elements of the time evolution, which are the purview of time-domain simulations. Also, although the signal is 3-dimensional, the background ionosphere is assumed horizontally uniform. Hence, the results are best understood as baseline, fundamental results that inform our thinking broadly.

2 Electric Circuit Theory and Electrostatics

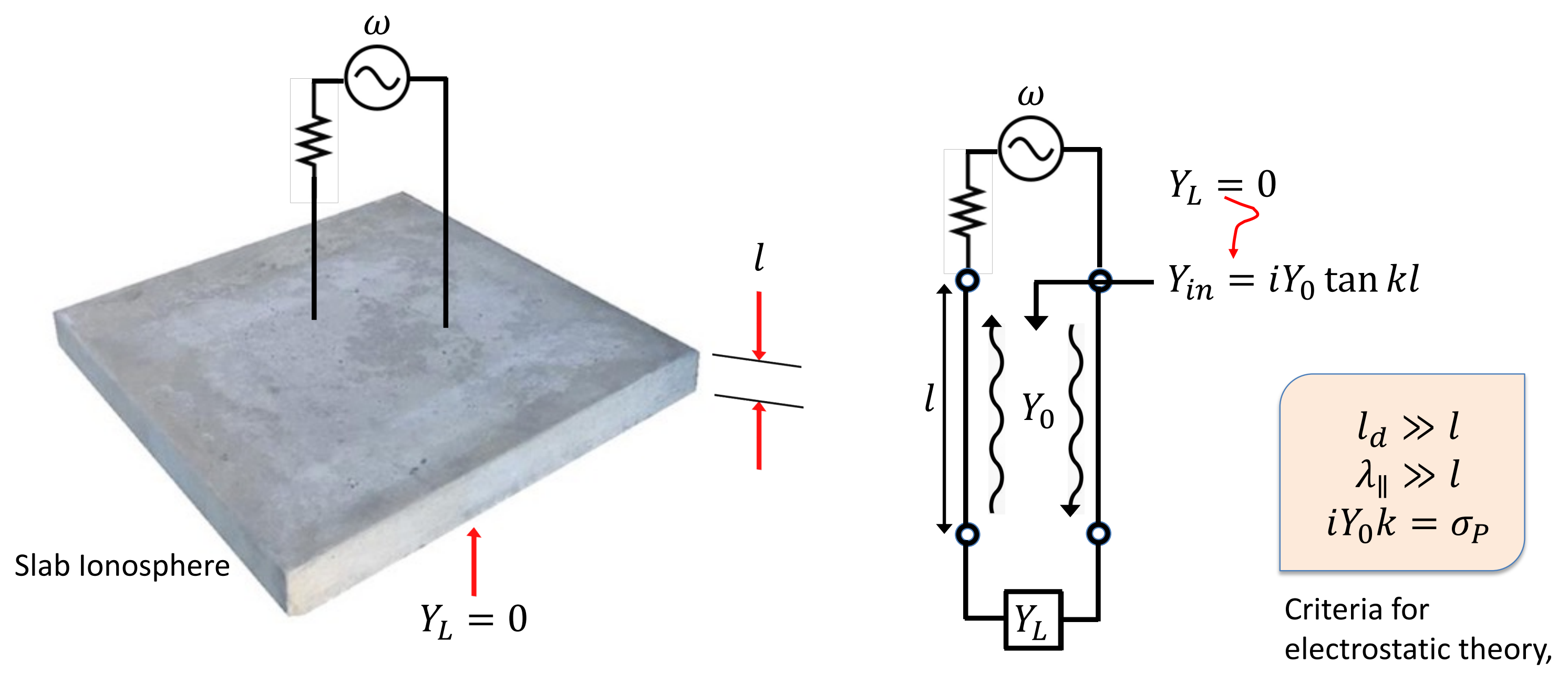


$$Y_L = G + i\omega C + \frac{1}{i\omega L}$$

Transmission Line formula:
Superpose the waves, match Y_L

$$Y_{in} = Y_0 \frac{iY_0 \tan kl + Y_L}{iY_L \tan kl + Y_0}$$

3 Gedankenexperiment: Slab Ionosphere



$$Y_L = 0$$

$$Y_{in} = iY_0 \tan kl$$

$$l_d \gg l$$
$$\lambda_\parallel \gg l$$
$$iY_0 k = \sigma_P$$

Criteria for electrostatic theory, where,

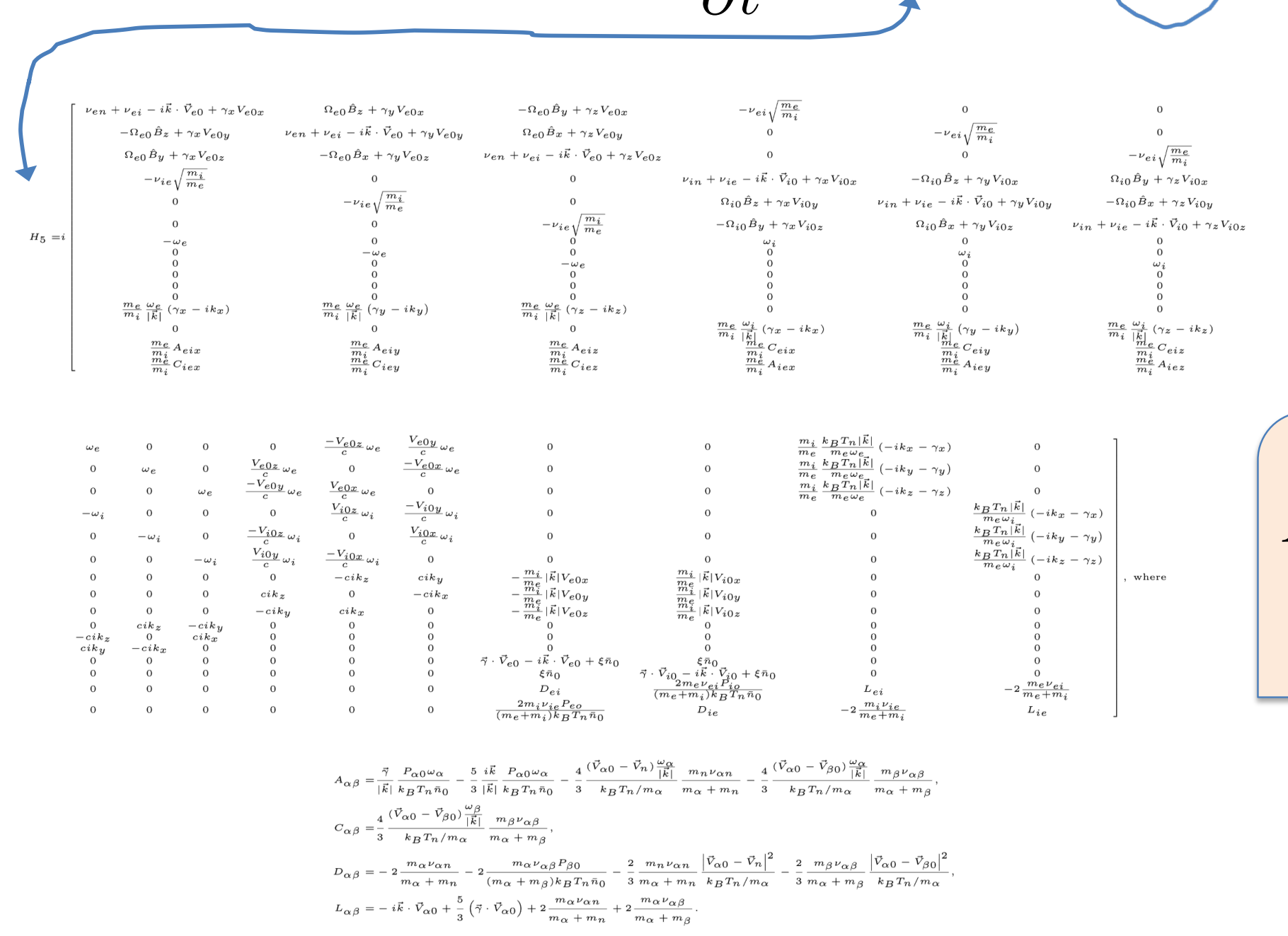
$$k = 2\pi/\lambda_\parallel - i/l_d$$

4 Making This Rigorous: 5-Moment Fluid Eqns.

$$\frac{\partial \vec{X}}{\partial t} - iH_5\vec{X} = \vec{F}(t)$$

Nonlinear terms we will drop, like electrostatic theory.

- momentum equations
- continuity equations
- energy equations
- Maxwell equations
- electrons and ions



arbitrary coefficients

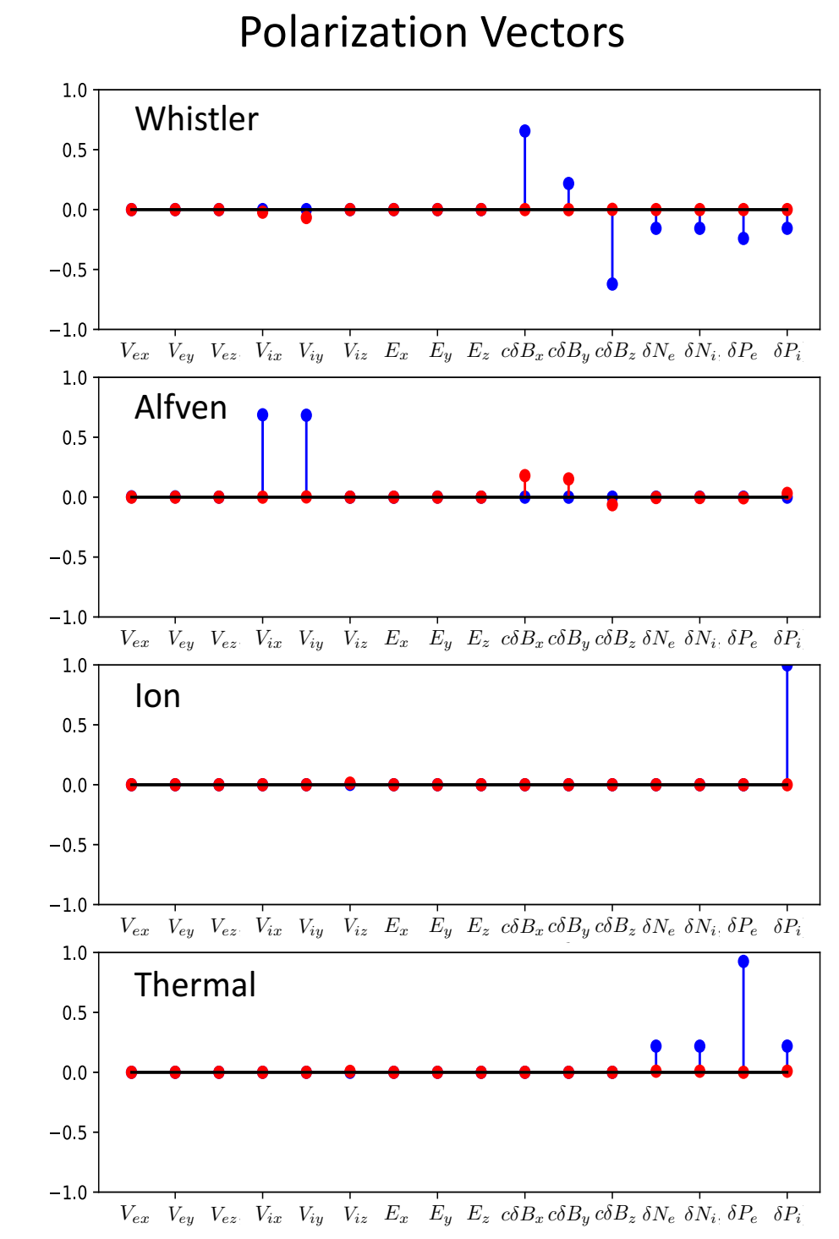
eigenvectors

eigenvalues

$$\vec{X}\left(t,\vec{k}\right) = \sum_{j=1}^{16} a_{0j}(\vec{k})\,\vec{h}_j(\vec{k})\,e^{i\omega_j(\vec{k})t}$$

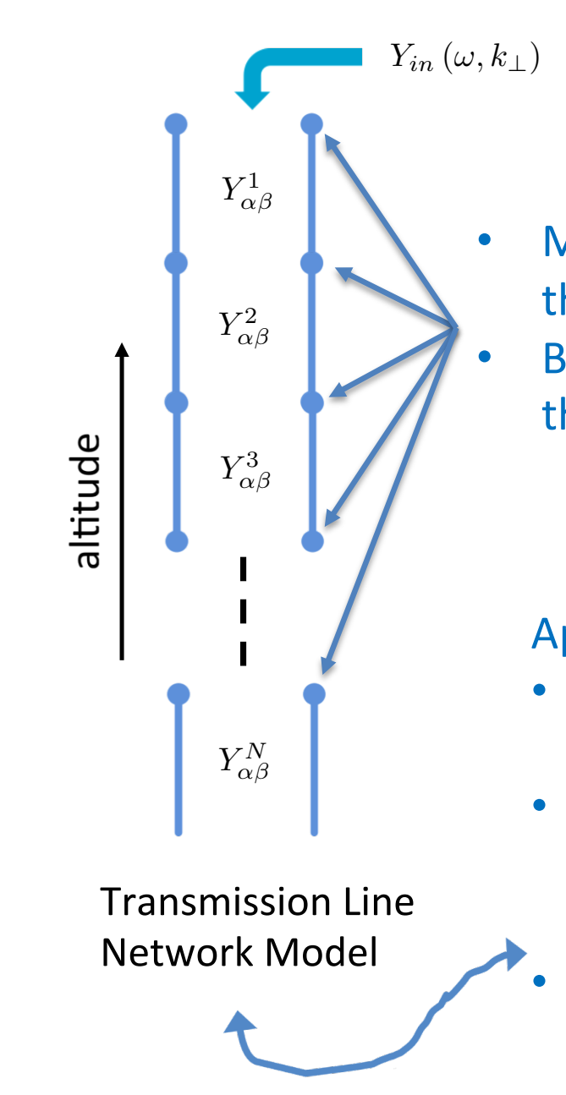
Exact Initial-Value Solution.

5 Use Two Wave-Modes and Stack Slabs, to make a Realistic Model



- 16 equations gives 8 potential waves.
- The X-mode, O-mode and Z-mode waves are too high in frequency.
- There is also a non-propagating pair.
- That leaves 4 waves.
- Only the Whistler and Alfven can travel more than 10 km.
- Whistler is cutoff in F region.

Stack Slabs and Use Two Modes



Transmission Line Network Model

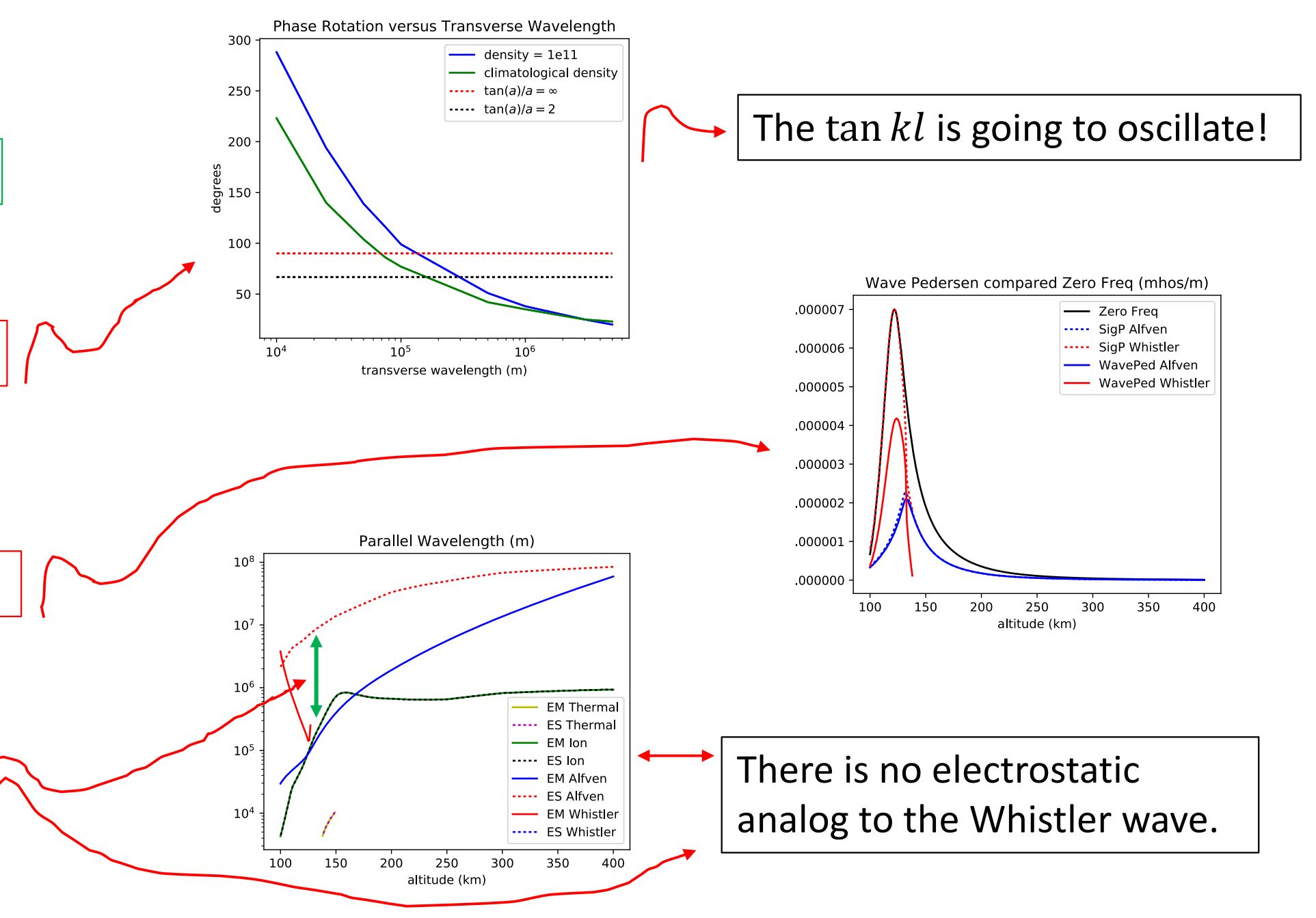
- Match across the boundaries.
- Bootstrap from the last section.

Approximations:

- Keep only the Alfven and Whistler terms.
- Usual interpretation of a propagating wave packet.
- Model ionosphere as stack of thin homogeneous slabs.

6 Back to the Gedankenexperiment, Check the Electrostatic Criteria for the Two Waves

- Are the dissipation scale lengths long enough to ignore? **Yes.**
- Are the parallel wavelengths long enough to ignore? **No!**
- Do the Wave-Pedersen conductivities equal the usual Pedersen conductivity? **No!**
- Can we substitute electrostatic waves? **No!**

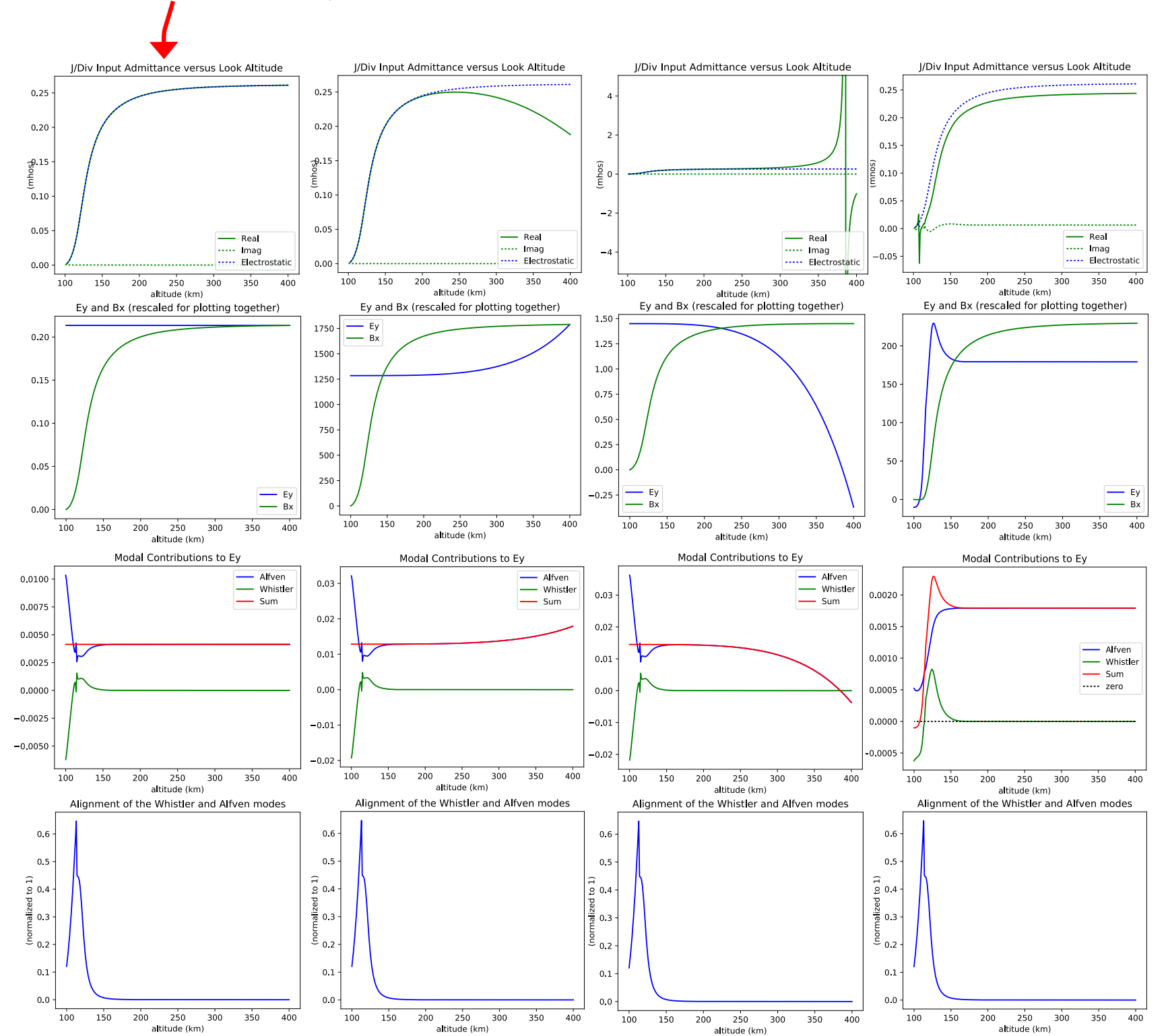


The tan kl is going to oscillate!

There is no electrostatic analog to the Whistler wave.

7 Model Validation and Three Types of Non-Electrostatic Effect

Exact reproduction of electrostatic theory



Ionospheric conductance versus altitude compared to electrostatic.

Mapping of E-field (blue) and B-field (green).

E-field contributions from Alfven (blue) and Whistler (green) modes.

Alignment of Alfven and Whistler modes. There is a near degeneracy!

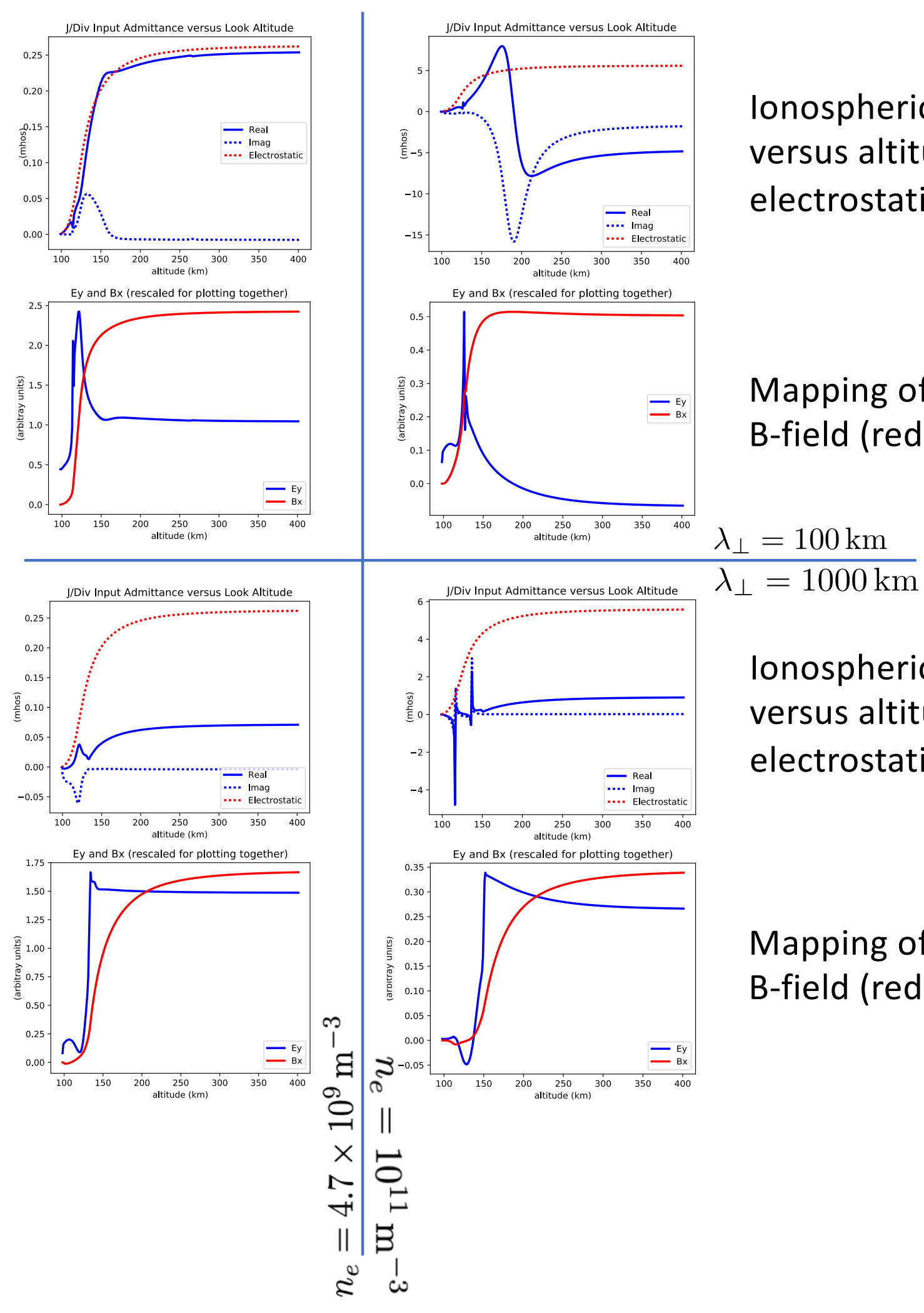
Modify the waves to satisfy the criteria for electrostatic theory.	Decrease only the dissipation scale length.	Decrease only the wavelength.	Make wavelengths unequal without decreasing.

$\lambda_\perp = 100\,\mathrm{km}$

$n_e = 4.7 \times 10^9\,\mathrm{m}^{-3}$

One kilometer thick slabs.

8 Summary of Theoretical Results

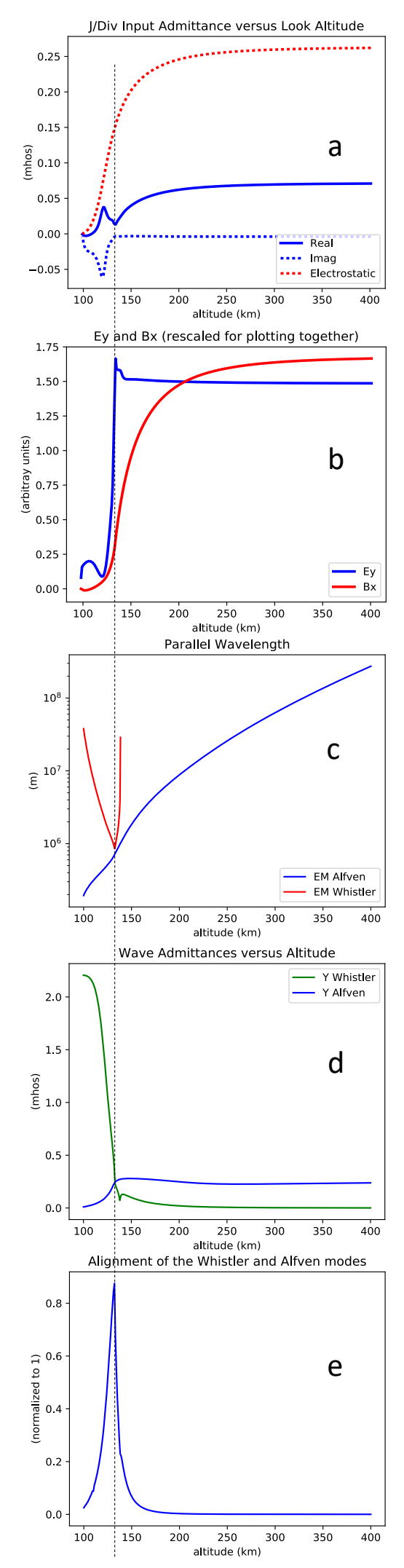


Ionospheric conductance versus altitude compared to electrostatic.

Mapping of E-field (blue) and B-field (red).

Ionospheric conductance versus altitude compared to electrostatic.

Mapping of E-field (blue) and B-field (red).



a Conductance compared to electrostatic

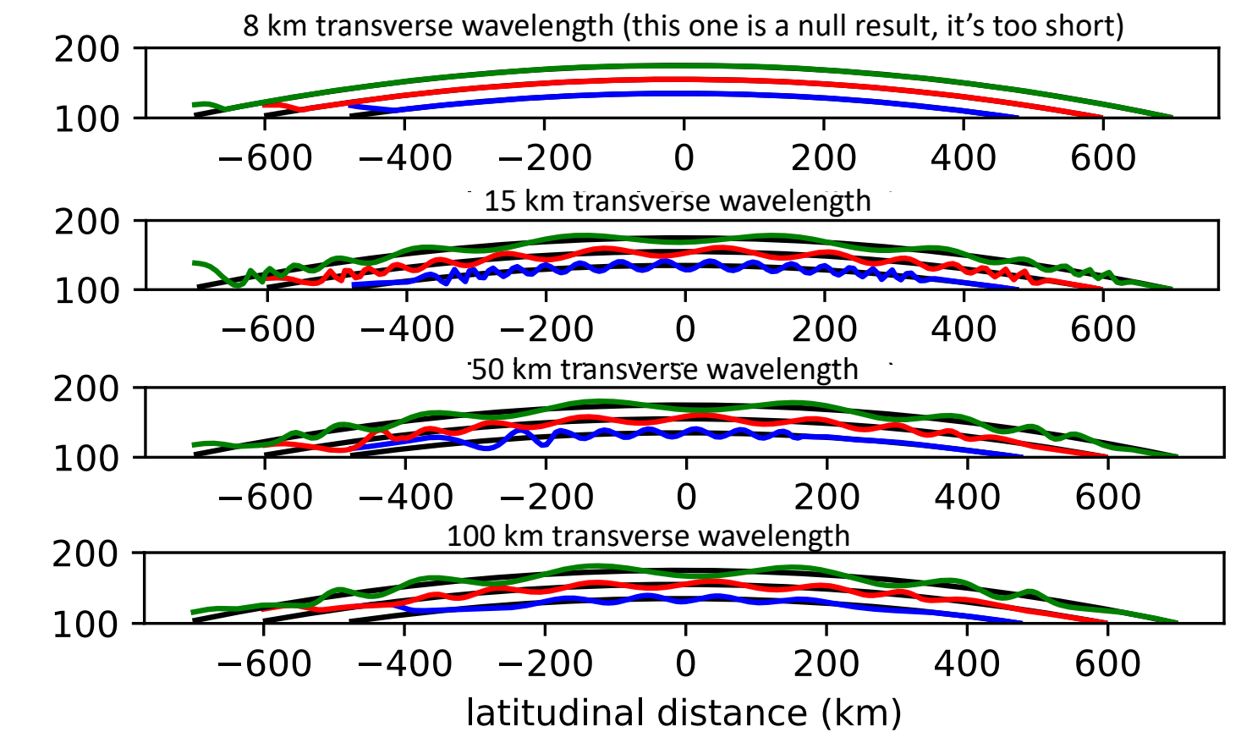
b E field and field-aligned current

c Parallel wavelength of modes

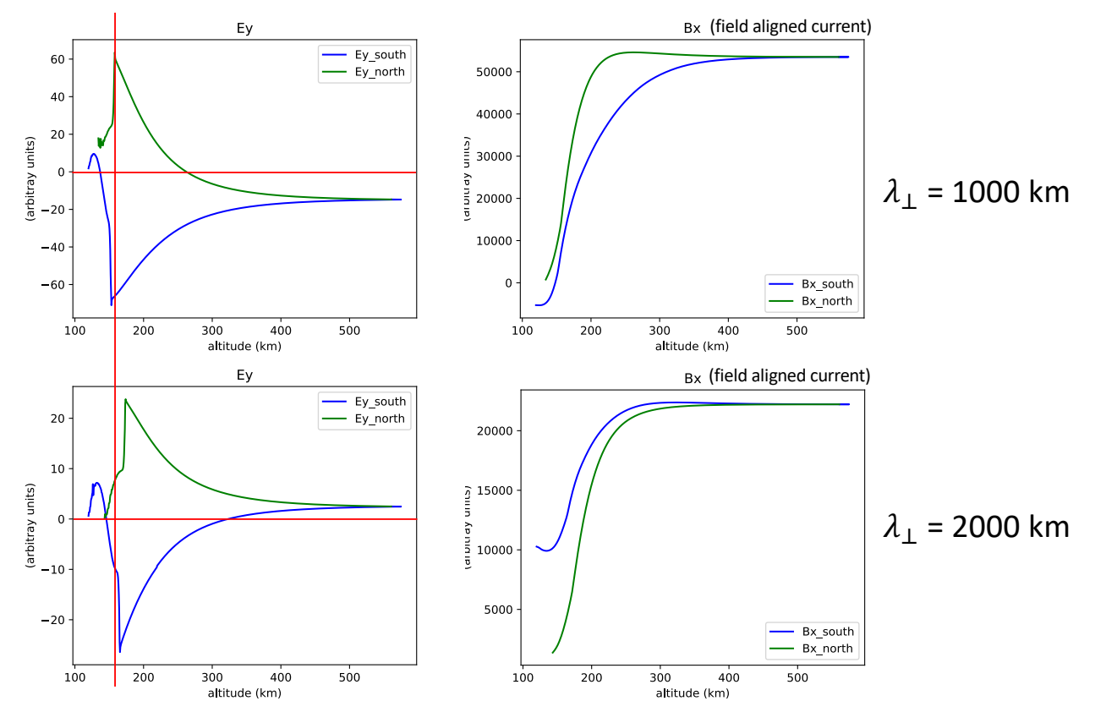
d Characteristic admittance of modes

e Degeneracy of modes

Equatorial:
Field aligned current for source at 103 km alt., for three field lines with peak altitudes 135 km, 155 km, and 175 km



Low Latitude:
Analyzing the field line from Tonga to IKON during eruption



9 Major Deviations from Electrostatic Theory

- The "wave-Pedersen conductivity" is the conductivity that contributes to ionospheric conductance, and it is roughly half the usual Pedersen conductivity. A large part of the difference can be attributed to inclusion of the imaginary part of ω in the conductivity equations, which is missing in electrostatic treatments.
- For transverse wavelengths of a few hundred kilometers and below, the parallel wavelength is often too short to ignore. To get the parallel wavelength correct, it is important to evaluate the dispersion relation for complex ω as a function of real k, rather than the reverse.
- Modal interaction becomes very important in the lower E region, where a second mode arises and there is a (near?) degeneracy of the two modes. The signal reflects strongly at the altitude of the degeneracy, so the conductivity at lower altitudes does not contribute to the ionospheric conductance seen from above.
- Altogether, the ionospheric conductance at longer transverse wavelengths is reduced by 70%-80%, and resonance is possible at shorter wavelengths.
- Substituting electrostatic waves for electromagnetic waves will likely give unphysical results for transverse scales larger than a couple of hundred meters. The wavelength for the Alfven wave is too long by more than an order of magnitude, and there is no analogue for the Whistler wave.

10 References

Cosgrove, R. B. (2016), *J. Geophys. Res. Space Physics*, 121, doi:10.1002/2015JA021672

Cosgrove, R. B. (2024), *Sci Rep*, 14(7701), doi:10.1038/s41598-024-58512-x, https://rdcu.be/dDjuf

The Paper

My Blog

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[1] Center for Geospace Studies, SRI International, Menlo Park, CA russell.cosgrove@sri.com