Ionospheric radio beacon signal analysis and parameter estimation using automatic differentiation

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Scientific Goal

A model that employs ray tracing and has been enhanced through automatic differentiation (AD) is presented to examine the estimation of regional electron density within a volume.

Motivations

- Hẏsell et al. (2021) described a method for specifying the ionospheric electron number density regionally in a three-dimensional volume. Although effective, the algorithm was very complicated due to the complexity in variational sensitivity analysis required for the ray amplitudes and to solve the two-point boundary problem for each ray.
- The new optimized model (with AD) was used to investigate the events of September 01, 2022, particularly because the ne ISR estimates were unavailable at times.

Introduction

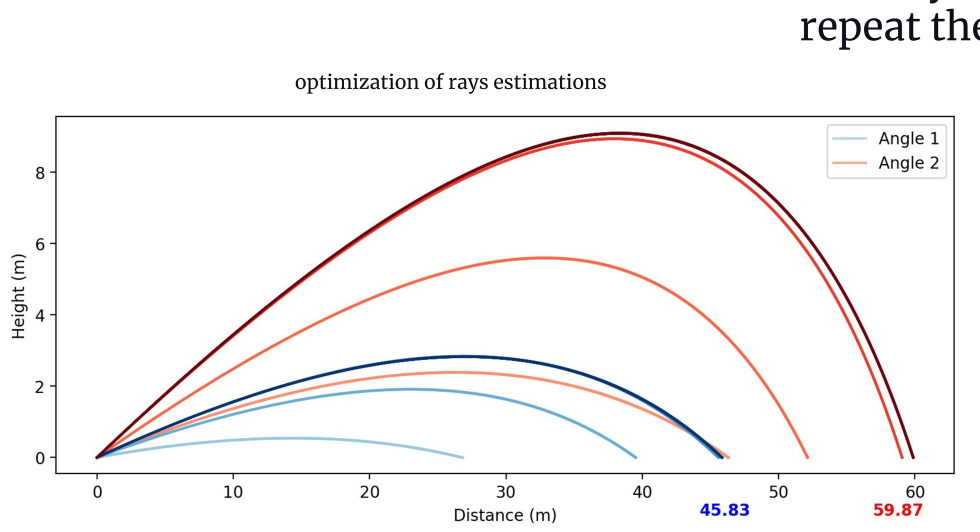
Let's consider a simpler example to understand how the model works.

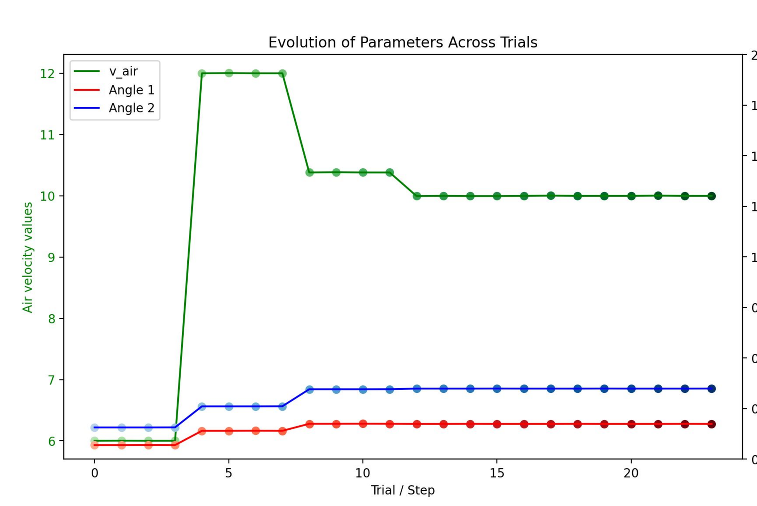
Set up:

- Model equation for predictions: parabolic trajectory equation
- Parameters to estimate:
  From the ray: shooting angles(angle 1 and angle 2)
  From the medium: air velocity
- Samples: final position from each ray
- Observables: Time delay

Estimating parameters to reconstruct the medium:

1. Set initial values for angles and air velocity.
2. Employing the least squares method and sensitivity analysis(AD used for this), we optimize ray predictions by minimizing the difference between the position prediction and the final position.
3. After discovering the optimal rays, estimate the air velocity by minimizing the discrepancy between the estimated time delay and the observable. If the difference is not too significant, retain the previous air velocity estimated; otherwise, adjust the parameters and repeat the process.





Methodology

Set up:

- Model equation for predictions: Ray tracing equations based on geometric optics (Jones and Stephenson, 1975)

$$\dot{r} = -\left(\frac{\partial H}{\partial \omega}\right)^{-1} \frac{\partial H}{\partial \boldsymbol{k}}$$

$$\dot{k} = \left(\frac{\partial H}{\partial \omega}\right)^{-1} \frac{\partial H}{\partial \boldsymbol{r}}$$

H: Hamiltonian, $\dot{r}$ and $\dot{k}$ are the equations of motion.

- Parameters to estimate:
  From the ray: initial ray bearings
  From the medium: ionosphere parameters
- Sample: final ray coordinates
- Observables:
  - **HF beacon data** : group delay, Doppler shift, and amplitude measurements (1-min cadence).
  - **The non-HF data** GNSS TEC measurements and electron density profiles from ISR(Incoherent scatter radar)

Station	Latitude (north)	Longitude (east)	Altitude (masl)
Jicamarca	−11.950	−76.873	52
Huancayo	−12.042	−75.323	3119
Mala	−12.666	−76.628	31
La Merced	−11.126	−75.368	817
Barranca	−10.760	−77.760	55
Oroya	−11.551	−75.942	3790
Ancon	−11.777	−77.150	51
Sicaya	−12.040	−75.296	3330
Ica	−14.089	−75.736	402

Table 1. HF Beacon Station Locations. The first 6 stations are receivers and the last 3 are transmitters.

Estimating parameters to reconstruct electron densities within a volume:

1. Set initial parameter values.

2. Minimize the difference between the position prediction and the final ray coordinates to optimize ray predictions.

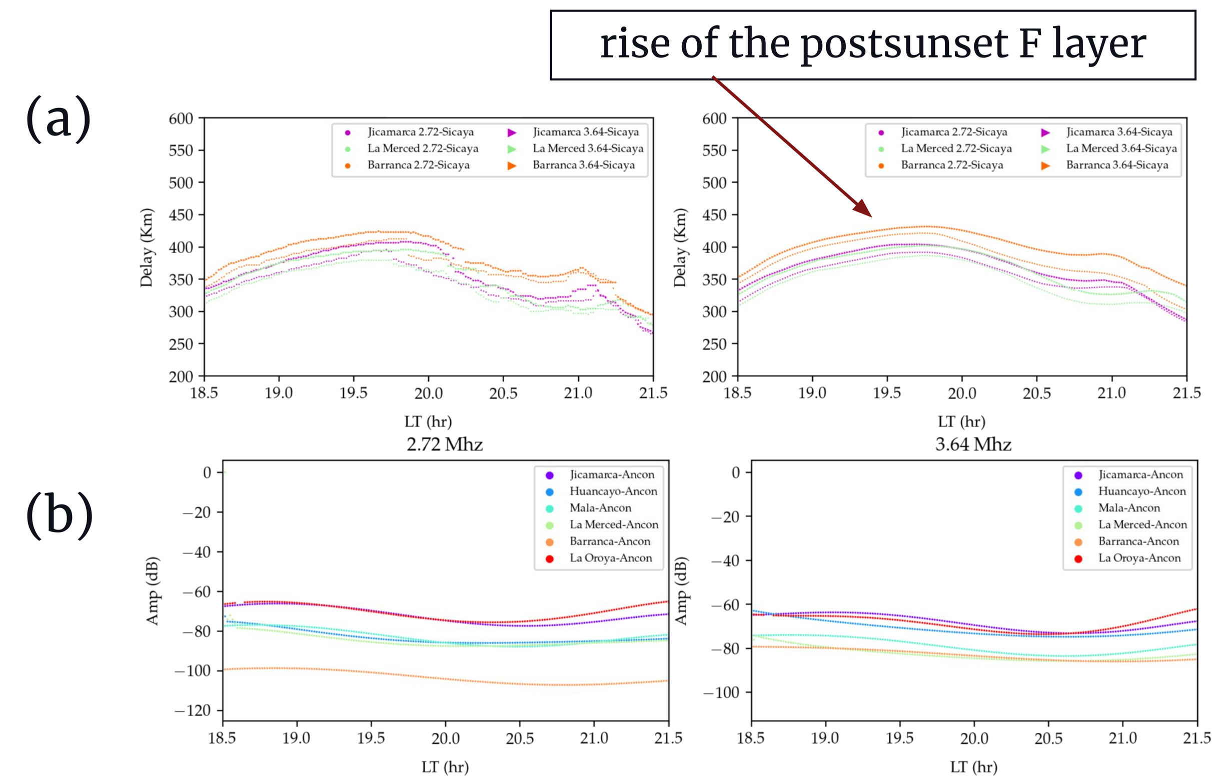
   Sensitivities equations are required in the minimization function. The sensitivities are also needed to predict the terminal ray amplitudes affected by ray focusing.

   This time, Automatic differentiation (Baydin et al., 2018) is used to calculate the sensitivities instead of another method (like the adjoint state method). This method can compute the gradients without writing code to solve sensitivities equations.

3. After discovering optimal rays, estimate the medium parameters by minimizing the discrepancy between the model estimations and observables. If the difference is not too significant, retain the previous medium parameters; otherwise, adjust the parameters and repeat the process.

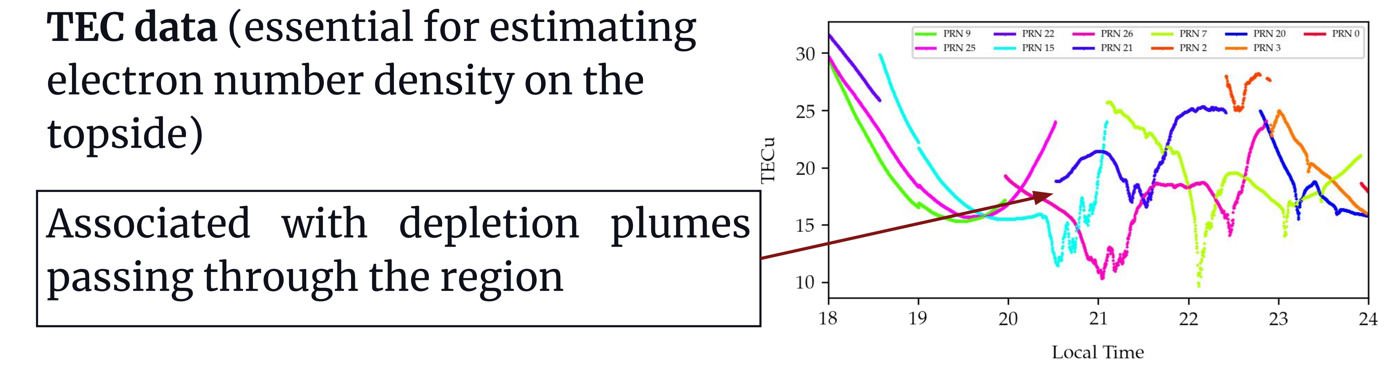
Data analysis

For **September 01, 2022 from 18:30 to 21:30 LT**

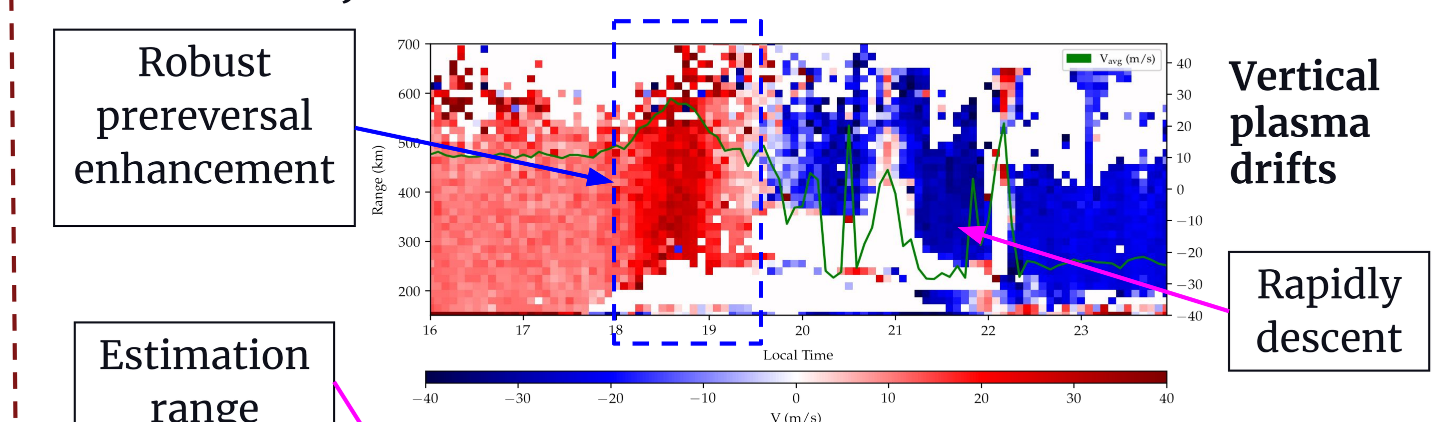


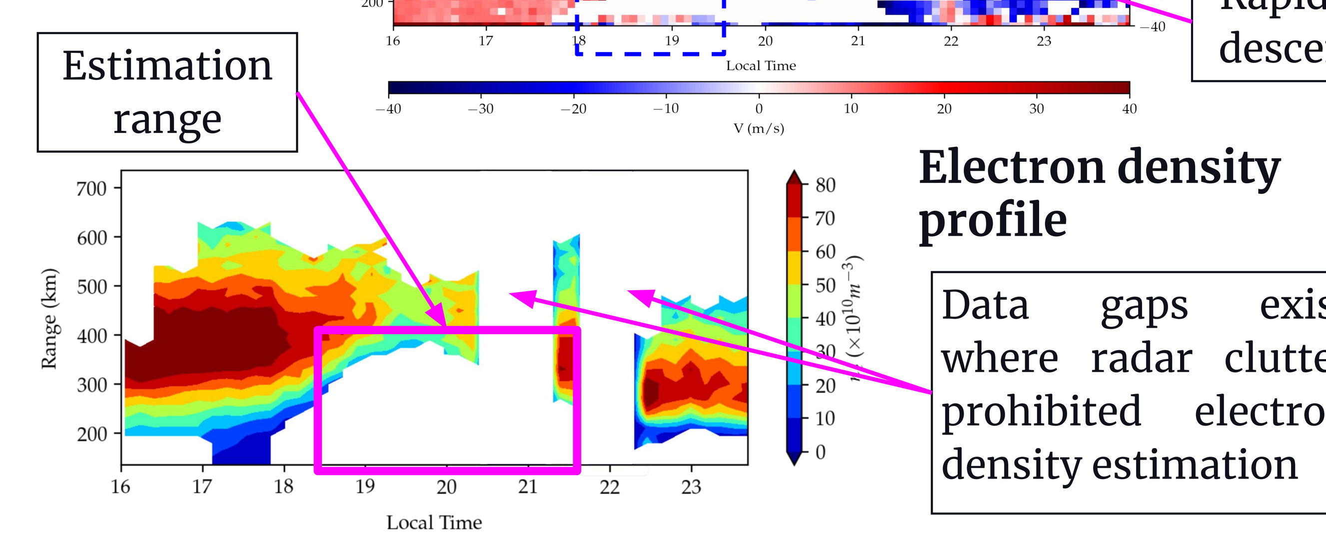
(a)Group delay (left) and accumulated carrier phase (right), (b)Relative received signal power estimates for 2.72 and 3.64 MHz frequencies for the Jicamarca **HF beacon receiver**.

TEC data (essential for estimating electron number density on the topside)



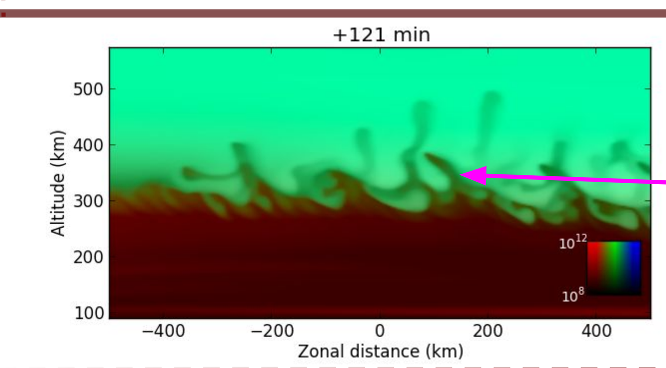
Measured with **Jicamarca ISR:**





Results

Numerical simulation of convective plume development over Jicamarca. The figure shows molecular ion, atomic ion, and proton number densities versus altitude and horizontal distance.

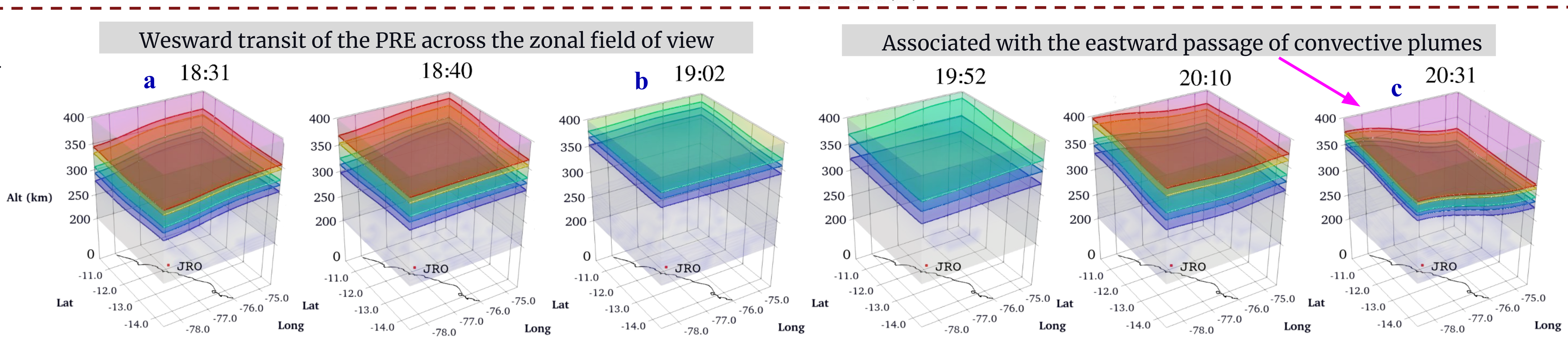


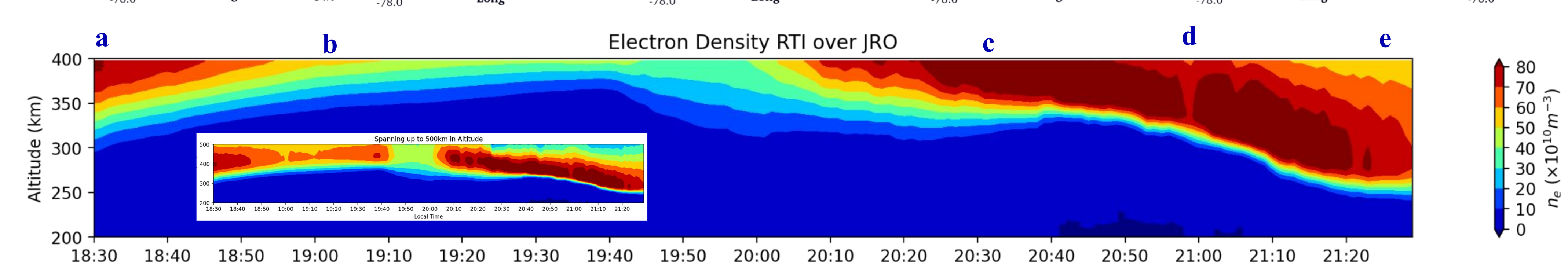
Plumes' size and impact were limited by the stabilizing effects of the westward background electric field associated with descent.

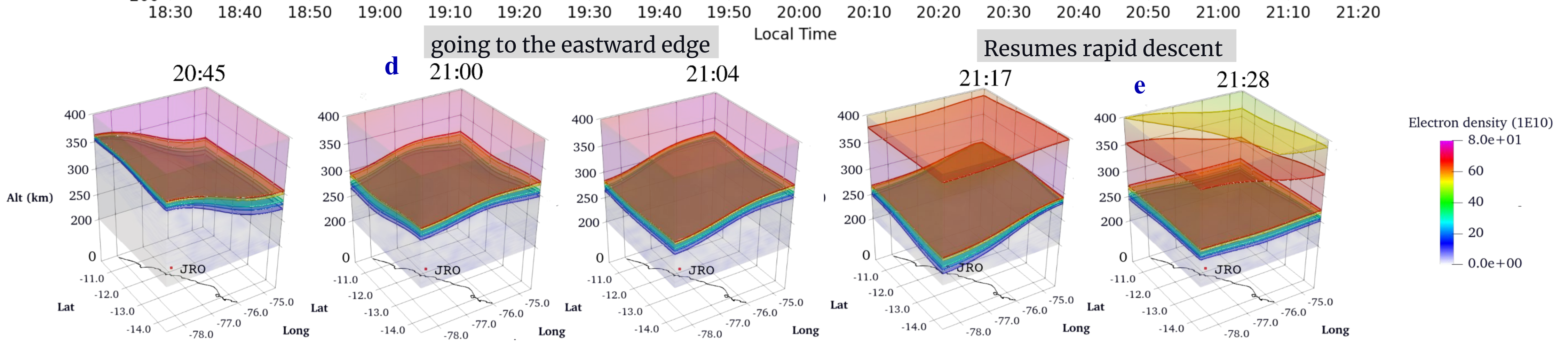
Reconstructed electron density isosurfaces

The results exhibited no significant north-south gradients throughout the event.

It seems the rapid descent of the ionosphere following the PRE (prereversal enhancement) was responsible for delaying and arresting the growth of the depletions







Conclusions and future work

- Automatic differentiation afforded a dramatic simplification of the numerical code without introducing penalties to computation speed or accuracy.
- HF data inversion method provided regional electron density measurements during a period when ionospheric irregularities were present, rendering ISR-derived Ne estimates unavailable at times.
- The regional electron density estimates revealed the passage of the prereversal enhancement of the zonal electric field through the region with the solar terminator followed by the rapid descent of the F region throughout the rest of the evening. Small radar plumes developed, but their size and impact were limited by the stabilizing effects of the westward background electric field associated with descent.
- We plan to apply the inversion method to HF beacon data acquired in Alaska, where additional kinds of ionospheric diagnostic information is available to incorporate into the analysis.

References

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