A Theory for Modeling Partially Magnetized Electrons under Influence of External Electric Field in Geophysical Applications using Fluid Coefficients Corresponding to Non-magnetized Plasma

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1. Abstract

In the presence of an external electric field and magnetic field, electron transport and rate coefficients vary as a function of the reduced electric field, reduced electron gyrofrequency, and the angle between electric field and magnetic field vectors [Starikovskiy et al., *Phys. Rev. E*, 103, 063201, 2021; Janalizadeh and Pasko, *J. Geophys. Res.: Space Phys.*, 128, e2022JA031009, 2023]. We present a theory based on solution of the Boltzmann equation in non-magnetized plasma to obtain electron transport and rate coefficients in magnetized plasma by using an effective electric field [Janalizadeh et al., *Plasma Sources Sci. Technol.*, DOI:10.1088/1361-6595/acdaf1, 2023]. This reduces the original problem with the three input parameters mentioned above to an equivalent problem with single input parameter, the effective reduced electric field E_{eff}. Comparisons between exact coeffcients obtained via BOLSIG+ [Hagelaar and Pitchford, *Plasma Sources Sci. Technol.*, 14, 722-733, 2005] and approximate coeffcients of the proposed method are presented for pure carbon dioxide, air, and a mixture of molecular hydrogen and atomic helium representing the giant gas planets of the solar system [Janalizadeh et al., 2023]. A study of the convergence criteria of the fixed-point iteration method, which we use to solve for E_{eff}, shows that the method always converges to a unique solution, provided a judicious choice of the initial value of the electric field for fixed-point iterations. In particular, a simple choice that ensures convergence is the initial value equal to the applied field. The modeling framework that we report in this work is applicable to a broad range of natural phenomena and gas mixtures including electrical gas discharges in the atmospheres of the Earth and Jupiter (i.e., sprites and elves).

2. Motivation

Characterizing the influence of an external magnetic field on weakly ionized plasma is of significant interest for both space physics and laboratory applications, such as sprite streamers assoicated with atmospheric dischargers, and laboratory streamer discharges.

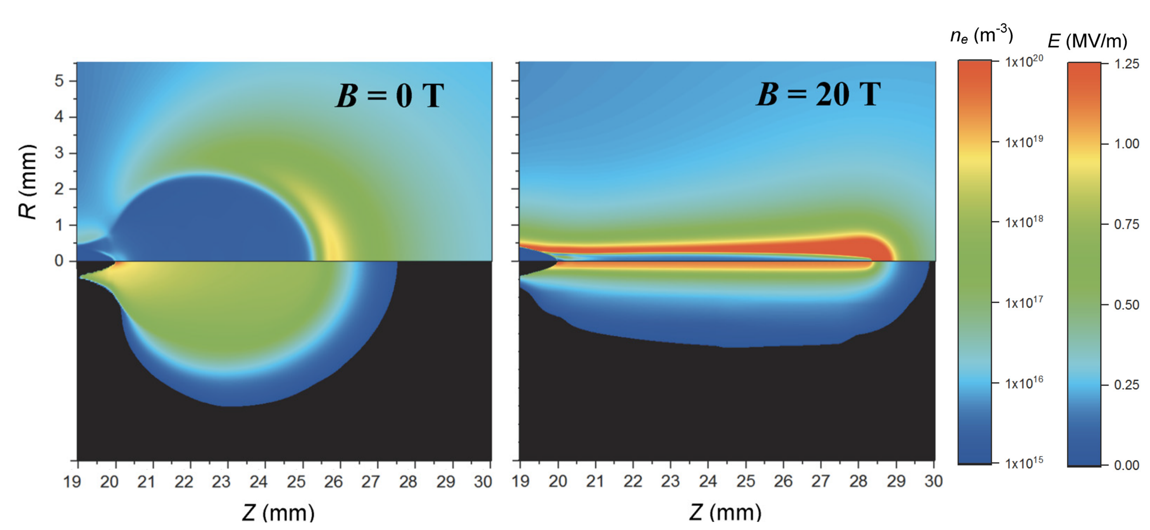

Figure 1. Spatial distributions of the electric field E (upper half panels) and electron density n_e (lower half panels) for a positive streamer in the absence (left) and presence (right) of magnetic field [Starikovskiy et al., 2021].

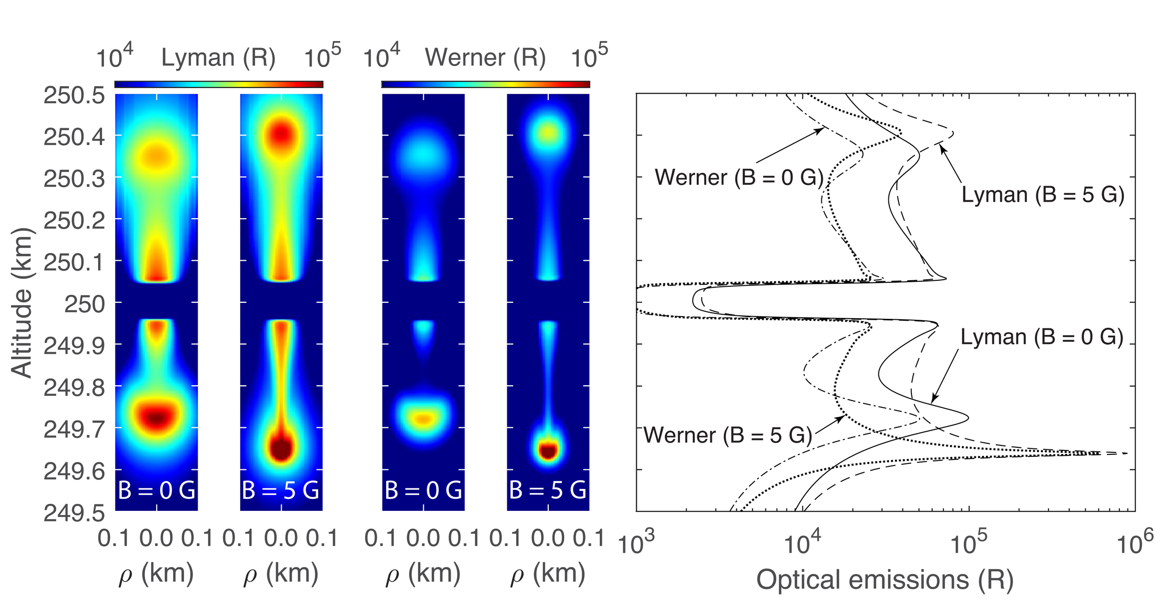

Figure 2. Optical emmision intensity of a sprite streamer in Jupiter's atmosphere in units of Rayleigh (R) for Lyman and Werner band systems of H_2 in the absence (B = 0 G) and presence (B = 5 G) of magnetic field [Janalizadeh and Pasko, 2023].

The presence of a magnetic field results in streamer sharpening, and brighter optical emissions.

3. Model Formulation

Transcendental Method:

The isotropic part of the electron velocity distribution function (EVDF), $f_{0B}(v)$, in the two-term approximation to the Boltzmann equation, satisfies the differential equation:

$$\frac{1}{3}\left(\frac{q_e}{m_e}\right)^2 \frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{v^2}{\nu_m}\left(\frac{E_\perp^2}{1+\beta_H^2}+E_\parallel^2\right)\frac{\partial f_{0B}}{\partial v}\right]+\mathcal{C}(f_{0B})=0$$

where $\beta_H = \omega_{ce}/\nu_m(\varepsilon)$ is the Hall parameter. We wish to find the electric field E_{eff} and corresponding EVDF f_0 in the absence of a magnetic field which minimizes the residual:

$$R(v)=\frac{1}{3}\left(\frac{q_e}{m_e}\right)^2 \frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{v^2}{\nu_m}\left(\frac{E_\perp^2}{1+\beta_H^2}+E_\parallel^2-E_{\text{eff}}^2\right)\frac{\partial f_0}{\partial v}\right]$$

It can be shown [Janalizadeh et al., 2023] that minimizing this residual results in

$$E_{\text{eff}}^2 = E_\parallel^2 + \frac{E_\perp^2}{1+\beta_{\text{eff}}^2(E_{\text{eff}})}$$

Approximate Transcendental Method:

Additionally, it can be shown [Janalizadeh et al., 2023] that when $\nu_m(\varepsilon)$ is approximately constant and $\beta_H^2(\varepsilon)$>>1, E_{eff} takes the same exact form as shown above, only with an effective Hall parameter $\beta_{\text{eff}} = \omega_{ce}/\nu_m(E_{\text{eff}})$, where $\nu_m(E_{\text{eff}})$ is the momentum transfer collision frequency averaged over the electron energy distribution. This resembles the electric field experienced by a single electron [Starikovskiy et al., 2021]:

$$E_{\text{eff}}^2 = E_\parallel^2 + \frac{E_\perp^2}{1+\beta_H^2(\varepsilon)}$$

The introduced method proceeds in two steps:

1. Calculate E_{eff} for a given $(E/N, \omega_{ce}/N, \angle\vec{E},\vec{B})$ using above equation.
2. Calculate electron transport and rate coefficients using E_{eff}.

4. Numerical Method

If we define $x \equiv E_{\text{eff}}/N$ and

$$\phi(x) \equiv \frac{1}{N}\left[E_\parallel^2 + \frac{E_\perp^2}{1+\beta_{\text{eff}}^2(x)}\right]^{1/2}$$

then x is fixed point of the function $\phi(x)$, i.e., $x = \phi(x)$. It can be shown that a solution x always exists [Burden and Faires, Numerical Analysis, 2005, Theorem 2.2]. Additionally, if $d\phi/dx$ exists in the interval $(0, E/N)$ and a positive constant k < 1 exists with

$$|d\phi/dx| \le k, \quad \text{for all } x \in (0, E/N),$$

then the solution x is unique and the fixed-point iteration defined by

$$x_{n+1} = \phi(x_n), \quad \text{where } n = 0, 1, 2, \ldots$$

always converges to this unique solution.

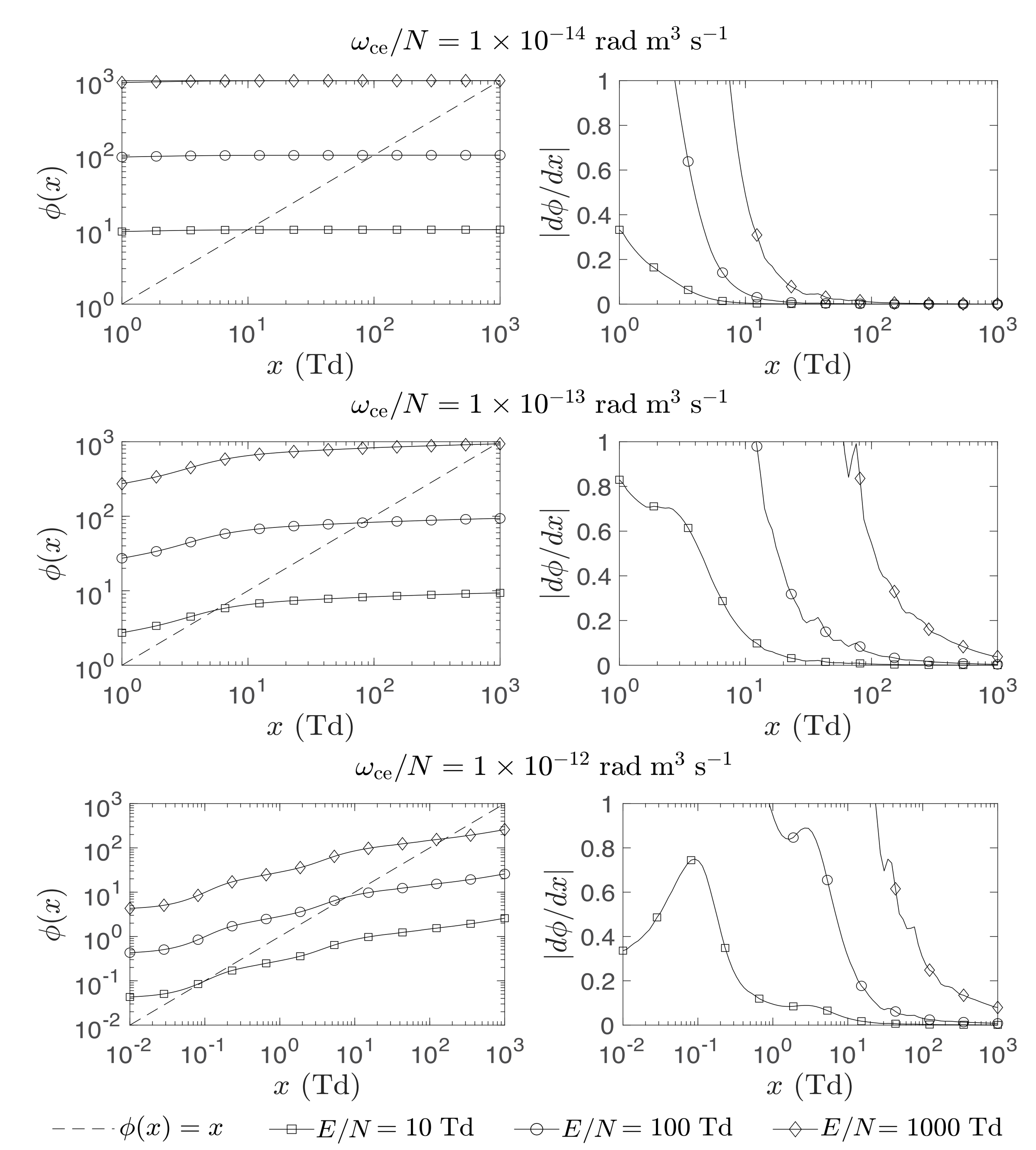

Figure 3. $\phi(x)$ and $|d\phi/dx|$ for weakly, partially, and highly magnetized plasma in air when E is perpendicular to B. 1 Townsend (Td) = 10^{-21} V m^2.

- Convergence is always achieved provided x_0 is chosen to lie towards the upper bound of the $[0,\ E/N]$ interval.
- Similar analysis for CO_2 and a (88% H_2, 12% He) mixture leads to the same conclusion.

5. Results for Air

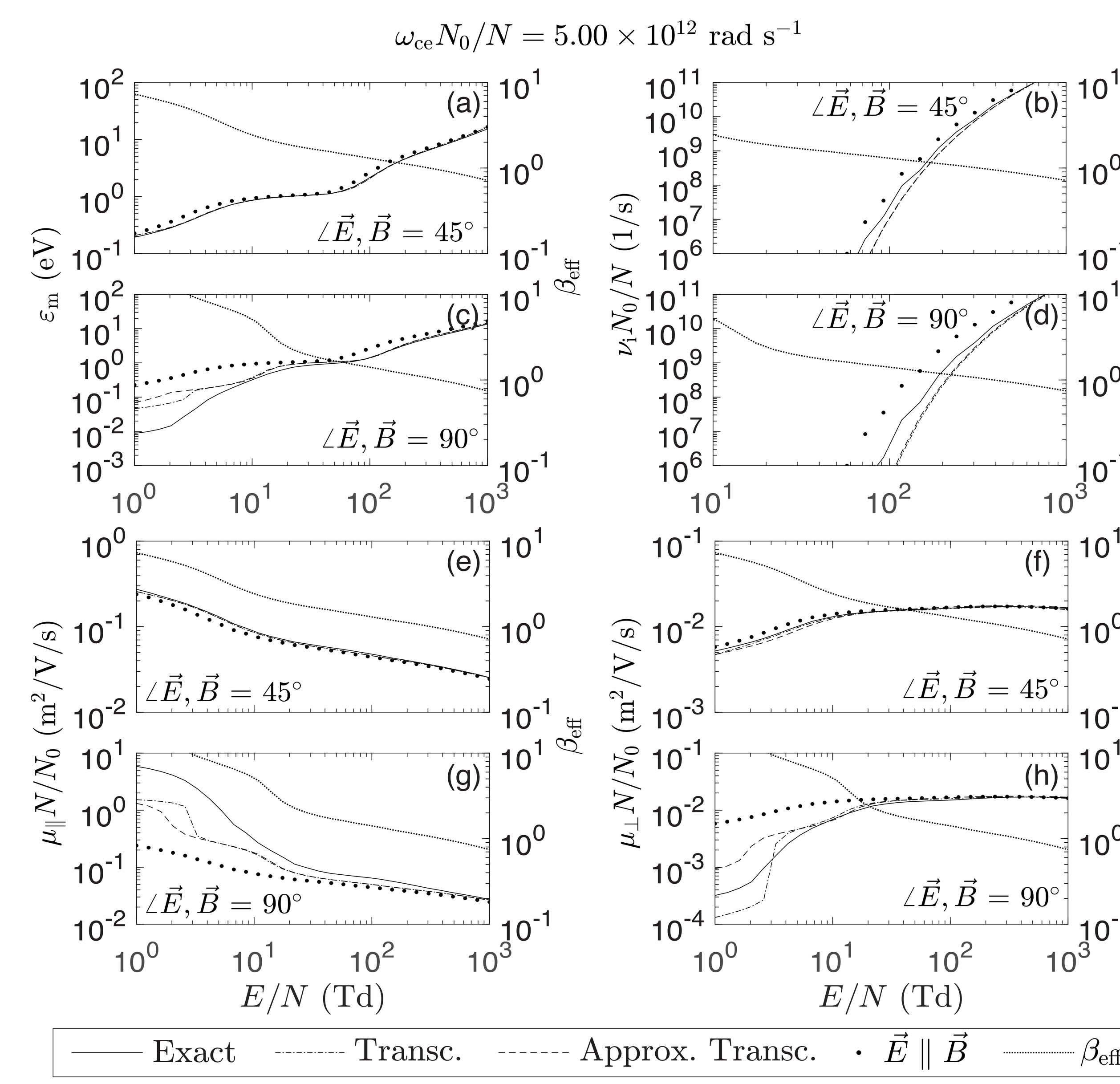

Figure 4. (a) and (c): Electron mean energy, (b) and (d): Reduced electron impact ionization frequency, (e) and (g): Electron mobility parallel to B, and (f) and (h): Electron mobility perpendicular to B. N_0 = 2.686 x 10^{25} m^{-3}

- The proposed method provides satisfactory agreement with exact solutions for major electron transport and rate coefficients, with small deviations where β_{eff} >> 1.
- Proposed method is of interest in the partially magnetized ($\beta_{\text{eff}} \sim 1$) regime, since when β_{eff} << 1, we have $E_{\text{eff}} \sim E$, and conversely when β_{eff} >> 1, we have $E_{\text{eff}} \sim E_\parallel$.

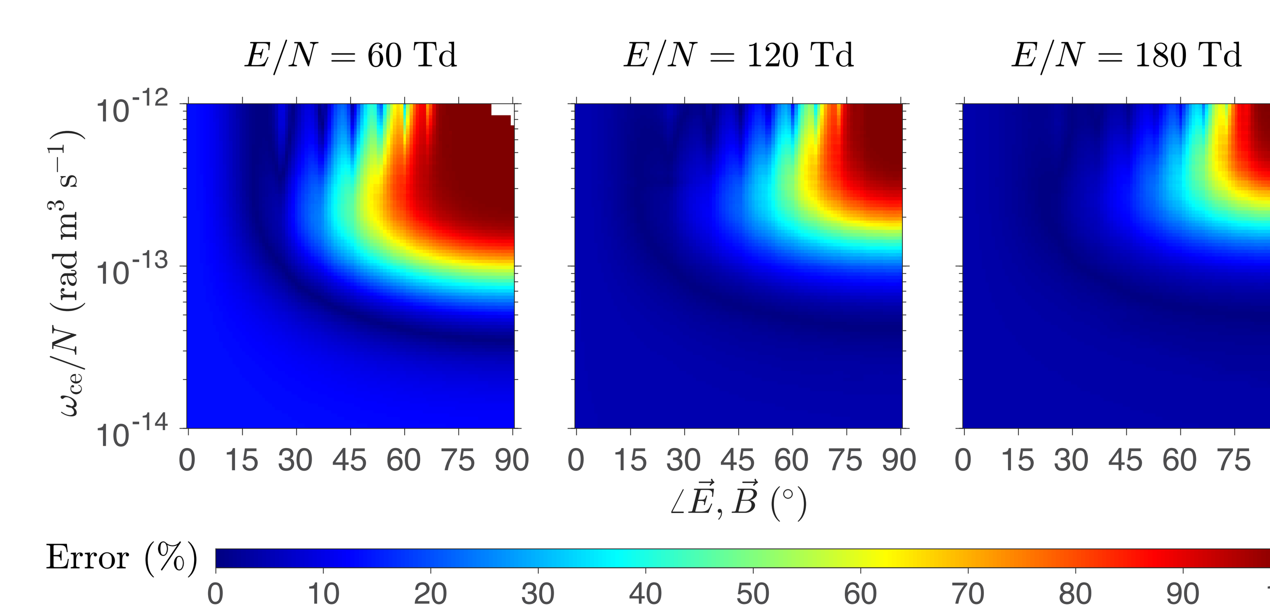

Figure 5. Percentage error of approximate transcendental method solutions for ionization frequency ν_i.

6. Conclusions

- The proposed modeling methodology leads to plasma fluid coefficients that are in satisfactory agreement with BOLSIG+'s exact calculations for air, a 88% H_2 and 12% He mixture (resembling the giant gas planets of the solar system), and pure CO_2.
- The numerical method always converges to a unique solution provided the initial value is chosen as the applied electric field.

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