Full Profile Incoherent Scatter Fitting of Arecibo Topside Echoes

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Introduction

Incoherent scatter radar with long transmitting pulse suffers from range mixing effect in measuring ionospheric state parameters such as temperatures, composition and densities. The range mixing distorts the spectrum generated from backscattered signals, introducing systematic error in spectral fitting.

For example, in the experiment in September 2016 at Arecibo, two distinct pulse lengths were used alternatively to probe the topside ionosphere at the same time. However, we found a systematic bias caused by range mixing between the two inversion results as shown in Figure 1.

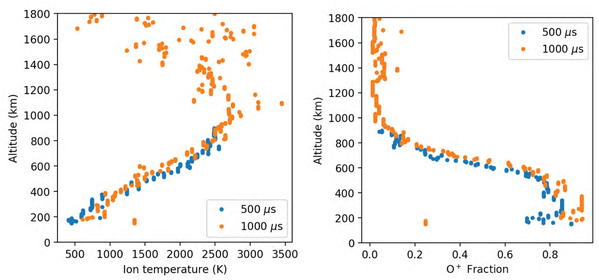


Figure 1: The ionospheric state parameters were inverted by single height fitting for both 500 μs and 1000 μs uncoded pulses with integration time of 1 minutes. The inversion difference is beyond the uncertainty of the measured parameters.

Such range mixing effect can be mitigated using full profile fitting technique as illustrated in Figure 2, which aims to model the range mixing effect and invert all the parameters from all altitudes at the same time.

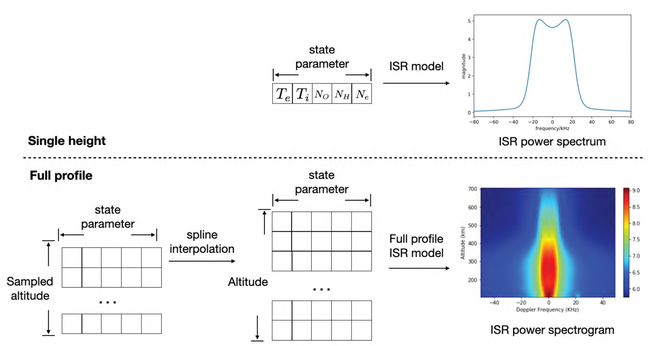


Figure 2: Comparison of the spectral model used in between single height fitting and full profile fitting. To reduce the number of fitting parameters, a common strategy is to apply spline to construct the parameter profiles with fewer hyperparameters. [Holt, 1992]

However, full profile fitting suffers from drawbacks such as:

1. Computational expensive: each iteration of the fitting requires generating intrinsic ISR spectrums at all altitudes.

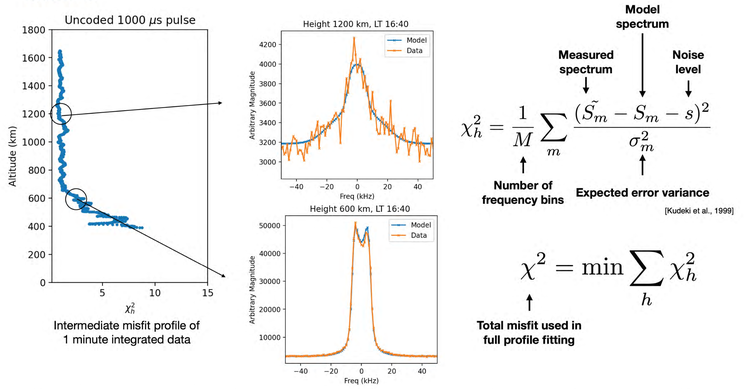
2. Sensitive to outliers: large misfit caused by outliers in the measured spectrogram (space debris or frequency interference) severely damages the convergence of the fitting.

In this work, we present full profile inversion technique with refinements to improve computational efficiency and outlier resilience.

Refinement 1: multi-pulse fusion

Full profile model is extended to include two pulse modes such that altitude coverage is combined and robustness against outliers is improved.

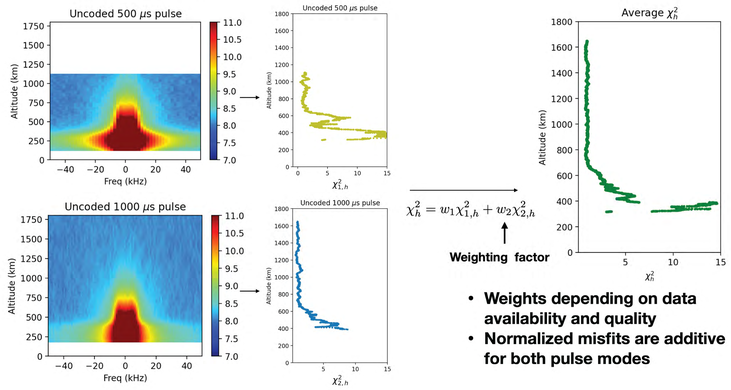
- A suitable kernel is applied to the intrinsic spectrogram to generate the spectrogram modeling range mixing by 500 μs or 1000 μs transmitted pulse.
- Misfit profile for a single pulse is composed by normalized misfits at all measured altitudes.



$$\chi_h^2 = \frac{1}{M}\sum_m \frac{(\tilde{S_m} - S_m - s)^2}{\sigma_m^2}$$

$$\chi^2 = \min \sum_h \chi_h^2$$

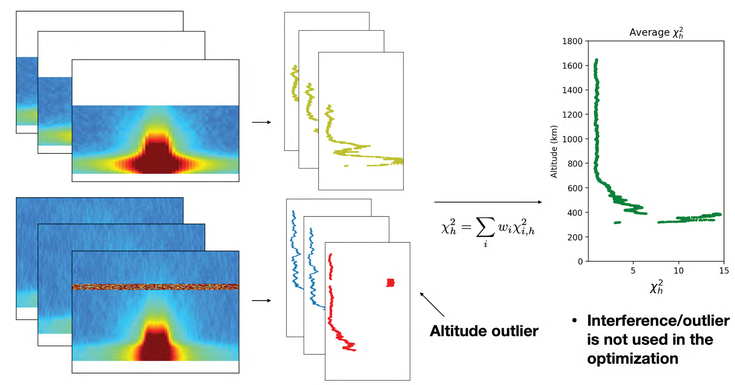
- The misfit at each altitude is normalized by its expected error variance at each frequency bin such that the misfit profiles become additive for both pulse modes.
- A weighted averaged misfit profile generated from individual misfit profiles is minimized in the full profile fitting.



$$\chi_h^2 = w_1\chi_{1,h}^2 + w_2\chi_{2,h}^2$$

- **Weights depending on data availability and quality**
- **Normalized misfits are additive for both pulse modes**

- The outliers are detected by majority vote from multiple misfit profiles within adjacent 5 minutes of the target time instance.



$$\chi_h^2 = \sum_i w_i \chi_{i,h}^2$$

- **Interference/outlier is not used in the optimization**

Refinement 2: IS spectrum model acceleration

Discretization errors caused by the chirp z-transform used in Gordeyev integral computations (shown in Figure 3) are reduced without increasing the number of samples in the integration, such that the incoherent scatter spectrum model is computed more efficiently and accurately.

$$J_s(\omega) \equiv \int_0^\infty d\tau e^{-j\omega\tau}\langle e^{j\mathbf{k}\cdot\Delta\mathbf{r}}\rangle$$

Gordeyev integral

- This was achieved by modeling and removing the discretization errors --- the error model was constructed after a series of empirical tests in which the incoherent scatter spectra were computed from theory using high and low resolution transforms.

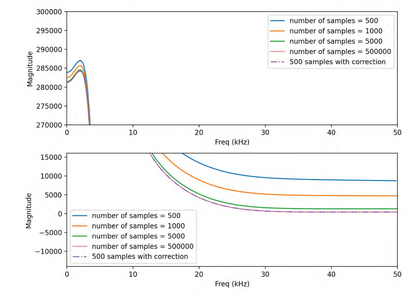
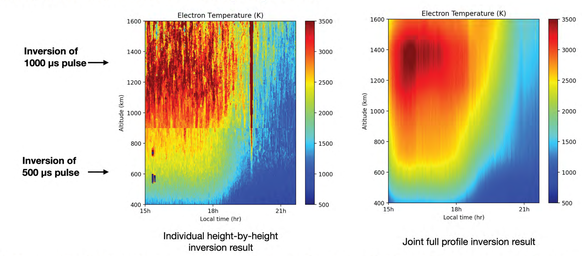


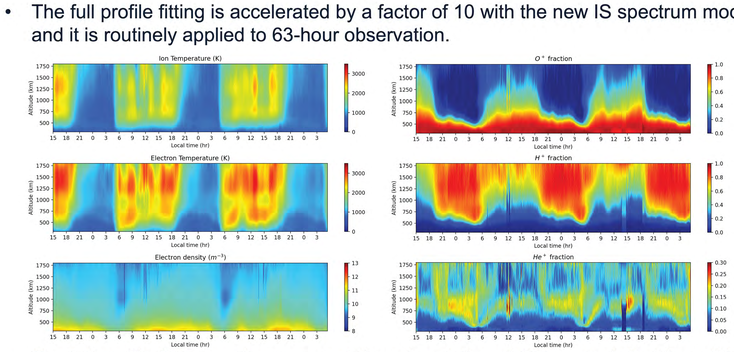
Figure 3: Comparison of the IS spectrum with the same parameters using different number of samples in evaluating Gordeyev integral with chirp-z transform.

Results

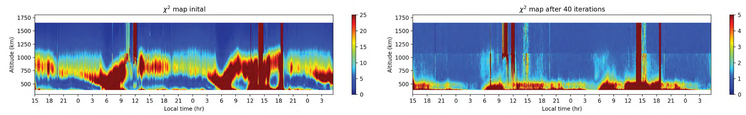
- Discrepancy in the spectral fitting results of using 500 μs and 1000 μs pulses is removed, and outliers are mostly rejected.



- The full profile fitting is accelerated by a factor of 10 with the new IS spectrum model and it is routinely applied to 63-hour observation.



- Our full profile fitting routine successfully reduced the misfit to a statistical "good" level. (A statistical "good" fit occurs when the normalized misfit is around 1.)



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