# Spherical Elementary Current System for Reconstructing Ionospheric Plasma Flow

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## Abstract

The Super Dual Auroral Radar Network (SuperDARN) is a network of radars which produce line of sight (LOS) velocities for plasma flows in the ionosphere. We implement a method to reconstruct the true velocity field of ionospheric plasma flow using a spherical elementary current system (SECS), developed from Amm [1]. The SECS method works by placing divergence free poles on the ionospheric shell and computing scaling factors at each pole which are a function of the observed LOS velocities. Once the scaling factors are known, velocity at any latitude/longitude point can be computed. We develop the SECS model and show in-depth the specific implementation and mathematical steps. We then show its general use on SuperDARN data by reconstructing ionospheric plasma flow. The SECS method is a powerful tool to reconstruct ionospheric plasma convection, and it offers a starting point for integration with other, more global ionospheric plasma flow models.

## Spherical Elementary Current System

Poles are placed on the Earth. These poles can be placed anywhere.
Define vectorfield solution to be divergence free. A resulting system of equations is formulated using this constraint.
For each pole and each velocity, a transfer matrix element is computed. The matrix system below is generated, where the number of input LOS velocities is represented by “i,” and the number of poles is represented by “j.”

$$\underbrace{\begin{bmatrix} T_{11} & & T_{1j} \\ \vdots & \ddots & \vdots \\ T_{i1} & & T_{ij} \end{bmatrix}}_{\text{Transfer Matrix}} + \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_j \end{bmatrix}}_{\text{Scaling Factors}} = \underbrace{\begin{bmatrix} Z_1 \\ \vdots \\ Z_i \end{bmatrix}}_{\text{Input Velocities}}$$

$$T_{ij} = \left(\frac{1}{4\pi R}\cot\frac{\phi}{2}\hat{\mathbf{e}}_\theta\right)\cdot\hat{\mathbf{e}}_i \quad \textit{(1)} \longleftarrow \text{Compute transfer matrix}$$

Transfer matrix is not necessarily square. Usually, more poles than input measurements, so an underdetermined system of equations
Compute scaling factors using truncated singular value decomposition
Amount of truncation defined by the parameter epsilon

$$\vec{V} = \sum_{j=1}^{poles}\left(\frac{I_j}{4\pi R}\cot\left(\frac{\phi}{2}\right)\hat{\mathbf{e}}_\theta\right) \quad \textit{(2)} \longleftarrow \text{Compute velocity at prediction points}$$

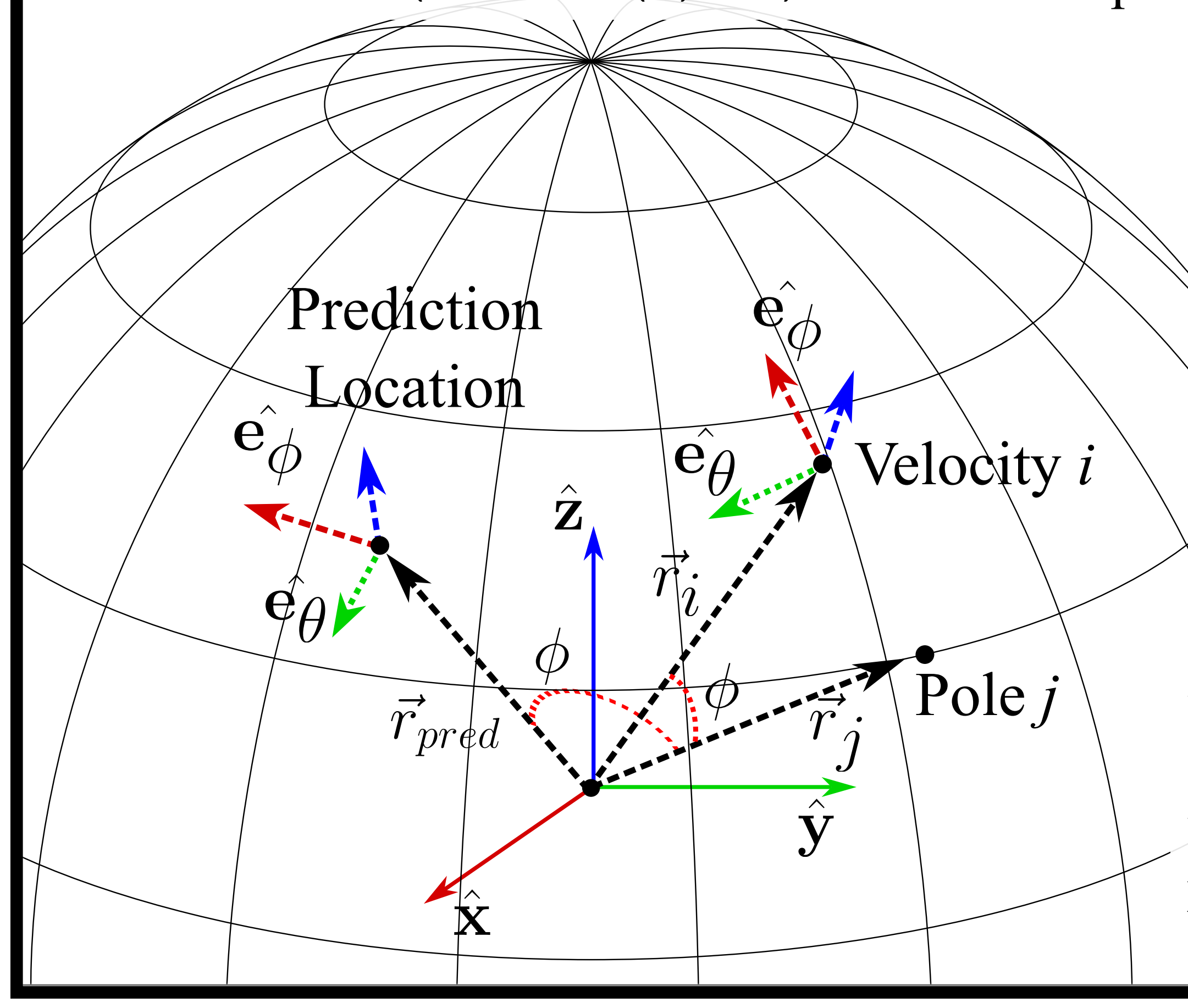

**Fig 1:** Graphic depicting the SECS coordinate systems. The input velocity locations, poles, and prediction locations are shown.

## Results – Applyng SECS to SuperDARN

Virginia Tech Plasma Convection

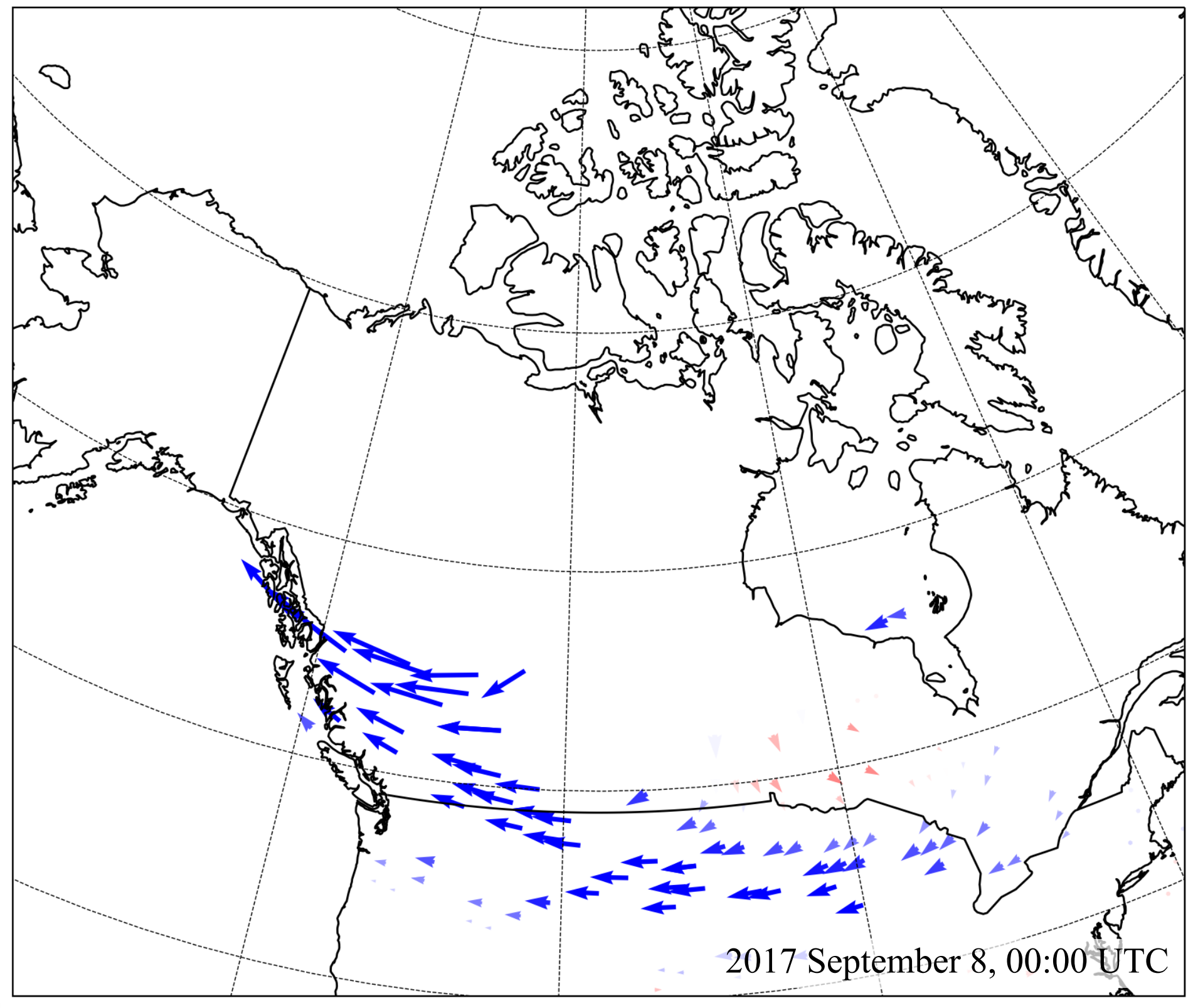

**Fig 2:** Virginia Tech plasma convection map for 2017 September 9, 00:00 UTC. The plasma convection map lines up with the other two maps.

Bristow Divergence-Free Map

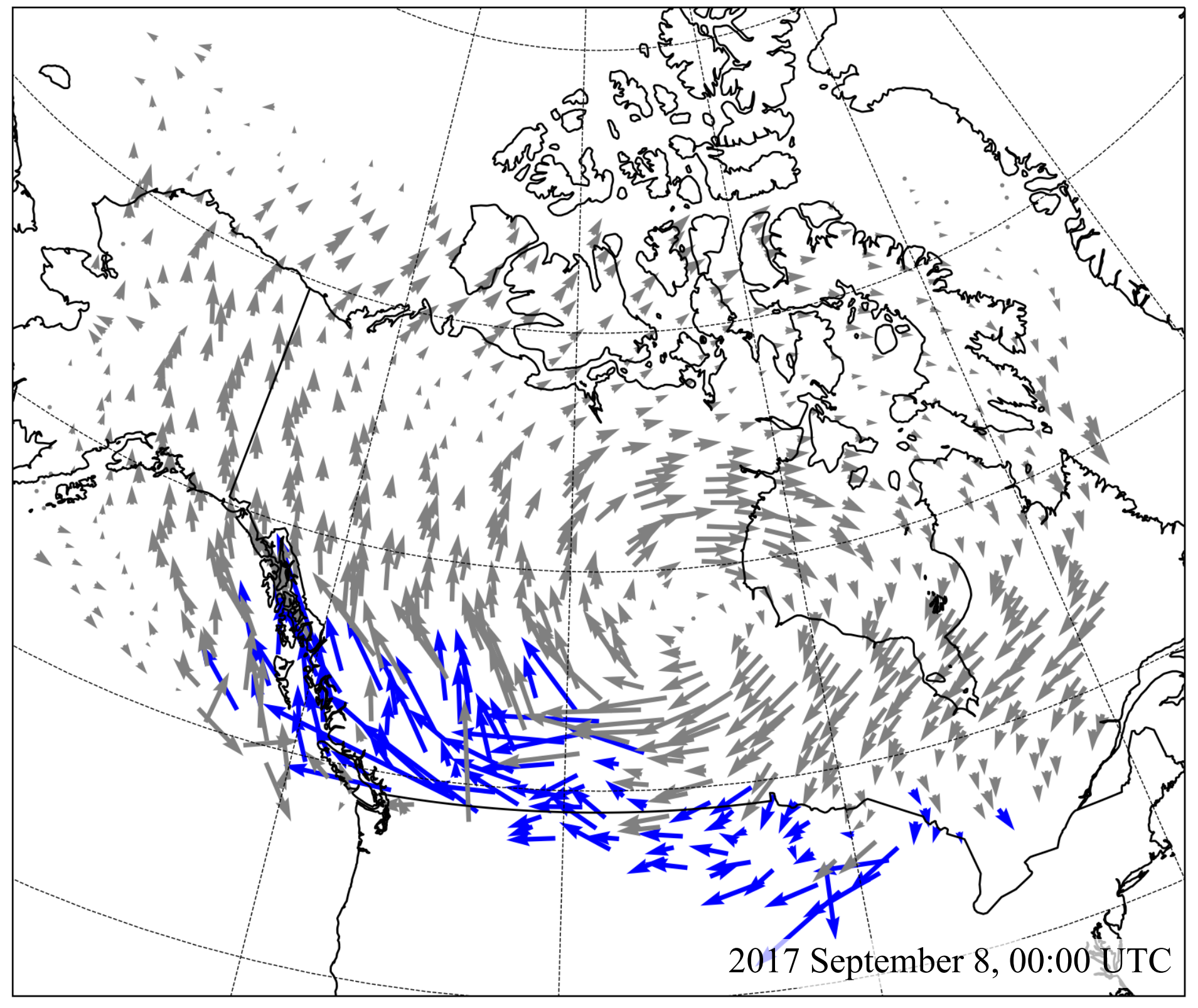

**Fig 3:** A divergence free solution obtained by Bill Bristow [2]. Blue vectors represent data that is within one degree of VT convection data.

Our SECS Implimentation

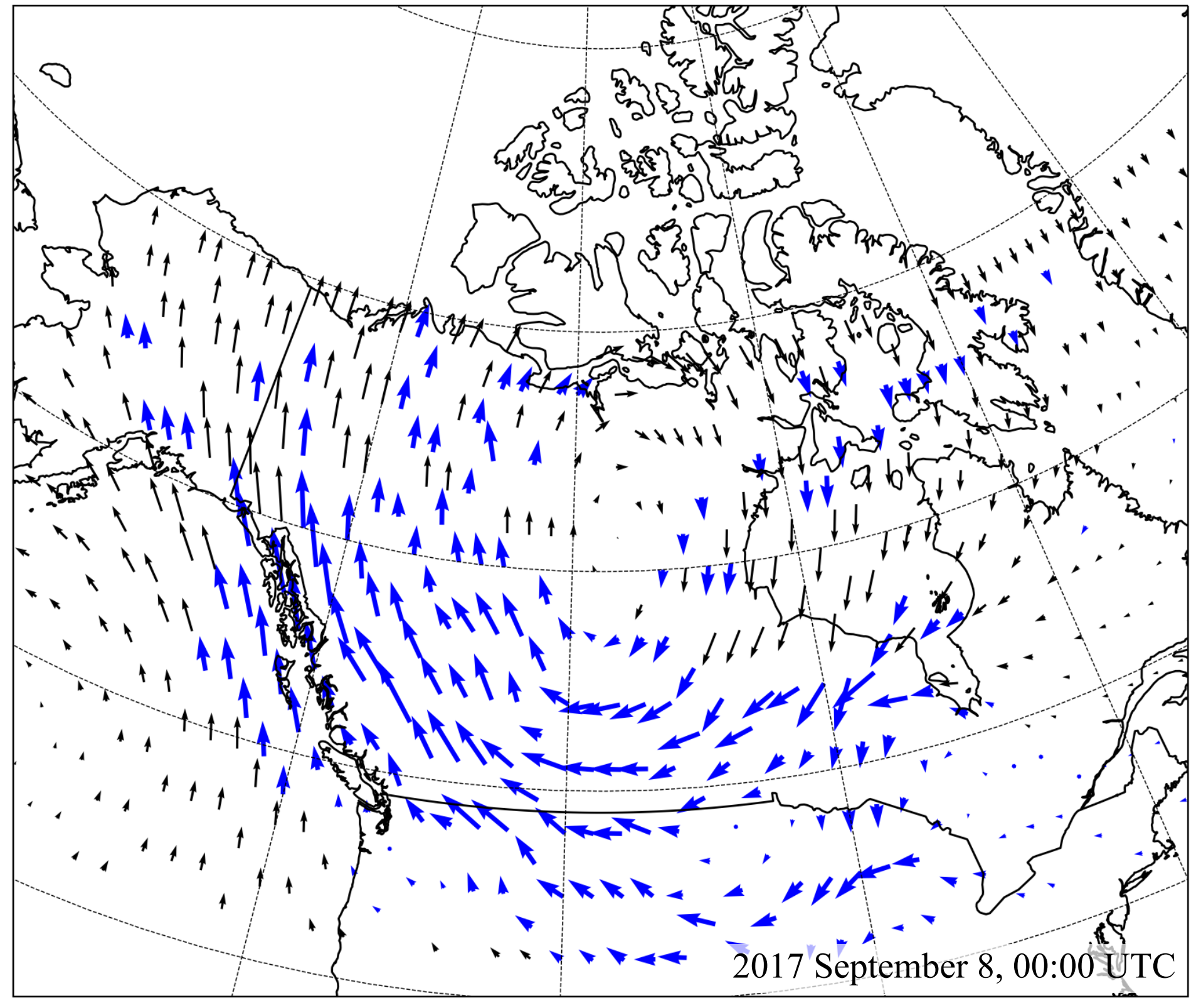

**Fig 4:** Plasma convection using the SECS method as developed here. The blue vectors represent prediction velocities that are within one degree of input velocity measurements. There are more blue vectors because more radars are being used to obtain a solution. The same general morphology matches both VT and Bristow’s convection maps.

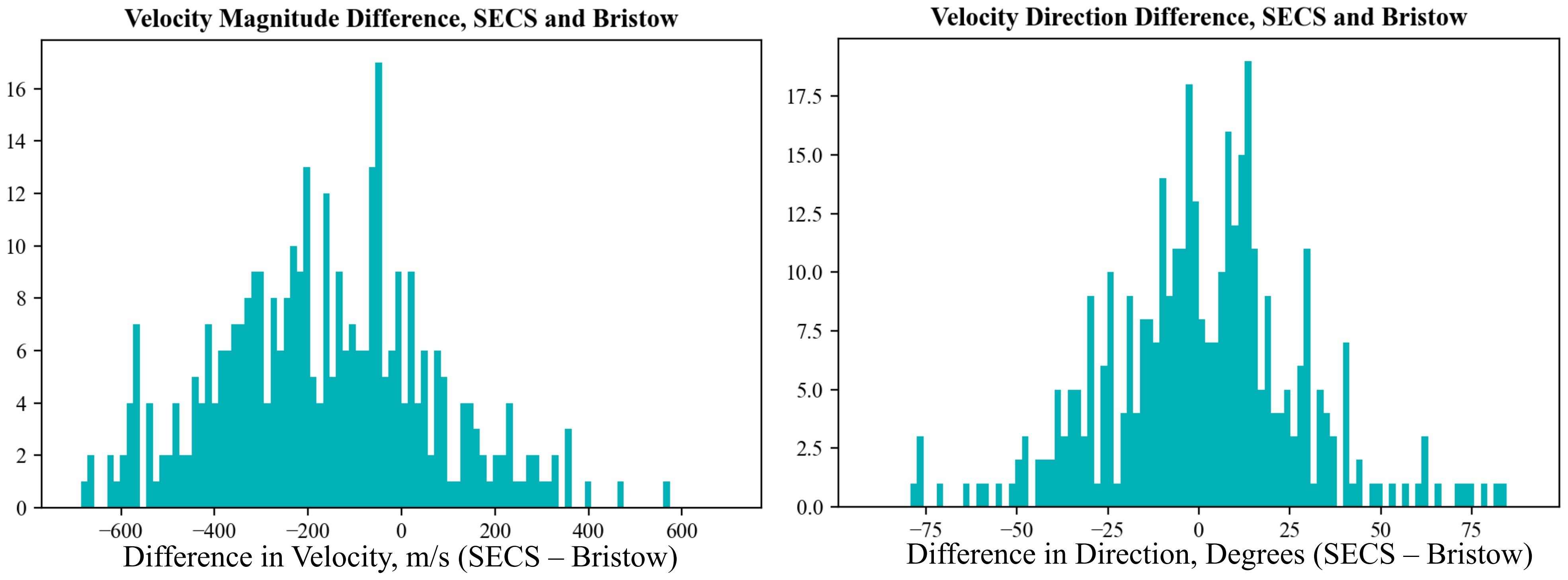

**Fig 5, 6:** Histograms showing differences between SECS solution and Bristow’s solution, showing differences in the magnitudes and the angle between the vectors. Notably, there is an ~100 m/s offset in mean velocity, as well as significant spread in both velocity magnitude and direction. Refer to **Fig. 4** and **Fig. 5** for qualitative comparison.

From **Fig. 5**, there is ~100 m/s difference in mean velocity, as well as significant spread. **Fig. 6** shows a spread in velocity directions, and while the mean is roughly around zero degrees, there is likewise a large spread. Visually inspecting **Fig. 3** and **Fig. 4**, there is much more variation in Bristow’s solution (blue arrows) compared to the same area in SECS. This suggests Bristow has a high model confidence to predict fine details where there are dense input measurements, whereas SECS outputs lower detail plasma flows.

## Discussion and Conclusion

The grid of poles acts as a proxy for maximum resolution. The denser the poles, the better the model can account for fine details such as shear flow in input measurements. However, more poles makes the system more underdetermined — i.e., more susceptible instability resulting from high condition number.

Future work includes refinement of pole locations. Poles do not need to be a grid; they can be placed wherever, and so clever placement could lead to improved modeling in areas with dense input measurements.

Value of epsilon, typically around 0.05, determines amount to truncate eigenspace. Smaller epsilon leads to less truncation, which can retain fine details. However, too small an epsilon will introduce spurious details into the solution, particularly in areas with sparse input measurements.

Future work includes quantitatively characterizing the effects of varying epsilon. Currently, an epsilon that produces a good fit visually is used, however studies can be conducted on truncated condition number and resulting measurement spread.

The SECS method allows regional ion drift reconstruction with divergence-free estimation provide mesoscale observations that provide invaluable context to optical observations of auroral activity captured by optical emissions.

We are working toward fusing these regional ion flow maps into the SuperDARN global convection (**Fig. 2**) that will be used to drive the GITM model for event studies.

[1] — Amm, O., Grocott, A., Lester, M., and Yeoman, T. K. (2010), Local determination of ionospheric plasma convection from coherent scatter radar data using the SECS technique, J. Geophys. Res., 115, A03304, doi:10.1029/2009JA014832.

[2] — Bristow, W. A., Hampton, D. L., and Otto, A. (2016), High-spatial-resolution velocity measurements derived using Local Divergence-Free Fitting of SuperDARN observations, J. Geophys. Res. Space Physics, 121, 1349– 1361, doi:10.1002/2015JA021862.

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