Exploring non-negative solutions for the radar imaging inverse problem at Jicamarca

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Abstract

The radar imaging technique has been applied at the Jicamarca Radio Observatory to study how ionospheric structures evolve as function of space and time for many years. The problem consists on estimating the inverse Fourier transform of spatial cross correlation measurements collected with a non-uniform distribution of antenna receivers. As a result we obtain the brightness function of the ionospheric structures that are of our interest. Among the different algorithms applied, Maximum Entropy has been widely used in the community showing a good performance. This technique naturally provides non-negative solutions of the brightness function which is desirable. In this work, we have explored some alternative algorithms to solve the radar imaging inverse problem imposing a non-negative constraint to the solution. Specifically, two alternative algorithms were implemented, one based on Tikhonov regularization, and the other applying compressed sensing using Daubechies basis functions. Imposing the non-negative constraint to both algorithms, we have obtained solutions very similar to the ones obtained with maximum entropy. A statistical comparison between these different approaches based on simulated data is presented to analyze their performance under different conditions.

Radar imaging problem

A radar transmits radiowave pulses that illuminate the structures in the ionosphere. The backscattered signals are detected by a set of antennas distributed on the ground. The correlation between the voltages v_i and v_j collected by a pair of antennas is called the visibility function and it is given by [1,2]

$$V(k\mathbf{d}_{i,j}) = < v_i v_j^* > = \int B_{\text{eff}}(\Omega) e^{jk\mathbf{d}_{i,j}\cdot\hat{\mathbf{r}}} d\Omega \,,$$

where k is the radar wavenumber, and $\mathbf{d}_{i,j}$ is the distance vector between the antennas i and j. This function is also the Fourier transform of the effective brightness $B_{\text{eff}}(\Omega)$ of the structures we want to image. Above, Ω is the solid angle, and $\hat{\mathbf{r}}$ is a unit vector in the direction of the returned signals (usually expressed in terms of direction cosines). If we consider that $B_{\text{eff}}(\Omega)$ is narrow in one direction (for instance in the case of the brightness of Equatorial plasma irregularities), we can reformulate the previous equation as a 1D problem,

$$g(kd) = \int e^{j\,k\,d\,\theta} f(\theta)\, d\theta \,,$$

where the visibility $g(kd)$ and the brightness $f(\theta)$ are a Fourier transform pair. This expression can be discretized and written in matrix form as follows,

$$\mathbf{g} = \mathbf{H}\,\mathbf{f} \,,$$

where $\mathbf{g}$ is a vector of the M cross-correlation samples measured by the radar, $\mathbf{f}$ is a vector of N elements that correspond to the brightness function we want to recover, and $\mathbf{H}$ is a $M \times N$ matrix that results from approximating the integral above as a summation. This system of equations is typically underdetermined ($M < N$), thus regularization techniques have to be applied in order to find a solution. In addition, we are imposing non-negative constraints to the standard regularization approaches to obtain appropriate $\mathbf{f}$ reconstructions.

Algorithms

1. Maximum Entropy

Considering a vector of errors $\mathbf{e}$ associated with the random nature of the visibility data, we can write $\mathbf{g} = \mathbf{H}\,\mathbf{f} + \mathbf{e}$, and thus maximize the entropy of the brightness samples [1]

$$\max_{f_i} \left(-\sum_i f_i \log(f_i/F) \right) \text{ subject to } \sum_j e_j^2/\sigma_j^2 = S \,,$$

where F is the total image brightness, σ_j^2 is the variance of the visibility samples and S is a constraint to the error norm.

2. Tikhonov Regularization + non-negative constraint

A standard formulation of the Tikhonov regularization is

$$\min_{\mathbf{f}} \|\mathbf{g} - \mathbf{H}\,\mathbf{f}\|_2^2 + \lambda \|\mathbf{f}\|_2^2 \,,$$

where λ is the regularization parameter. We can reformulate this problem imposing a constraint to the error norm and forcing the solutions to be non negative, i.e.,

$$\min_{\mathbf{f}} \|\mathbf{f}\|_2 \text{ subject to } \|\mathbf{g} - \mathbf{H}\,\mathbf{f}\|_2 \le \epsilon \text{ and } \mathbf{f} \ge \mathbf{0} \,.$$

In this formulation ϵ is a constraint equivalent to S in the ME approach, thus, similar solutions should be expected.

3. Compressed Sensing + non-negative constraint

Considering the transformation $\mathbf{f} = \mathbf{\Psi}\,\mathbf{s}$, such that $\mathbf{\Psi}$ is a basis in which $\mathbf{s}$ is sparse, we can formulate the compressed sensing problem [3] as a basis pursuit denoising problem [4]. Thus, if in addition we impose a non-negative constraint to the brightness solution, we have the following problem

$$\min_{\mathbf{s}} \|\mathbf{s}\|_1 \text{ subject to } \|\mathbf{g} - \mathbf{H\Psi s}\|_2 \le \epsilon \text{ and } \mathbf{\Psi s} \ge 0 \,.$$

This problem can be solved as a convex optimization problem. We are considering that $\mathbf{\Psi}$ is a Daub-16 basis transformation matrix.

Simulation and Results

Considering the Jicamarca antenna configuration described in [3], we have applied four methods (Capon, Tikhonov, Maximum entropy, and Compressed sensing) to reconstruct images from simulated visibility measurements. For this purpose, we have considered a Gaussian-shaped reference brightness function with an angular width of 0.015 rad. We have explored different values of SNR and incoherent integrations.

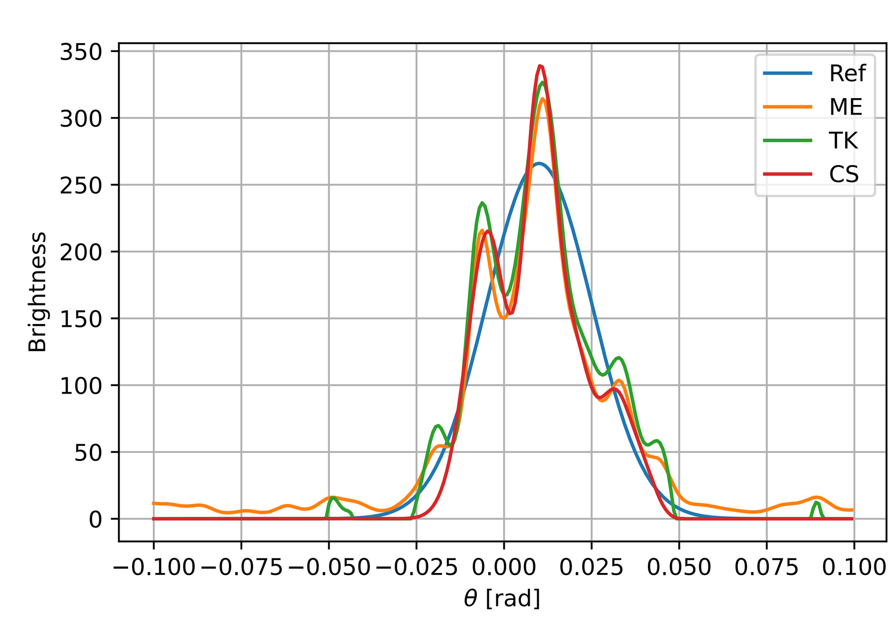

Figure 1: Reference and sample (non-negative) reconstructed images used in the comparison. The reference has a Gaussian shape with an angular width of 0.015 rad (~0.85 deg).

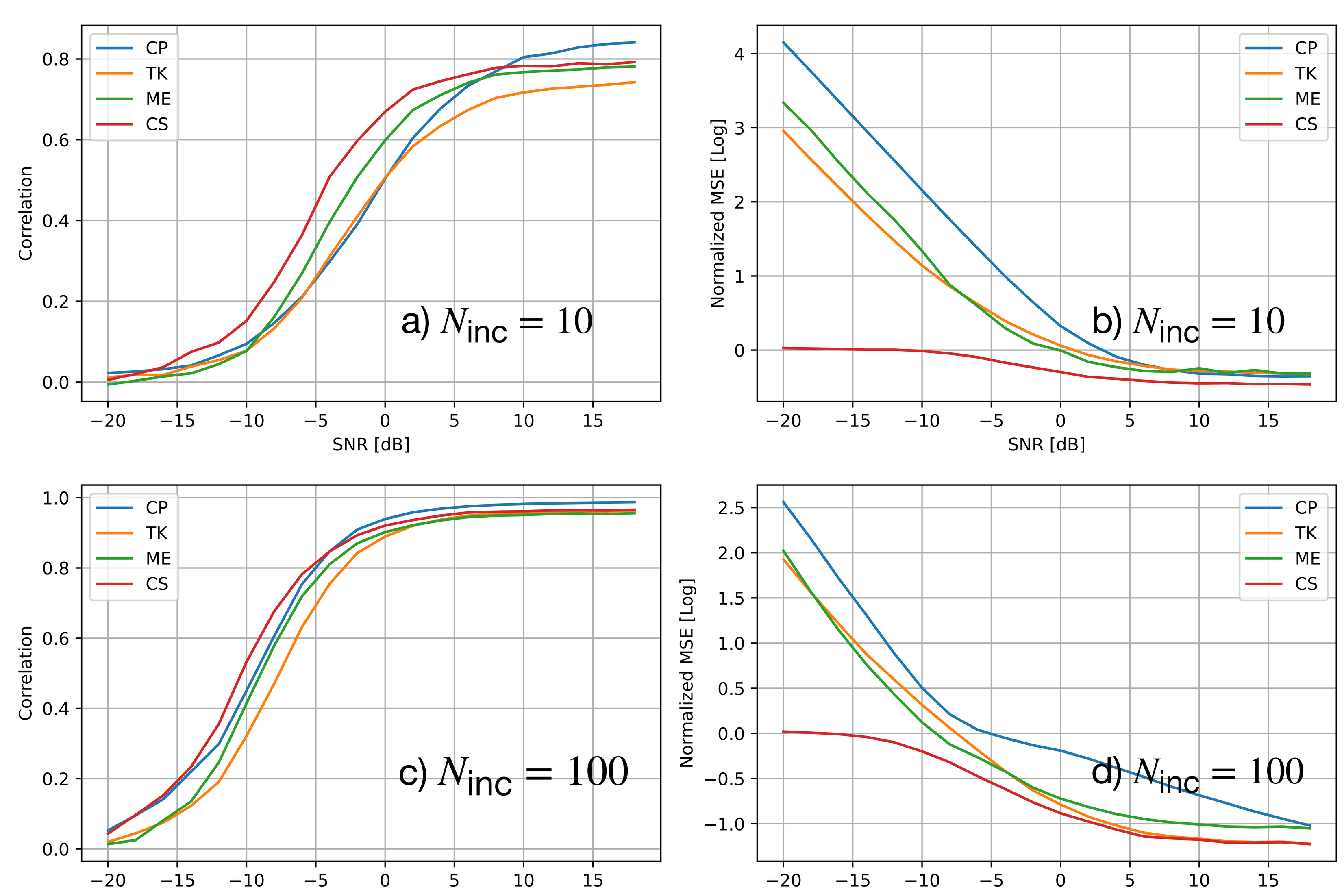

Figure 2: Comparison of the correlation and MSE of reconstructed images with respect to the truth brightness as function of SNR. Four different techniques were considered (Capon, Tikhonov, Maximum Entropy, and Compressed Sensing). The top panels correspond to $N_{inc} = 10$ while the bottom panels correspond to $N_{inc} = 100$.

Conclusions

Inverse problem algorithms with non-negative constraints were applied to the radar imaging problem. In general, the reconstructed images using the different methods have similar shapes. In the correlation analysis, we can verify that, at low SNR, the compressed sensing reconstructed images are more similar to the reference image. When SNR is high, all the different methods have similar performance. In the MSE analysis, we can also verify that compressed sensing reconstructed images are the ones that have less error, however, Maximum entropy and Tikhonov regularization have similar behavior. Further studies will be conducted for 2D brightness functions.

References

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