Study of the effects of Coulomb collisions on H^+, He^+ and O^+ plasmas for ISR applications at Jicamarca

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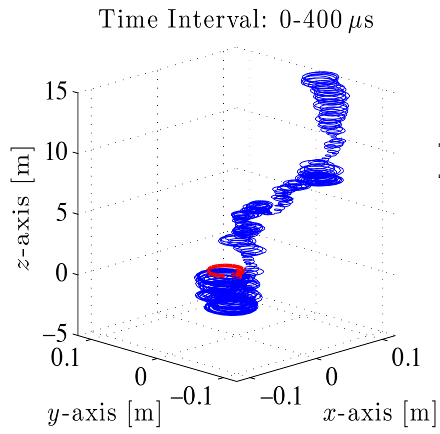
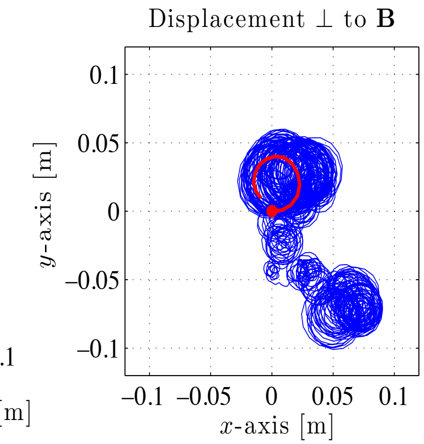
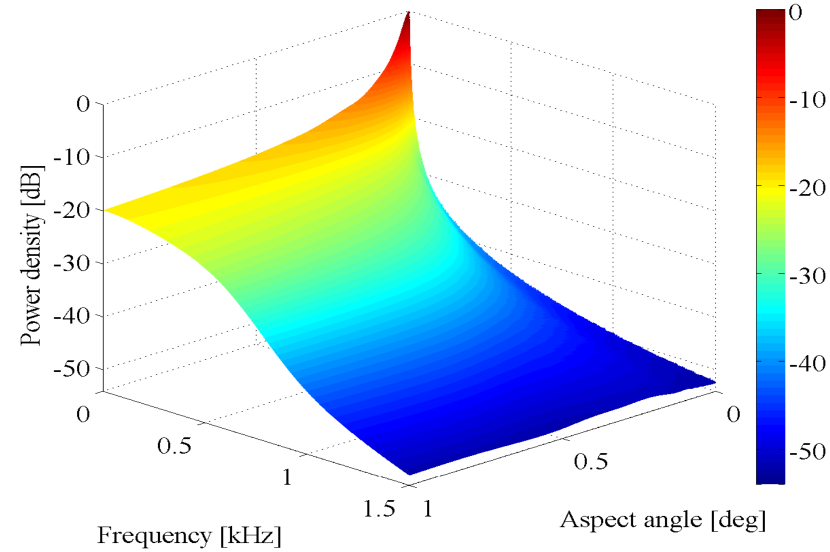
Project Outline

Coulomb collision effects on H^+, He^+, and O^+ plasmas

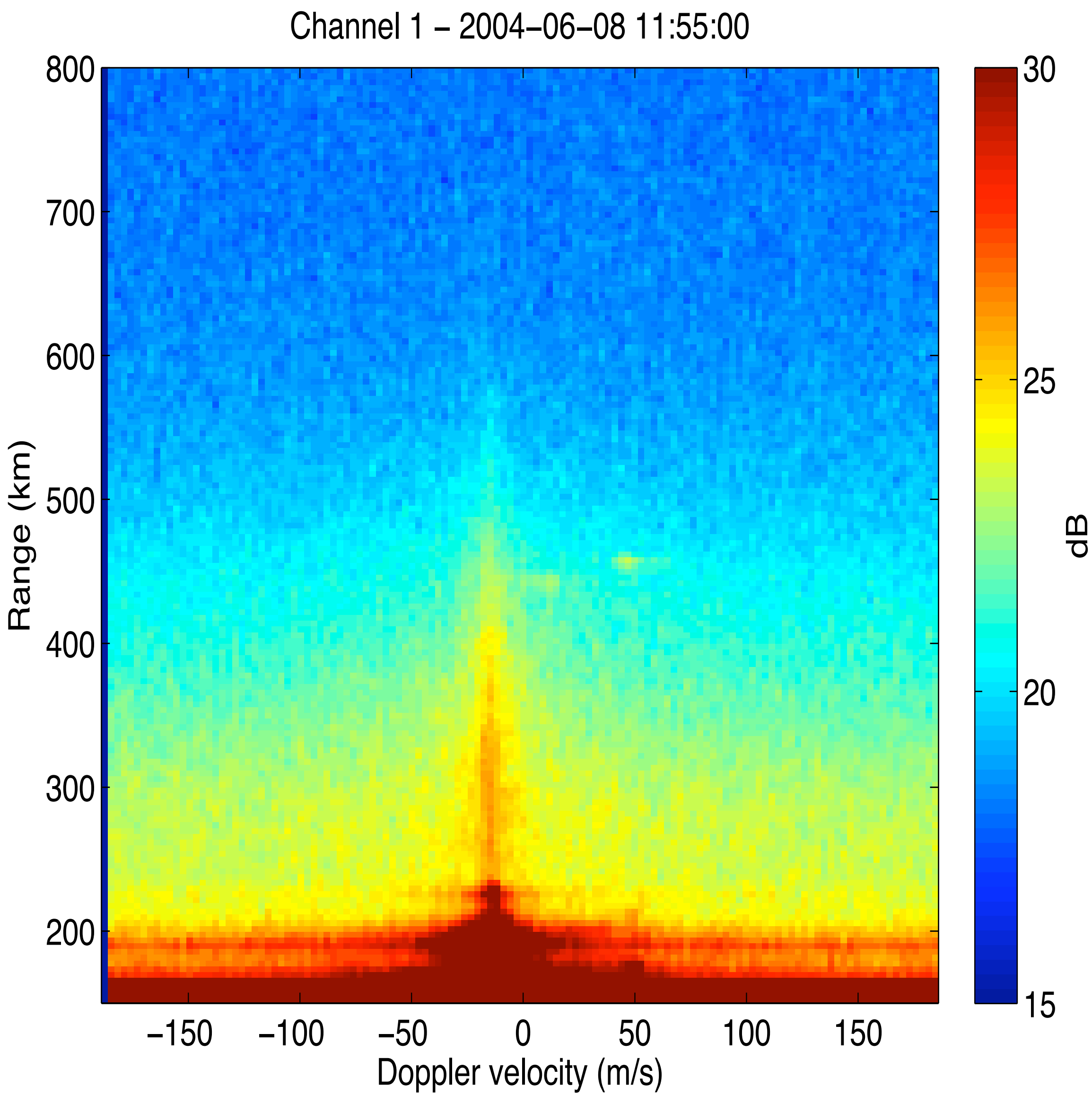
- Massive simulation of particle trajectories in H^+, He^+ and O^+ plasmas (Langevin equation and Fokker-Planck collision model).
- Statistical analysis of the simulated trajectories and construction of a numerical library of single-particle ACF's.
- Comparison of the collisional model with standard incoherent scatter theories.
- Application of the model to ISR experiments at Jicamarca.







Motivation: ISR spectrum perp. to B



To develop a model for the IS spectra measured with antenna beams pointed perpendicular-to-B at Jicamarca with the goal of estimating ionospheric physical parameters (e.g. densities, temperatures).

Motivation: ISR spectrum perp. to B

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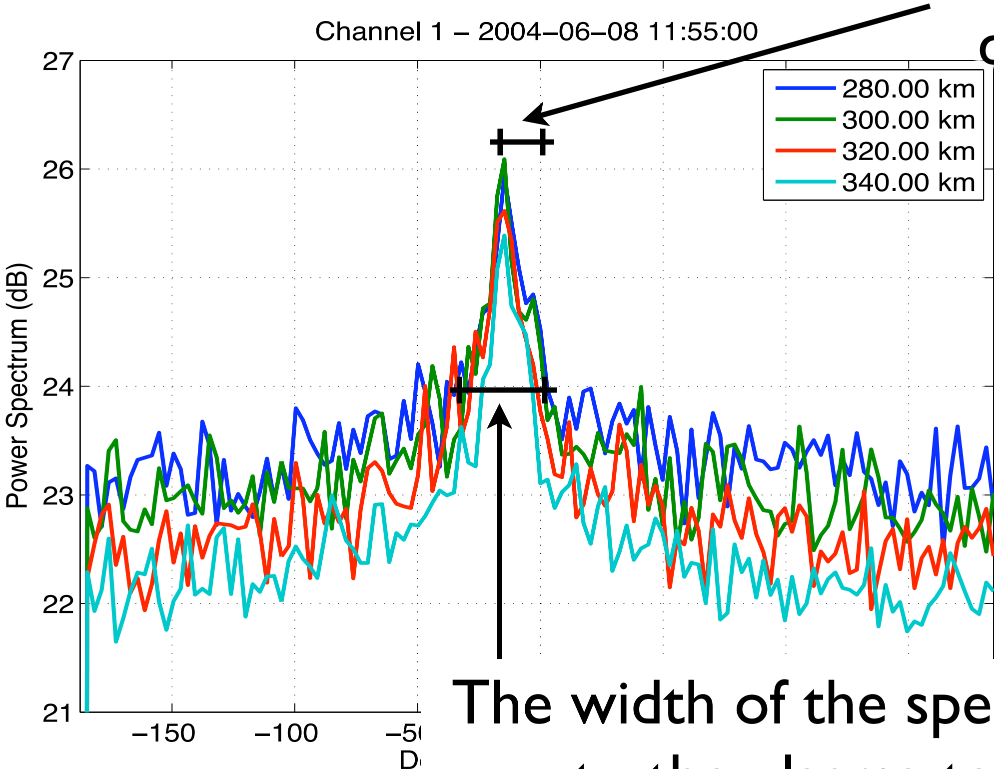
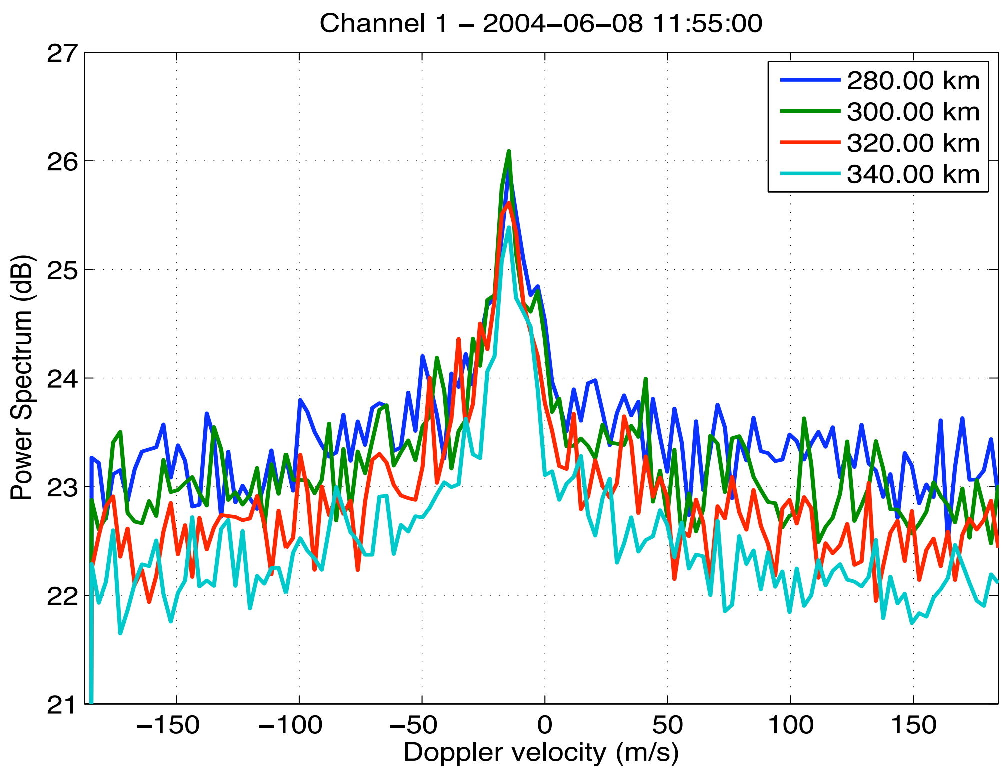
Doppler shift of the spectrum is a direct measurement of the drift.



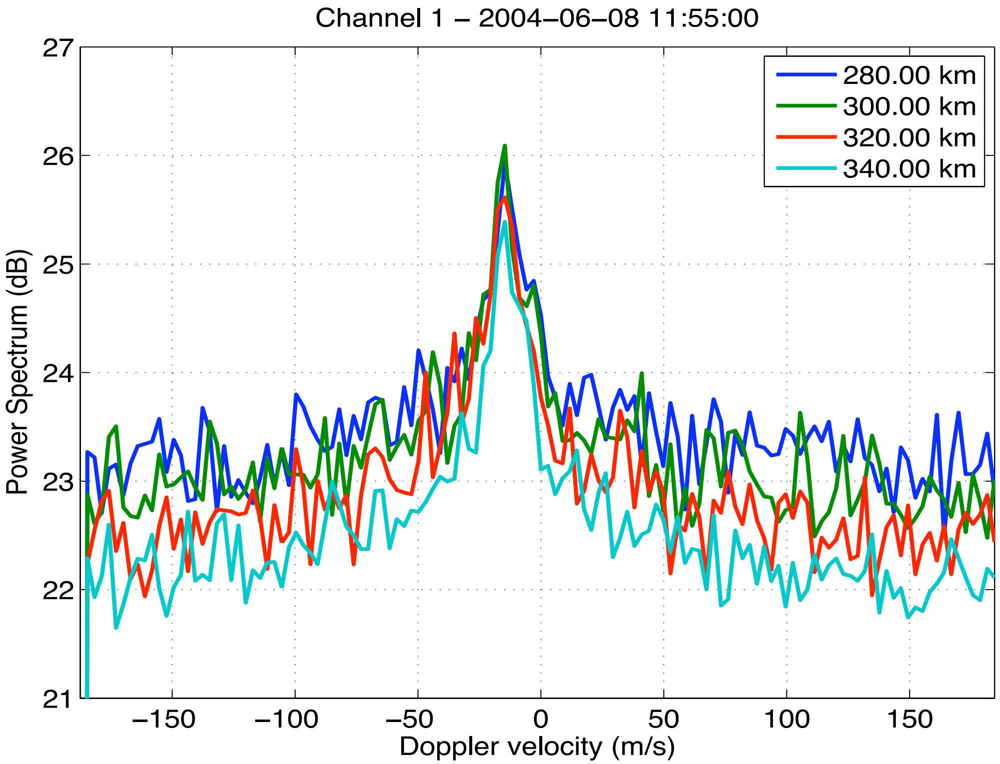
The width of the spectrum is related to the plasma temperature.

Motivation: ISR spectrum perp. to B



Kudeki et al (1999) fitted the measurements using a simplified spectral model. This model was developed based on the collisionless IS theory. But, the temperatures they obtained were about half of what is expected.

Motivation: ISR spectrum perp. to B



Kudeki et al (1999) fitted the measurements using a simplified spectral model. This model was developed based on the collisionless IS theory. But, the temperatures they obtained were about half of what is expected.

Sulzer and Gonzalez (1999) showed that, due to Coulomb collision effects, the IS spectrum becomes narrower than what the collisionless theory predicts at small magnetic aspect angles.

Simulation of particle trajectories based on Langevin equation

IS spectrum and Gordeyev integrals

- The power spectrum of the incoherent scatter signals is proportional to the spectrum of electron density fluctuations in a plasma (e.g., Kudeki & Milla, 2011)

$$\langle|n_e(\vec{k},\omega)|^2\rangle = \frac{|j\omega\epsilon_o + \sigma_i(\vec{k},\omega)|^2\langle|n_{te}(\vec{k},\omega)|^2\rangle + |\sigma_e(\vec{k},\omega)|^2\langle|n_{ti}(\vec{k},\omega)|^2\rangle}{|j\omega\epsilon_o + \sigma_e(\vec{k},\omega) + \sigma_i(\vec{k},\omega)|^2}$$

- Thanks to the fluctuation-dissipation (or Nyquist) theorem, the self-spectra of thermal density fluctuations and species conductivities are link to each other. Moreover they can be written in the following forms

$$\frac{\langle|n_{ts}(\vec{k},\omega)|^2\rangle}{N_s} = 2\mathrm{Re}\{J_s(\omega)\} \qquad \frac{\sigma_s(\omega,\vec{k})}{j\omega\epsilon_o} = \frac{1 - j\omega J_s(\omega)}{k^2 h_s^2}$$

where $J_s(\omega)$ denotes the so-called Gordeyev integral for each particle species.

Gordeyev integral, Fokker-Planck collision model and Langevin equation

- The Gordeyev integrals are effectively Fourier transforms of the electron or ion particle ACFs (Hagfors & Brockelman, 1971)

$$J_s(\omega) = \int_0^\infty d\tau e^{-j\omega\tau} \langle e^{j\vec{k}\cdot\Delta\vec{r}_s} \rangle \qquad \langle e^{j\vec{k}\cdot\Delta\vec{r}_s} \rangle = \langle e^{j\vec{k}\cdot(\vec{r}_s(t+\tau)-\vec{r}_s(t))} \rangle$$

- Instead of computing the integrals solving a kinetic equation with the Fokker-Planck collision operator, we decided to compute them from simulated electron and ion trajectories.
- The trajectories are simulated using a Generalized version of the Langevin equation in which Coulomb collisions are modeled by a deterministic friction force and random diffusion forces acting on a test particle.

$$\begin{aligned} \frac{d\vec{v}(t)}{dt} &= \frac{q}{m}\,\vec{v}(t)\times\vec{B} - \beta(v)\,\vec{v}(t) + \sqrt{D_{\|}(v)}\,\mathcal{W}_1(t)\,\hat{v}_{\|}(t) \\ &\quad \sqrt{\frac{D_{\perp}(v)}{2}}\,\mathcal{W}_2(t)\,\hat{v}_{\perp 1}(t) + \sqrt{\frac{D_{\perp}(v)}{2}}\,\mathcal{W}_3(t)\,\hat{v}_{\perp 2}(t) \end{aligned}$$

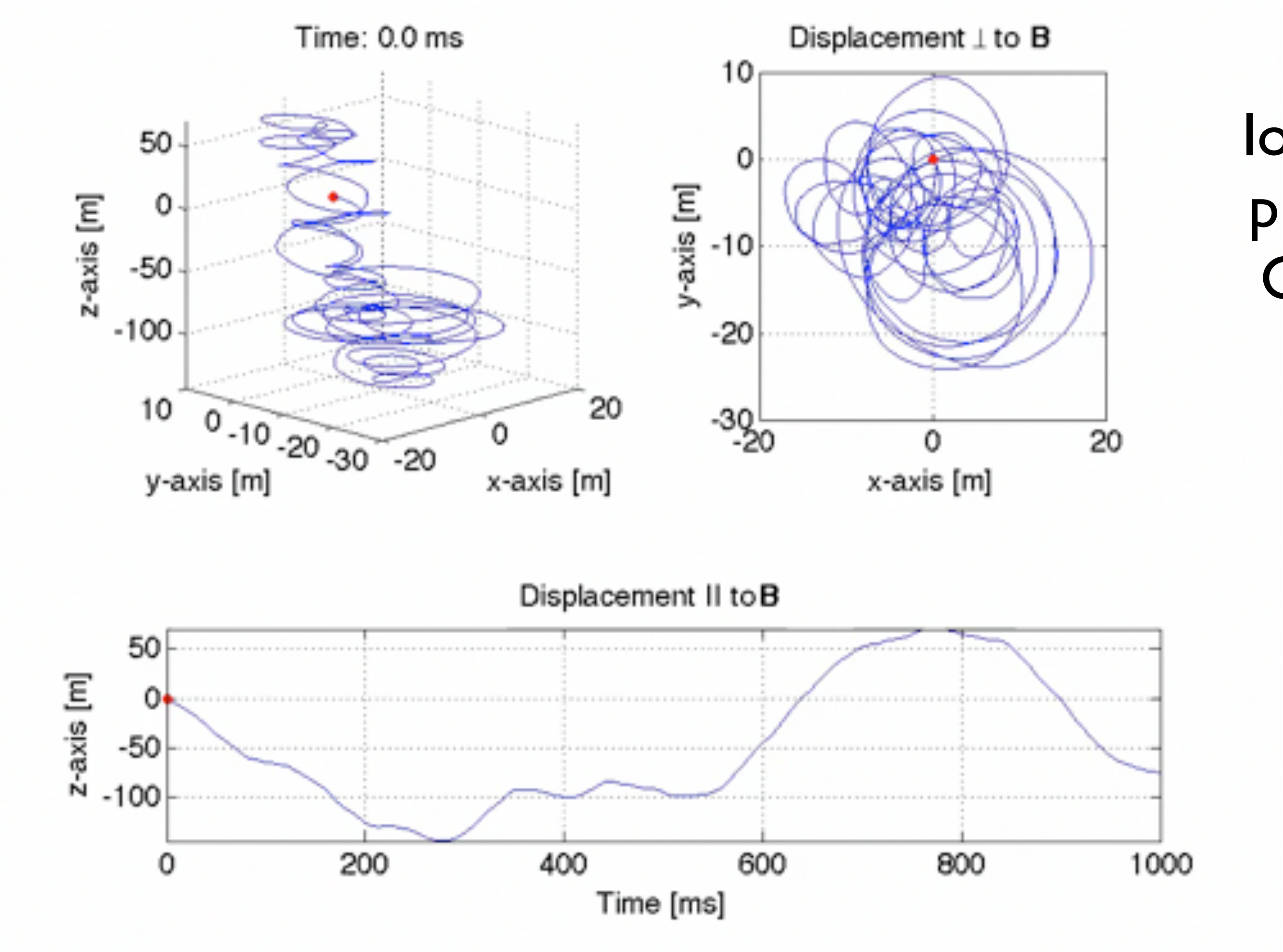
3D particle trajectory sample

Ion moving in an O+ plasma experiencing Coulomb collisions

$N_e = 10^{11}$ m^{-3}
$T_e = 2000$ K
$T_i = 2000$ K
$B = 20000$ nT

10^4 sequences of 2^{17} samples are generated (~30 GB), however, only the statistics (pdfs and ACFs) are stored (~60 MB).

3D particle trajectory sample



Ion moving in an O+ plasma experiencing Coulomb collisions

$N_e = 10^{11}$ m^{-3}
$T_e = 2000$ K
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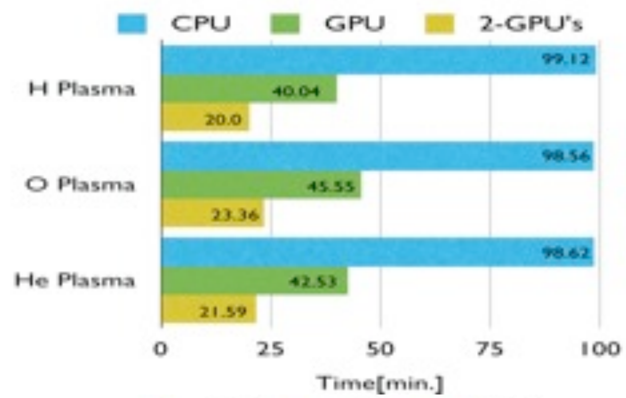
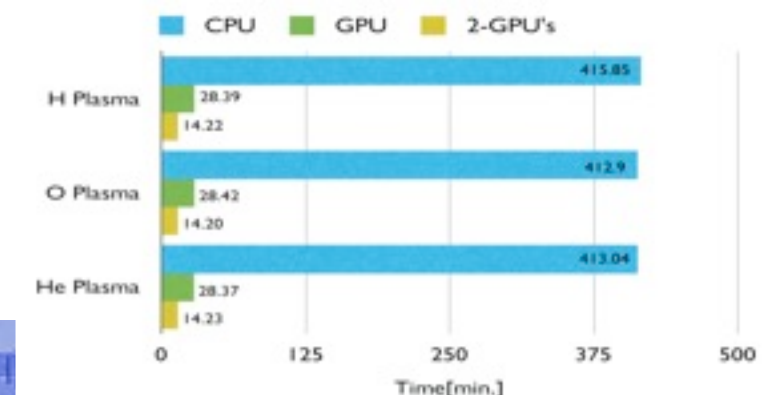
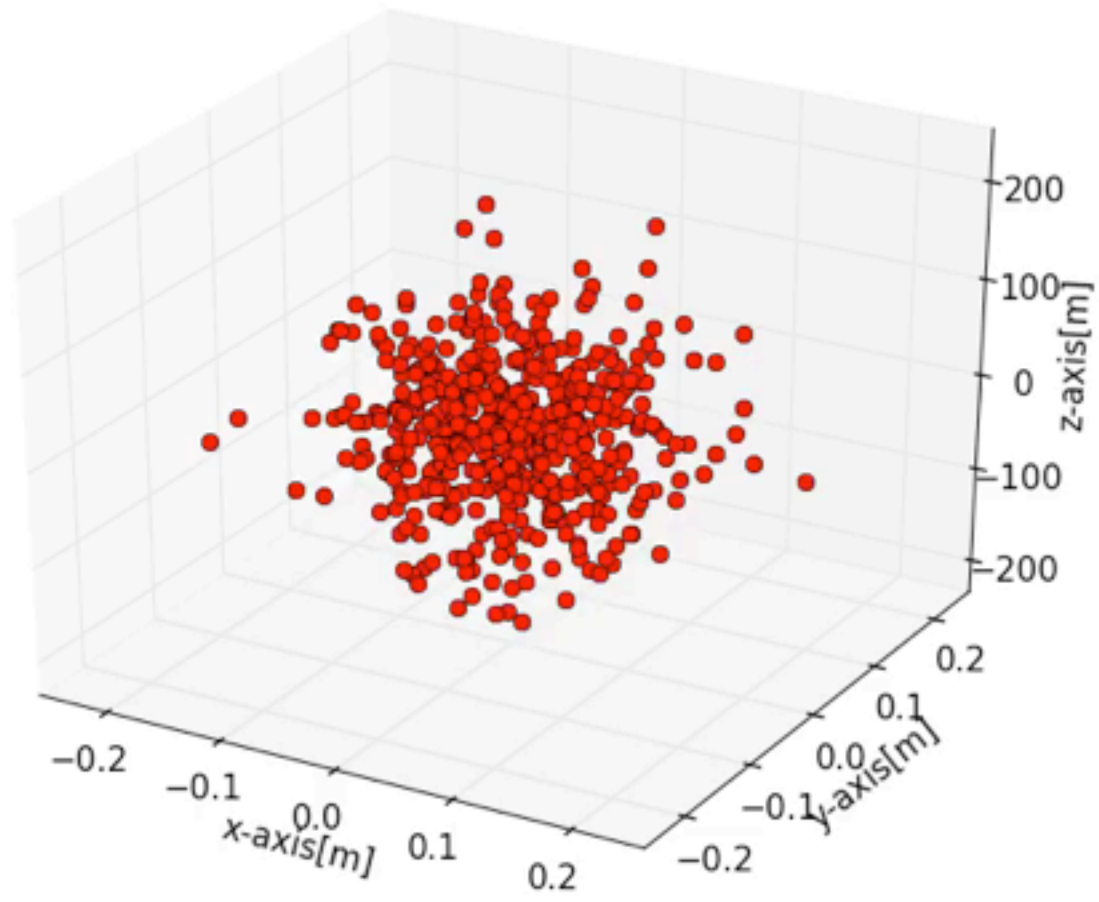
Simulation of multiple trajectories using CUDA

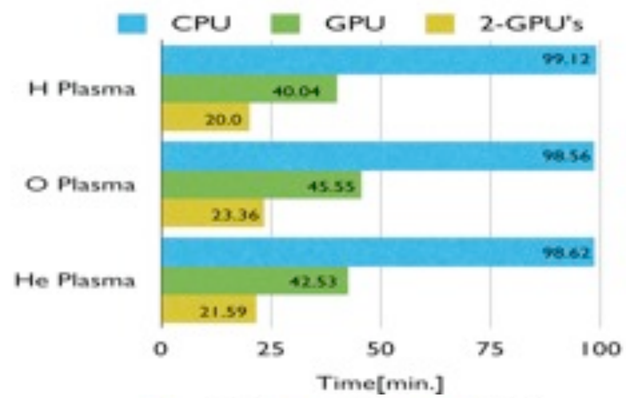


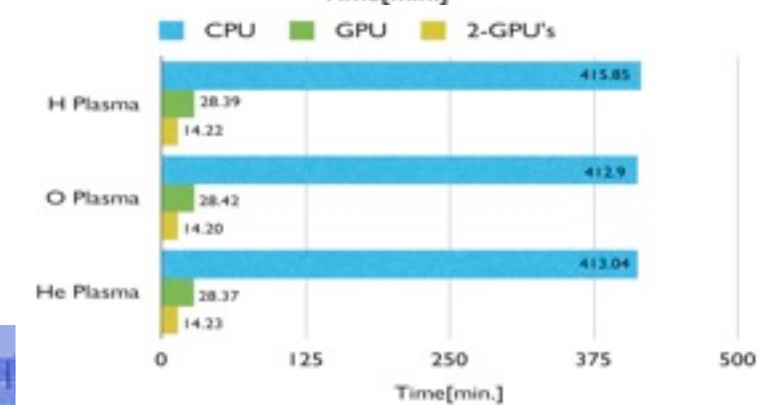


Significant saving in simulation and processing time.

Simulation of multiple trajectories using CUDA



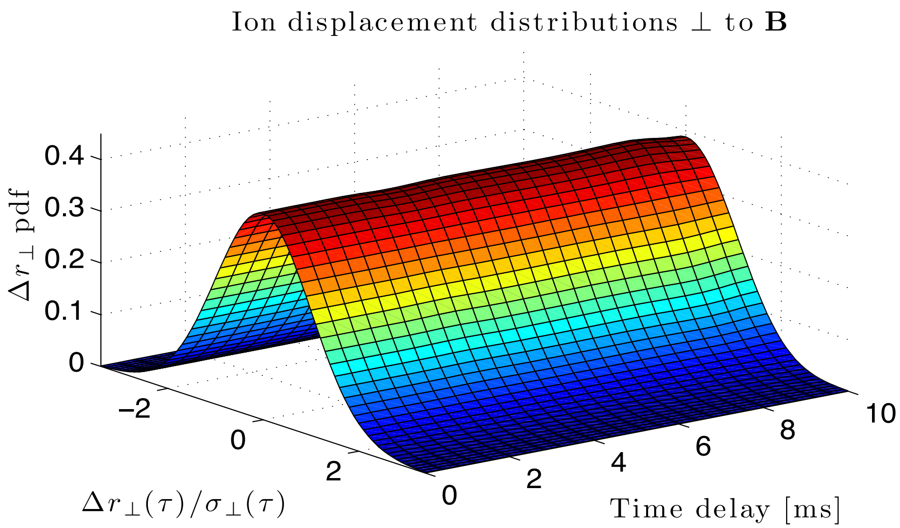


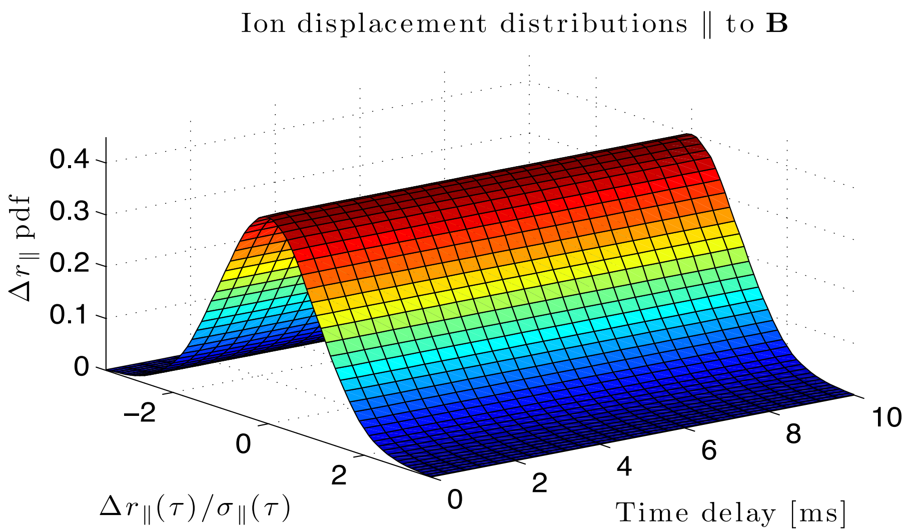


Significant saving in simulation and processing time.

Statistics of test-ion displacements in H^+, He^+, and O^+ plasmas and comparison to the Brownian model

Ion displacement distributions

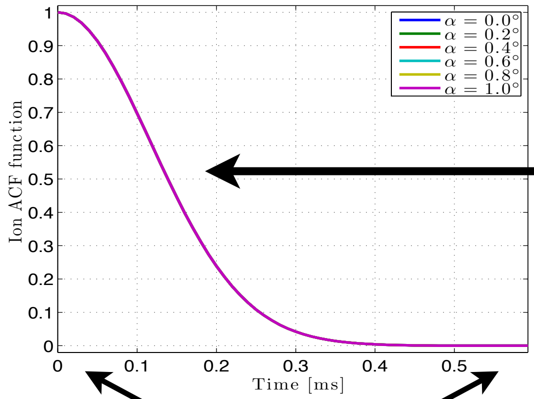




- H^+, He^+, and O^+ ion displacement distributions are Gaussian in the direction perpendicular to B as function of delay τ.
- In the parallel direction, the distributions also look gaussian.
- A Brownian motion model with Gaussian trajectories is a good representation of the ion process (Woodman, 1967).
- The single-ion ACF can be approximated by

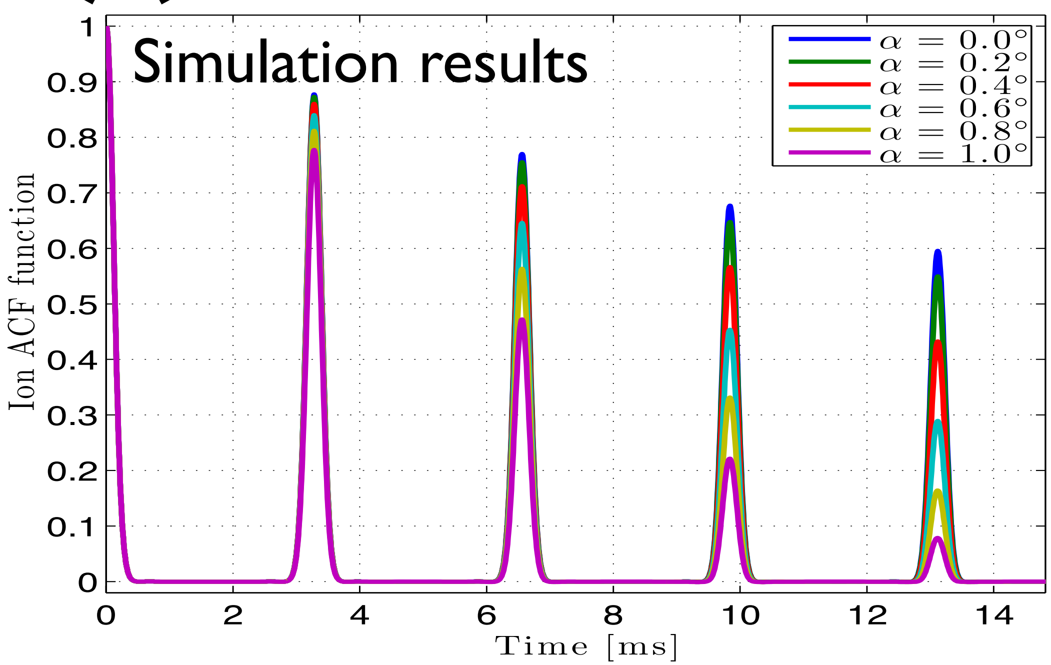
$$\left\langle e^{j\vec{k}\cdot\Delta\vec{r}}\right\rangle = e^{-\frac{1}{2}k^2\sin^2\alpha\langle\Delta r_\parallel^2\rangle} \times e^{-\frac{1}{2}k^2\cos^2\alpha\langle\Delta r_\perp^2\rangle}$$

H^+ single-ion ACFs



Correlation time proportional to

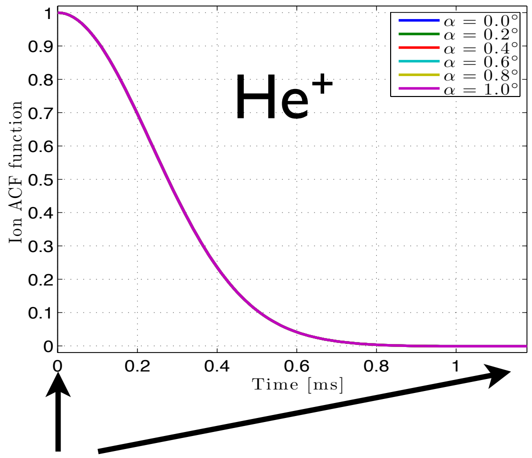
$$\frac{1}{k_B C_i}$$



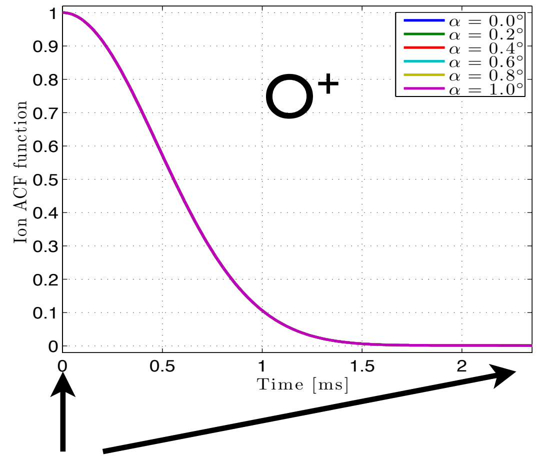
At perpendicular to B, periodicity of the H^+ ion ACF is damped by ion-ion collisions. At larger angles is a mixed effect of collisional and non-collisional damping.

Brownian-motion model captures well all the details of the ion ACFs.

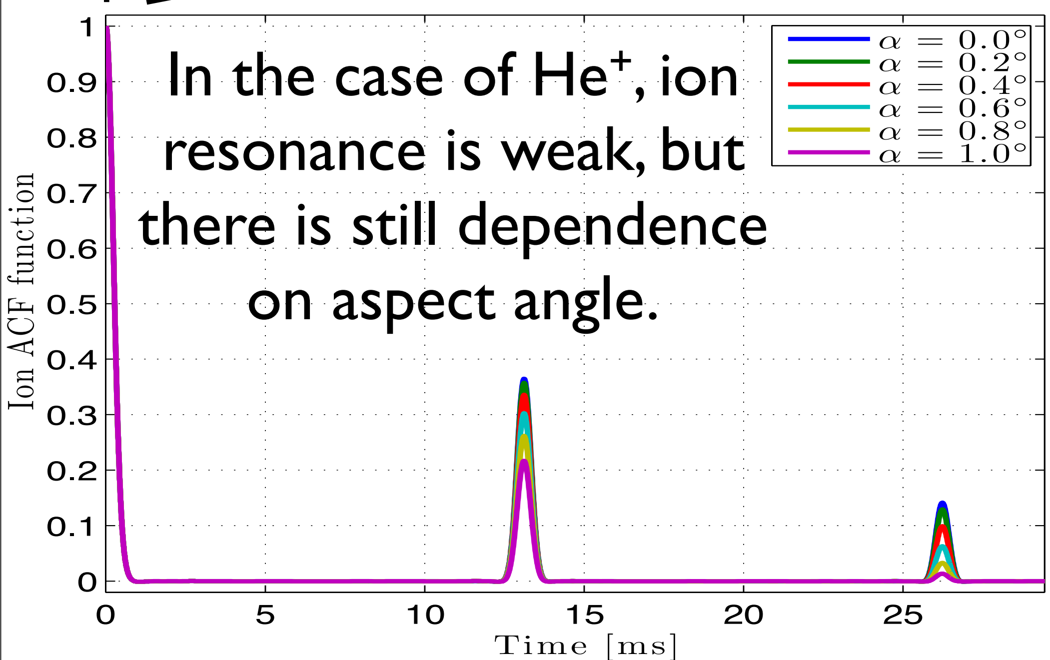
He^+ and O^+ single-ion ACFs



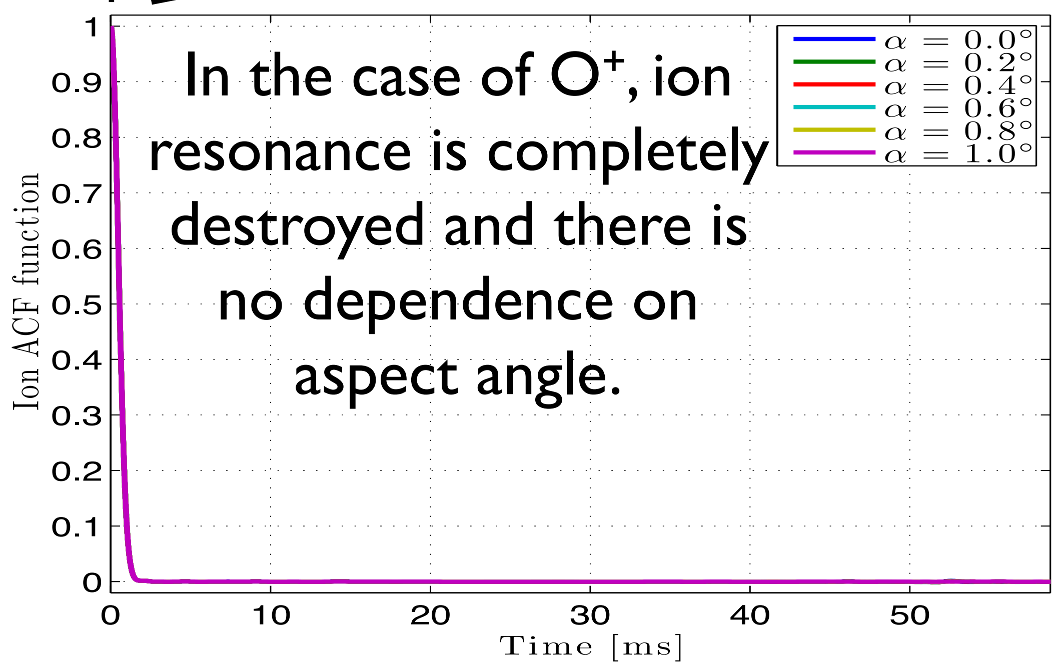
Correlation times are proportional to $\dfrac{1}{k_B C_i}$



In both cases, the Brownian motion model captures all the details of the ion ACFs.



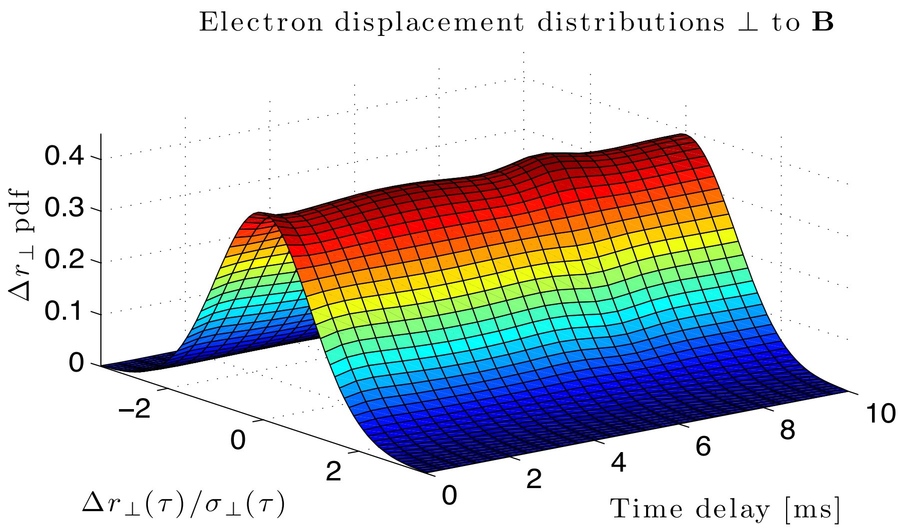
In the case of He^+, ion resonance is weak, but there is still dependence on aspect angle.

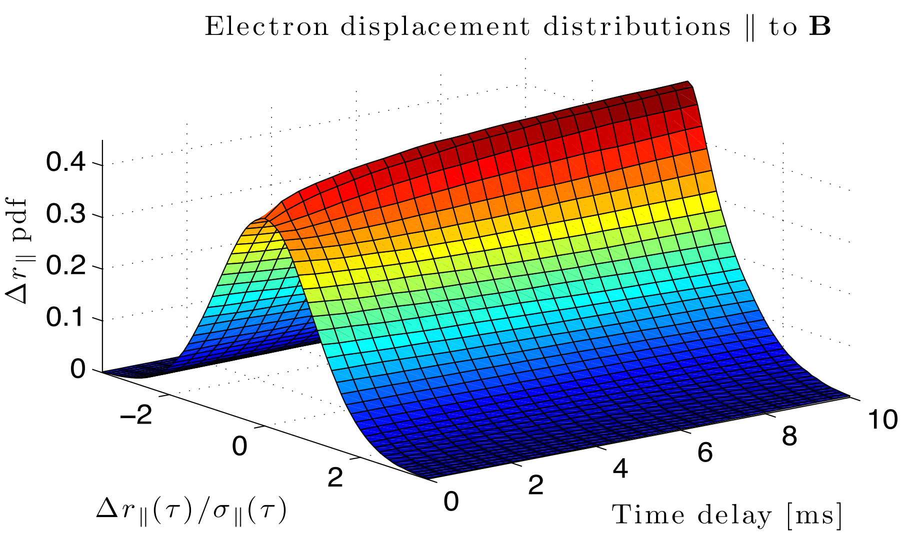


In the case of O^+, ion resonance is completely destroyed and there is no dependence on aspect angle.

Statistics of test-electron displacements in H^+, He^+, and O^+ plasmas and comparison to the Brownian model

Electron displacement distributions



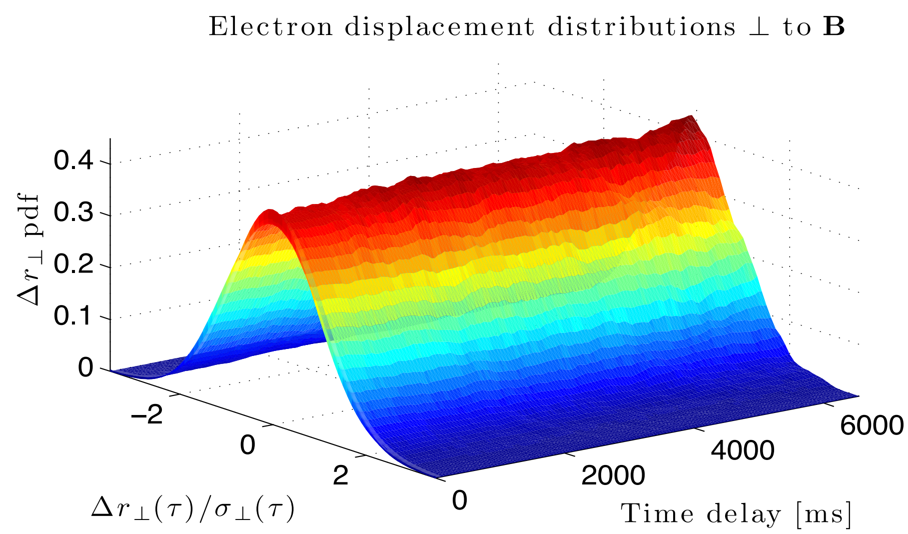


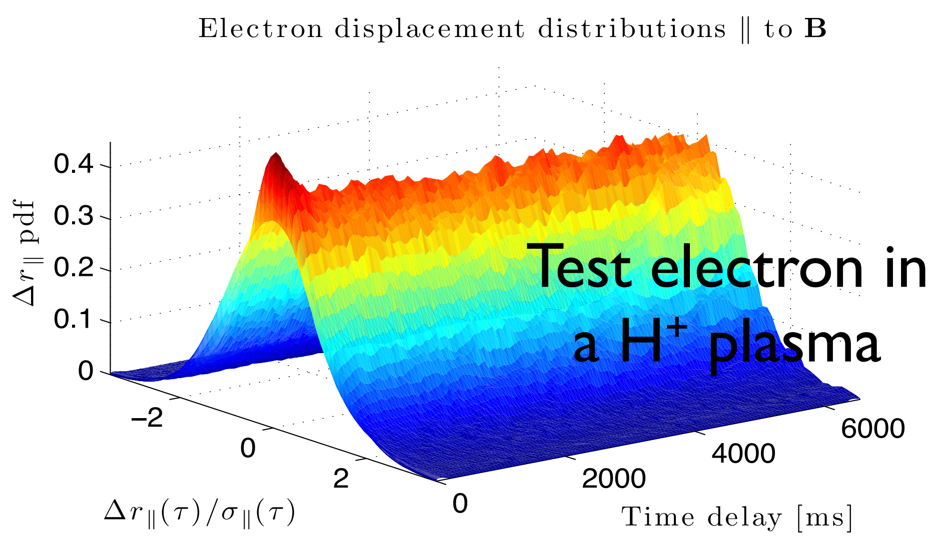
- Electron distributions look the same for H^+, He^+, and O^+ plasmas.
- In the direction perp. to B, the distribution is approximately Gaussian as function of time delay.
- In the parallel direction, the distribution looks Gaussian at short delays, but becomes narrower within a "collision time".
- Brownian motion (gaussian displacements) is not a good model for the electron motion.

$$\cancel{\left\langle e^{j\vec{k}\cdot\Delta\vec{r}}\right\rangle = e^{-\frac{1}{2}k^2\sin^2\alpha\langle\Delta r_\parallel^2\rangle} \times e^{-\frac{1}{2}k^2\cos^2\alpha\langle\Delta r_\perp^2\rangle}}$$

More on electron displacement distributions

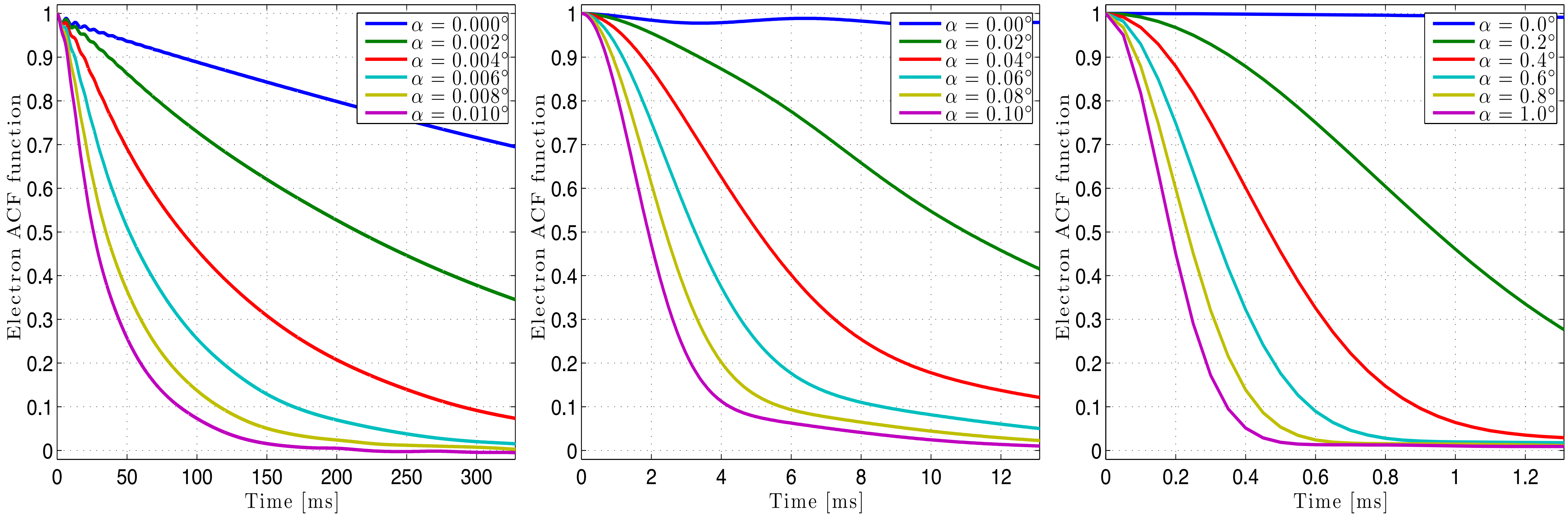
- The distribution in the direction perpendicular to B remains Gaussian for long time delays.
- In the parallel direction, the distribution becomes Gaussian again after a few "collision times".
- The electron distribution is independent of the ion type.
- For ISR applications around perpendicular to B, the correlation times are of the order of the electron "collision time", therefore the choice of the collision model does matter to define the shape of the spectrum.





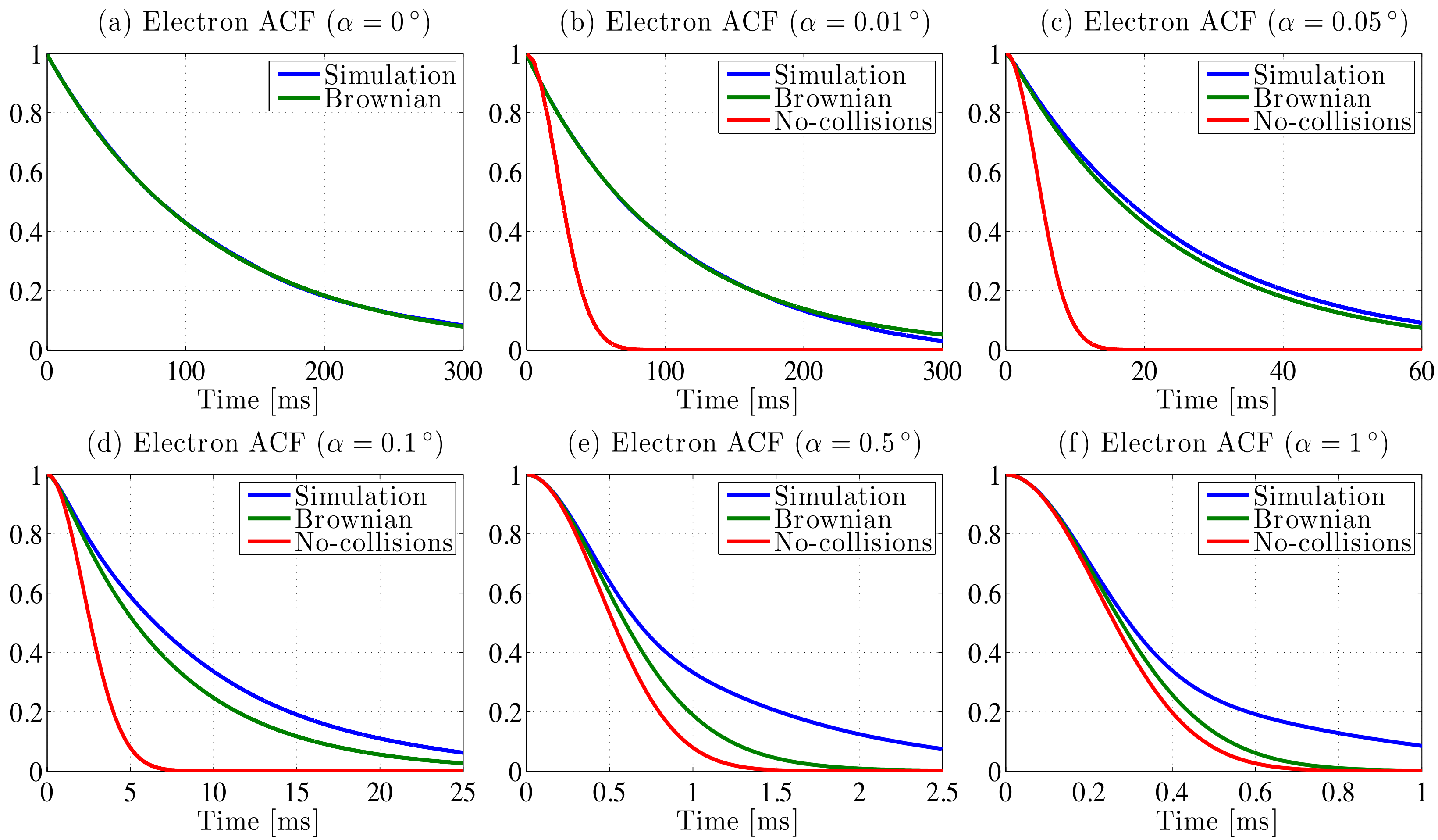
Simulated single-electron ACFs

0°<α<0.01° 0°<α<0.1° 0°<α<1°



The simulated electron ACFs are effectively the same despite the plasma configuration (i.e., despite the type of ions to which the electrons collide). Very close to perp. to B (α<0.01°) the electron ACFs have long correlation times (of the order of a "collision time"), while as the angle increases, correlation times decrease very rapidly (100 times within 1 deg).

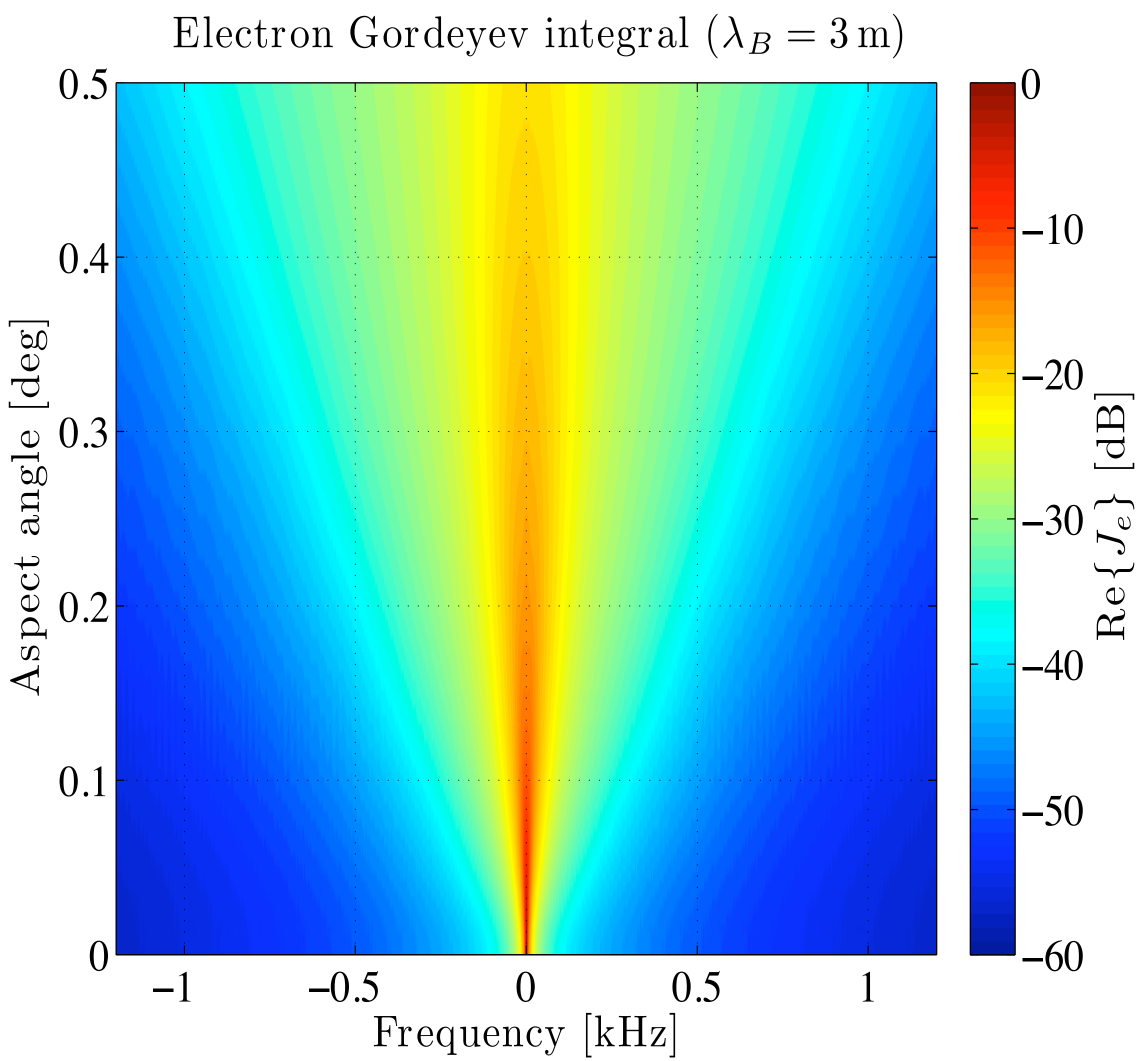
Comparison of single-electron ACFs



The Brownian motion model does not capture all the details of the electron ACFs.

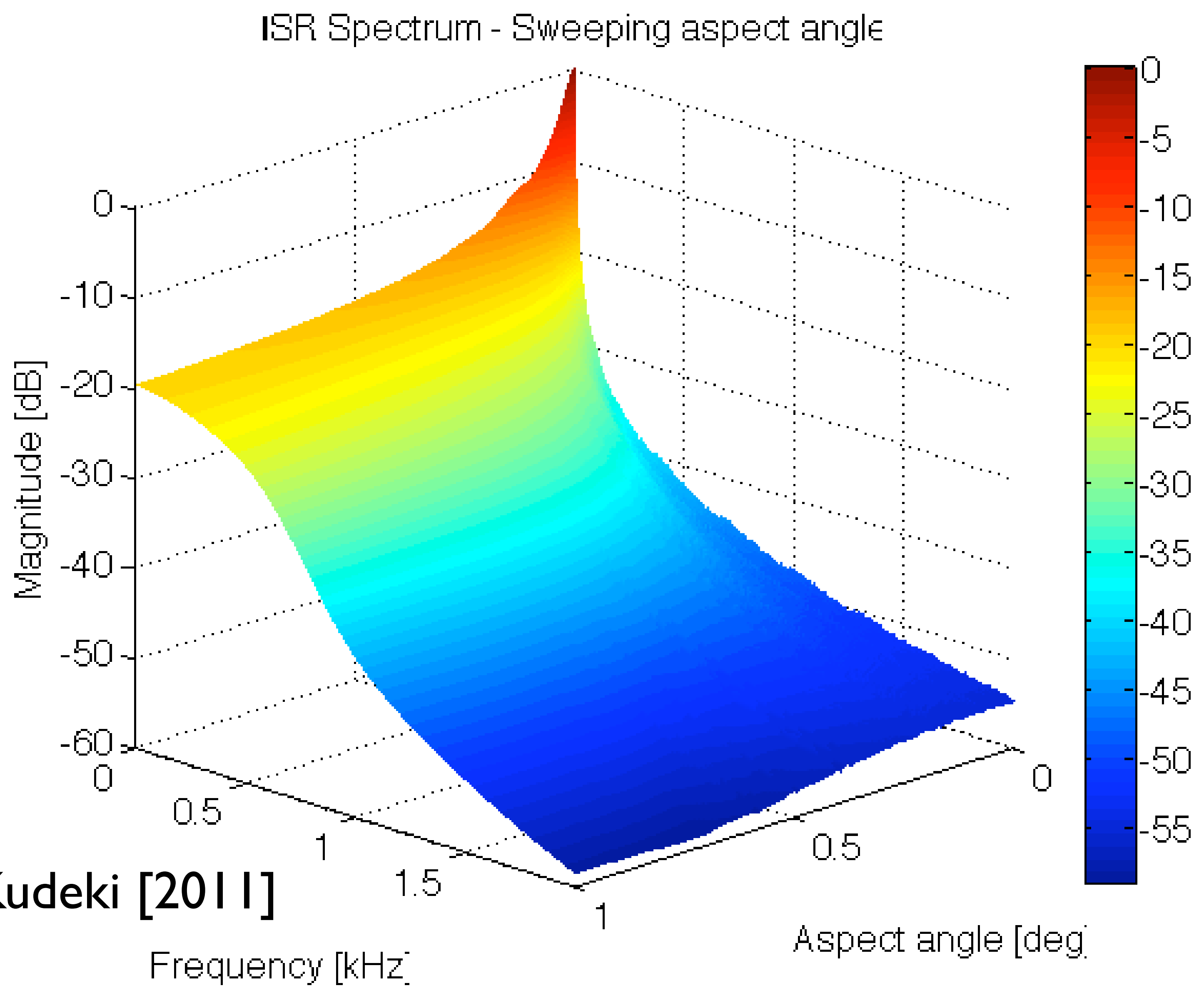
Simulated electron ACF's for λ_B=3m at different magnetic aspect angles: (a) α=0°, (b) α=0.01°, (c) α=0.05°, (d) α=0.1°, (e) α=0.5°, and (f) α=1°.

Database of single-electron ACFs and Gordeyev integrals



- Before, we built a library of single-electron ACFs (and corresponding Gordeyev integrals).
- The library spans different values of Ne, Te, B, and α.
- As the electron ACFs are independent of the plasma configuration, there is no need to develop another library but to parametrize it in order to use it for ISR applications.

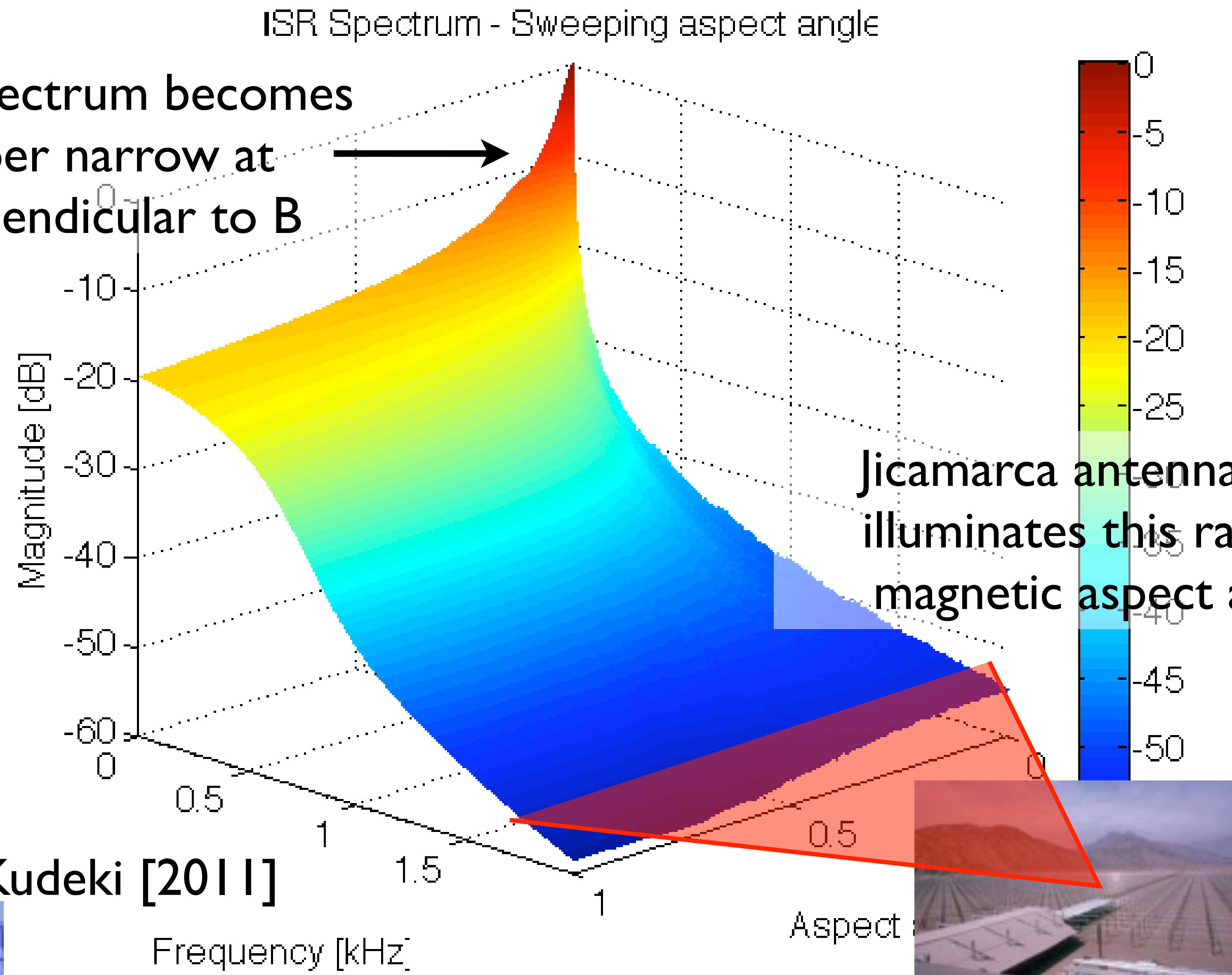
Collisional IS Spectrum



Milla & Kudeki [2011]

Collisional IS Spectrum

The spectrum becomes super narrow at perpendicular to B



Jicamarca antenna beam illuminates this range of magnetic aspect angles

Milla & Kudeki [2011]

Conclusions

- Coulomb collision effects (as modeled by the Fokker-Planck equation with Spitzer coefficients) on the ion motion can be approximated as a Brownian motion process for H^+, He^+, and O^+ (ionospheric) plasmas.
- In the case of the electrons, Brownian motion does not capture all the details of the electron ACFs because the electron displacement distributions are not Gaussian. The approximation is not appropriate in this case.
- Electron displacement statistics are independent of the plasma configuration, therefore, electron ACFs are the same for H^+, He^+, and O^+ plasmas. We expect the same to happen in multiple-ion component plasmas.

Current Work

ISR radar experiments and data analysis

- Validation of the collisional ISR spectrum model with standard experiments at Jicamarca.
- Multi-beam radar experiments to measure perpendicular-to-B and off perpendicular ISR data from the topside ionosphere.
- Analysis of radar data and inversion of ionospheric parameters (Densities, temperatures, and drifts).