# **The Information Content of the Aurora**

Presented by Joshua Semeter SRI International CEDAR June 26, 2000

## **PHAZE 2 Movie**



Arnoldy et al., JGRA, 1999; Semeter et al., JGRA, 2001

## **Color Separation**

Brightness ratio (e.g. red/blue) is also • indeterminate from single station







-250 -200 -150 -100

-300

0

### **Geographic Projections** Assumes thin layer at constant altitude





2.5

Kilo Rayleighs

٥

## The Rayleigh: A Tomographic Unit

"Because of a lack of general understanding of the units, there was some confusion and misuse of the observational data."

Joseph Chamberlain, 1961

$$I = \text{Surface brightness} = \frac{1}{4\pi} \int_{0}^{\infty} v(r) dr \left[ \frac{\text{photons}}{\text{cm}^2 \text{ s ster}} \right]$$

$$4\pi I = 1 \text{ Rayleigh} = \int_{0}^{\infty} v(r) dr \left[ \frac{\text{photons}}{\text{cm}^2 \text{ s(column) ster}} \right]$$

 $4\pi l$  is not a flux

 $4\pi I$  is the integrated volume emission rate when the source is isotropic and there is no scattering or extinction in the lower atmosphere. In that case it is also then the integrated rate of atomic or molecular energy transitions related to some excitation process.



# **Forward Model of Auroral Optical Emissions**



# **Overview**

- Energy transport, energy conversion, and auroral production
- Atmospheric response to energetic particles
- Inferring the incident energy spectrum from ground-based observations
  - $\Rightarrow$  The tomographic approach
  - $\Rightarrow$  The multispectral approach
- Simultaneous spatial-spectral-temporal analysis

# **Energy Conservation in Auroral Magnetosphere**

$$\frac{\partial}{\partial t}(\frac{1}{2}\rho u^2 + U) + \nabla \cdot \left[(\frac{1}{2}\rho u^2 + U)\mathbf{u} + p\mathbf{u} + \mathbf{q}\right] = \mathbf{j} \cdot \mathbf{E} + \rho \mathbf{u} \cdot \mathbf{F}_g/m$$

Consider the steady state simplification:

$$\nabla \cdot \mathbf{Q} = \mathbf{j} \cdot \mathbf{E}$$
 where  $\mathbf{Q} = (\frac{1}{2}\rho u^2)\mathbf{u} = \mathbf{j} \cdot \mathbf{E}$ 

and electromagnetic energy conservation (Poynting's Theorem):

$$\nabla \cdot \mathbf{P} = -\mathbf{j} \cdot \mathbf{E}$$
 where  $\mathbf{P} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_o}$ 

Energy is either carried by the electromagnetic field or by the particles. An electric field appears wherever conversion is taking place such that

$$abla \cdot (\mathbf{P} + \mathbf{Q}) = 0$$

# Large Scale Magnetospheric Energy Transport



### **Energy Transport in the Auroral Zone**



### **Velocity Distribution Functions for Energetic Aurora**



### Current-Voltage Relationship of the Acceleration Region

It is difficult to directly measure the energy conversion rate  $j_{||}E_{||}$  in the acceleration region.

But the related quantities  $V = -\int E_{\parallel} \partial l$  and  $\phi_E$  can be estimated from the measured f.

A relationship between  $\phi_E$  and V was derived by Fridman and Lemaire [1981] based on the kinetic model of Knight [1973]. To first order:

$$\phi_E = 0.846 \frac{N_e (E_{0\parallel})^{1/2}}{E_{0\perp}} V^2 \tag{1}$$

The proportionality was observed experimentally by Lyons et al. [1979]. Does it hold in general?

# $\phi_E/v^2$ for PHAZE 2



The simple relationships predicted by kinetic theory do not hold in regions where  $\phi_E(E)$ is non-Maxwellian, i.e.,

- When the aurora is very weak
- When the aurora is very energetic
- When the aurora is very turbulent

## **Atmospheric Response to Energetic Particles**



### **Volume Production Profiles**



# **The Auroral Tomography Problem**



#### **Continuous problem**

$$b(\theta) = \int L(\mathbf{r}, \theta) v(\mathbf{r}) d\mathbf{r}$$
  
$$\Omega$$



#### **III-posed problem**

$$b(\theta_k) = \int k(\mathbf{r}, \theta_k) v(\mathbf{r}) d\mathbf{r}$$
  

$$\Omega$$



#### **III-conditioned problem**

$$b(\theta_k) = \sum_j L(\mathbf{r}_j, \theta_k) v(\mathbf{r}_j) \Delta \mathbf{r}_j$$
$$b = Lv$$

### Forward Auroral Tomographic Equation with Pixel Basis

Canonical forward equation:

 $d(\theta) = \int_{\Omega} L(\mathbf{r}, \theta) v(\mathbf{r}) d\mathbf{r} + n(\theta)$ 

 $M = I \times J =$ # unknowns

 $N = K_1 + K_2 + K_3 =$ # measurements

 $N > M \rightarrow$  no solution  $\rightarrow$  least squares

 $M < N \rightarrow$  infinite solutions  $\rightarrow$  "best" one satisfies a side constraint (e.g., least squares minimum norm)



### A Unifying Perspective: Tikhonov Regularization

$$\hat{\mathbf{x}}_{\mathrm{Tik}} = \underset{\mathbf{x}}{\operatorname{argmin}} \quad \underbrace{\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_{2}^{2}}_{\operatorname{Data Fidelity}} + \underbrace{\gamma^{2} \|\mathbf{L}\mathbf{x}\|_{2}^{2}}_{\operatorname{Prior Info}}$$

- $\mathbf{L} = \mathbf{I}, \gamma = 1 \Rightarrow$  Least squares minimum norm solution
- $\mathbf{R}_x = r_{\mathbf{x}} \mathbf{I}, \mathbf{R}_n = r_{\mathbf{n}} \mathbf{I}, \gamma = r_{\mathbf{n}} / r_{\mathbf{x}}, \mathbf{L} = \mathbf{I} \Rightarrow \text{MAP}$  with prior probability  $p_{\mathbf{X}}(\mathbf{x}) \propto e^{-\gamma^2 \mathbf{x}^T \mathbf{L}^T \mathbf{L} \mathbf{x}}$
- $\mathbf{L}\mathbf{x} = \ln \mathbf{x} \Rightarrow$  Maximum Entropy or MART

# Estimating $\phi_n(E)$ from $\hat{n}_e(z)$

Consider a classic inverse problem associated with the left-hand side of this chart, namely

Can  $\phi_n(E)$  be recovered from a measured E-region auroral ionization profile?

- UNTANGLE

(Vondrak & Baron, 1975)

- CARD (Brekke et al., 1989)



### **Forward Model of Arc-Related Ionization**

$$q_e(z) = \int_{\text{all } E} Q(E, z)\phi_n(E)dE$$
(2)

Appropriate discretization in altitude and energy gives us

$$\mathbf{q} = \mathbf{Q}\phi \tag{3}$$

in steady state with  $n_{\text{ions}} = n_e$ 

$$q = L = \alpha n_e^2 \tag{4}$$

### **CARD** Analysis

Solve linear inverse problem

$$\mathbf{n_e} = \mathbf{ ilde Q} \phi + \mathbf{n}$$

where

 $\tilde{\mathbf{Q}} = \sqrt{\mathbf{Q}/\alpha}$  and **n** is noise.

How many energies can be uniquely resolved?



### **Direct Estimation of** $\phi_n(E)$ from Ground-Based Photometry

Suppose v(x, z) is related to  $\phi_n(E)$  by

$$v(x,z) = \int_{\text{all } E} \phi_n(E,x) Q(E,z) dE$$
(1)

Combining this with the tomographic equation gives

1

$$d(\theta) = \int_{\Omega} K(x, z, \theta) \underbrace{\int_{\text{all } E} \phi_n(E, x) Q(E, z) dE dx dz + n(\theta)}_{\phi_n(E, x) \Rightarrow v(x, z)} (2)$$

Or in vector form

$$\mathbf{d} = \mathbf{L}\mathbf{M}\phi + \mathbf{n} \tag{3}$$

### Experimental test of $\phi_n(E)$ from $n_e(z)$



## Single Site Measurements: Perspective Effects



### Optical Characteristics of Nonlinear Current/Voltage Relationships



# **Considerations for Red Line Modeling**

Unlike prompt emissions redline does not vary linearly with energy flux:

- The low excitation energy (1.96 eV) of  $O({}^{1}D)$  means that heating results in significant production.
- The long radiative lifetime (126 s) of  $O({}^{1}D)$  causes hysteresis in the emission.
- $O(^{1}D)$  is excited in both proton and electron precipitation.



# Red Aurora of April 11, 1997





### C OTIF Tomographic Analysis of April 11, 1997



# **Explanation for Low Redline Emission Altitude**



# The Simultaneous Multispectral Imager (SMI)

