Simulating Ionospheric Plasma Physics Using Millions and Millions of Particles

By

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Talk Outline

- What are simulations?
- What Ionospheric problems require simulations?
- Introduction to kinetic plasma physics
- Introduction to Particle-In-Cell (PIC) simulations
- Example problems
- Limitations of these methods

What are simulations?

- The Encyclopedia Britannica says "the use of a computer to represent the dynamic responses of one system by the behavior of another system modeled after it. A **simulation** uses a mathematical description, or model, of a real system in the form of a computer program."
- Contrasts with "Model" – which we use to mean any precise description

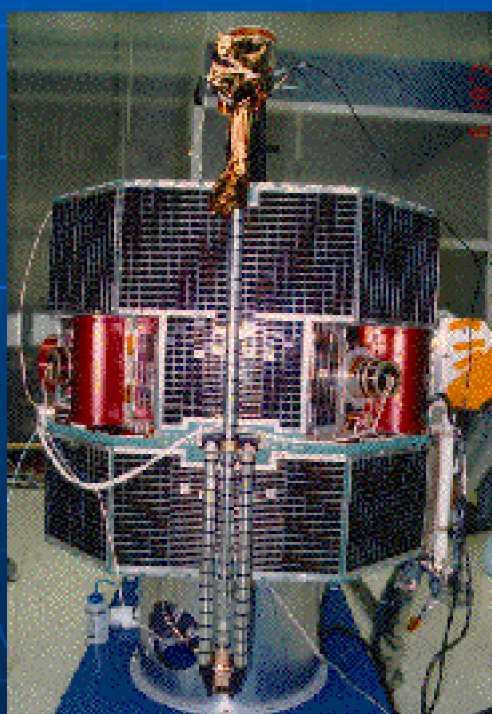
What use are simulations?

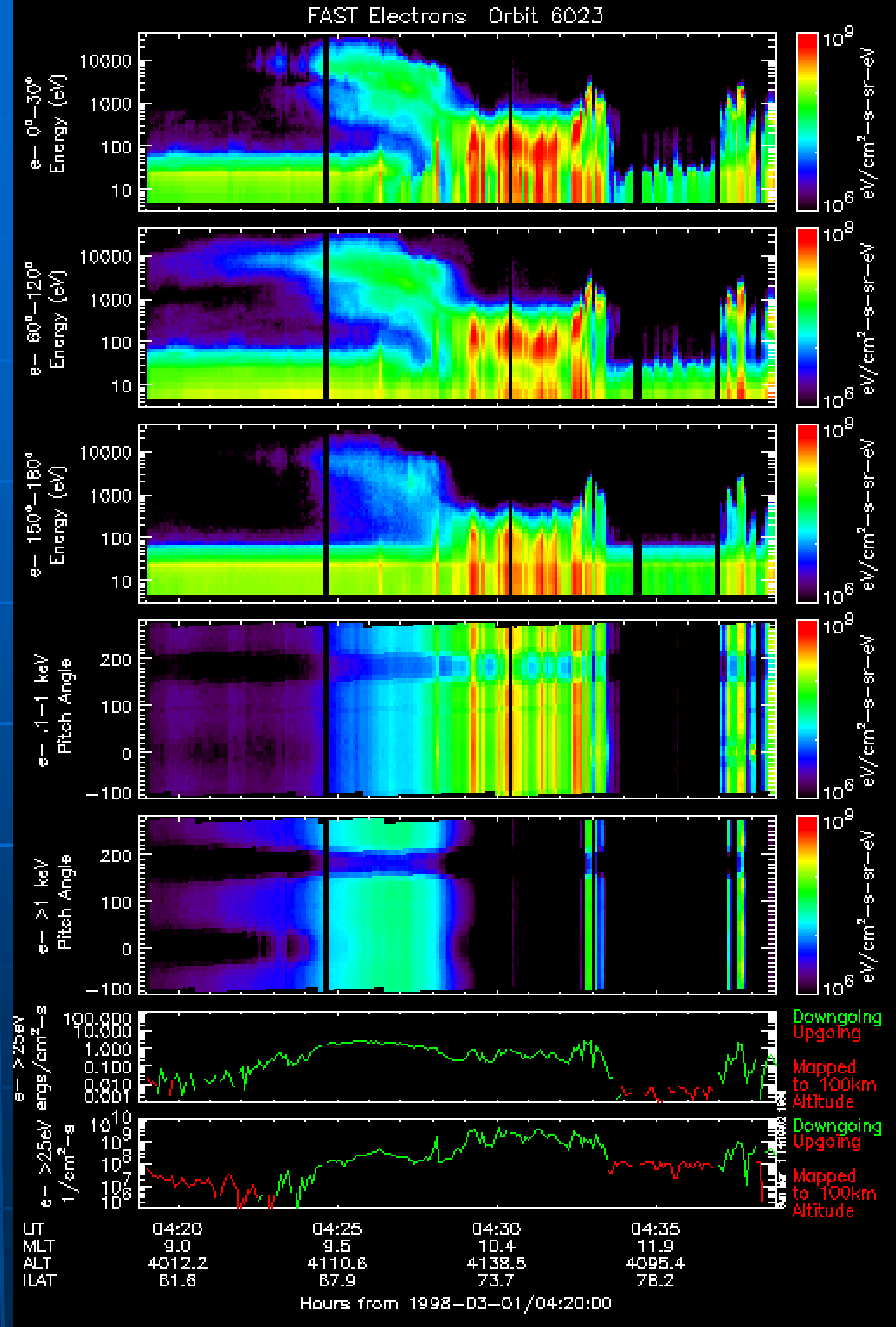
- Views of nature:
  - Physicists think that equations approximate the real world.
  - Engineers think that the real world approximates equations.
  - Mathematicians don’t care...
- Simulations explore the behavior of systems too complex for analytical theory
  - Inhomogeneous systems
  - Nonlinear systems
  - Turbulence
- Simulations give insight into complex systems!

Where does one need simulations?

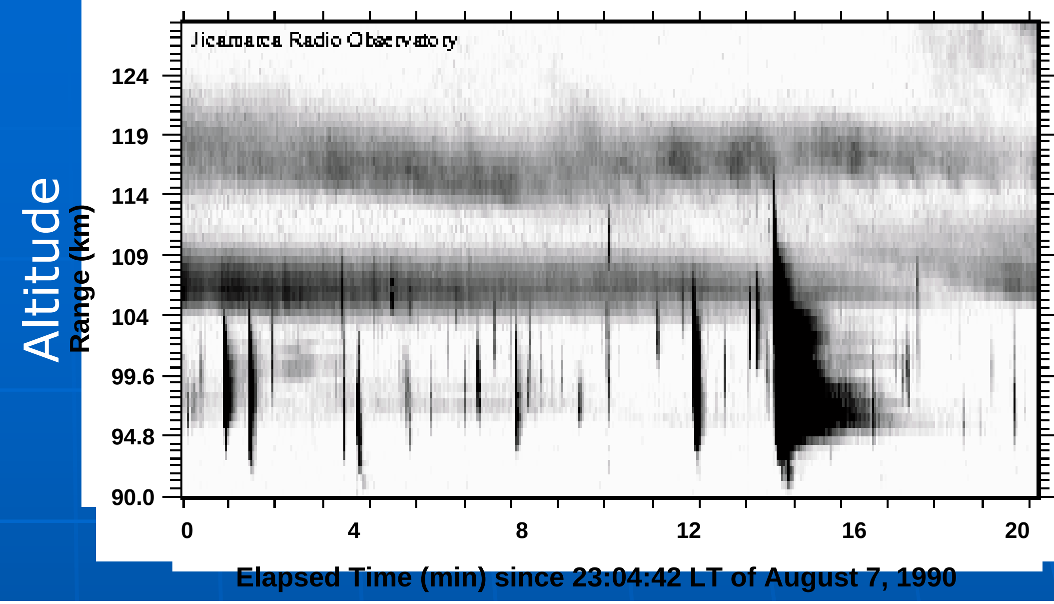
- The Auroral Ionosphere: Electrons accelerate from 3000-1500 km altitude by unknown mechanisms

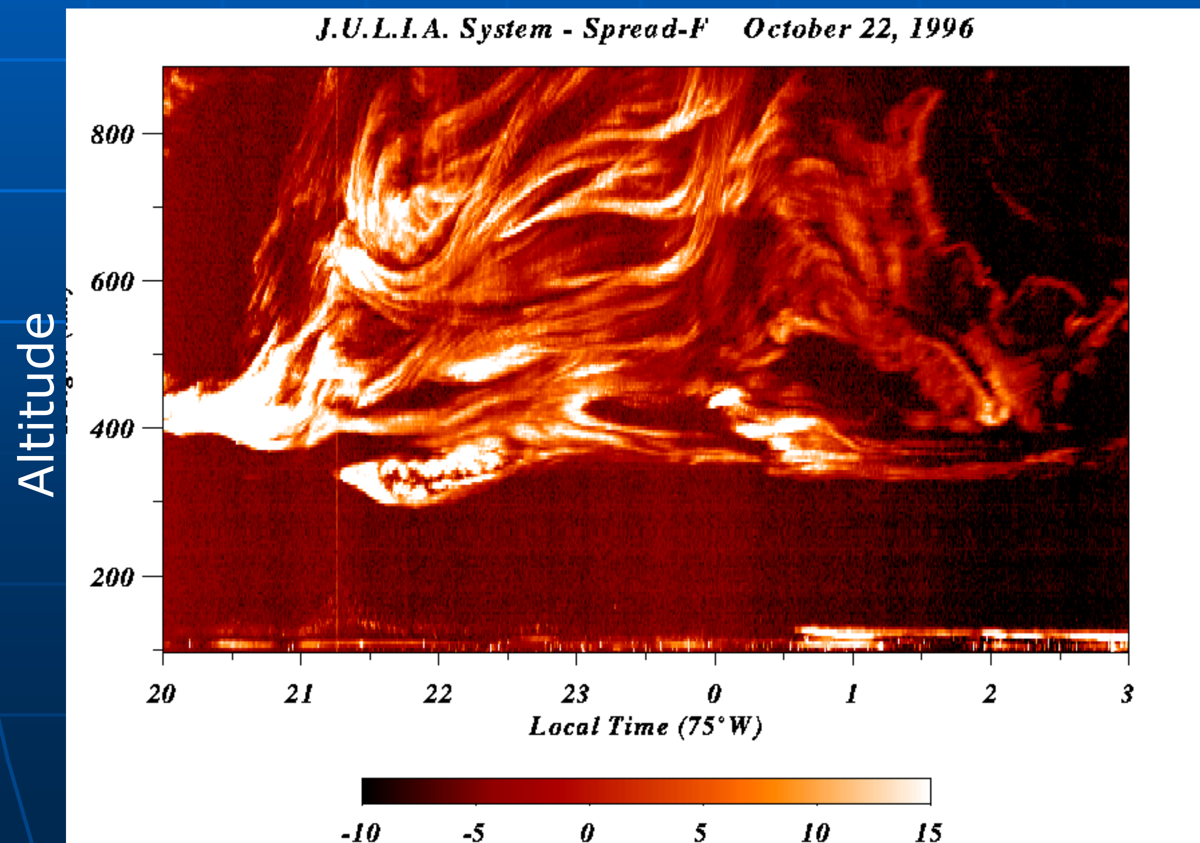
FAST Spacecraft measures turbulent auroral plasmas





Radars Measure Electron Density Irregularities in E & F Regions





Plasma Theory in 5 Minutes

1. Charged particles create fields: Maxwell's Equations
2. Lorentz Force Accelerates Particles:

$$\vec{\nabla} \bullet \vec{E} = \frac{e}{\varepsilon_0}(n_i - n_e) \quad \vec{\nabla} \bullet \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \vec{\nabla} \times \vec{B} = \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \varepsilon_0 \vec{J}$$

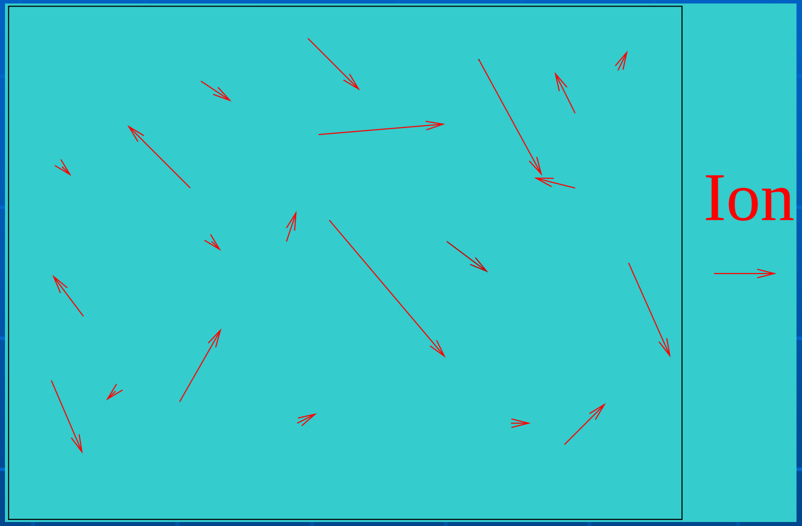
$$\frac{d\vec{v}_i}{dt} = \frac{q_i}{m_i}\left[E(\vec{x},t) + \vec{v}_i \times B(\vec{x},t)\right]$$

3. Collisions deflect particles (important in the lower ionosphere)

3. Too many particles – Need simplifications!

Particle Simulations

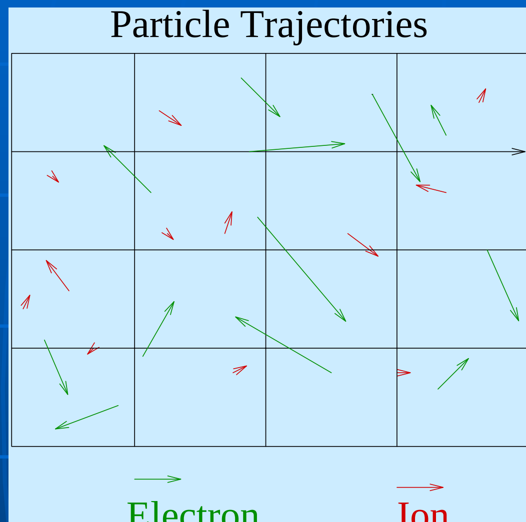
- Particles move within a box:
  - Position: $\boldsymbol{x}_i$
  - Velocity: $\boldsymbol{v}_i$
- Particles generate fields which accelerate other particles
- Too Slow! Speed proportional to the number of particles squared.



$$\vec{F}_{ij} = \frac{q_i q_j}{4\pi\varepsilon_0 (\vec{x}_i - \vec{x}_j)^2}$$

Particle-In-Cell (PIC) Steps:

1. Gather to determine charge density, ρ

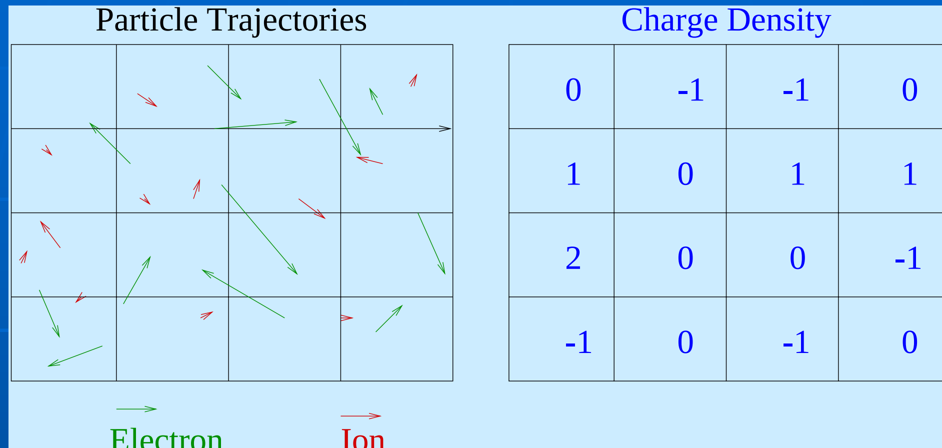


Charge Density

0	-1	-1	0
1	0	1	1
2	0	0	-1
-1	0	-1	0

2. Calculate Electric field: $\vec{\nabla} \bullet \vec{E} = \rho / \varepsilon_0$
3. Update velocities: $\dfrac{d\vec{v}_i}{dt} = \dfrac{q_i}{m_i}\left[E(\vec{x}_i, t) + \vec{v}_i \times B(\vec{x}_i, t)\right]$
4. Update Positions: $\dfrac{d\vec{x}_i}{dt} = \vec{v}_i$
5. Go to Step 1

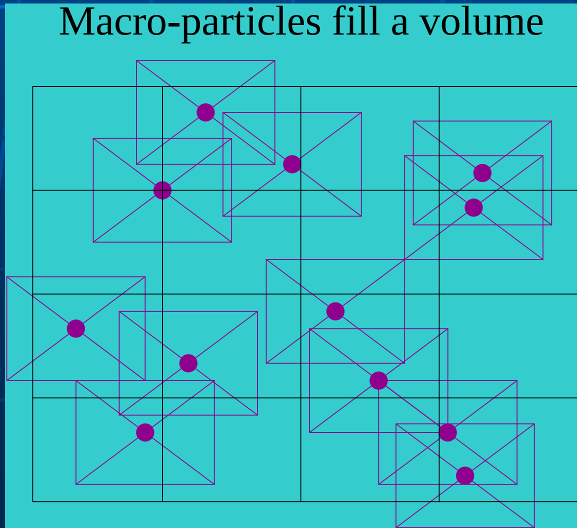
Assumptions made by PIC



- Short range interactions eliminated
  - Simulators with a meshes cannot model behavior smaller than the mesh
  - Features must be bigger than the mesh
- Each PIC particle models the behavior of more than 10^6 real particles
- Full kinetic physics represented
  - Particle trapping – resonant acceleration
  - Landau damping – resonant wave damping
- Fluid Simulators also use a mesh
  - Only one velocity in one location (unlike kinetic simulators)
  - Misses some physics but is less costly (per cell)

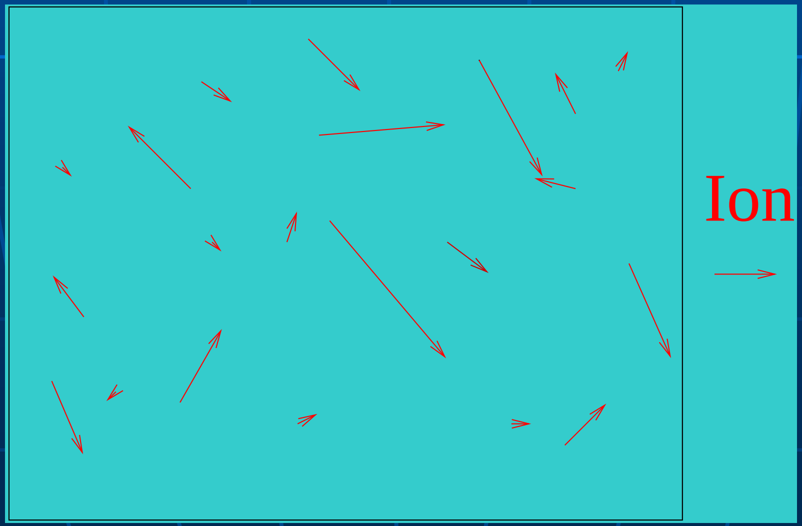
One Problem with PIC

- Particle noise from limited numbers of particles
  - Random walk statistics: $\sigma \propto \sqrt{n_{particles/cell}}$
  - Example n=144 particles/cell -> σ_n=8.3%
  - Fixes:
    - Nature reduces this through electrostatic shielding
    - Use non-point particles
    - Use millions and millions of particles
    - Use super computers!



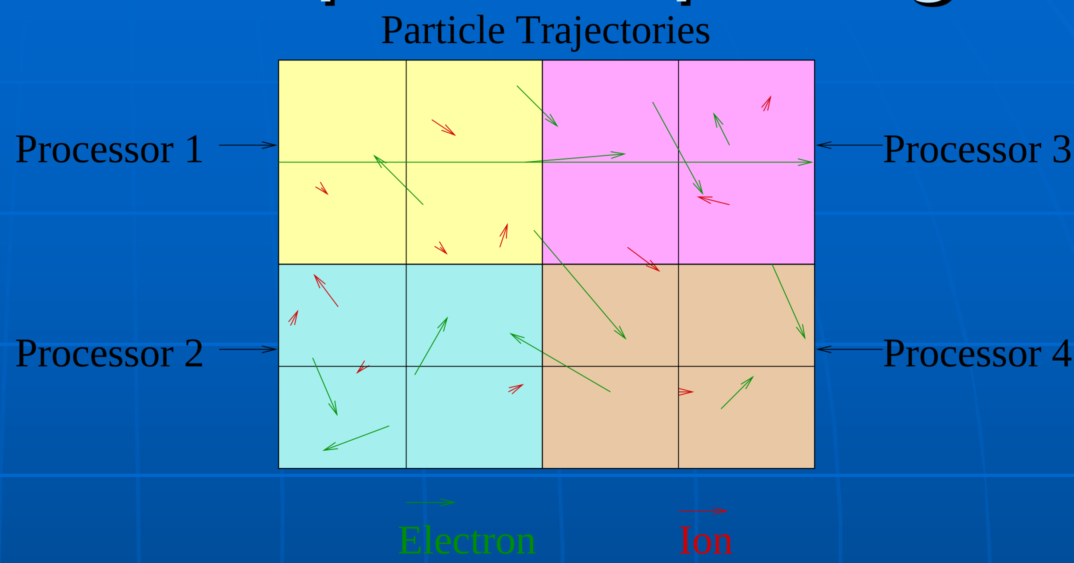
Boundary Conditions (BC)

- Simulations of all types require BC
- BC introduce limitations and, sometimes, error
- Example: *Periodic* is the simplest BC
  - The right side connects to the left
  - The top to the bottom
  - Particles leaving the Left reenter on the Right and visa versa
  - Particles leaving the top -> bottom …

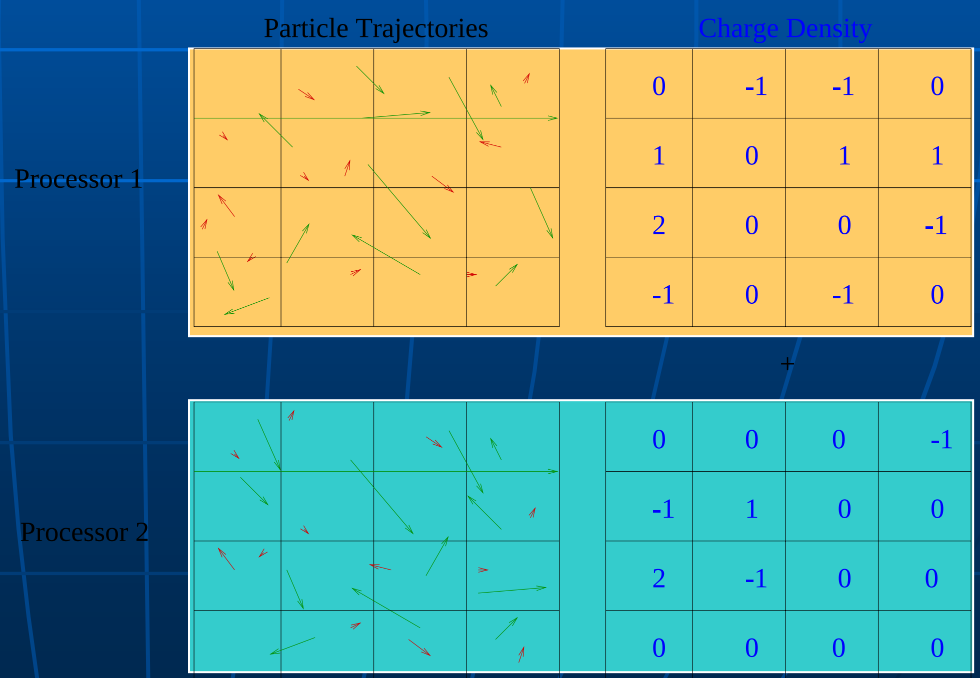


Solution: Parallel Supercomputing

- Domain Decomposition



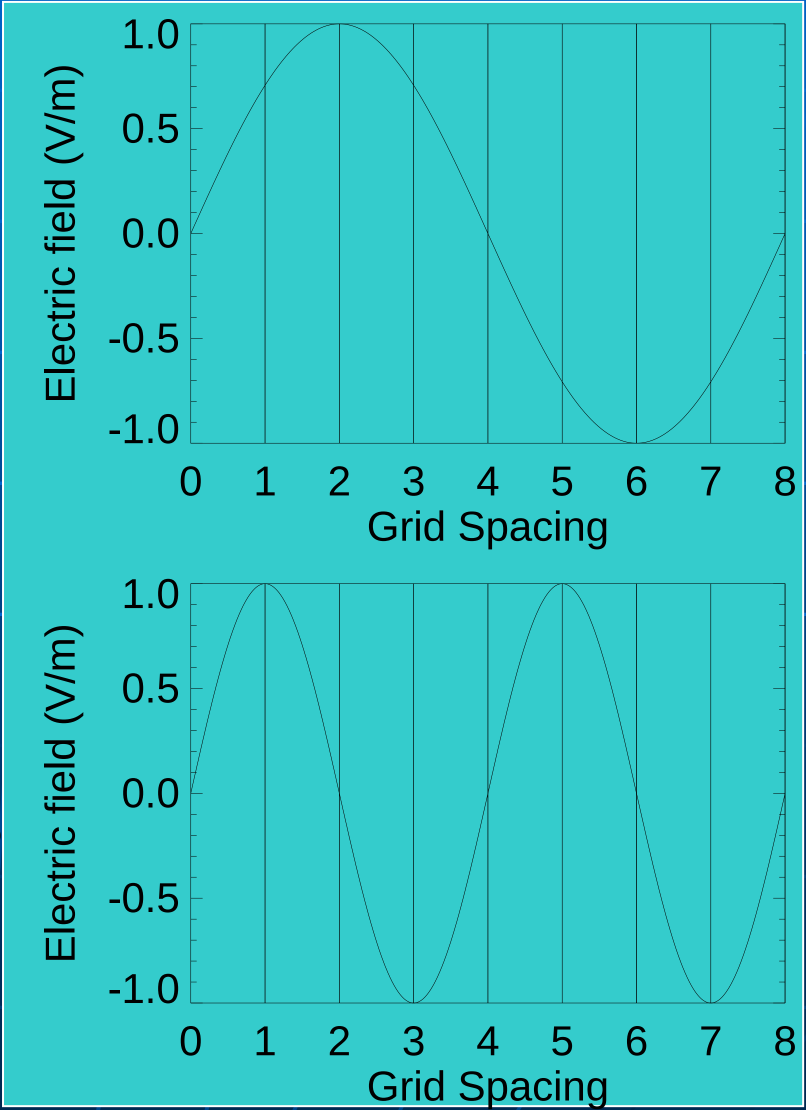
- Mesh Parallelization



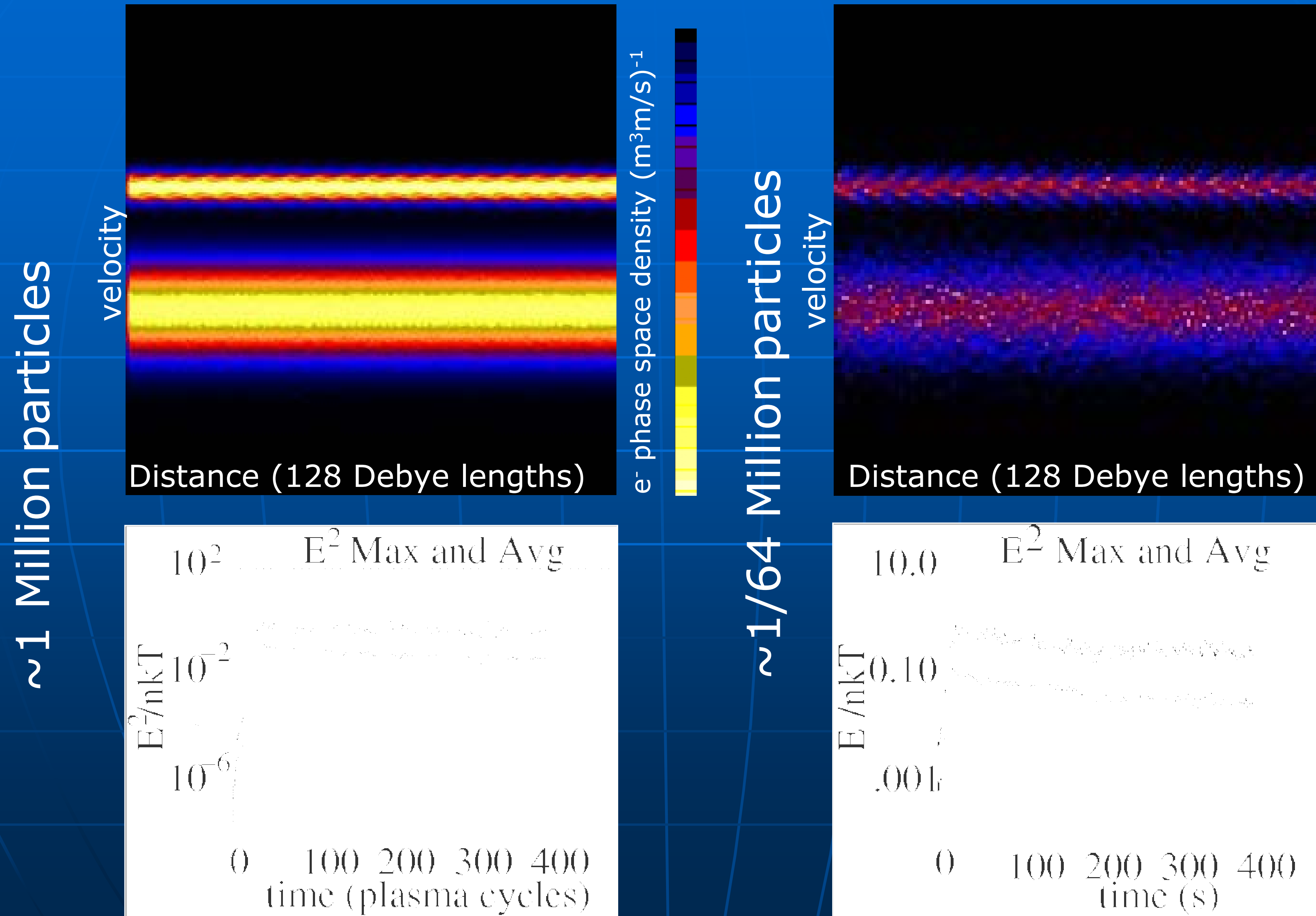
Boundary Conditions Cause Limitations

- Periodic boundaries quantize the simulation:
  - Only a full wave or
  - Integer multiples allowed
  - Simulations must not focus on waves spanning the system
- Other BC have other issues
- True in fluid simulators as well

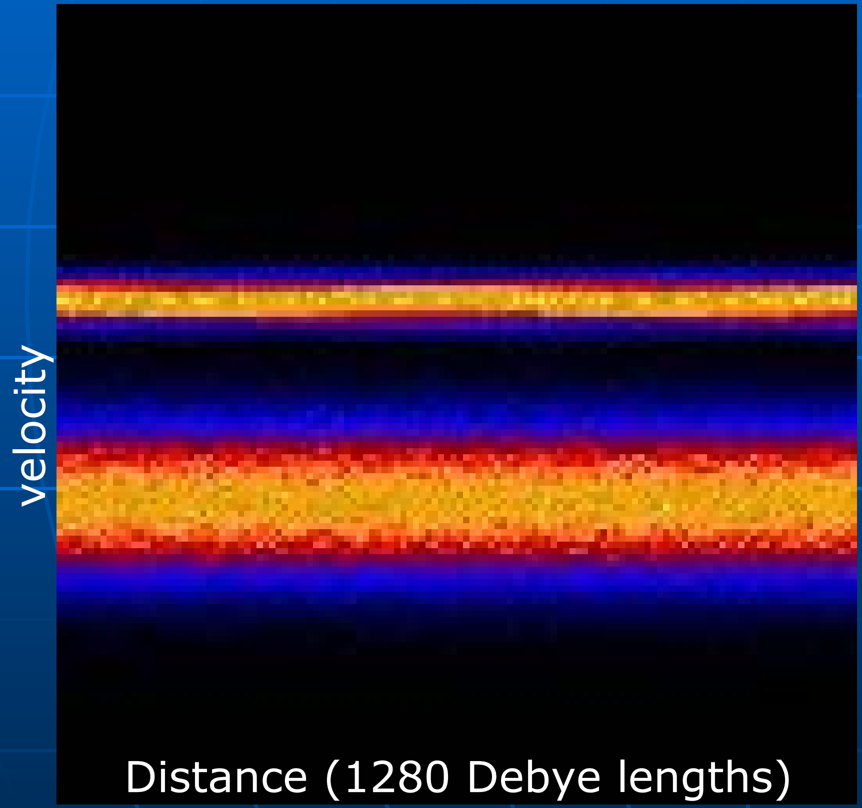
Example in 1D



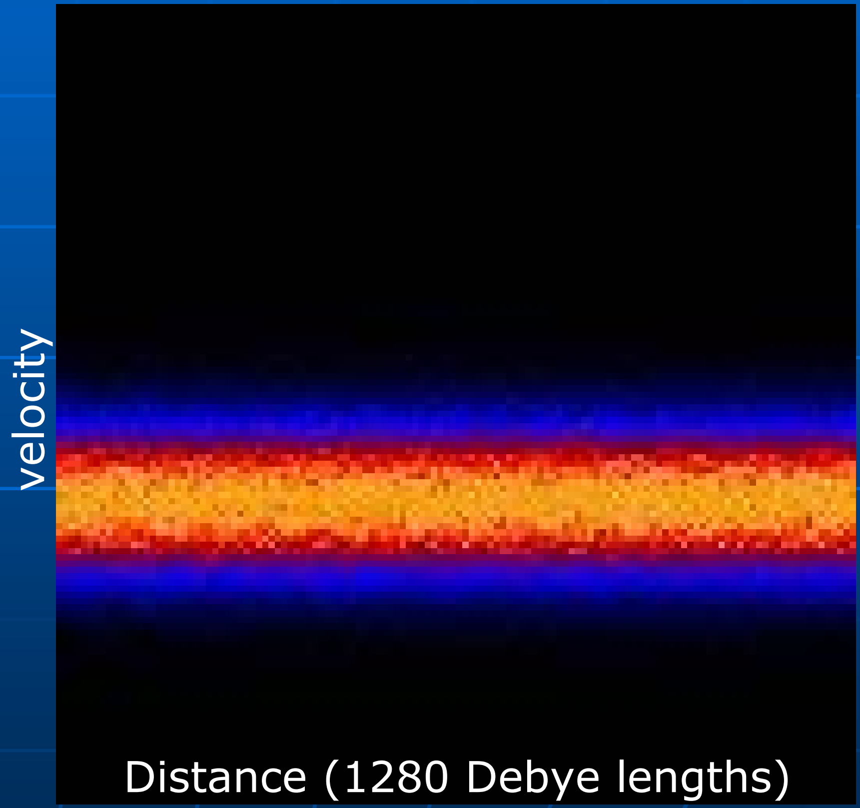
Example: 1D electron two-stream Instability



Expand grid spacing 10X



Eliminate Beam

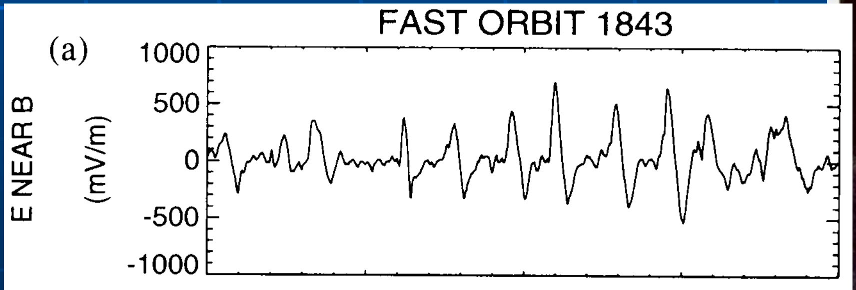


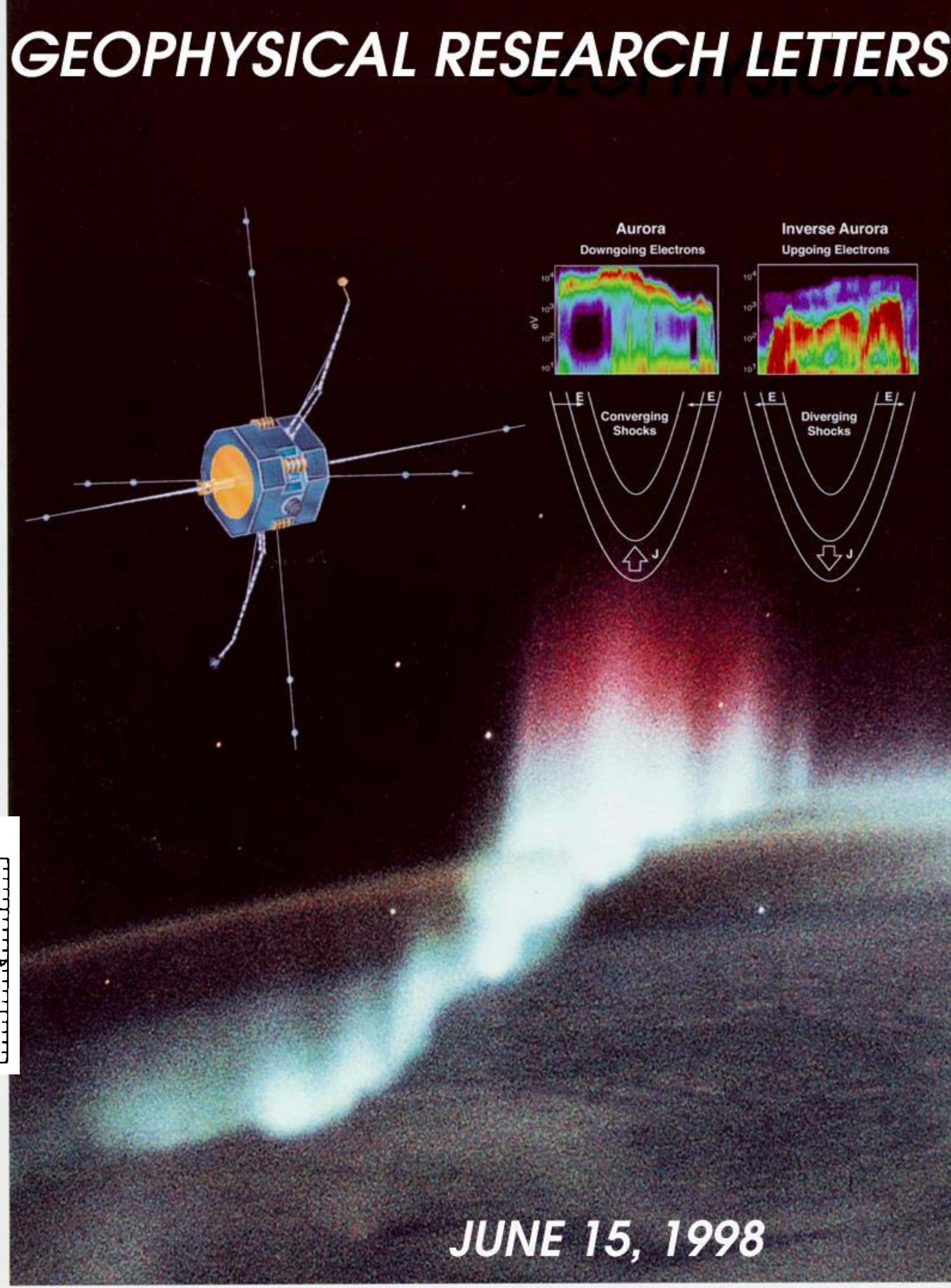
(8x longer simulation in time, shown 16x as fast)

Simulation Limitations

- Systematic:
  - Do the equations represent the physics?
  - Do you resolve the important scales?
- Numerical:
  - Stability
  - Accuracy

Electron Holes in auroral ionosphere





Electron Holes in 2D

Electric Field

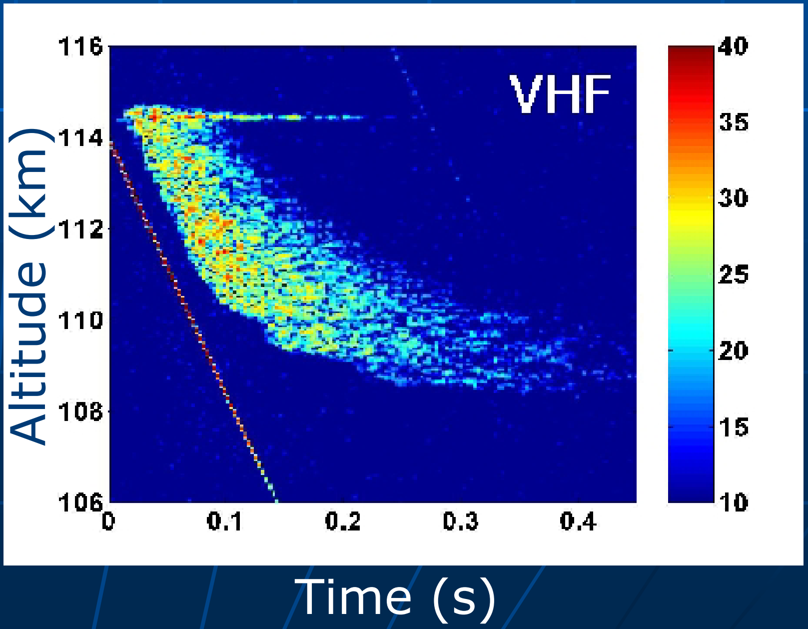
Simulations enable us to:

- Understand dynamics of plasmas and fluids
- Study energy and momentum flow
- Characterize Turbulence

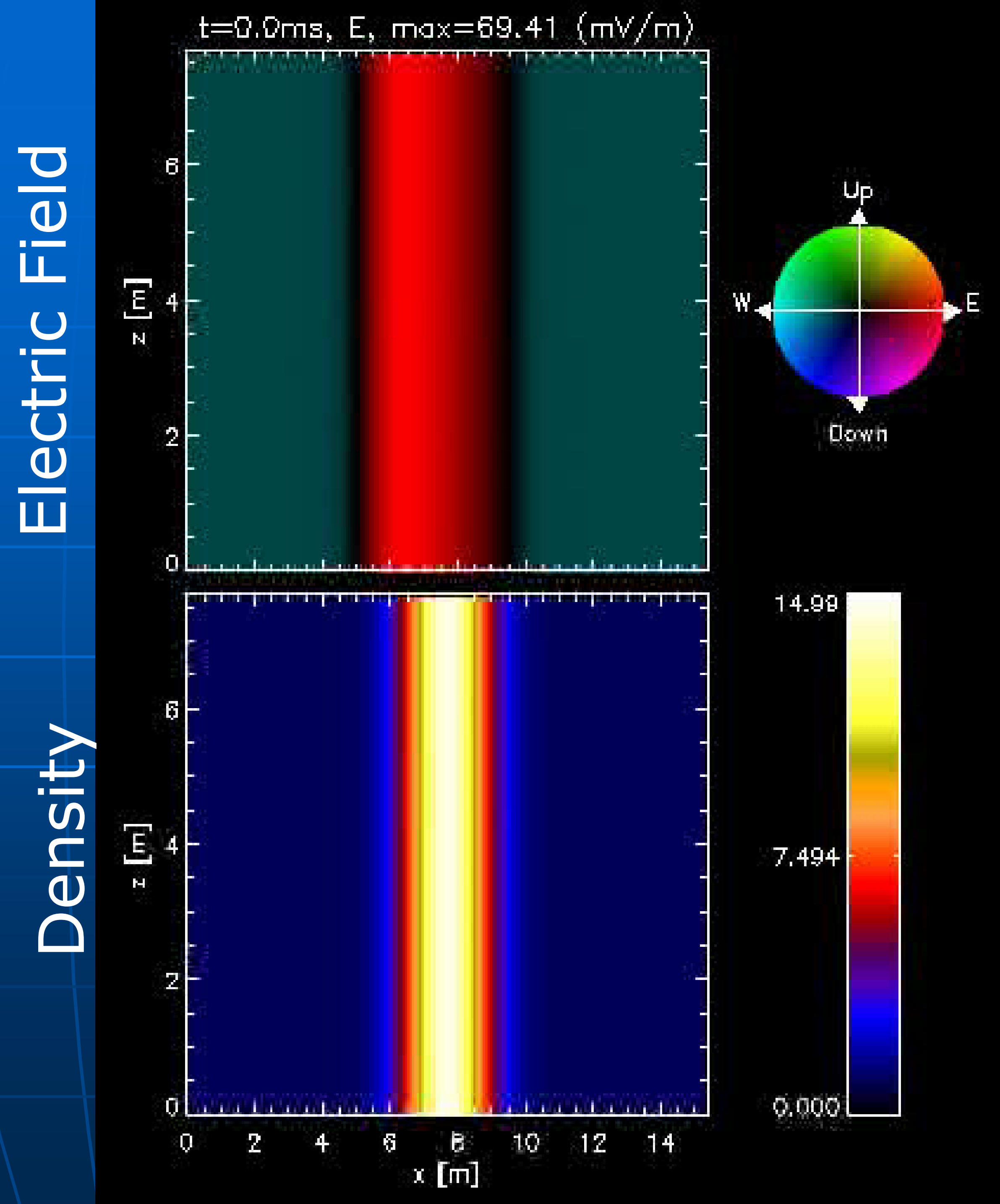
Meteor Plasma waves

Leonids picture from the shuttle

Large Aperture Radar Detection of a Meteor



Meteor Plasma Simulation



Conclusions

- Simulations enable us to explore nonlinear systems
- Simulations subject to systematic limitations and numerical errors
- Enable us to better understand:
  - our models and
  - nature