2001 CEDAR-SCOSTEP

Longmont, Colorado

June 17-22, 2001

Tutorial Lecture #5

by Fabrizio Sassi/Rolando Garcia

CGD/ACD, NCAR, USA

Dynamics of the Equatorial Middle Atmosphere

Dynamics of the Equatorial Middle Atmosphere

1. Introduction
    - Why tropical dynamics is special
    - An example: zonal mean wind evolution in the stratosphere

2. Climatology of the tropical middle atmosphere
    - The QBO and SAO
    - Waves in the Tropics

3. Coupling between the tropical lower atmosphere and middle atmosphere
    - Observations
    - Modeling the SAO

4. Conclusions

Extratropical vs. Tropical Dynamics

1. Zonal Mean Equations

$$
\begin{aligned}
\bar{u}_t + \bar{v}\bar{u}_y + \bar{w}\bar{u}_z - f\bar{v} &= \mathcal{F} && \text{zonal momentum} \\
\bar{v}_y + \rho^{-1}(\rho\bar{w})_z &= 0 && \text{continuity} \\
f\bar{u} &= -\bar{\phi}_y && \text{meridional momentum} \\
\bar{\phi}_{zt} + \bar{v}\bar{\phi}_{zy} + \bar{w}S &= -\alpha\bar{\phi}_z && \text{thermodynamics}
\end{aligned}
$$

2. Quasi-steady motions outside Tropics

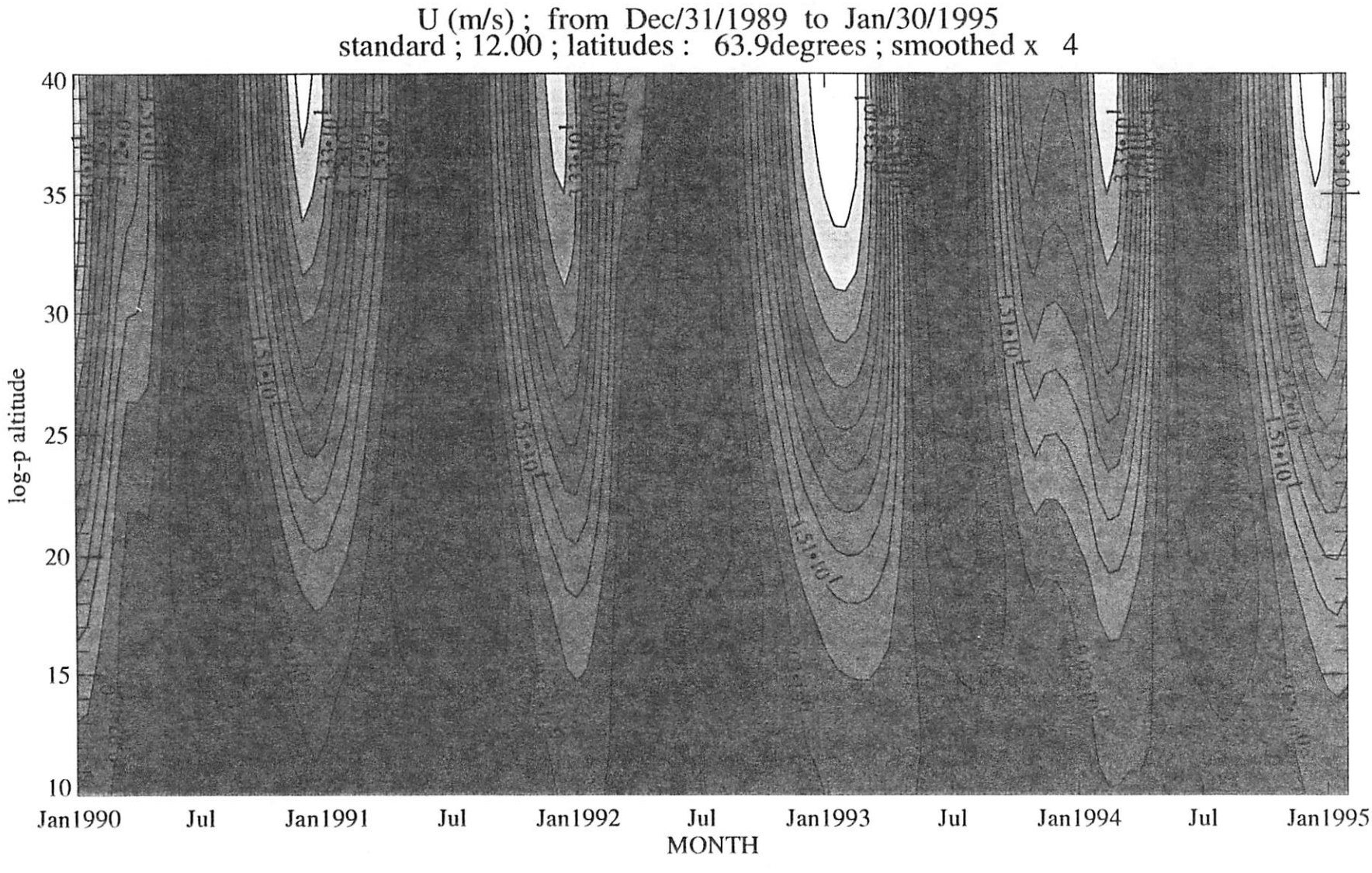
- the momentum balance is $-f\bar{v} = \mathcal{F}$
- which implies $(\bar{v}, \bar{w})$ are determined by $\mathcal{F}$
- in turn this determines $\bar{\phi}$ via $\bar{w}S = -\alpha\bar{\phi}_z$
- and $\bar{u}$ via $f\bar{u} = -\bar{\phi}_y$

3. Low-frequency motions in Tropics ($f \to \beta y$)

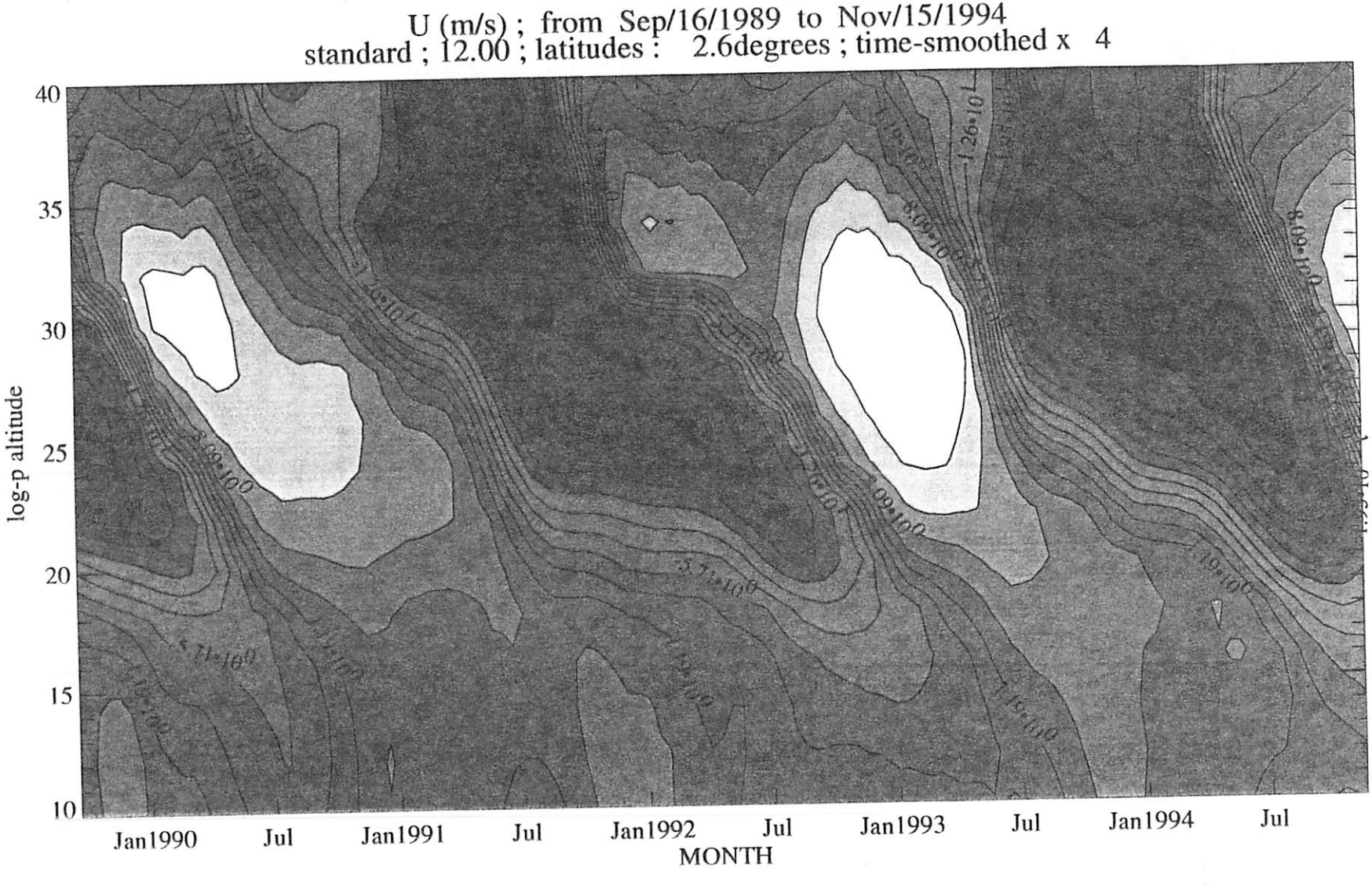
- the Coriolis force does not dominate, so all terms in the zonal momentum equation are potentially important

$$\bar{u}_t + \bar{v}\bar{u}_y + \bar{w}\bar{u}_z - \beta y\bar{v} = \mathcal{F}$$

- then $\mathcal{F}$ can give rise to long-period oscillations in $\bar{u}$ (SAO, QBO) instead of a quasi-steady-state
- $\bar{u}$ remains in geostrophic balance $\beta y\bar{u} = -\bar{\phi}_y$



min = -3.12367 ; max = 42.4286 ; Contour from -3.12367 to 42.4286 by 3.03682
GS-2D output from file fxx.nc


U (m/s) ; from Sep/16/1989 to Nov/15/1994
standard ; 12.00 ; latitudes : 2.6degrees ; time-smoothed x 4
log-p altitude
40
35
30
25
20
15
10
Jan1990
Jul
Jan1991
Jul
Jan1992
Jul
Jan1993
Jul
Jan1994
Jul
MONTH
min = -33.3177 ; max = 18.4392 ; Contour from -33.3177 to 18.4392 by 3.45046
GS-2D output from file fxx.nc

Circulation of the Tropical Middle Atmosphere

- The zonally-averaged circulation
  - QBO
  - SAO
  - wind and temperature oscillations
- Waves
  - equatorially-trapped waves
  - small-scale waves

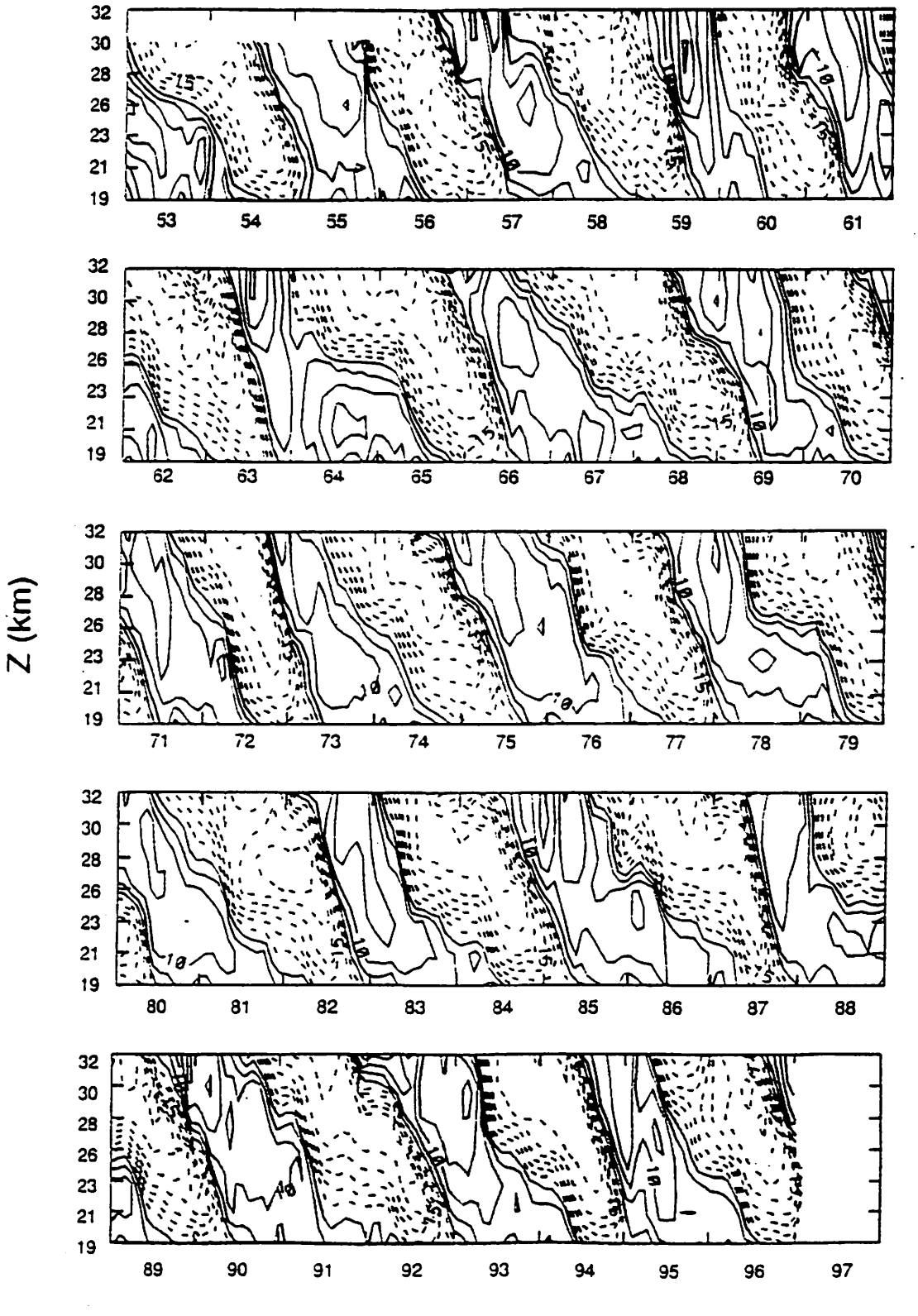


Figure 2.32. Time-height cross section of the monthly average zonal wind in the lower stratosphere at Singapore, with the seasonal cycle removed (courtesy of S. Pawson, Stratospheric Research Group, Free University of Berlin).

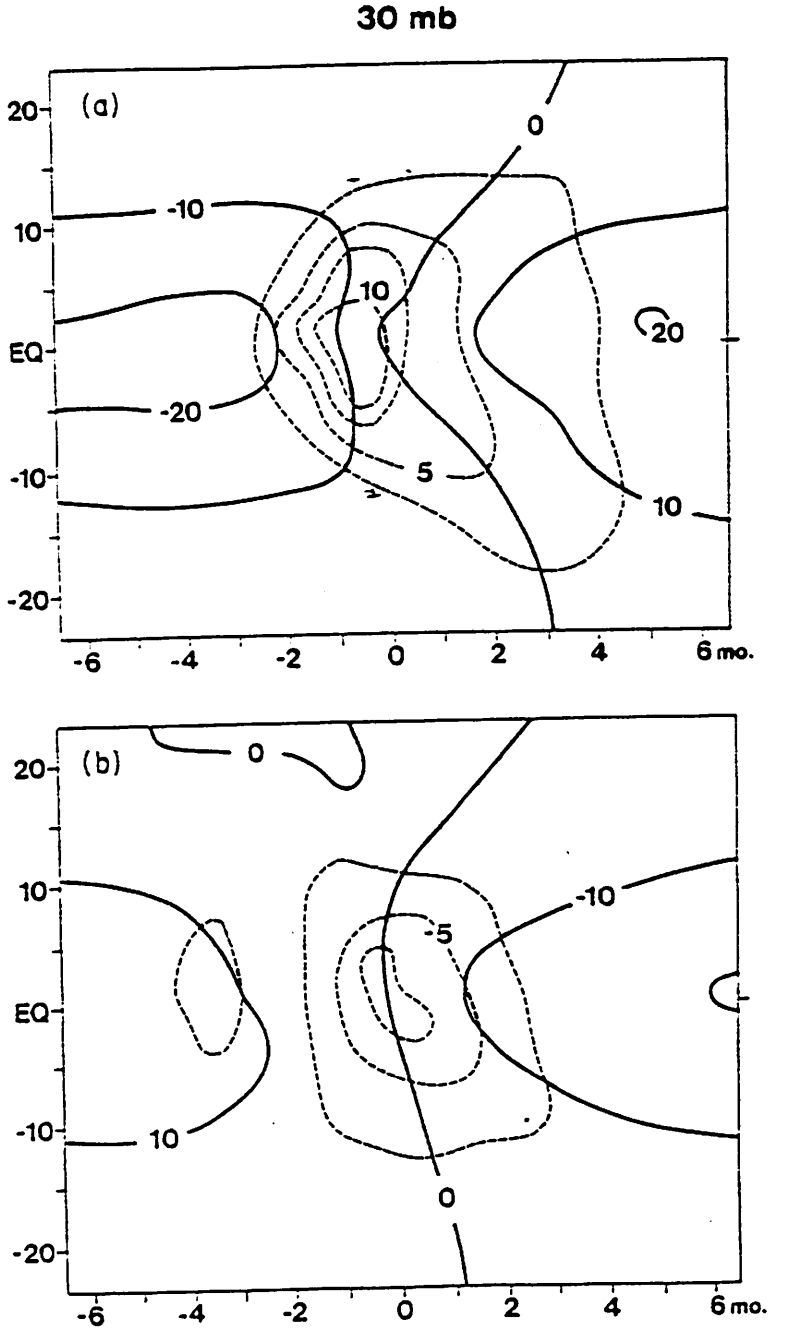


Fig. 8.3. Composite latitude-time section of zonal wind at 30-mb for westerly (upper) and easterly (lower) phases of the QBO. Zonal winds (solid lines) in m s^{-1}; acceleration (dashed lines) in m s^{-1} $month^{-1}$. [From Dunkerton and Delisi (1985). American Meteorological Society.]

RICHARD J. REED (JGR, 1966)

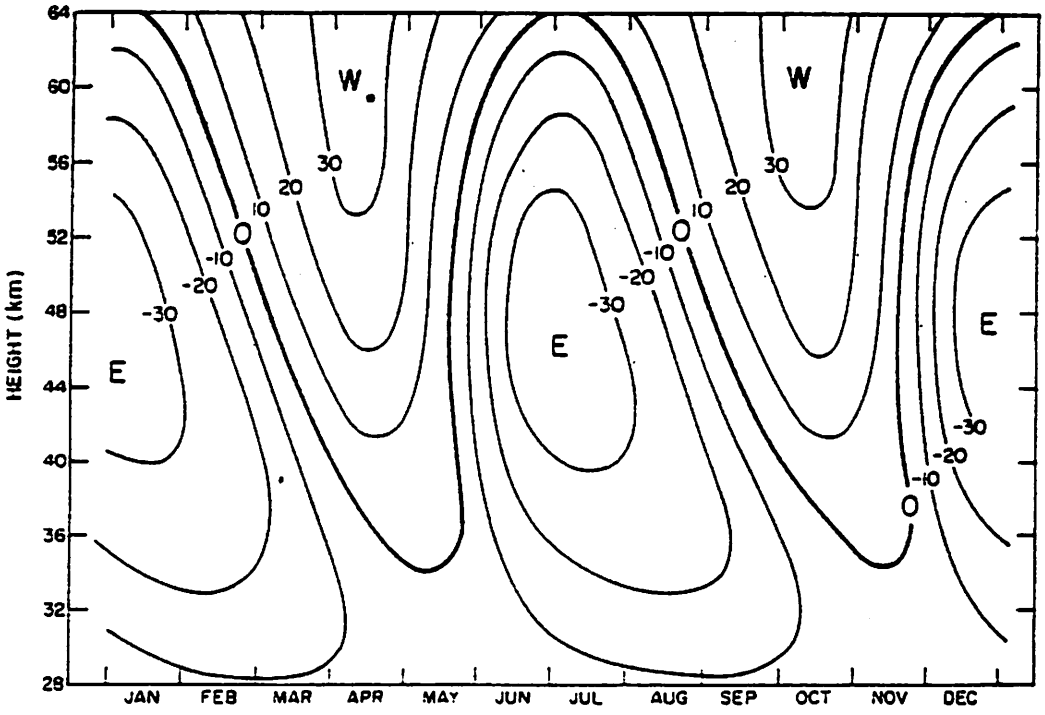
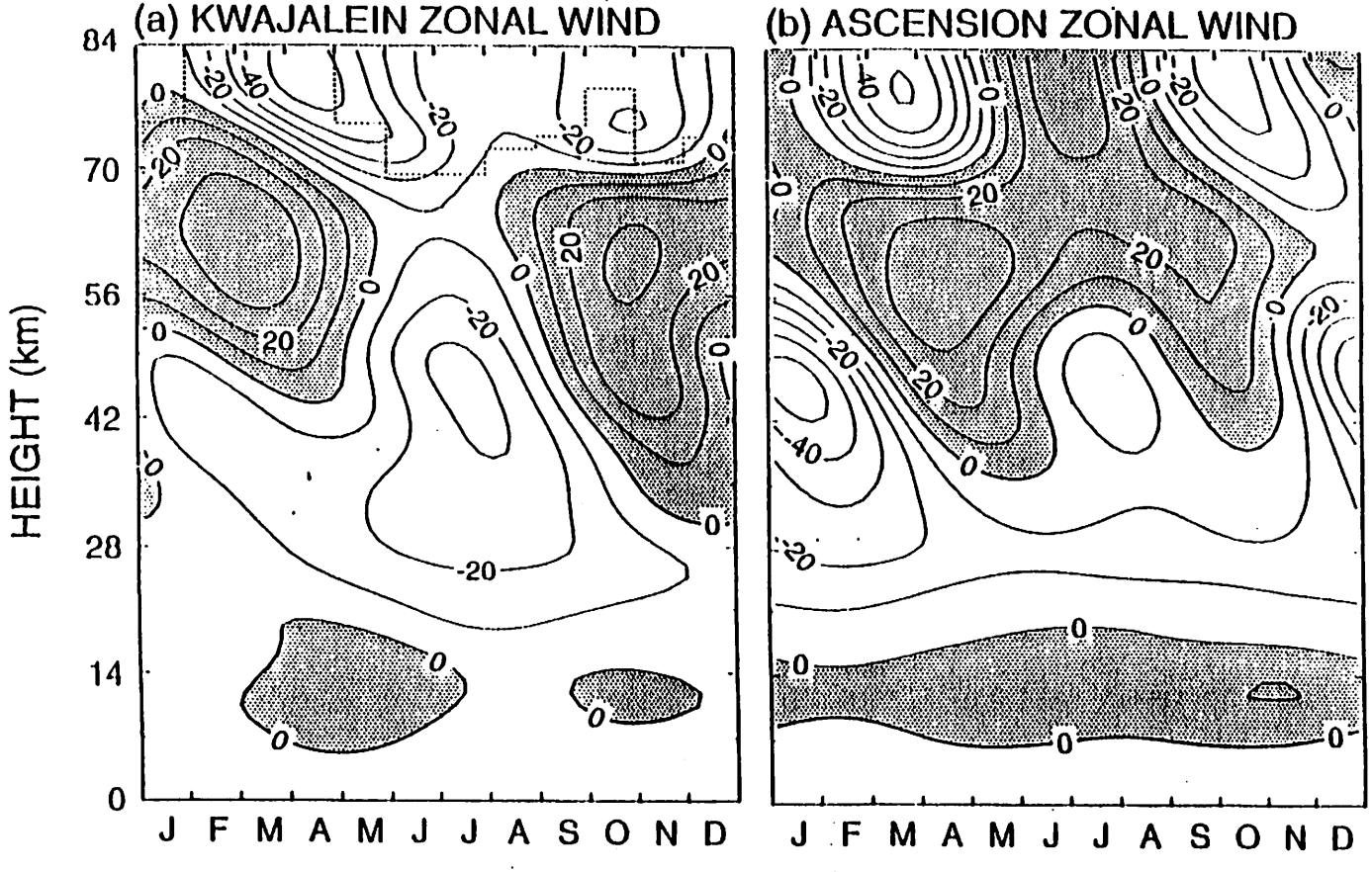


Fig. 3. Estimated variation of zonal wind (in m sec^{-1}) with month and altitude at the equator.

(Ascension I. rocketsondes)



(Ascension + Kwajalein Rocketsondes) Annual Composite

- Semiannual Harmonic -

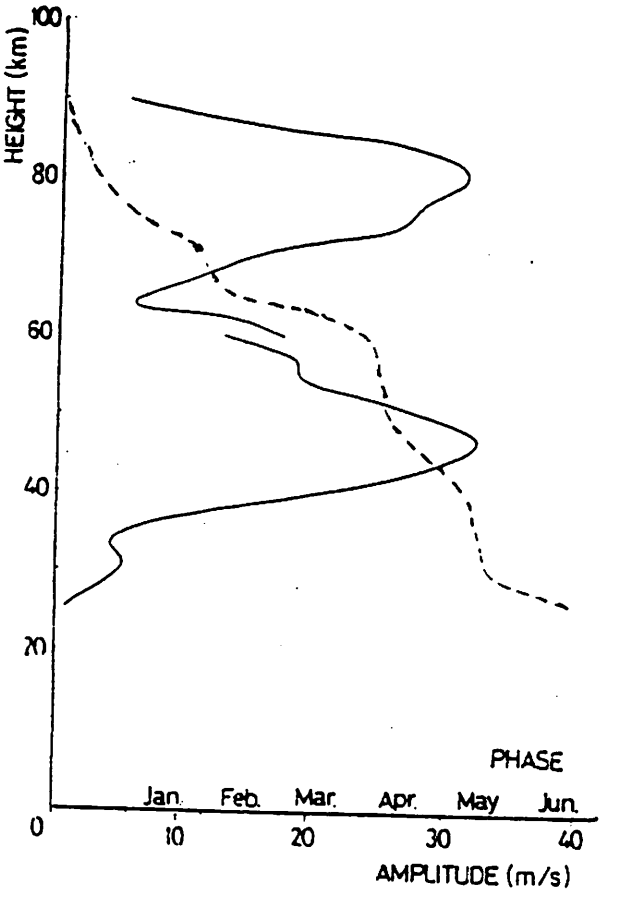
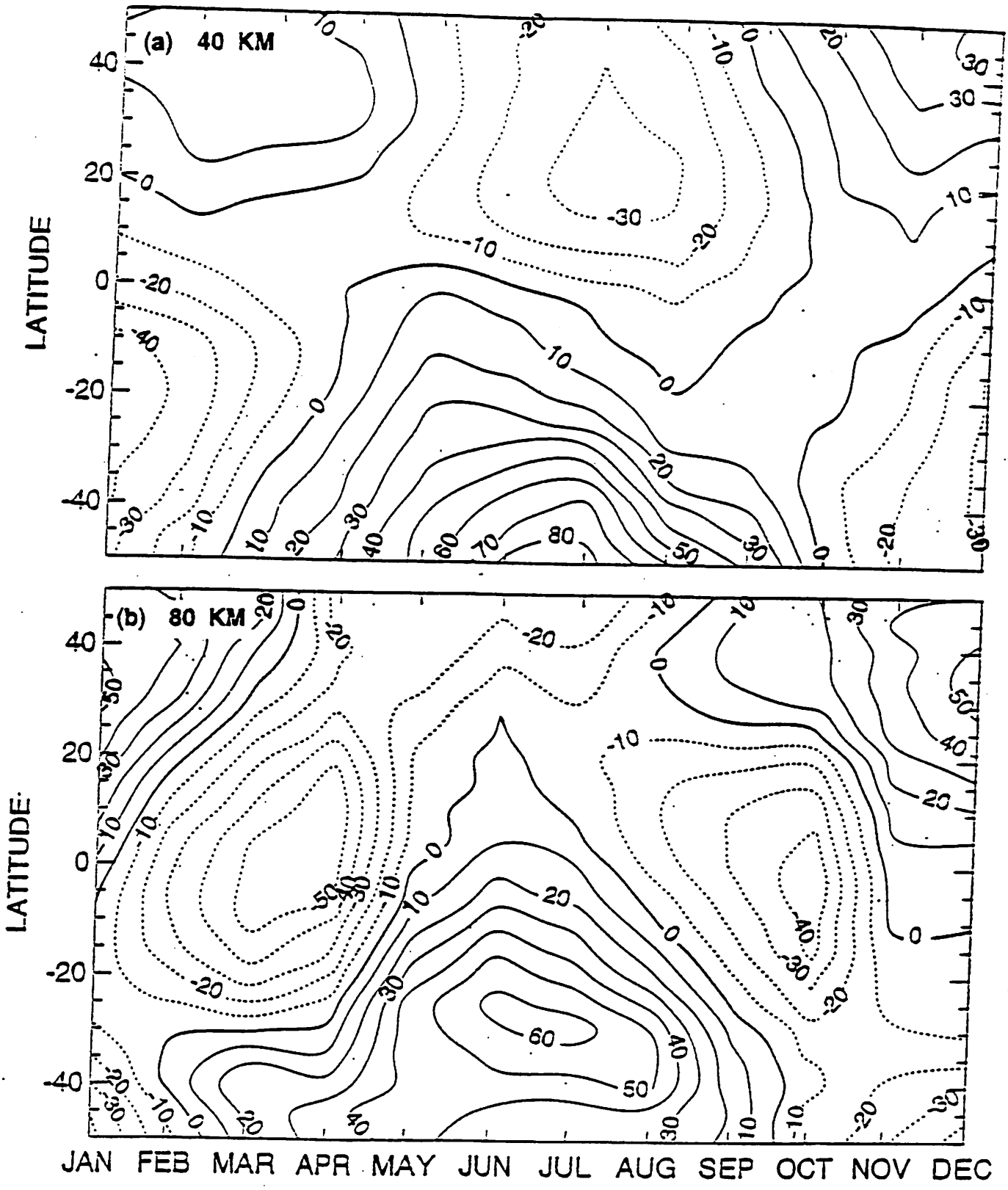
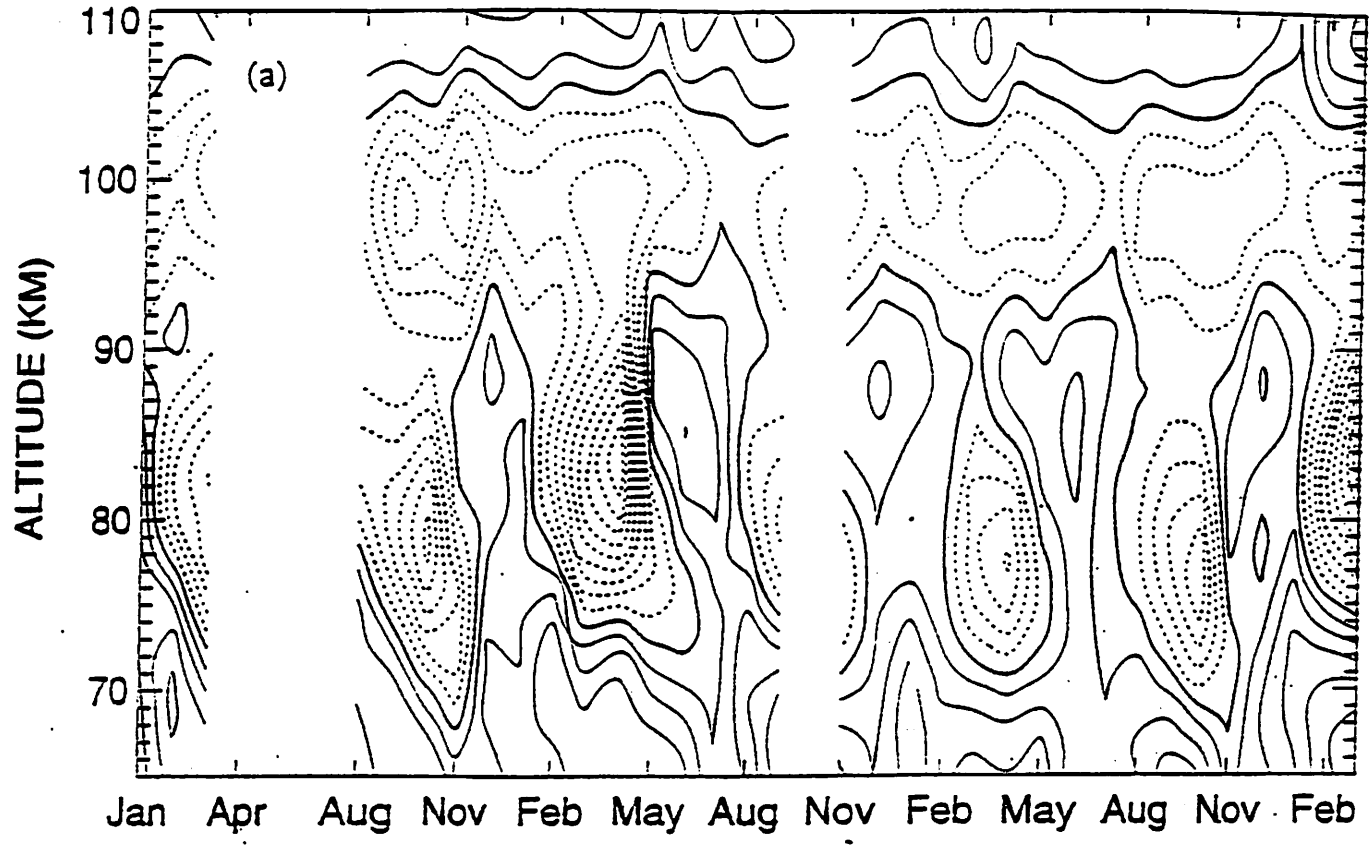
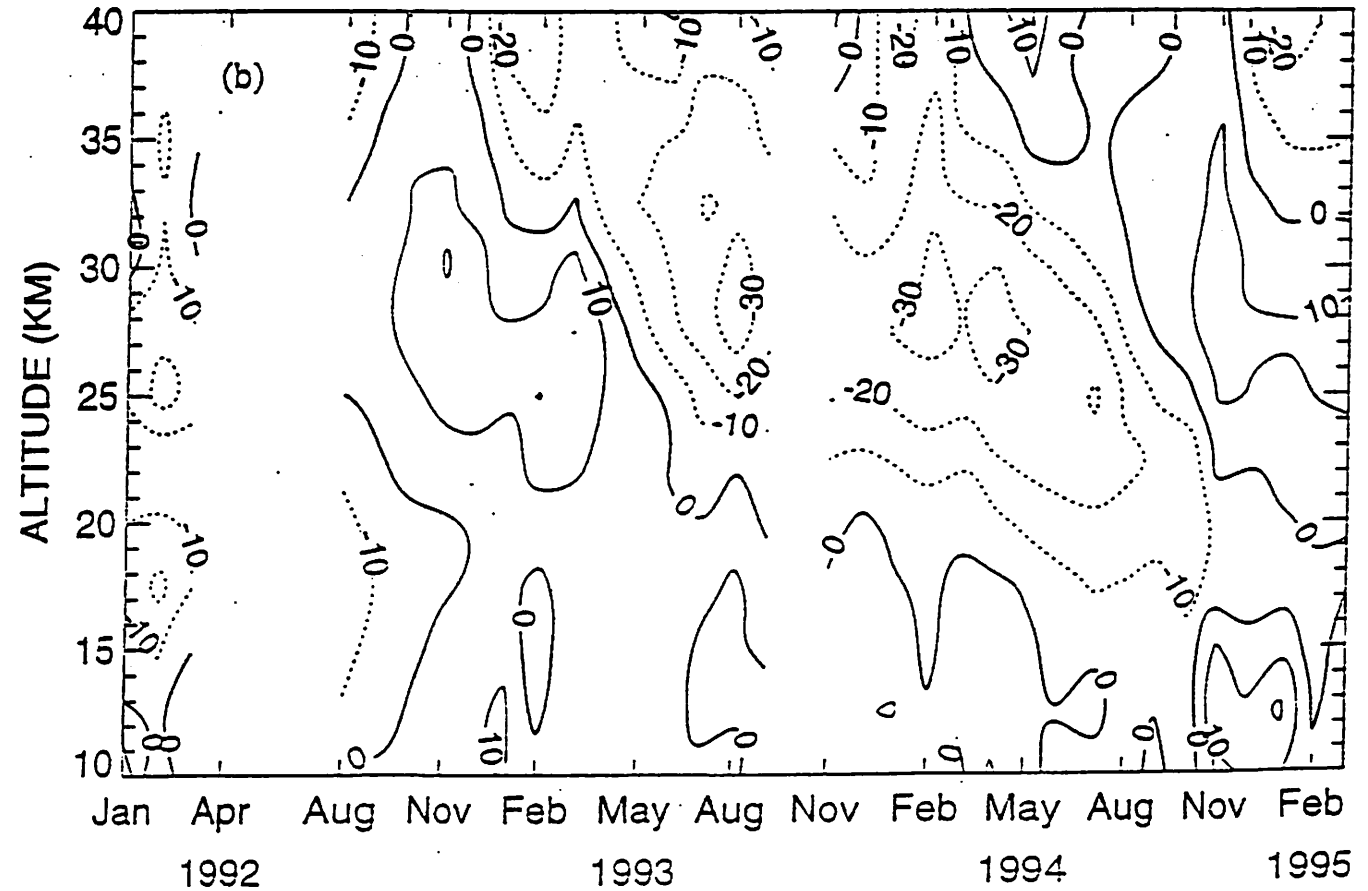


Fig. 8.15. The SAO at Ascension Island (8°S); amplitude (solid lines), phase (dashed line). Phase refers to time of first maximum westerly component in the calendar year. Break near 60 km is caused by separately fitting data above and below that level to sinusoidal curves. [From Hirota (1978). American Meteorological Society.]


(a) 40 KM
(b) 80 KM
LATITUDE
LATITUDE
40
20
0
-20
-40
JAN FEB MAR APR MAY JUN JUL AUG SEP OCT NOV DEC

(HRDI wind)


(a)
ALTITUDE (KM)
110
100
90
80
70
Jan
Apr
Aug
Nov
Feb
May
Aug
Nov
Feb
May
Aug
Nov
Feb


(b)
ALTITUDE (KM)
40
35
30
25
20
15
10
Jan
Apr
Aug
Nov
Feb
May
Aug
Nov
Feb
May
Aug
Nov
Feb
1992
1993
1994
1995

(HRDI wind)

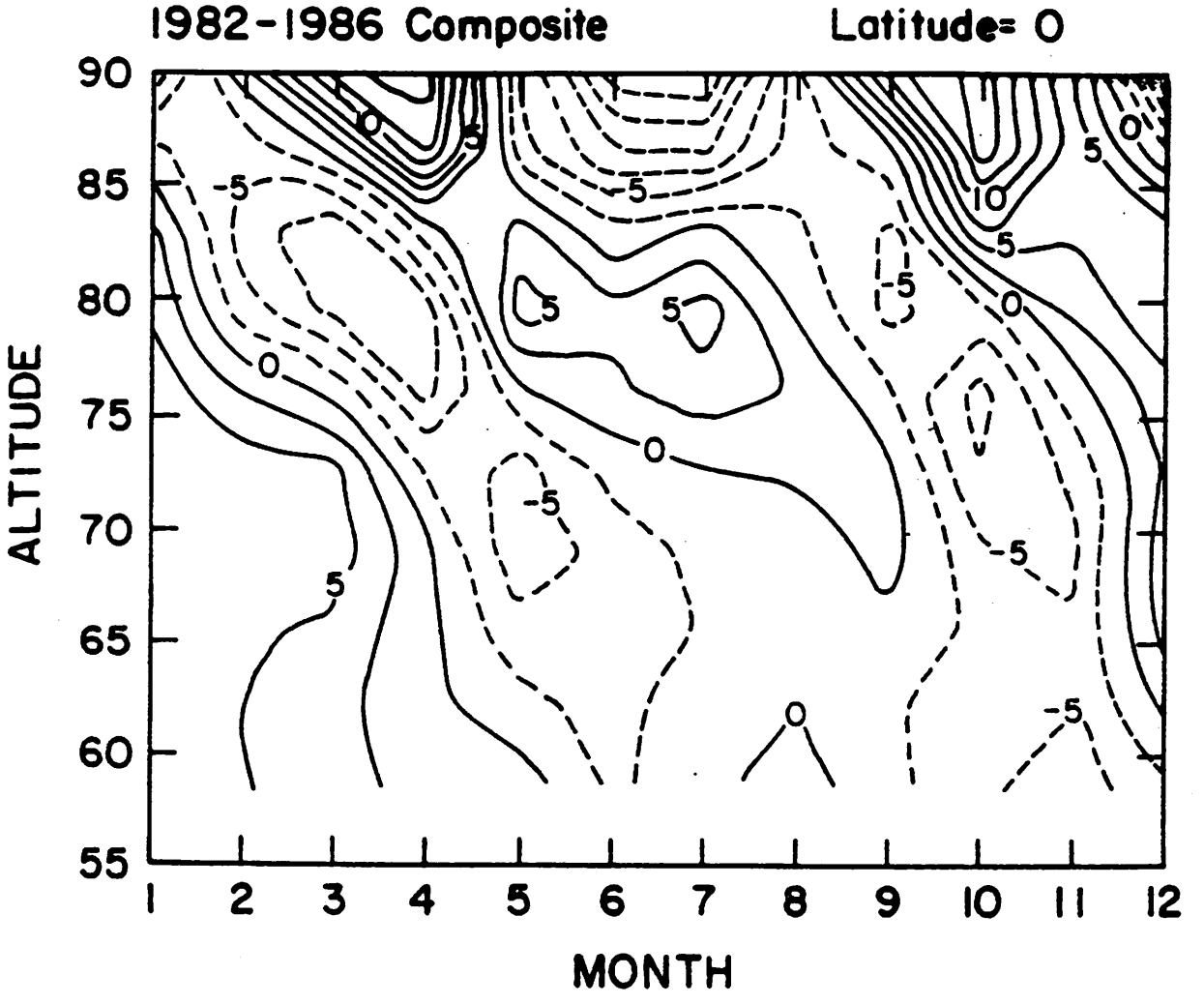


FIG. 3. Annual composite of temperature deviations (K) from the series mean for 1982–86 at the equator.

Temperature Oscillations

- The QBO and SAO in zonal mean wind are accompanied by oscillations in zonal mean temperature

- These result from the adiabatic effects of the secondary circulation $(\bar{v}, \bar{w})$ that is forced in conjunction with the zonal mean wind

- From geostrophic balance:

$$\beta y \bar{u} = -\bar{\phi}_y$$

which by hydrostatic balance, $T = (H/R)\phi_z$, is equivalent to

$$\beta y \bar{u}_z = -\frac{R}{H}\bar{T}_y$$

so that

$$\bar{u}_z = -\frac{R}{\beta H}\bar{T}_{yy} \simeq \frac{R}{\beta H L^2}\bar{T}$$

- This implies

$\bar{u}_z > 0 \rightarrow \bar{T} > 0$ warm anomaly in westerly shear

and

$\bar{u}_z < 0 \rightarrow \bar{T} < 0$ cold anomaly in easterly shear

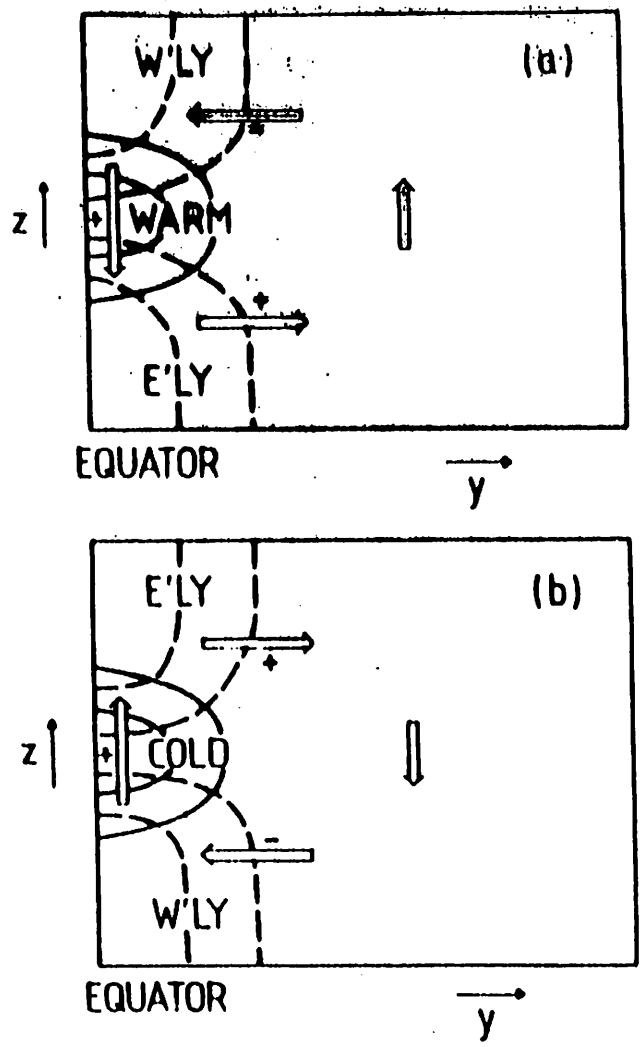


Fig. 8.5. Schematic latitude-height sections showing the mean meridional circulation associated with the equatorial temperature anomaly of the QBO. Solid contours show temperature anomaly isotherms, dashed contours are zonal wind isopleths. Plus and minus signs designate signs of the zonal wind accelerations driven by the mean meridional circulation. (a) Westerly shear zone, (b) easterly shear zone. [From Plumb and Bell (1982), with permission.]

Extratropical Rossby Waves vs. Equatorial Waves

1. Extratropical large-scale waves are quasi-geostrophic and quasi-non-divergent
   - their restoring force is due to the variation of f with latitude
   - gravity (which acts through the divergence field) is not of first order importance
   - the motion is described by the quasi-geostrophic vorticity equation:
   - the waves described by this equation are extratropical Rossby waves

2. Tropical large-scale waves respond both to variations in f and to gravity forces
   - divergence is important for tropical waves
   - tropical waves are equatorially trapped, the trapping being most effective at low frequencies

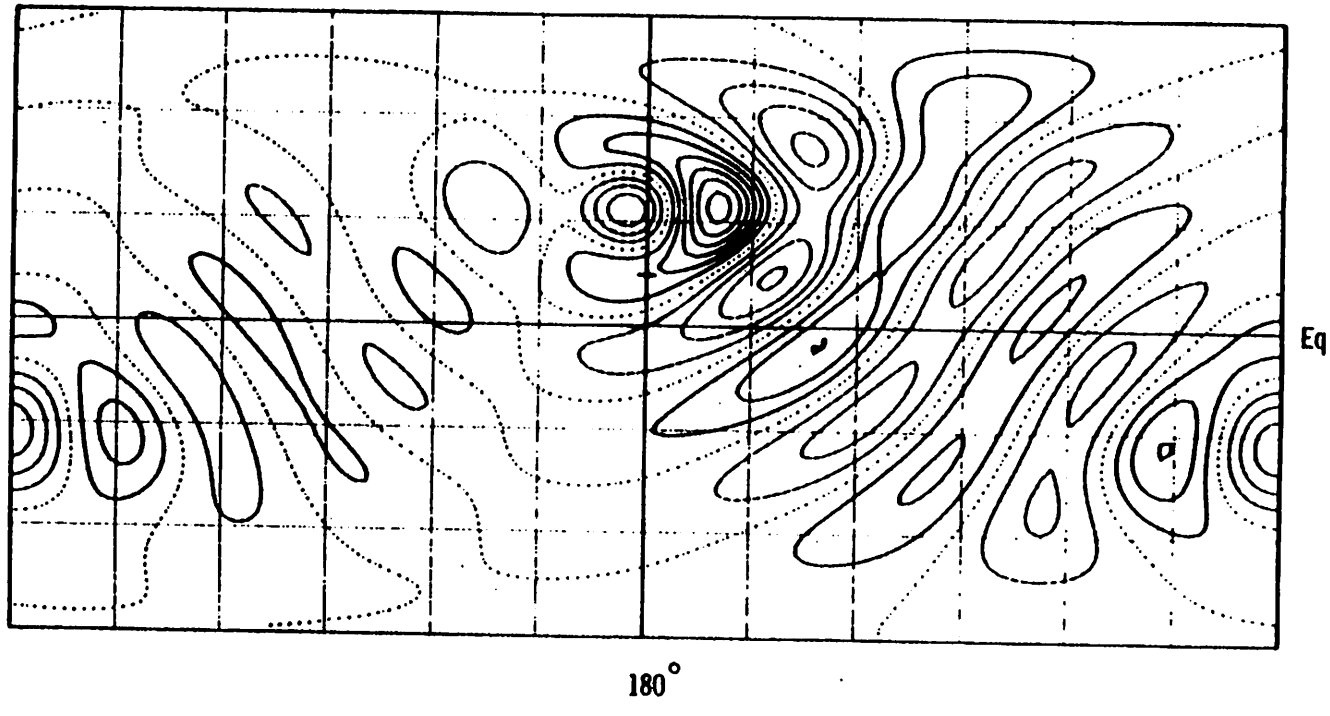


Fig. 11.6. Planetary wave propagation on a sphere, as found in a numerical experiment of Grose and Hoskins (1979). Contours are of perturbation vorticity, and disturbances to a superrotation zonal flow (i.e., an eastward flow with uniform angular velocity about the earth's axis) are produced by a circular mountain centered at 30°N and 180° longitude, and with radius equal to 22.5° of latitude. Waves travel backward and forward across the equator along ray paths that are curved because of variation in the Coriolis parameter f with latitude. The equatorial trapping effect is evident. The amplitude of the wave decays with distance because of dissipative effects included in the model. [From Grose and Hoskins (1979, Fig. 3a).]

An example: The Equatorial Kelvin Wave

- a special case, wherein $v' = 0$
- equations of motion (equatorial beta-plane):

$$\begin{aligned}
\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right) u' &= -\phi'_x \\
\beta y u' &= -\phi'_y \\
u'_x + \rho^{-1}(\rho w')_z &= 0 \\
\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right) \phi'_z + w' S &= 0
\end{aligned}$$

- note that ϕ'_x is balanced by advection of momentum (as in a pure gravity wave) while ϕ'_y is balanced by the Coriolis force (as in Rossby waves)
- the equations have the well-known solution

$$\phi' \propto \exp\left[-\frac{y^2}{2L^2}\right] \cdot \exp i(kx + mz - \omega t) \cdot \exp(z/2H)$$

$$L^2 = \frac{(\omega - k\bar{u})}{\beta k}$$

where ω, k are the frequency and zonal wavenumber, and the width L is a measure of equatorial trapping

- the dispersion relation is:

$$\omega^2 = (kN/m)^2$$

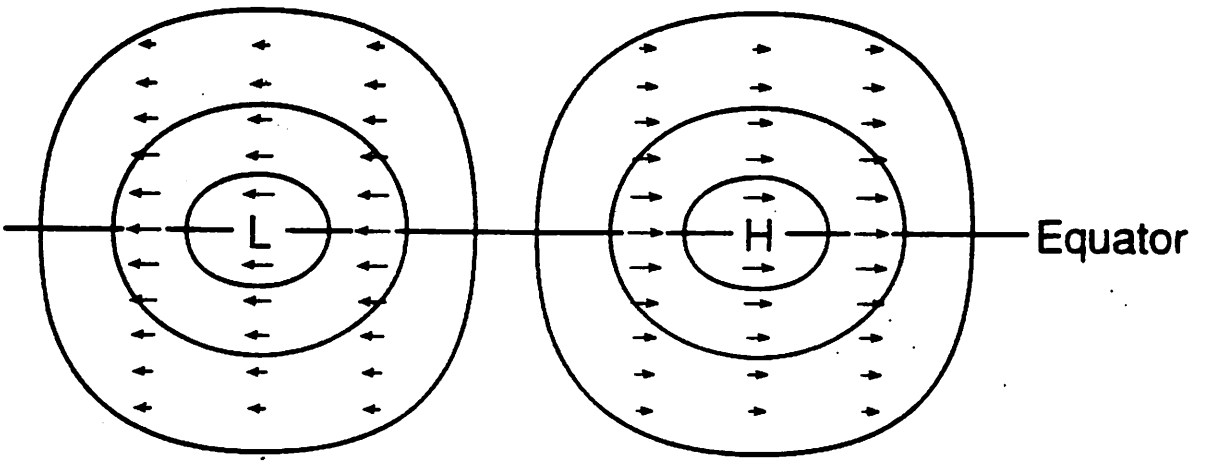


Figure 2.15. *Geopotential and wind structure of an equatorial Kelvin wave. The zonal Coriolis torque on the velocity field balances the geopotential gradient (after Matsuno, 1966).*

Other Equatorial Waves

- Are general solutions of the equatorial β-plane equations (analogous to Hough modes for $\epsilon \to \infty$)

- The β-plane equations can be combined to give:

$$\left[\frac{d^2}{dy^2} + \left(\frac{m^2\omega^2}{N^2} - k^2 - \frac{k\beta}{\omega}\right) - \frac{\beta^2 m^2}{N^2} y^2\right] \hat{v} = 0$$

  where $\hat{v} \cdot \exp i(kx + mz - \omega t) \cdot \exp(z/2H) = v'$

- The solutions $\hat{v}$ are of the form

$$\hat{v} \propto \exp\left(-\frac{y^2}{2L^2}\right) H_n(y/L)$$

  where H_n are Hermite polynomials, whose index n satisfies the dispersion relation:

$$\frac{m^2\omega^2}{N^2} - k^2 - \frac{\beta k}{\omega} = (2n+1)\frac{\beta|m|}{N}$$

- These solutions are of three kinds:
  - eastward-propagating gravity waves (the Kelvin wave is similar to these, but has $v' = 0$) and westward-propagating gravity waves (the migrating tides are of this kind)
  - westward-propagating equatorial Rossby waves
  - Rossby-gravity waves (these resemble Rossby waves for $\omega < 0$ and Kelvin waves for $\omega > 0$)

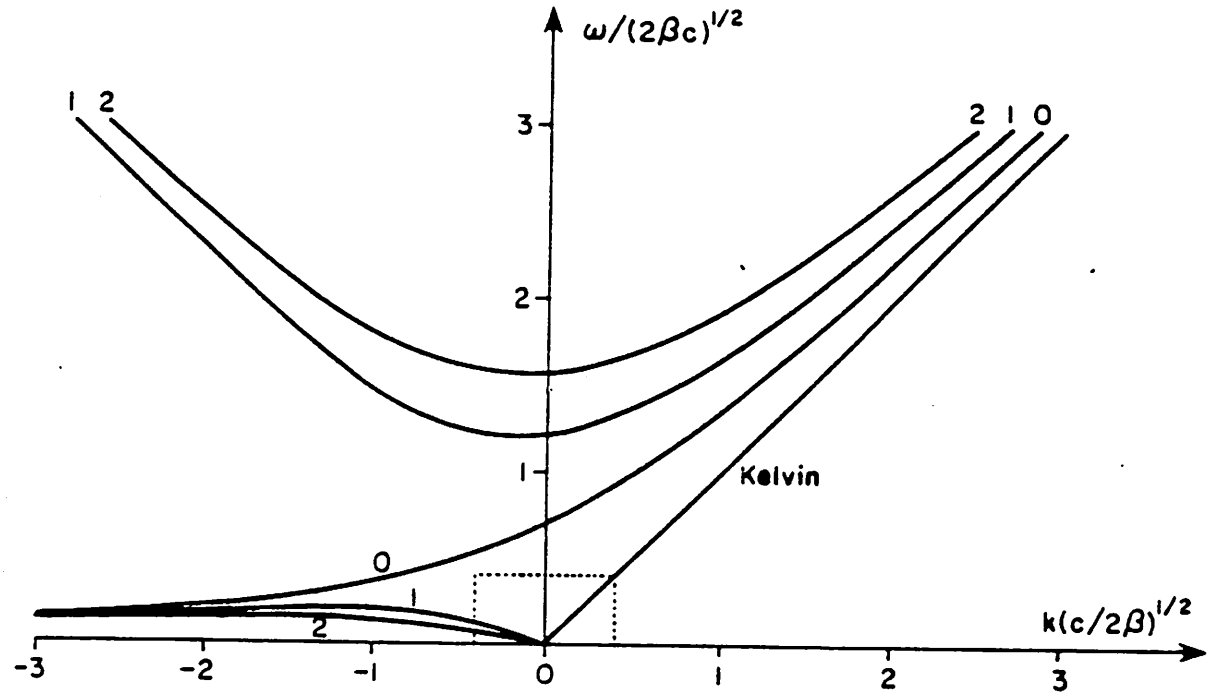


Fig. 11.1. Dispersion curves for equatorial waves. The vertical axis is the frequency in units of $(2\beta c)^{1/2}$ and the horizontal axis is the east–west wavenumber in units of $(2\beta/c)^{1/2}$. The curve labeled 0 corresponds to the mixed planetary–gravity wave. The upper curves labeled 1 and 2 are the first two gravity wave modes and the corresponding lower curves are the first two planetary wave modes. [Reproduced from "Numerical Models of Ocean Circulation," 1975, by permission of the National Academy of Science, Washington, D.C.]

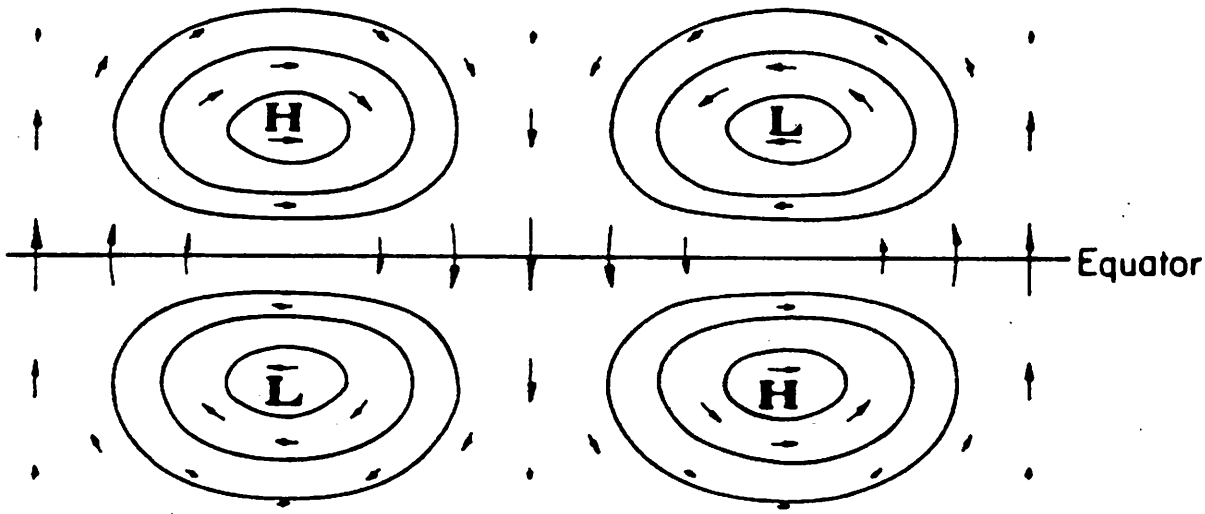
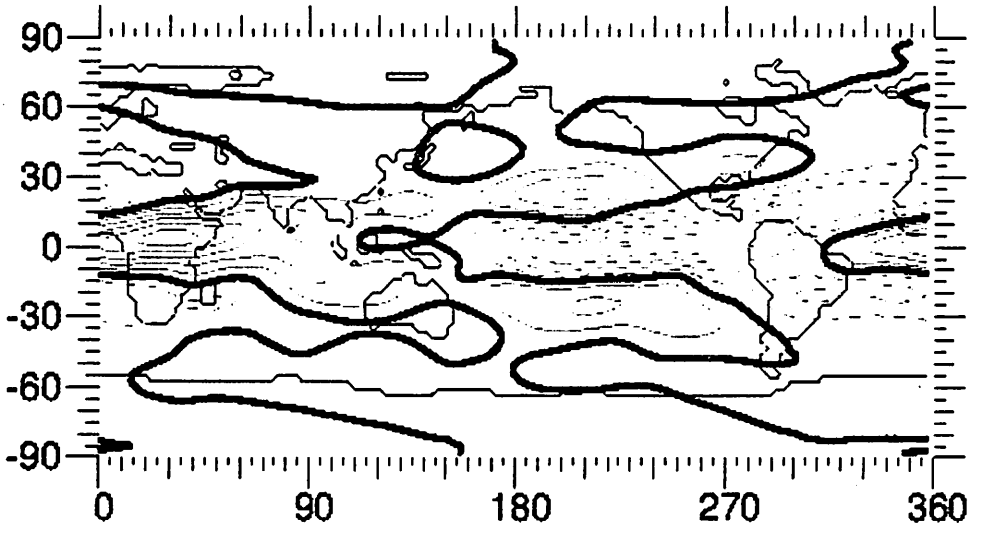


Fig. 4.22. Schematic illustration of geopotential and horizonal wind fluctuations for t Rossby-gravity wave of westward phase speed. [Adapted from Matsuno (1966).]

Diurnal Temperature Tide: Spring Equinox. 90 km



ctr. interval 2K

Small-scale gravity waves

- Spatial and temporal scales are small enough that Coriolis force is not important

- Restoring force is gravity, acting through convergence/divergence

- Governing equation is

$$\frac{\partial^2}{\partial t^2}\left(\phi'_{zz} - \frac{\phi'_z}{H}\right) + N^2\left(\phi'_{xx} + \phi'_{yy}\right) = 0$$

- Solutions are of the form

$$\phi' \propto e^{i(kx+ly+mz-\omega t)} e^{z/2H}$$

- And the dispersion relation is

$$\omega^2 = \frac{\left(k^2 + l^2\right) N^2}{\left(m^2 + \frac{1}{4H^2}\right)}$$

- Small-scale gravity waves are ubiquitous in the Earth's atmosphere
  - they play a central role in extratropical dynamics
  - they may also play a leading role in tropical dynamics

Coupling between the lower and middle atmosphere in the Tropics

- Wave excitation inferred from observations of tropical convection

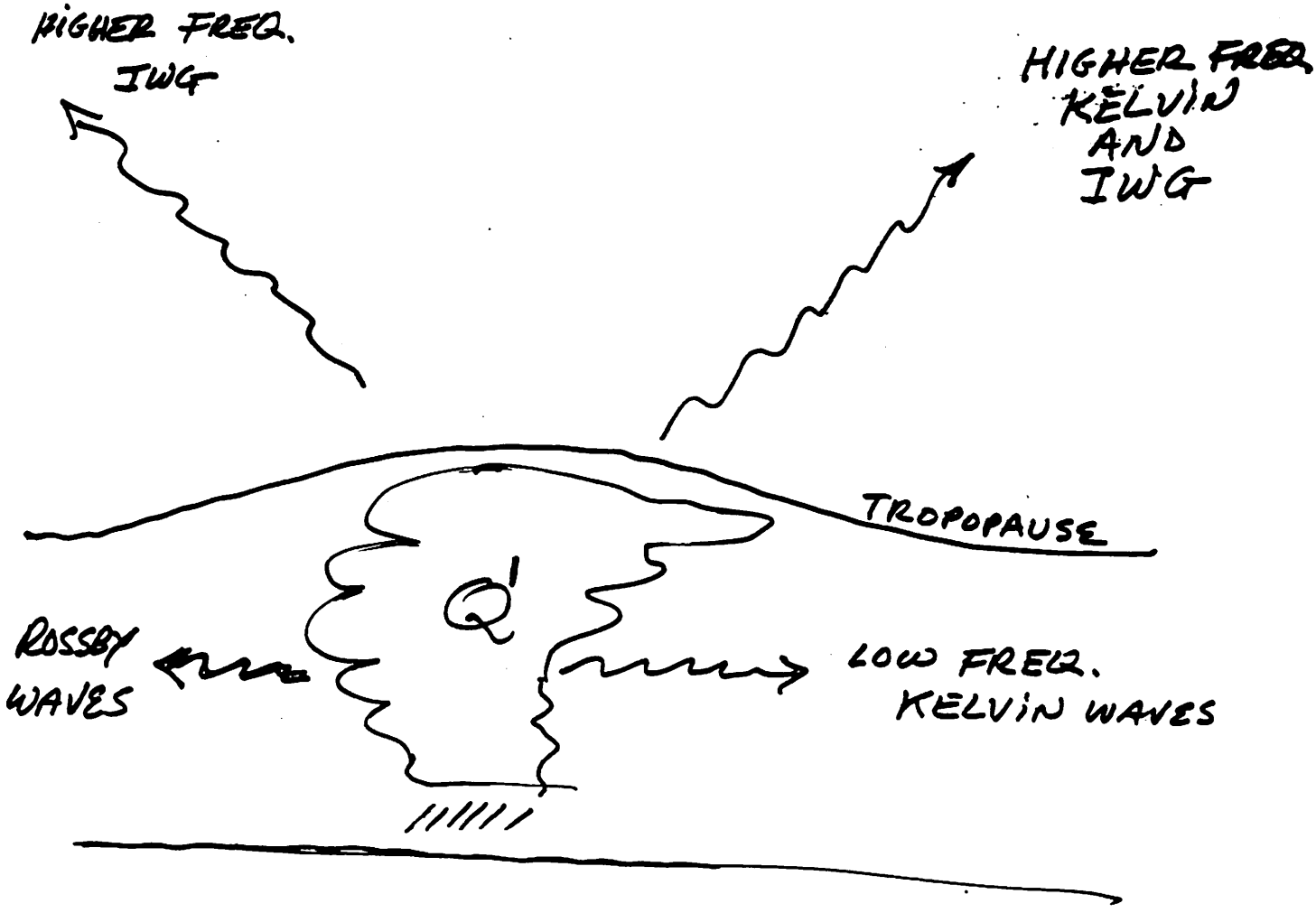
- Mechanism of wave-mean flow interaction

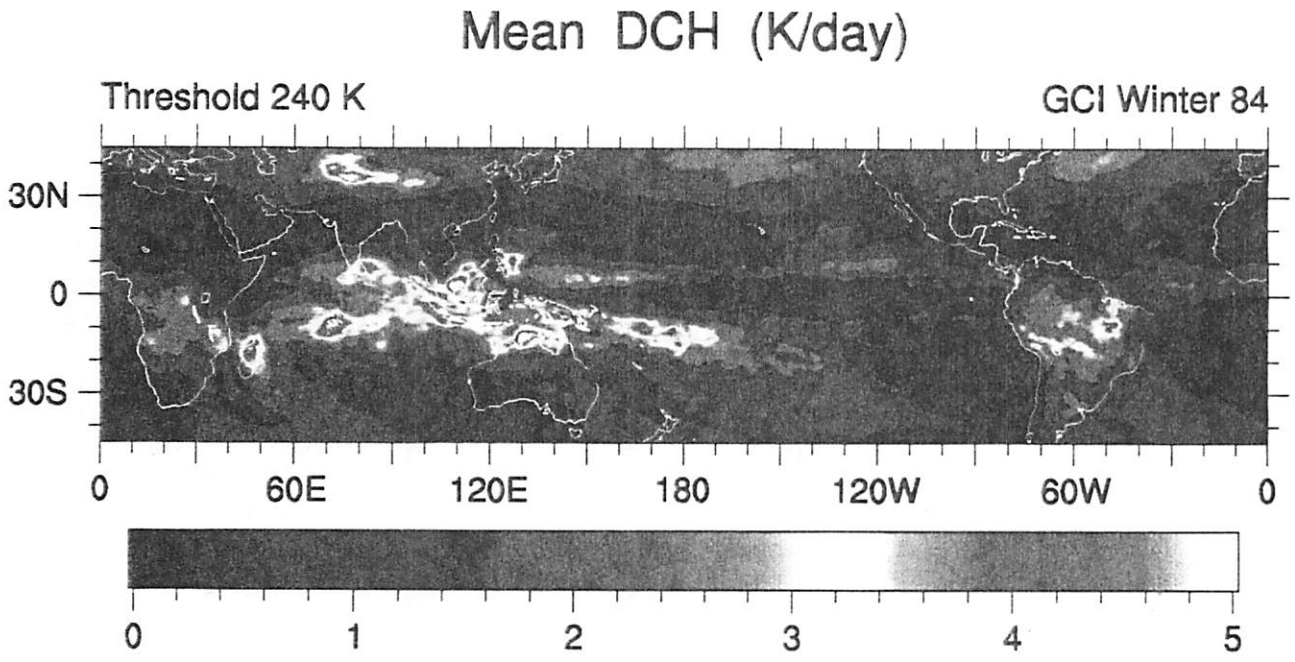
- Modeling the coupling

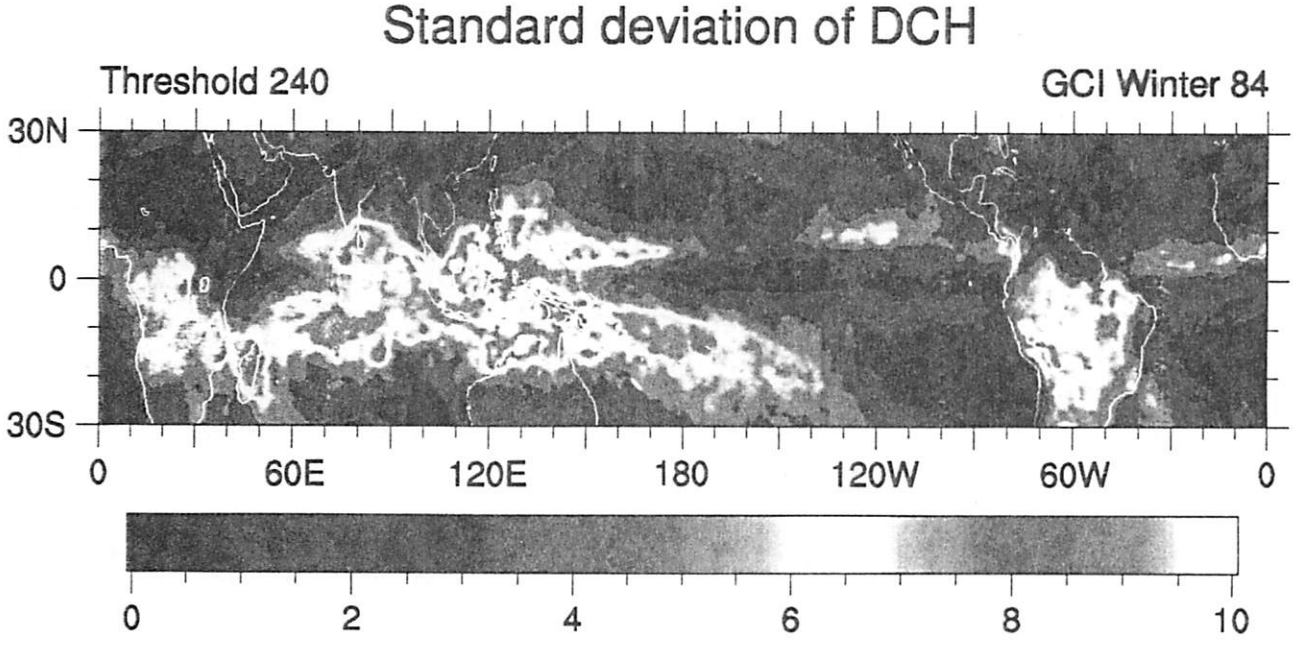
  – An example: the SAO

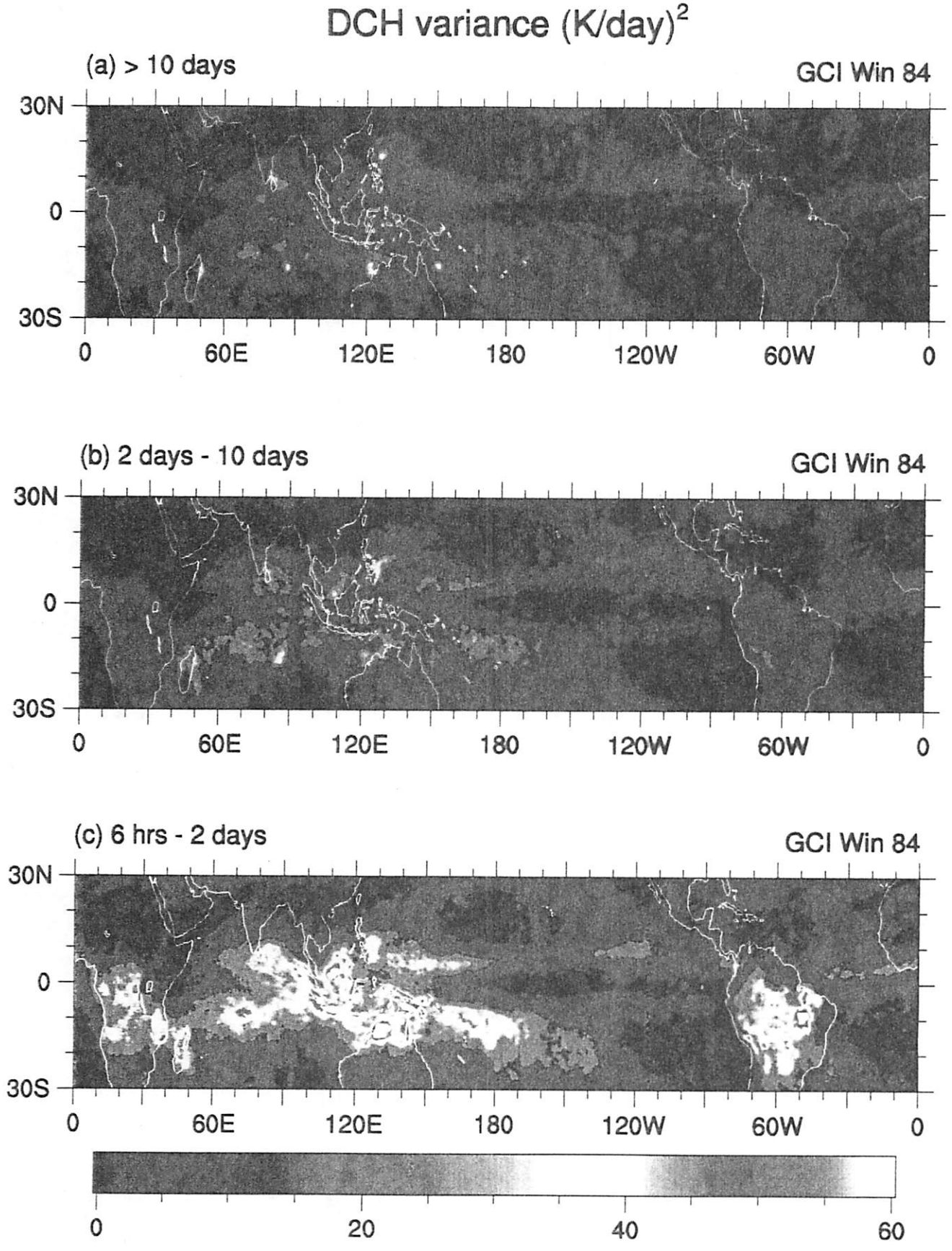
WAVE FORCING BY CONVECTIVE HEATING

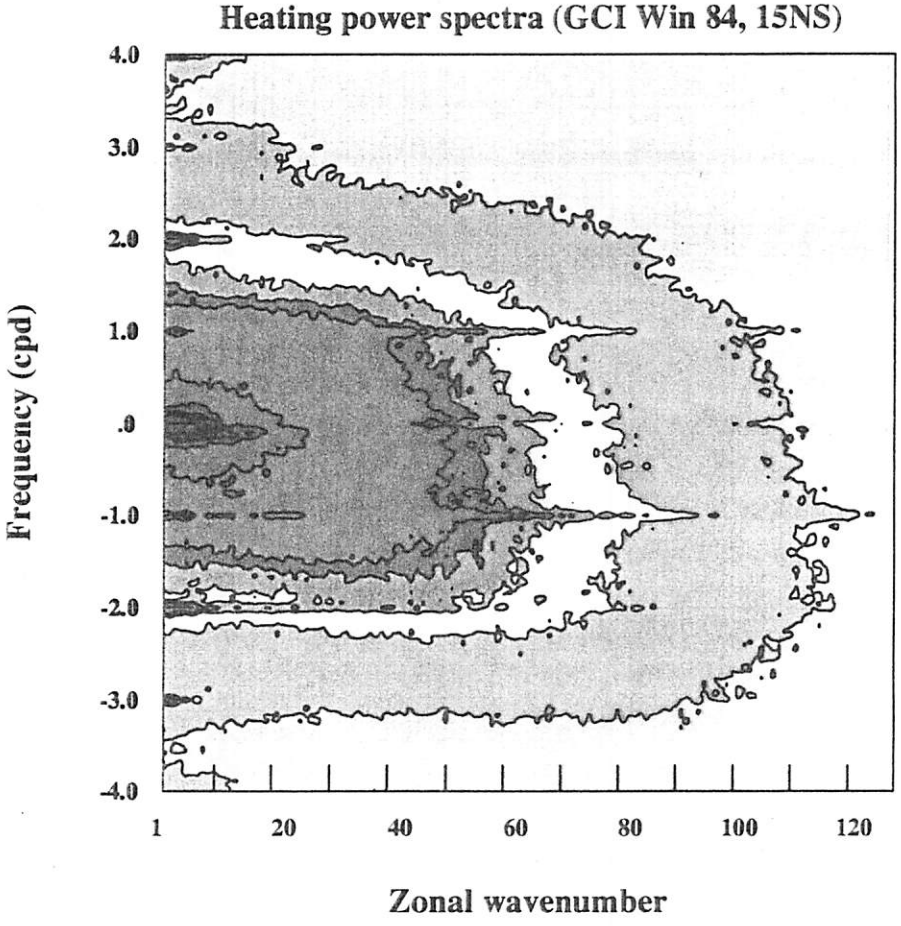
$$\left[\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right]T' + w'\frac{HN^2}{R} = Q'_{conv}$$



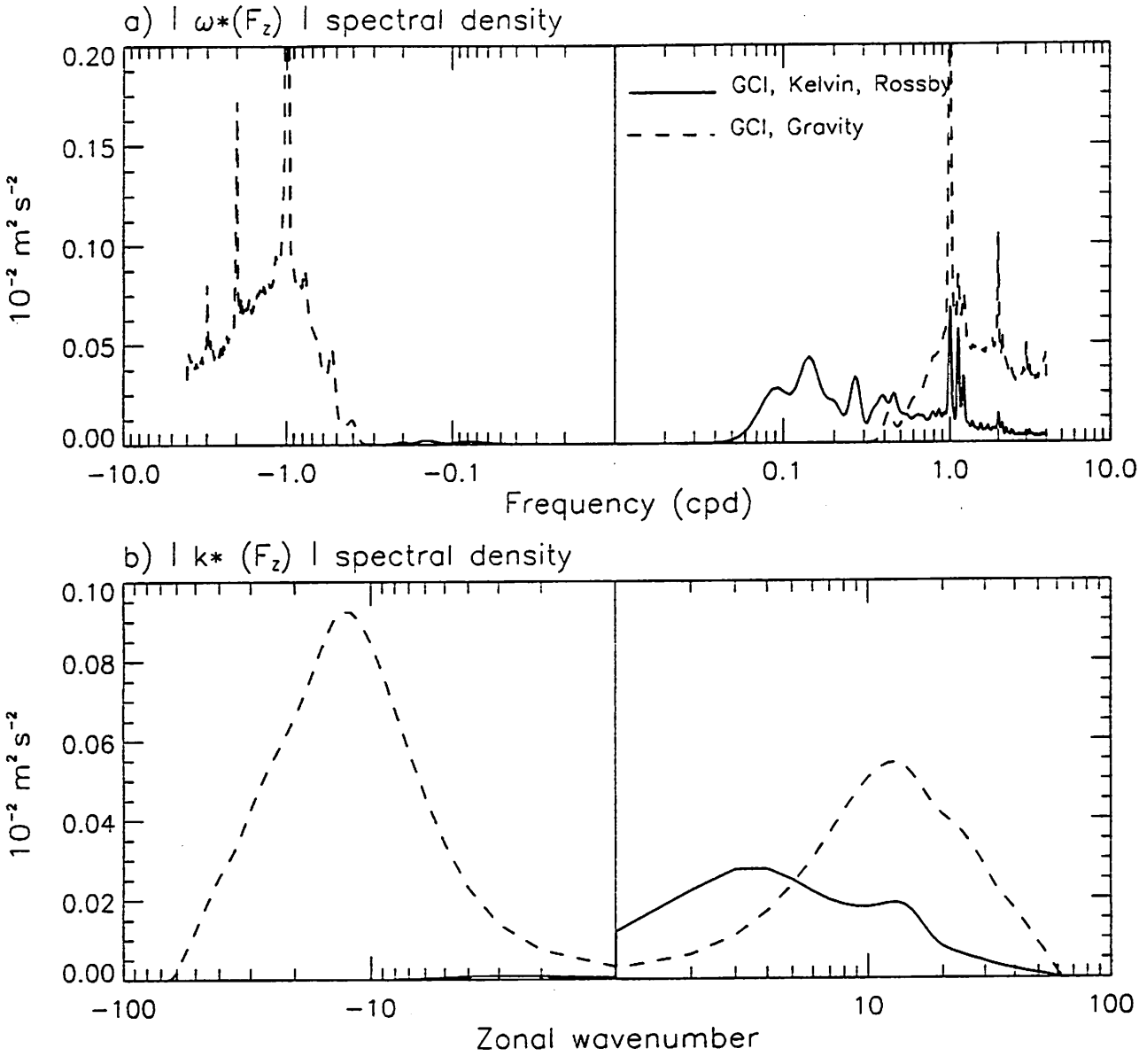

Mean DCH (K/day)
Threshold 240 K
GCI Winter 84
30N
0
30S
0
60E
120E
180
120W
60W
0
0
1
2
3
4
5

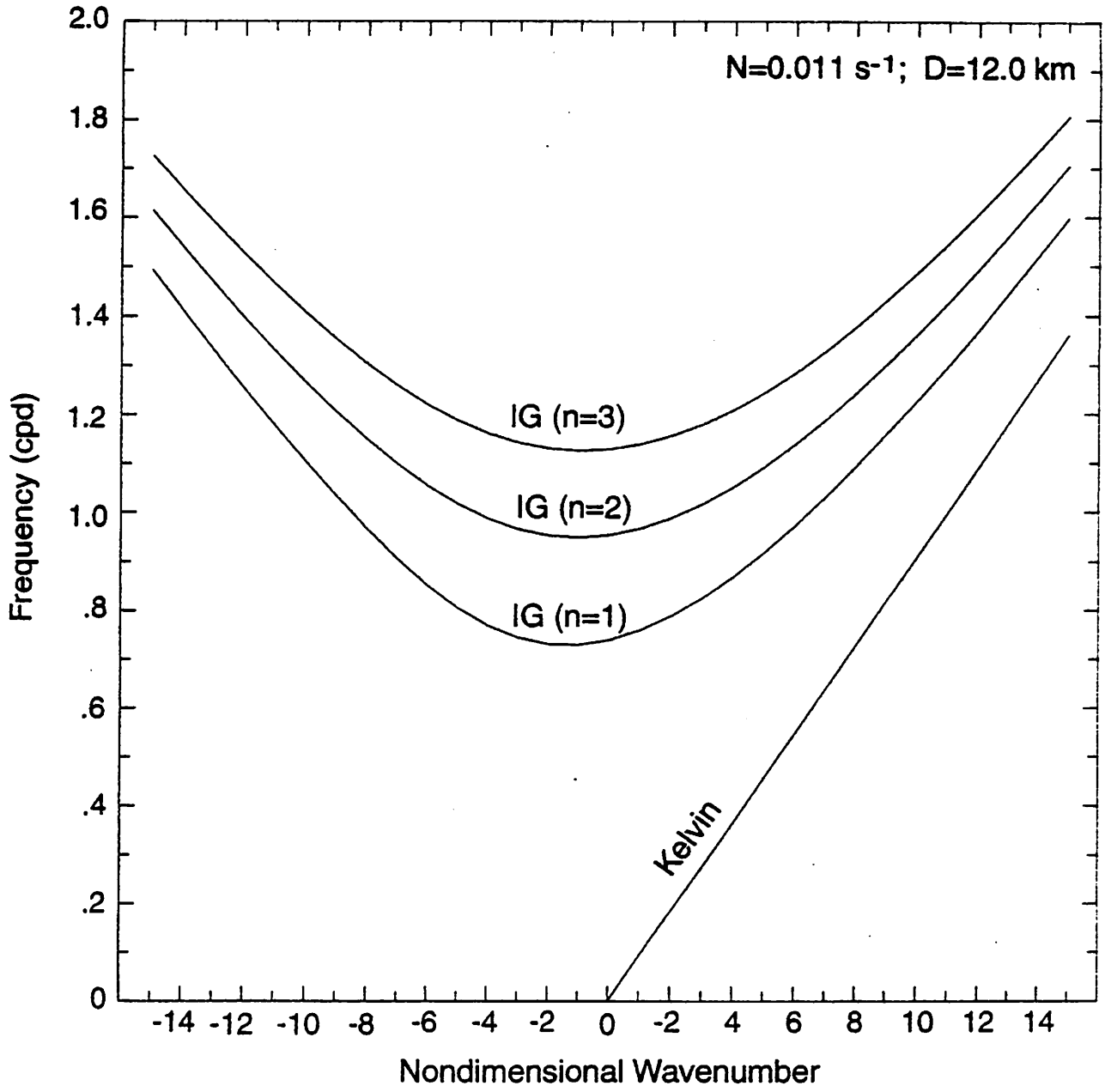

Standard deviation of DCH
Threshold 240
GCI Winter 84
30N
0
30S
0
60E
120E
180
120W
60W
0
0
2
4
6
8
10


DCH variance (K/day)^2
(a) > 10 days
GCI Win 84
30N
0
30S
0
60E
120E
180
120W
60W
0
(b) 2 days - 10 days
GCI Win 84
30N
0
30S
0
60E
120E
180
120W
60W
0
(c) 6 hrs - 2 days
GCI Win 84
30N
0
30S
0
60E
120E
180
120W
60W
0
0
20
40
60


Heating power spectra (GCI Win 84, 15NS)
Frequency (cpd)
4.0
3.0
2.0
1.0
.0
-1.0
-2.0
-3.0
-4.0
1
20
40
60
80
100
120
Zonal wavenumber

VERTICAL COMPONENT OF EP FLUX, C > 30 m/s




N=0.011 s-1; D=12.0 km
IG (n=3)
IG (n=2)
IG (n=1)
Kelvin
Frequency (cpd)
Nondimensional Wavenumber
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
-14 -12 -10 -8 -6 -4 -2 0 -2 4 6 8 10 12 14

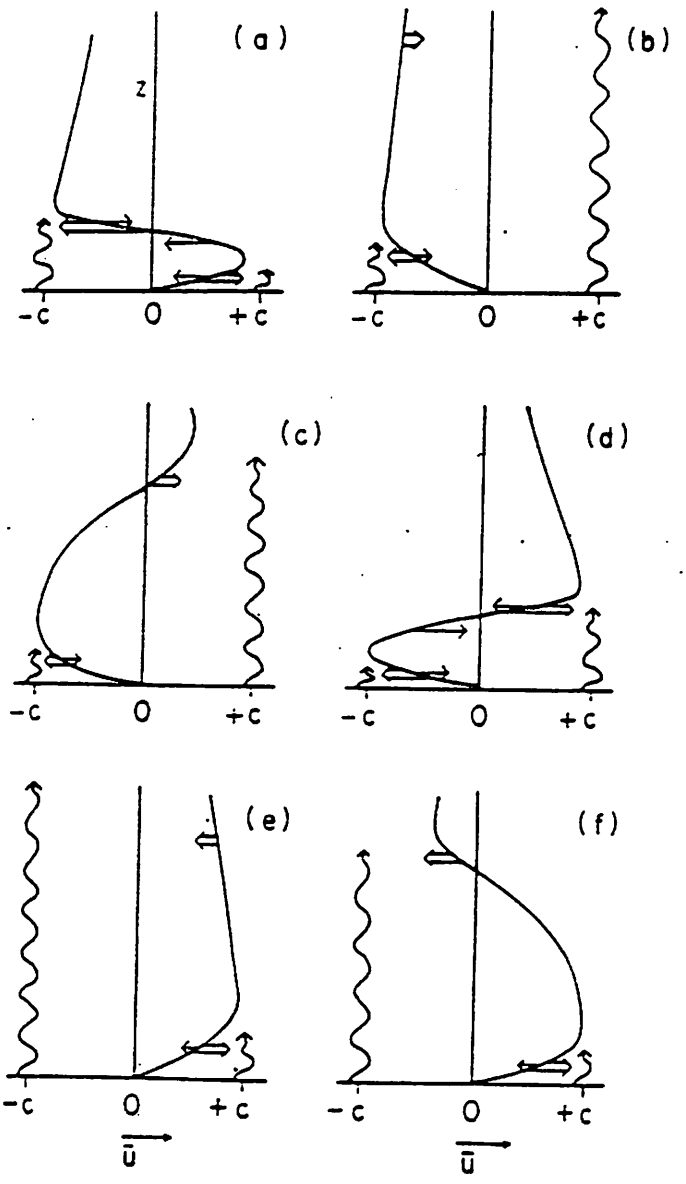
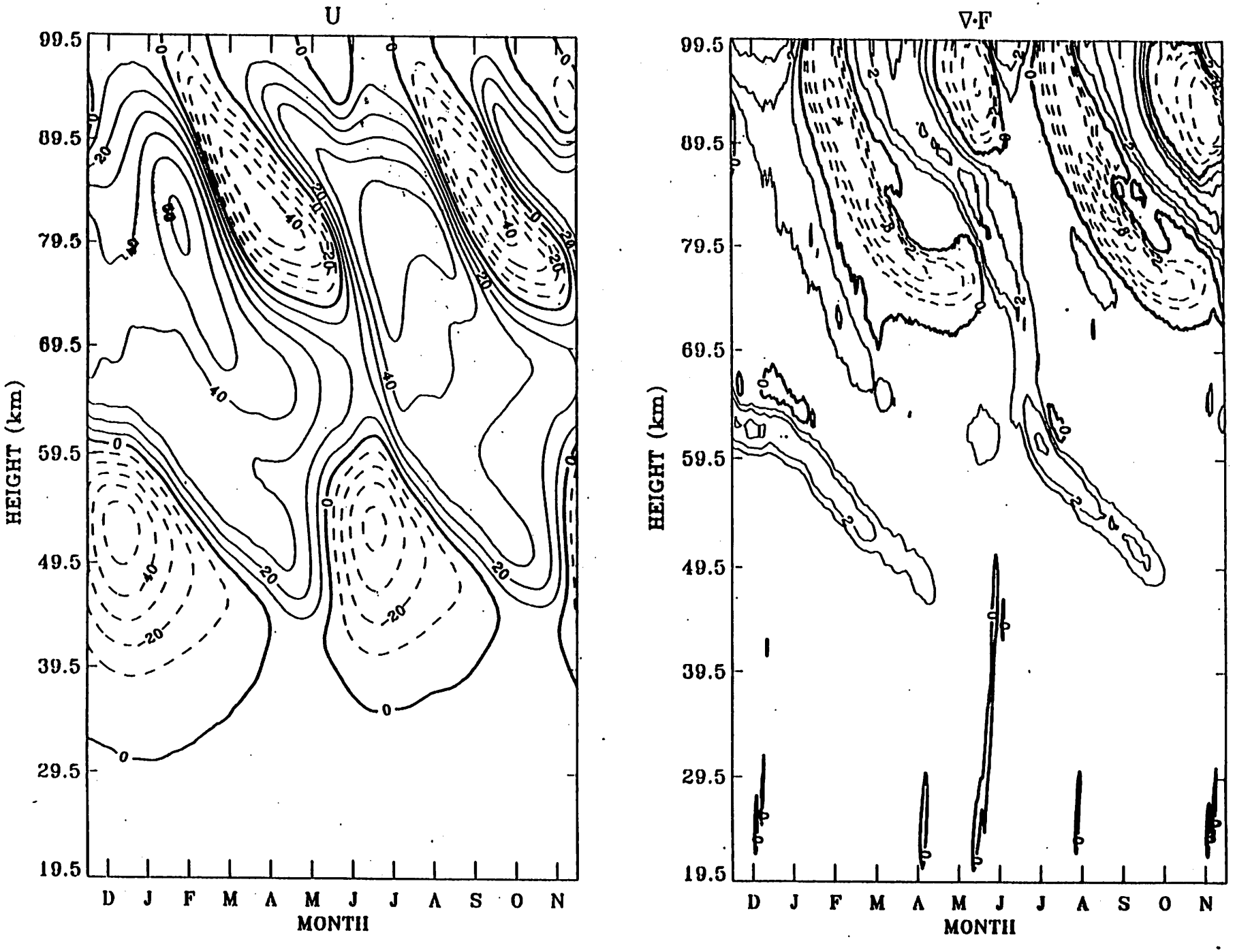


Fig. 8.7. Schematic representation of the evolution of the mean flow in Plumb's analog of the QBO. Six stages of a complete cycle are shown. Double arrows show wave-driven accelerations and single arrows show viscously driven accelerations. Wavy lines indicate relative penetration of easterly and westerly waves. See text for details. [After Plumb (1984).]

Andrews et al. (1987)

Modeling the effect of convectively excited waves

- Equatorial beta-plane model ($k = 1, 15$)
- Specify heating distribution consistent with tropical OLR observations
- Model calculations:
  - –Nominal case
  - –Planetary-scale waves only
  - –No diurnal forcing
  - –Interaction with the QBO


U
∇·F
HEIGHT (km)
99.5
89.5
79.5
69.5
59.5
49.5
39.5
29.5
19.5
D J F M A M J J A S O N
MONTH

Sassi + [illegible]

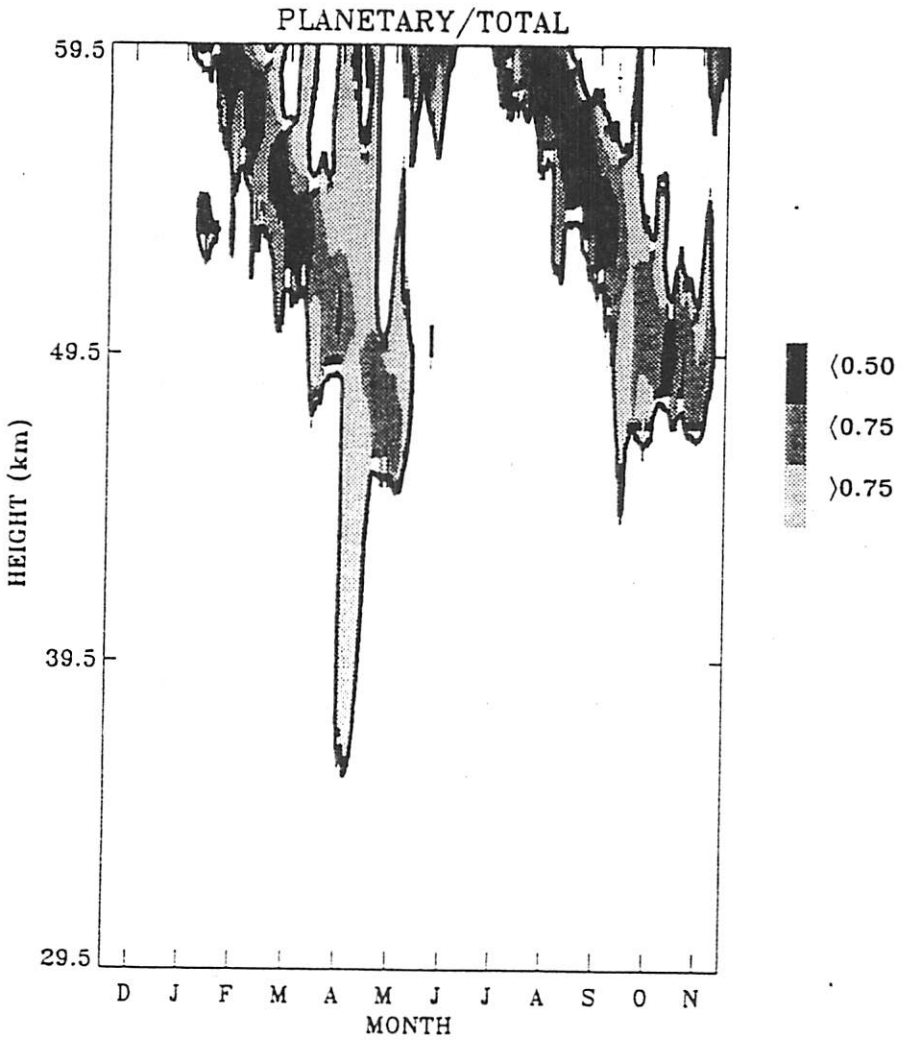
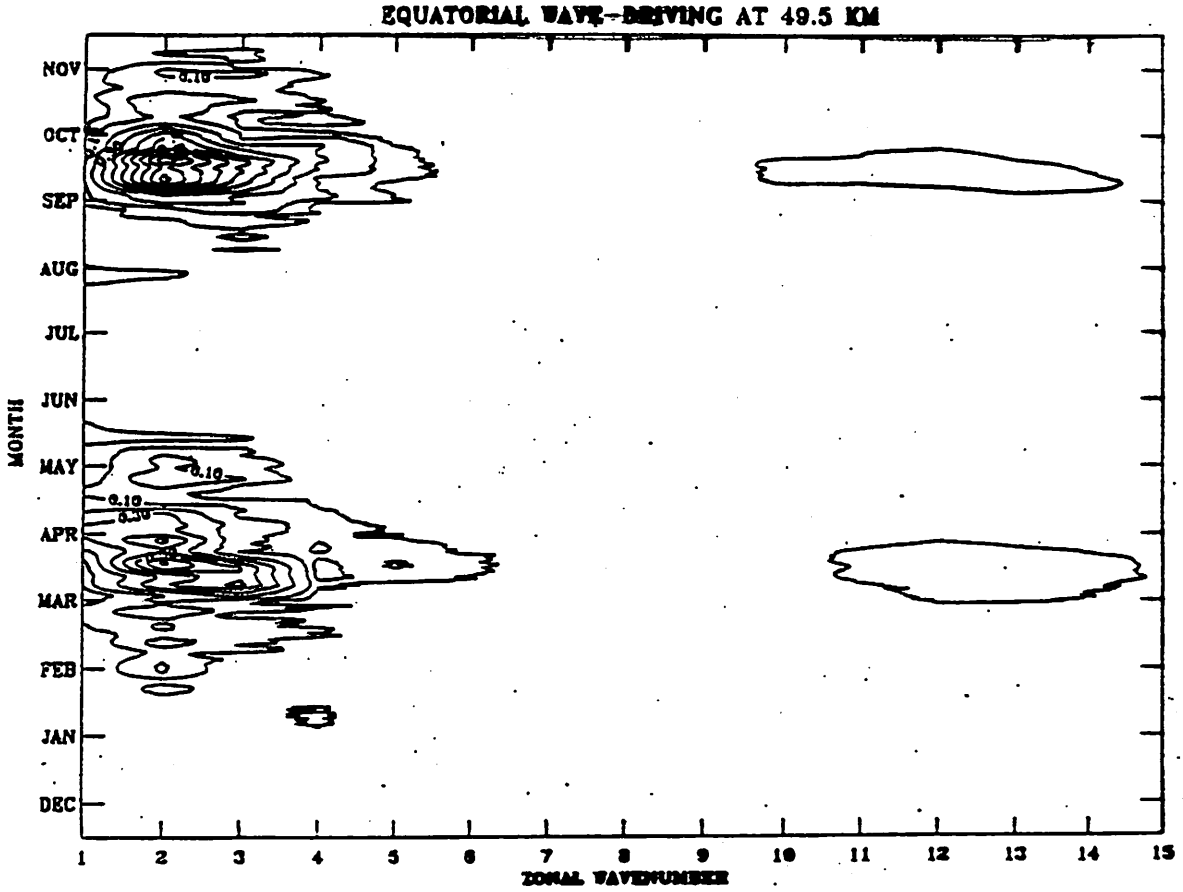
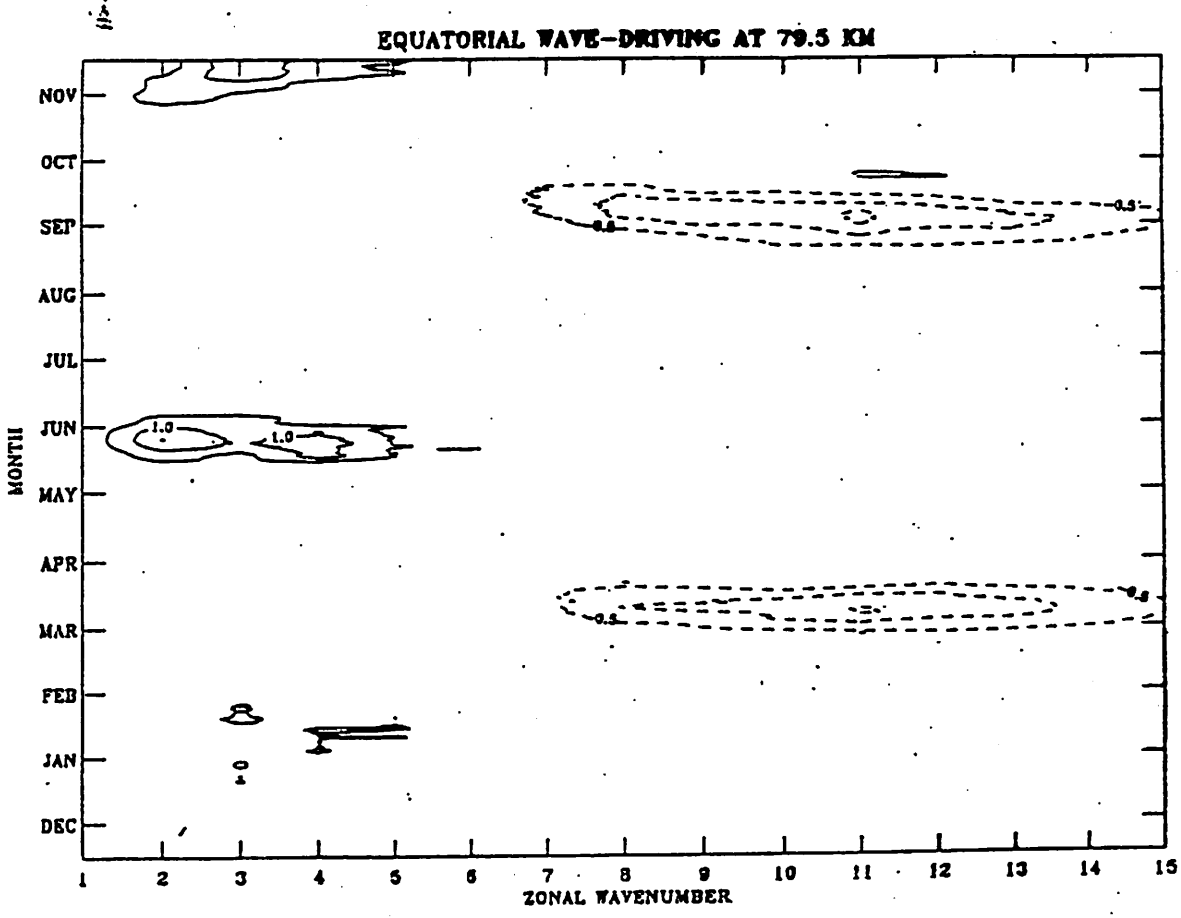


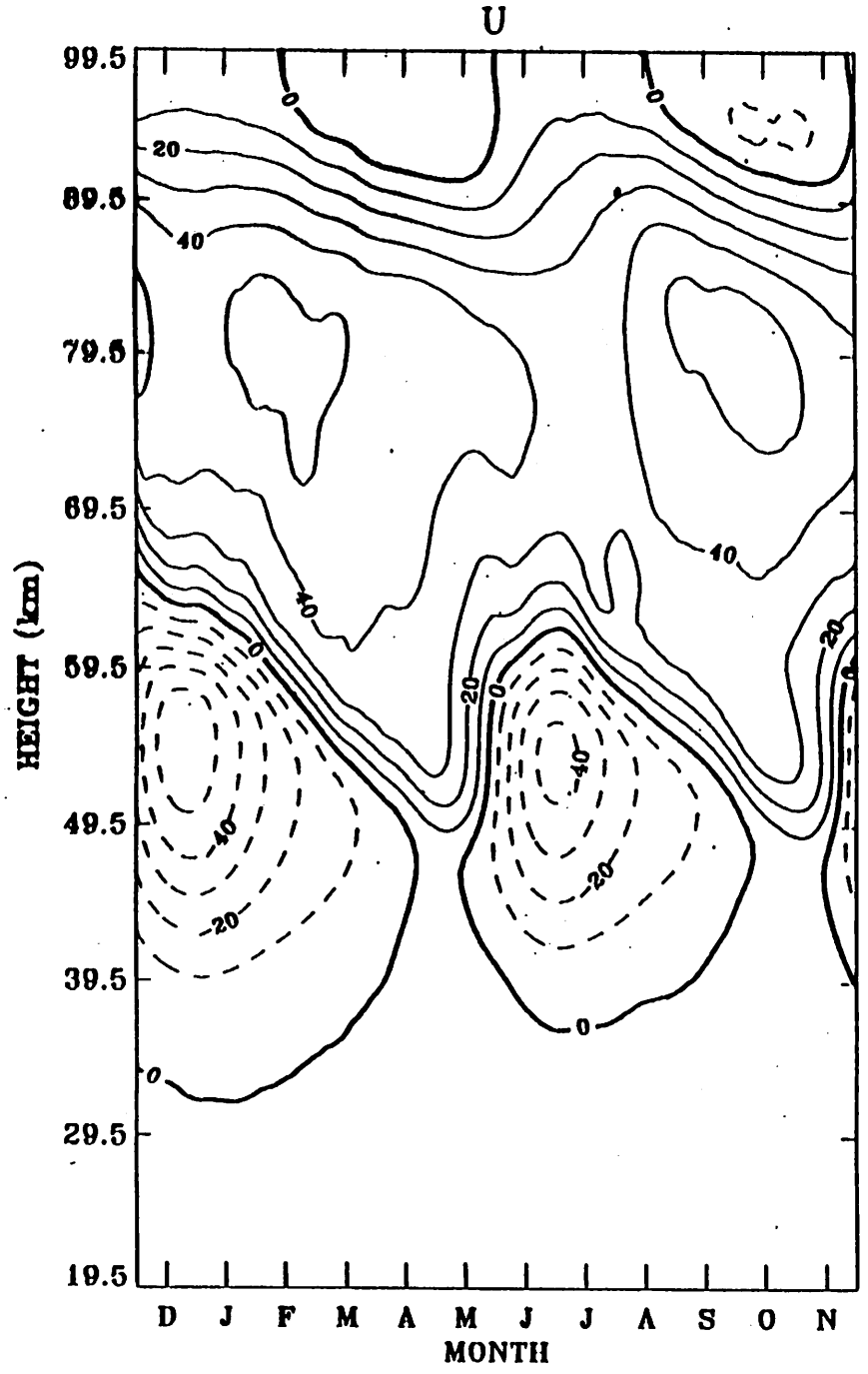
FIG. 5. Ratio of the force exerted by planetary-scale waves to force exerted by all waves. Three shadings are marked: less than 0.5 (the darkest), between 0.5 and 0.75 (medium dark), and more than 0.75 (light shade). The bold line encloses the region where the total force is larger than 0.25 m s^{-1} day^{-1}.

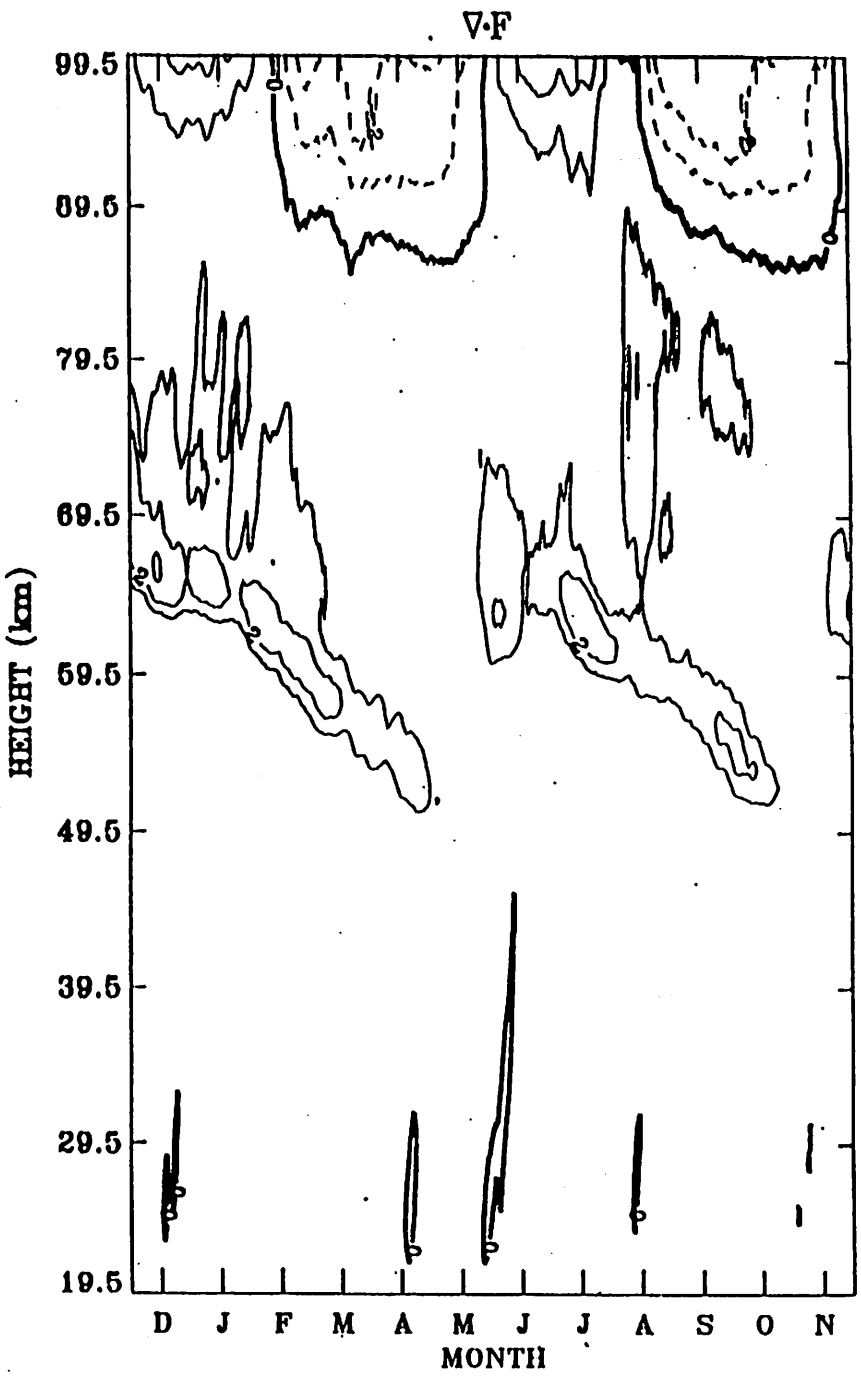

EQUATORIAL WAVE-DRIVING AT 49.5 KM
NOV
OCT
SEP
AUG
JUL
JUN
MAY
APR
MAR
FEB
JAN
DEC
MONTH
0.10
0.10
0.10
0.10
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
ZONAL WAVENUMBER


EQUATORIAL WAVE-DRIVING AT 79.5 KM
NOV
OCT
SEP
AUG
JUL
JUN
MAY
APR
MAR
FEB
JAN
DEC
MONTH
0.5
0.5
1.0
1.0
0.5
0.5
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
ZONAL WAVENUMBER

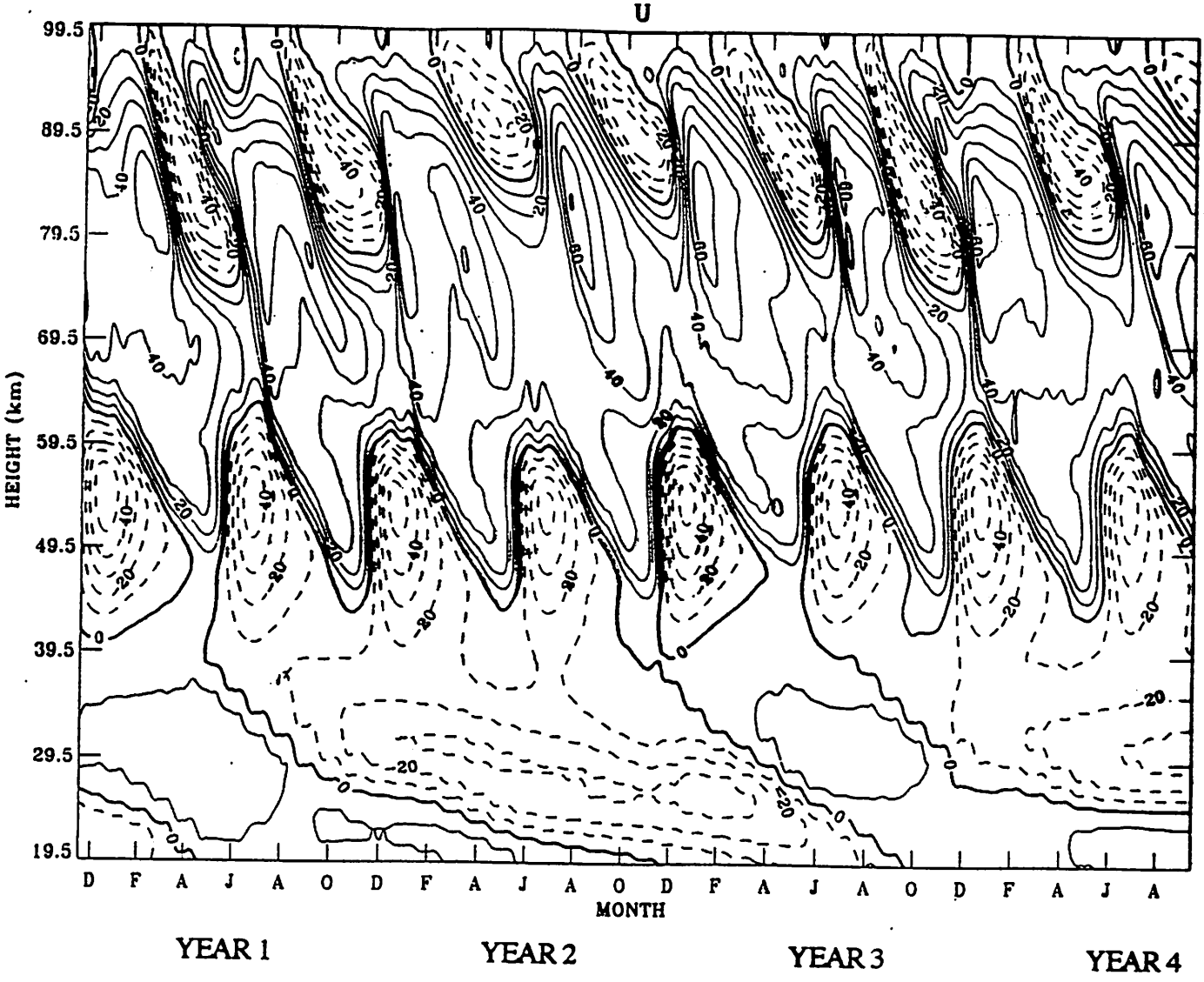
(12)

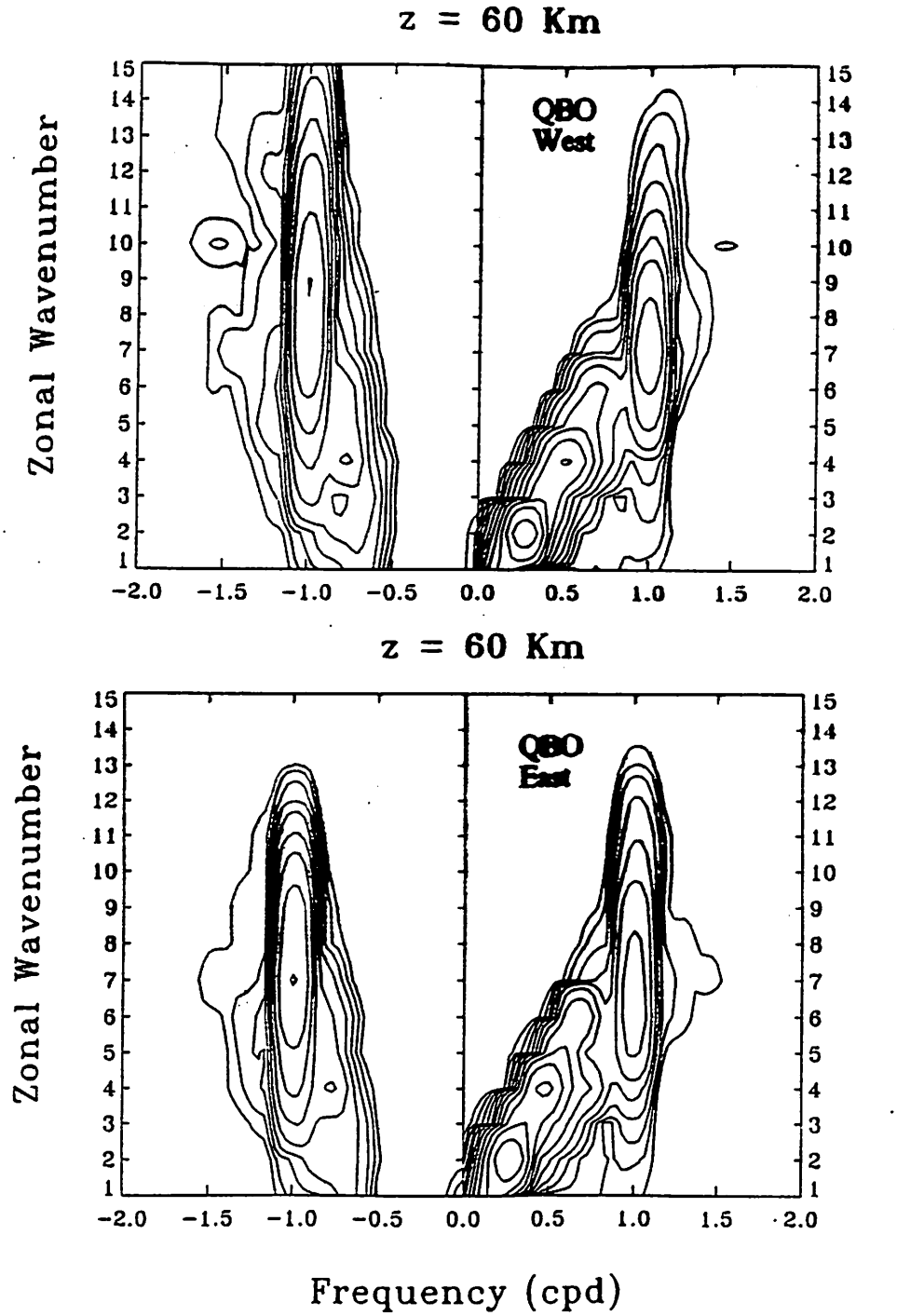
NO DIURNAL COMPONENT OF HEATING





Sassi + Garcia (1999)


U
99.5
89.5
79.5
69.5
59.5
49.5
39.5
29.5
19.5
HEIGHT (km)
D F A J A O D F A J A O D F A J A O D F A J A
MONTH
YEAR 1
YEAR 2
YEAR 3
YEAR 4


z = 60 Km
QBO
West
Zonal Wavenumber
z = 60 Km
QBO
East
Zonal Wavenumber
Frequency (cpd)

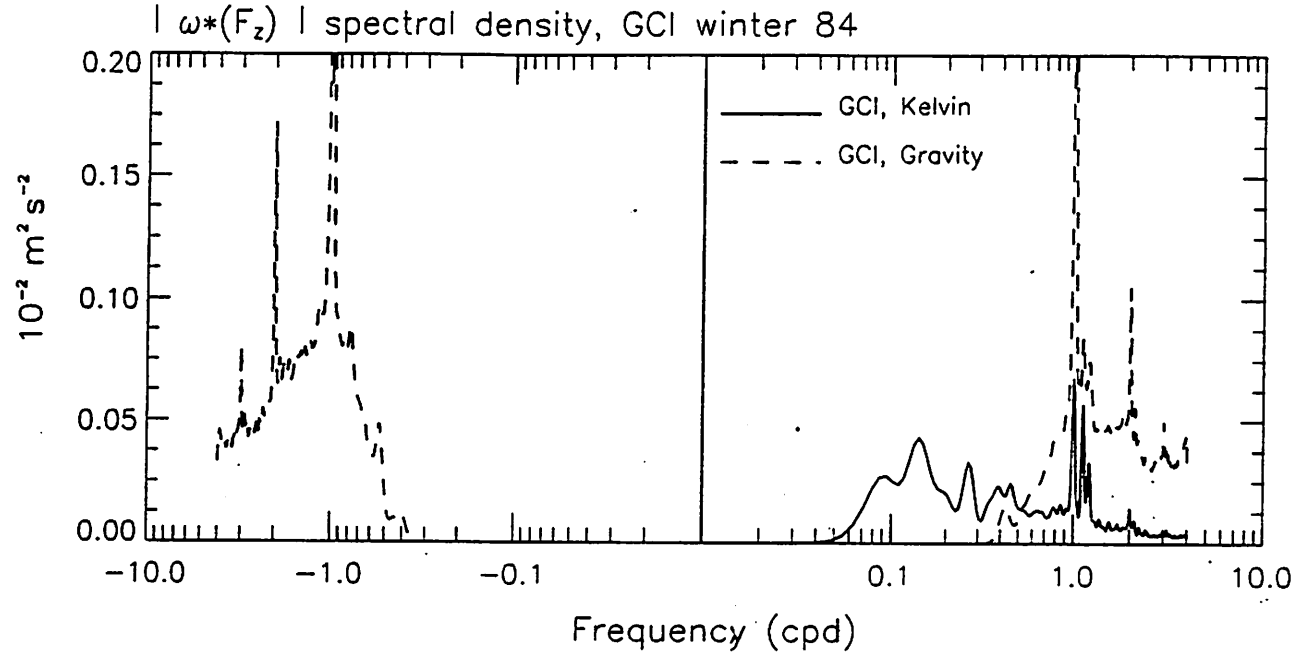
Conclusions

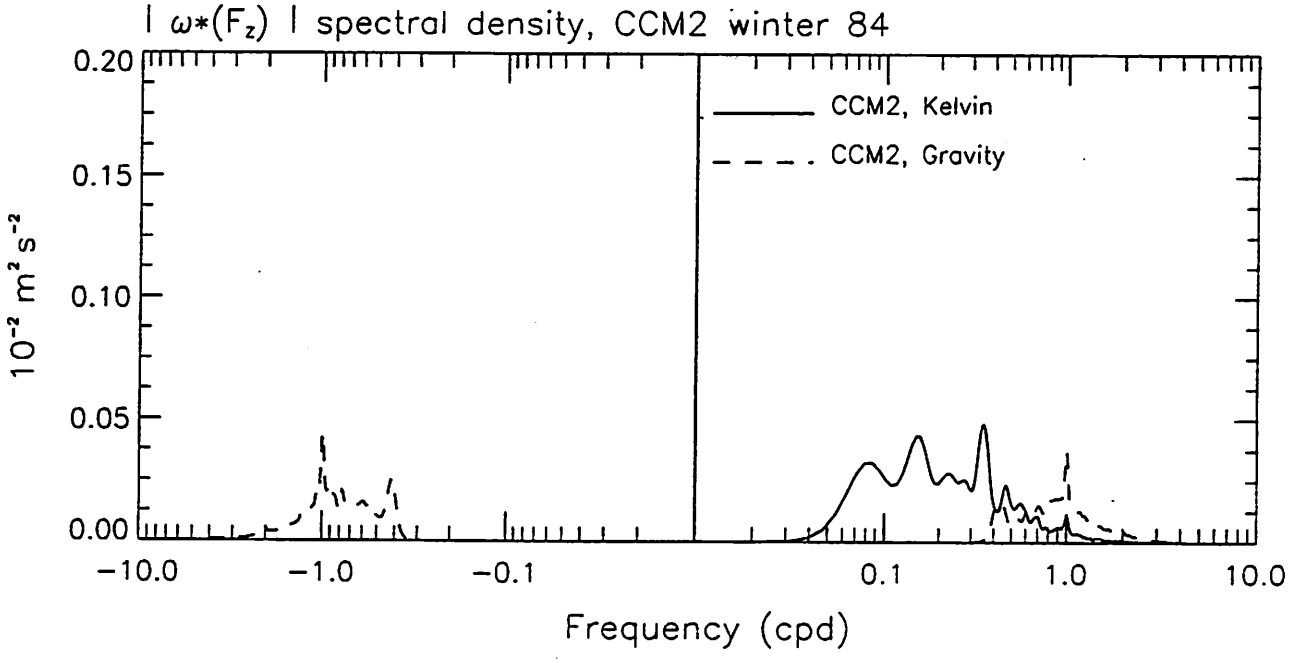
- Small f in the Tropics leads to oscillatory circulation systems

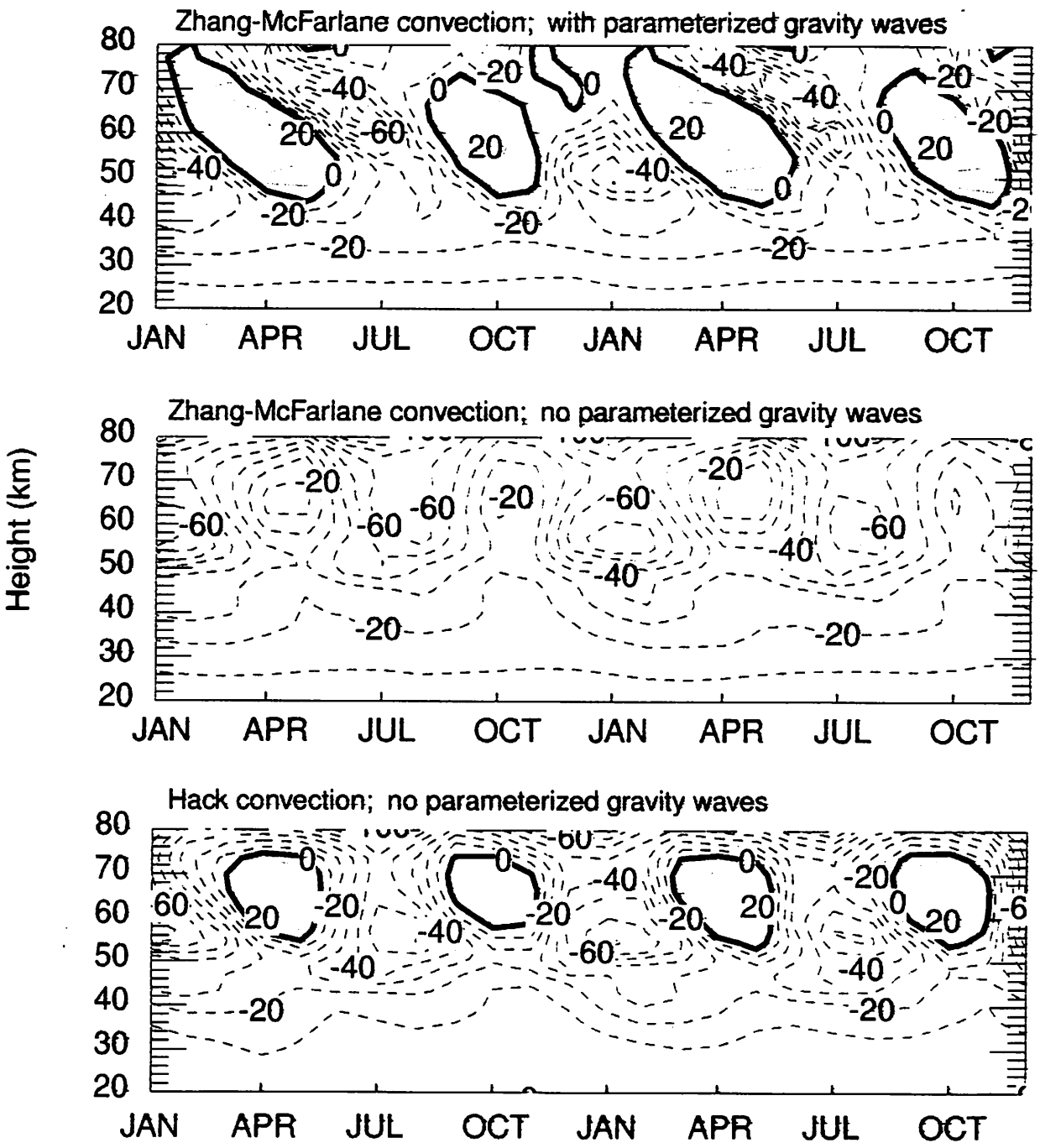
- Tropical oscillations are wave-driven
  - deep convection plays a major role in wave excitation
  - nature of waves is still ucertain
  - equatorially-trapped waves vs. small-scale gravity waves?

- Modeling and observations suggest equatorially-trapped waves are important
  - both planetary-scale and intermediate-scale
  - high-frequency (diurnal and even faster)
  - pose a challenge for interpretation of observations

VERTICAL COMPONENT OF EP FLUX, C > 30 m/s






Zhang-McFarlane convection; with parameterized gravity waves
Zhang-McFarlane convection; no parameterized gravity waves
Hack convection; no parameterized gravity waves
Height (km)
80
70
60
50
40
30
20
JAN
APR
JUL
OCT
JAN
APR
JUL
OCT