A Tutorial

Variational Assimilation of Meteorological Observations: How It Works in the Lower Atmosphere

by

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Variational Assimilation of Meteorological Observations:
How It Works in the Lower Atmosphere

Outline

- Purpose of data assimilation
- What information do we need to do the job?
- Common problems in data assimilation
- An analogy
- The variational approach - an inverse problem
- The *penalty* function
- Practical details and an example (GPS occultation)
- Summary

Purpose of Data Assimilation

To combine atmospheric measurements with our knowledge of atmospheric behavior, as codified in computer models, thus producing a "best" estimate of current conditions.

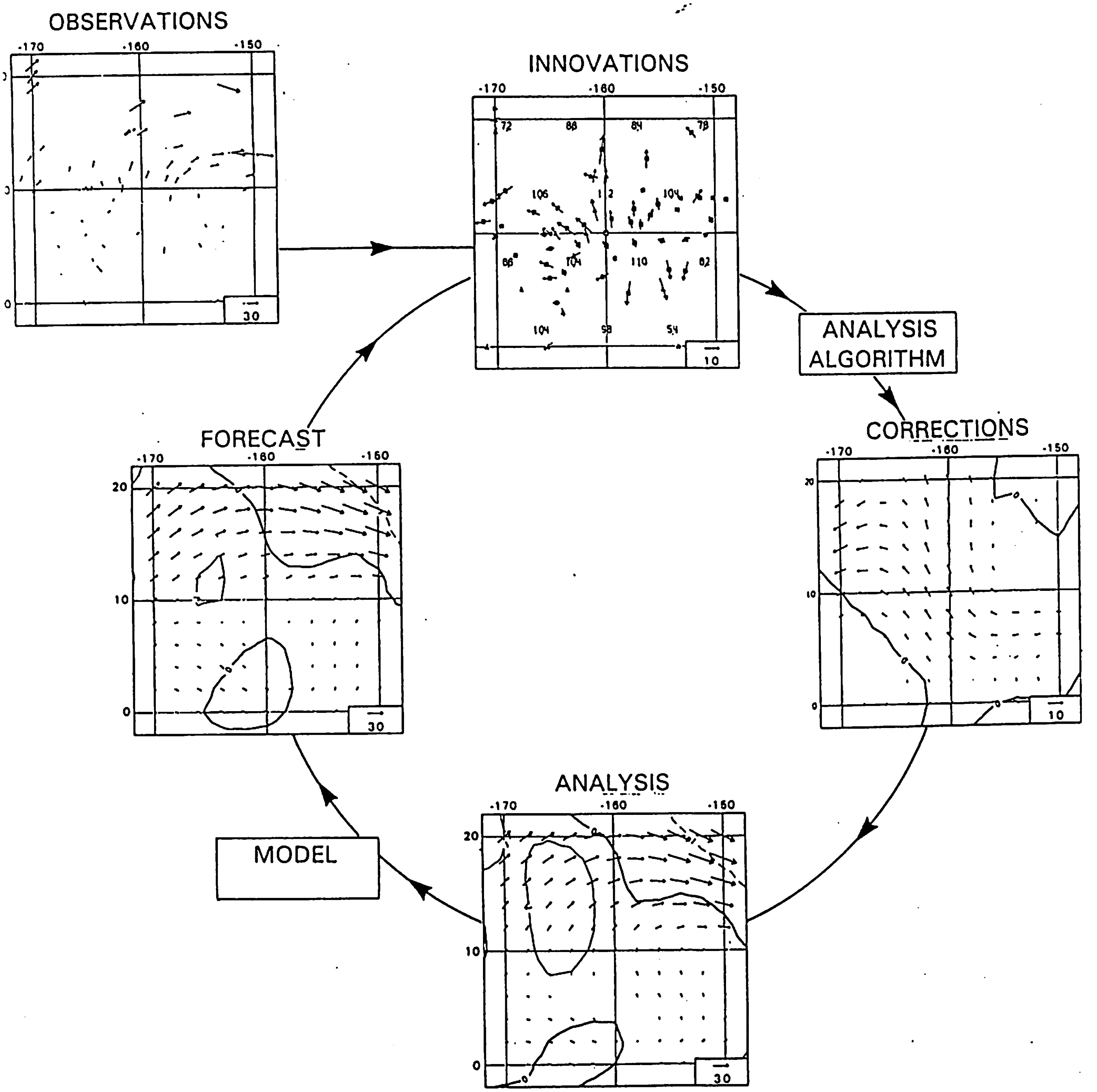
Such analyses have great diagnostic value and are also the basis for numerical prediction.

What information do we need to do the job?

- **Observations**
  - always imperfect, sometimes with gross errors
  - often indirect, or highly processed
  - never completely adequate in coverage or information content

- **Numerical Prediction Model**
  - also imperfect, *but*
  - model imposes dynamical consistency between mass and wind fields and approximates physical processes
  - model ensures smooth evolution of atmospheric conditions (temporal continuity)
  - Accurate model provides good first guess (a priori information partly dependent on earlier observations). First guess usually needs only minor corrections based on current observations.

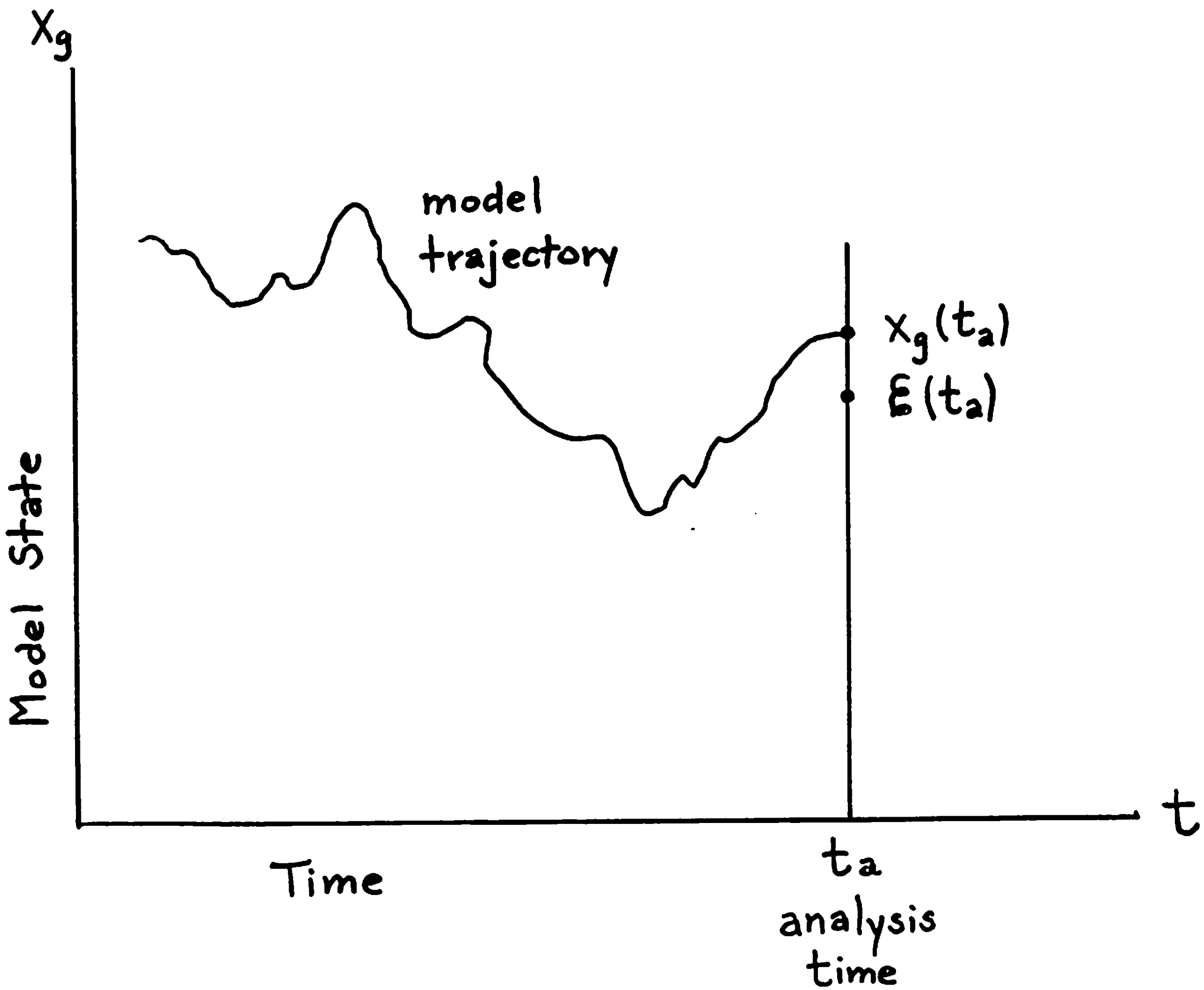
ANALYSIS/FORECAST CYCLE



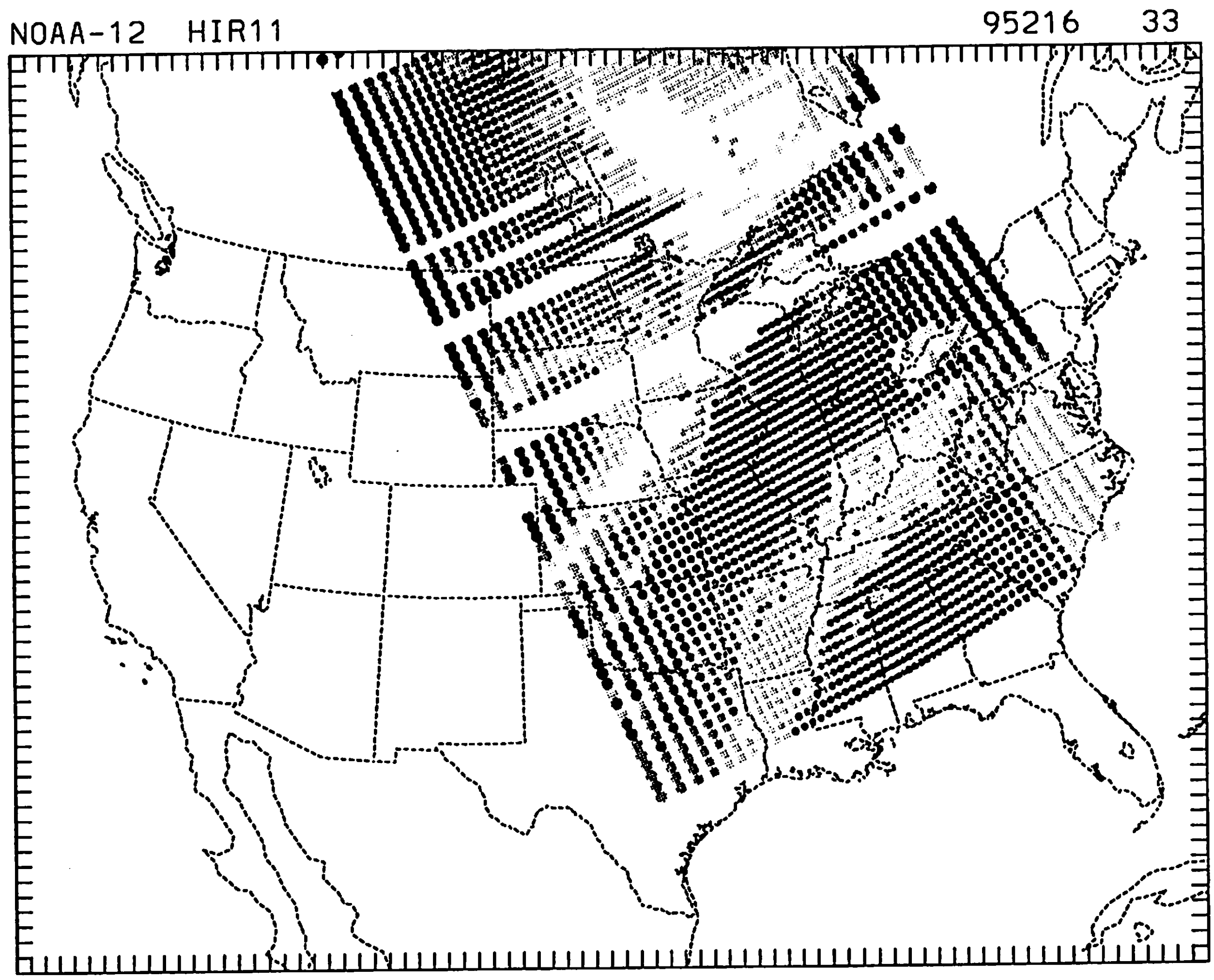
Daley, Roger, 1997:
Atmospheric Data Assimilation.
Journal of the Meteorological Society of Japan, Vol. 75, No. 1B, 319-329.
Special issue, distributed by Universal Academy Press, Inc.
A collection of papers presented at the WMO Second International Symposium on Assimilation of Observations in Meteorology and Oceanography, 13-17 March 1995, Tokyo, Japan.
Edited by M. Ghil, K. Ide, A Bennett, P. Courtier, M. Kimoto, M. Nagata, M. Saiki, and N. Sato.

Common Problems in Data Assimilation

- The analyzed field does not match a realizable model state.
- The distribution of observations is highly non-uniform.
- The observed variables do not match the variables predicted by the model.
- Observing systems are diverse, each subject to different kinds of error, sometimes poorly known.



x_g
model trajectory
$x_g(t_a)$
$\mathcal{E}(t_a)$
Model State
t
Time
t_a
analysis time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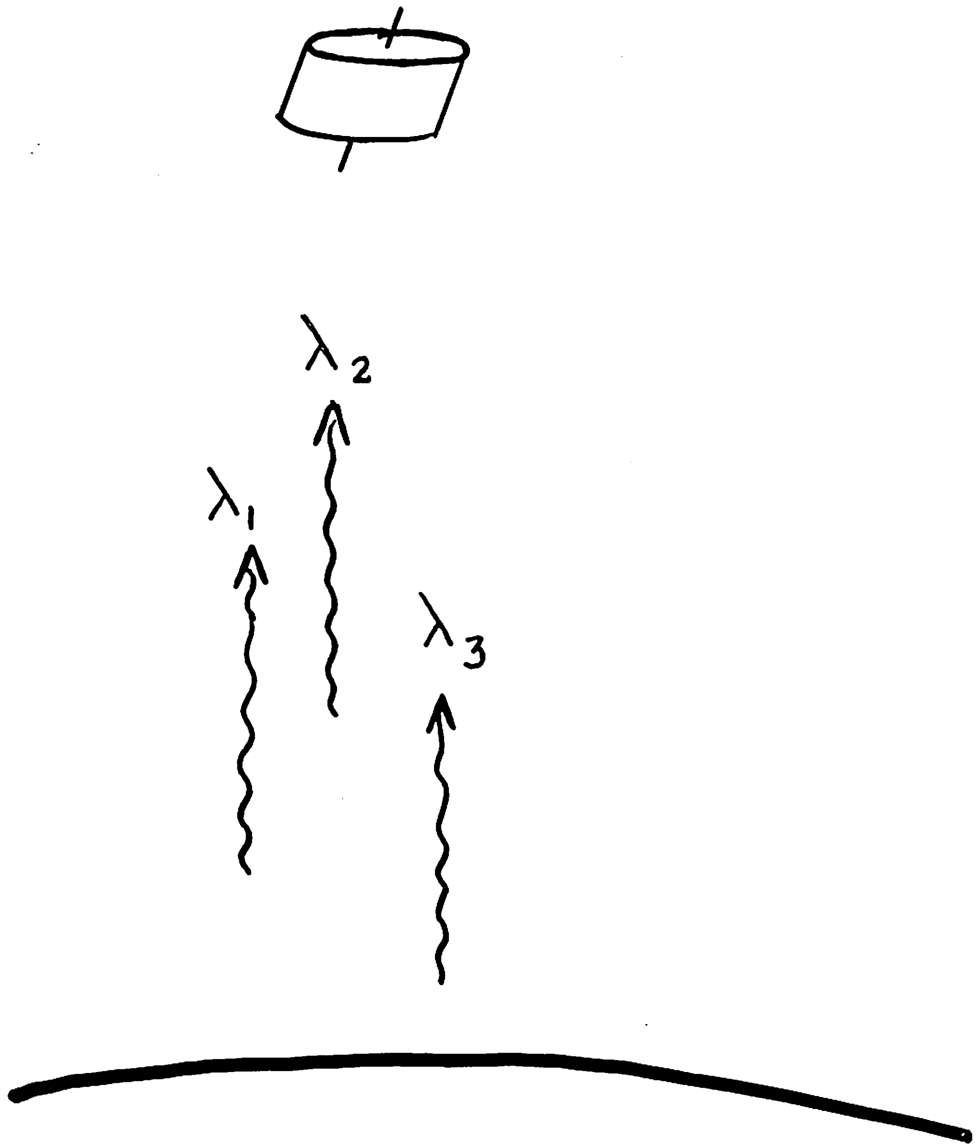
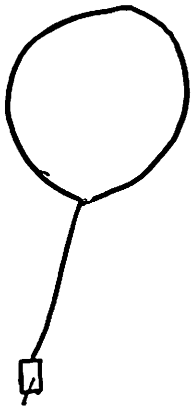


Direct Assimilation of Satellite Radiances

Different Measurement Techniques

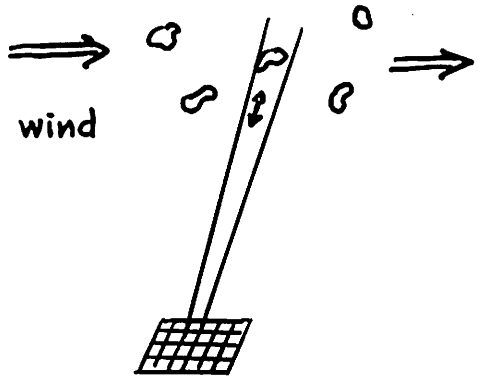
Rawinsonde

Carried by wind
Rising at ~ 1000 ft/min



Wind Profiler

Measuring radial velocity of turbulent blobs of air



Satellite Radiometer

Measuring upwelling radiation from deep layers

An Analogy

Human body $\leftrightarrow$ Numerical prediction model

Human digestion $\leftrightarrow$ Data assimilation

No single food group is adequate for good nutrition.

No single type of observation is adequate for a good forecast.

Too little food leads to malnourishment and poor health.

Too few observations leads to a poor forecast.

An Analogy (continued)

Some foods (raw fish) are distasteful, so we cook them first *or* we may change our tastes (Japanese like sushi).

Some observations are difficult to assimilate directly into the model, so we either process the observations to make them look more like model variables or we calculate what the observation would be, given the model state variables. Then assimilation becomes easier.

Tainted food makes us sick; sanitation helps.

Erroneous data make the model sick; quality control helps.

Simple problem

Estimate an unknown quantity x from two collocated measurements y_1 and y_2 subject to errors ε_1 and ε_2.

$$y_1 = x + \varepsilon_1$$

$$y_2 = x + \varepsilon_2$$

Assume $E(\varepsilon_1) = E(\varepsilon_2) = 0$ and $E(\varepsilon_1 \varepsilon_2) = 0$

$E(\cdot)$ is the statistical mean.

Define $\sigma_1^2 = E(\varepsilon_1^2)$ $\quad \sigma_2^2 = E(\varepsilon_2^2)$

Form the linear estimate

$$x' = a_1 y_1 + a_2 y_2$$

subject to $E(x' - x) = 0$

This implies that $a_1 + a_2 = 1$.

Finally, we require that $\sigma^2 = E[(x'-x)^2]$ be minimized.

Solution:

$$a_1 = \sigma_2^2/(\sigma_1^2+\sigma_2^2) \qquad a_2 = \sigma_1^2/(\sigma_1^2+\sigma_2^2)$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

Equivalent problem:

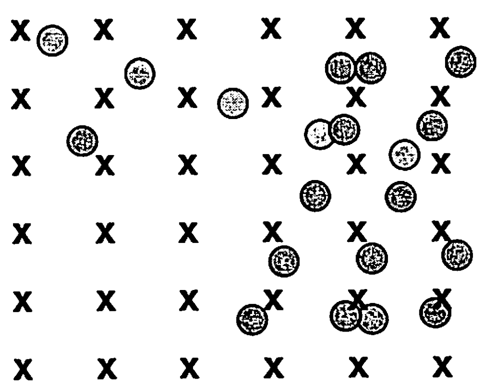
Find an estimate ξ of x that is close to the observations. Do this by minimizing the "distance" between ξ and the observations:

$$J(\xi) = \frac{(\xi - y_1)^2}{\sigma_1^2} + \frac{(\xi - y_2)^2}{\sigma_2^2}$$

The ξ which minimizes J is the same as the estimate x', just discussed.

Next, consider the more complicated situation:

- **many observations**
- **different kinds of instruments measuring the same quantities**
- **observations in different places**
- **different quantities measured, but all related to the variable to be estimated**
- **independent information from a model**
- **spatially correlated errors**



temperature measured by rawinsonde

temperature measured by aircraft

radiance measured by satellite

3-D Variational Analysis

Basic idea: Minimize cost (penalty) function.

$$\boxed{J(x) = (x-x_b)^T B^{-1}(x-x_b) + (y-K(x))^T (O+F)^{-1}(y-K(x))}$$

The first term fits the analysis to the background.
The second term fits the analysis to the observations.

Explanation of terms:

$\underline{x}$ gridpoint variables (variables carried in the model); the analyzed state to be determined (a vector)

$\underline{x_b}$ the background (first guess) provided by a short-range numerical forecast (a vector)

$\underline{y}$ observations of all kinds (a vector)

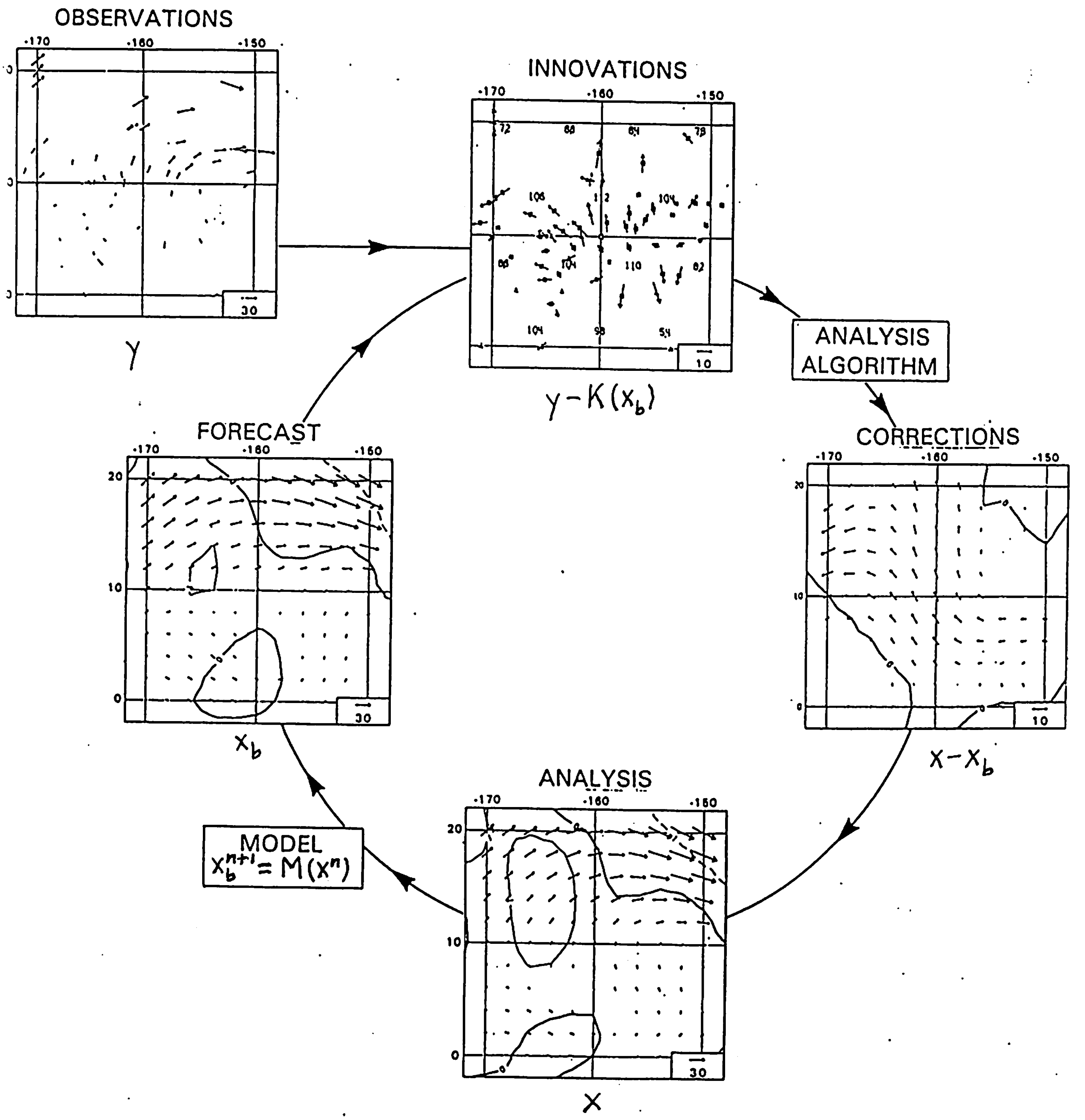
$\underline{\underline{K}}$ forward linear model, which interpolates from the model grid to the observation location and, if necessary, converts from the model variables to the observed variables (a matrix).

$\underline{\underline{B}}$ background error covariances; B contains statistical information about errors in the short-range forecast that provides the background (a matrix).

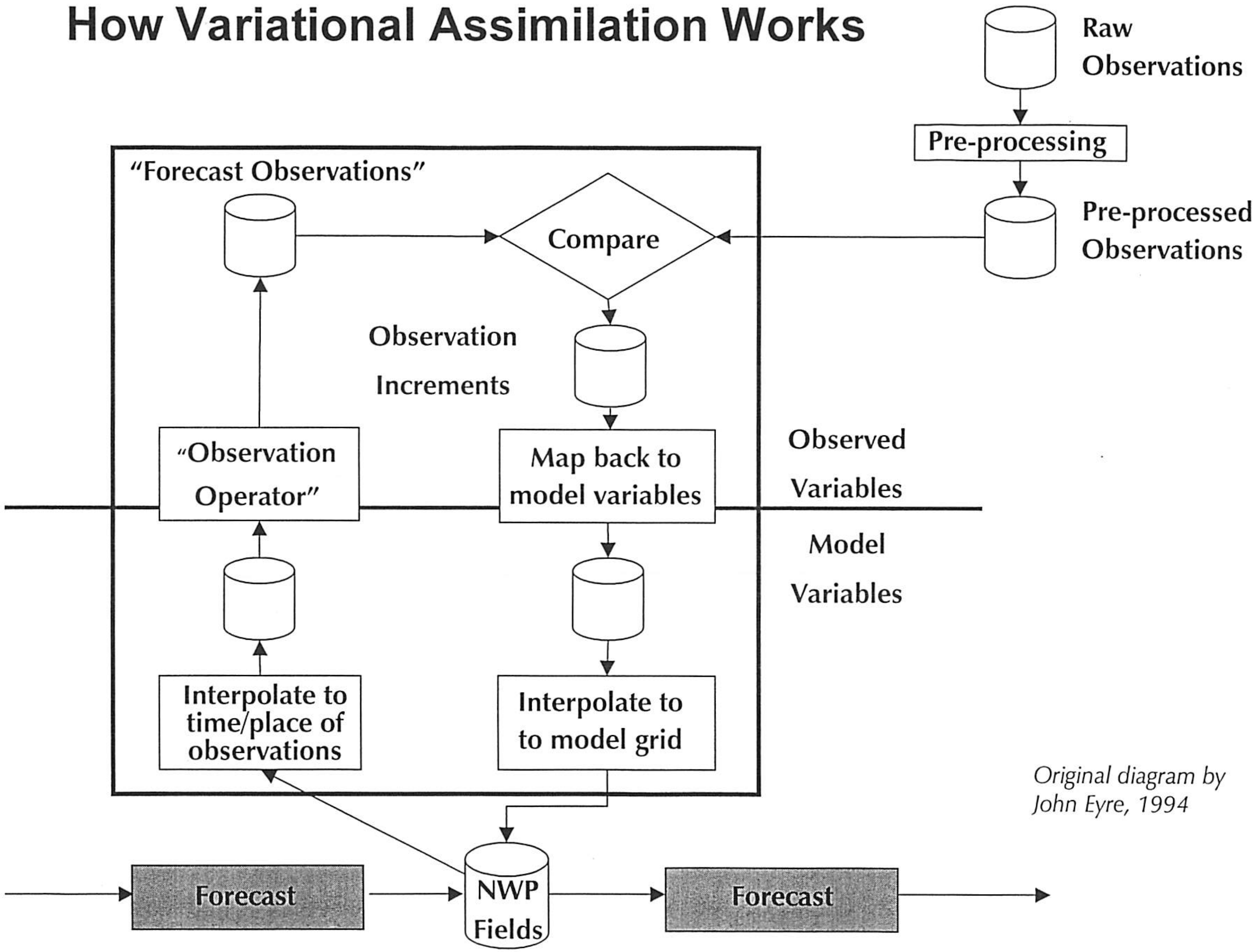
$\underline{\underline{O}}$ observation error covariance; contains statistical information about errors in the observations (matrix)

$\underline{\underline{F}}$ covariance of errors in the forward model (matrix)

ANALYSIS/FORECAST CYCLE



How Variational Assimilation Works

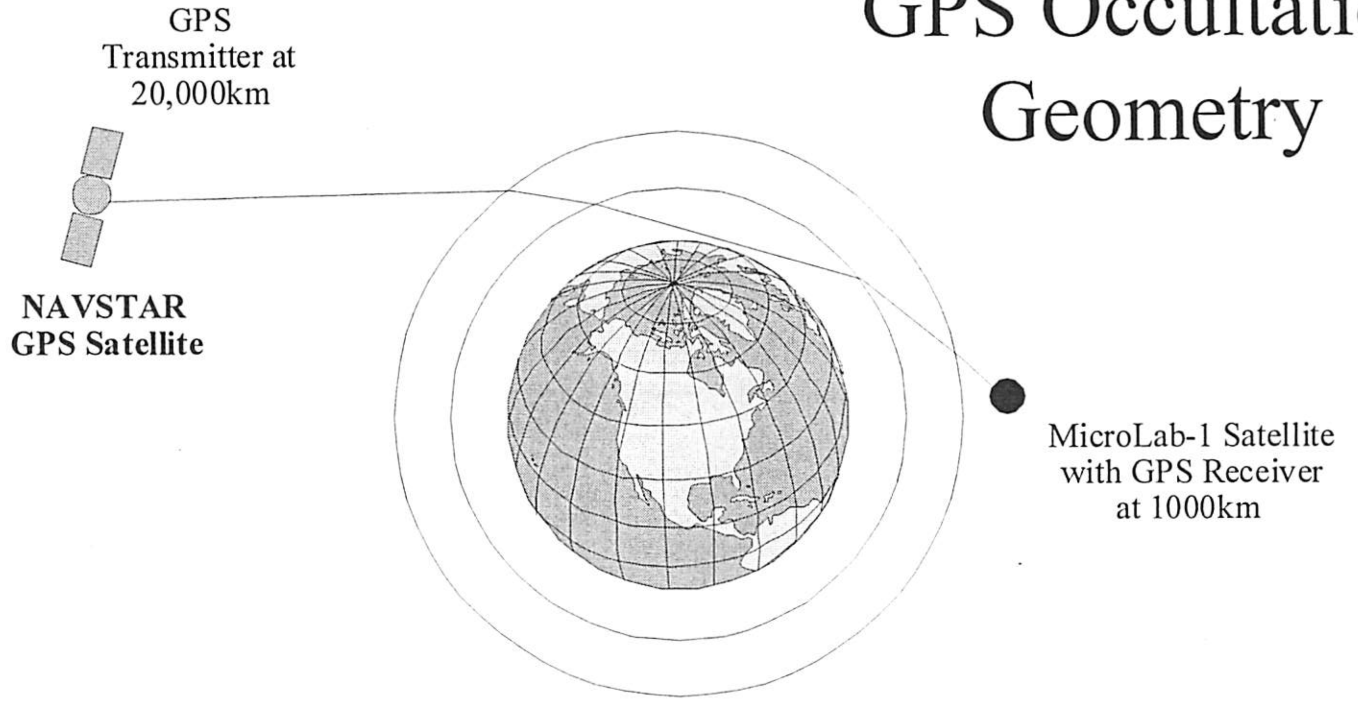


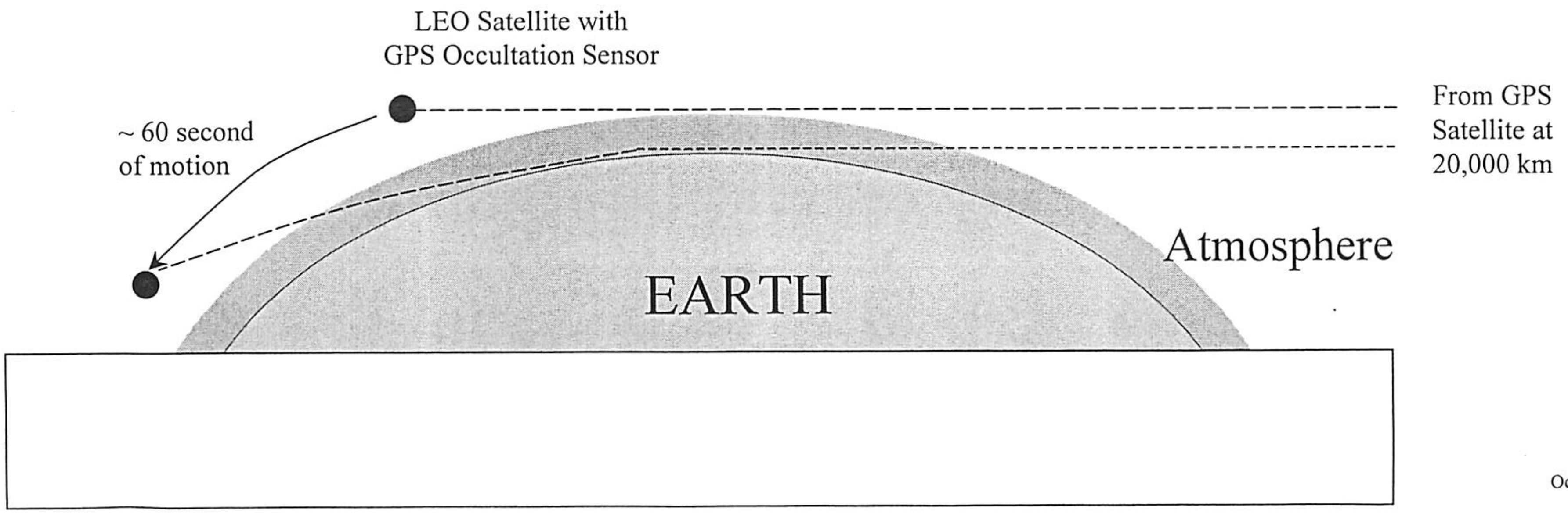
Original diagram by John Eyre, 1994

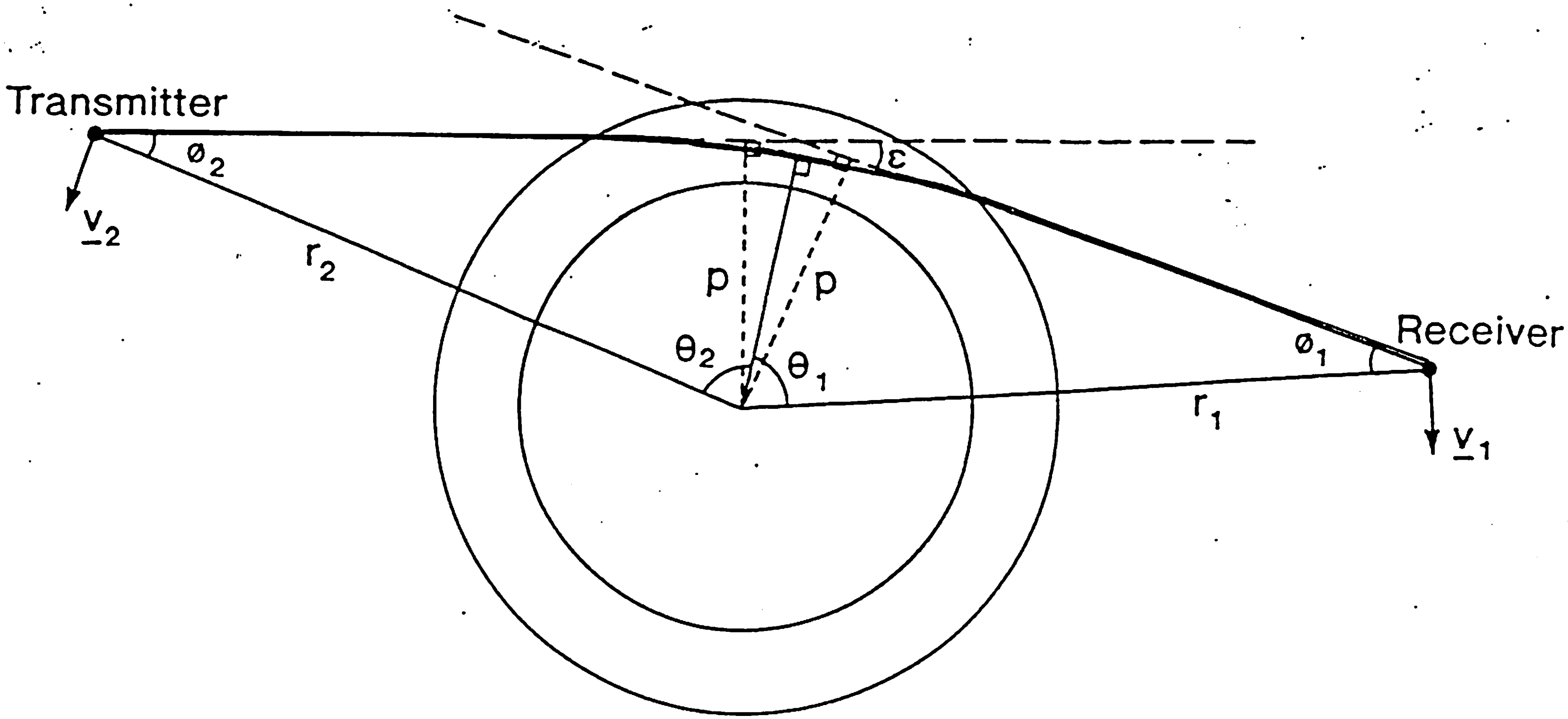
Practical Details

- Large dimensionality of the problem
- Determination of error covariances not easy
- Effect of background error covariance matrix approximated by filters
- Minimum of penalty function determined iteratively using conjugate gradient method (steepest descent method more popular in Europe)
- Preconditioning helps to accelerate convergence toward solution.
- Choice of vertical coordinate on which to analyze deserves careful consideration.
- Variational constraints (relating the mass and wind fields) may be added to the penalty function to force more balanced flows in the analysis.

GPS Occultation Geometry







For each limb path, the frequency data, averaged over a suitable time interval, and the information on the orbital geometry should be converted to values of:

- impact parameter $p = r_1 \sin \phi_1 = r_2 \sin \phi_2$
- angle of refraction ε
- latitude and longitude of tangent point
- direction on the earth's surface of the plane of the measurement
- adjustment of ε and p for ionospheric effects

John Eyre, 1994

Eyre, J.R., 1994:
Assimilation of Radio Occultation Measurements into a Numerical Weather Prediction System. Research Department Technical Memorandum No. 199 (May), European Centre for Medium Range Weather Forecasts, 22 pp. + 4 tables + 10 figs.

JPL

OBTAINING TEMPERATURE AND PRESSURE FROM REFRACTIVITY

$$N=(n-1)\times10^6=\underbrace{77.6\frac{P}{T}}_{\text{Dry}}+\underbrace{3.73\times10^5\frac{P_W}{T^2}}_{\text{Moist}}-\underbrace{40.3\times10^6\frac{n_e}{f^2}}_{\text{Ionosphere}}$$

+ higher order ionospheric terms

- Equation of state

$$\rho=0.3484\frac{P}{T}$$

- Hydrostatic equilibrium equation

$$\frac{\partial P}{\partial h}=-g\rho$$

density

n = index of refraction
N = refractivity
P = pressure
T = temperature
P_W = water vapor pressure
n_e = electron density
f = operating frequency
ρ = density
h = height
g = gravitational acceleration

Three Options for the Assimilation of Radio Occultation Data

(Eyre, 1994)

Option 1: Direct assimilation of bending angle

- Observations assimilated in “raw” form; error characteristics fairly simple
- Forward operator K is fairly complicated
  - Interpolate model temperature, humidity, and pressure at each level into the plane containing both satellites and the earth.
  - for a given value of the impact parameter p and from the model variables interpolated into the plane of occultation, calculate the bending angle ε

Option 2: Assimilation of retrieved profiles of refractivity

- Bending angles ε are inverted prior to assimilation using the Abel Transform, which gives refractivity profile at the tangent point (assumes spherical symmetry)
- Retrieval errors associated with refractivity profile difficult to specify
- Interpolate model fields to point of tangency and compute refractivity profile from temperature and moisture (forward operator).

Option 3: Assimilation of retrieved profiles of temperature or humidity

- In neutral atmosphere, refractivity depends upon temperature and humidity, and the effects of each are hard to separate. Could use low-level moisture profile from model to infer temperature profile from occultation; could retrieve temperature directly from refractivity in high, cold upper troposphere and stratosphere
- Error characteristics of these retrievals very difficult to figure
- Forward operator very simple: linear interpolation of temperature or moisture sounding to point of tangency.

Extension of variational assimilation to the time dimension

- Four-dimensional variational assimilation (4DVAR)
  - Fits a sequence of model states to a time sequence of observations
  - Penalty function has background term valid at t_o; observation term contains model states and observations at times between t_o and $t_o + \Delta t$, the assimilating interval.
  - Model equations relate the state at x^{n+1} to the state at x^n.
  - Wait until the end of the assimilation interval to do 4DVAR; initial state that is the solution to 4DVAR is at the beginning of the interval.
- Extended Kalman Filter
  - Sequential correction of model state for each new set of observations
  - Background error covariances evolve in time.
  - System is integrated forward in time.
- Both methods *very* expensive computationally.

Summary

The basis of variational assimilation is to find an estimate of the model state x that minimizes the penalty function

$$J(x) = (x-x_b)^T B^{-1} (x-x_b) + (y-K(x))^T (O+F)^{-1} (y-K(x))$$

Makes the estimate close to the back-ground (the a priori estimate that comes from the prediction model	Makes the estimate close to the observations

Most of the work in variational assimilation involves

- Making the problem computationally efficient
- Estimating or approximating B, the background error covariances
- Estimating O, the observation error covariances, and F, the errors of representativeness