

**Testing Theories of Gravity Wave
Saturation and Dissipation
in the Middle Atmosphere**

by

Chet Gardner

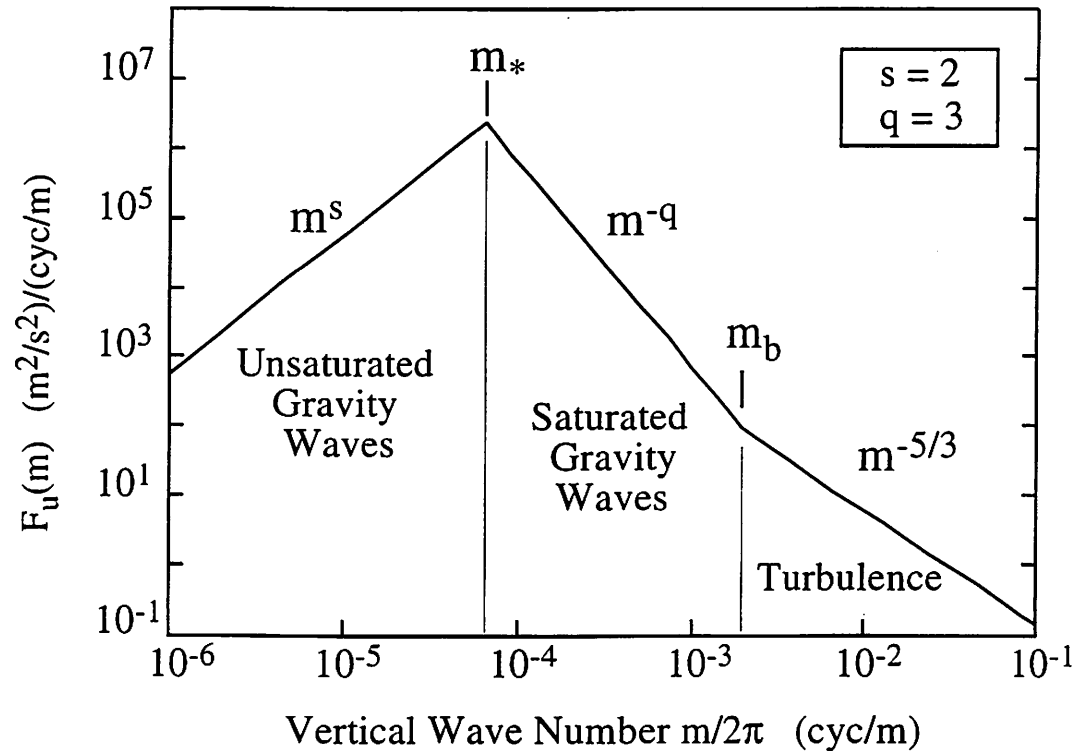
University of Illinois at Urbana-Champaign

1995 Annual CEDAR Meeting
Boulder, CO

26-30 June 1995

- **Several groups have begun to develop gravity wave parameterization schemes for inclusion in global circulation model codes.**
- **In spite of the enormous amount of theoretical and experimental research conducted during the past three decades on atmospheric gravity waves, there remains considerable disagreement about which processes dominate wave dissipation in the atmosphere.**
- **Much of this controversy could be eliminated by conducting definitive experimental tests of the fundamental physics underpinning the leading theories.**

Vertical Wave Number Spectrum



Current models partition the m -spectrum into an unsaturated region $m < m_*$ which is dominated by source effects and a saturated region $m_* < m < m_b$ which is controlled by various saturation and dissipation processes. The region $m > m_b$ is dominated by turbulence. m_* is the wave number of the largest scale saturated wave while m_b marks the transition between saturated gravity waves and turbulence.

Wave Saturation and Dissipation Theories

- **Ed Dewan and Earl Good [JGR, 91, 2742, 1986]**
***m*-spectrum is controlled by shear and convective instabilities. Wave packets saturate more or less independently (linear instability assumption). By assuming saturated waves obey the polarization and dispersion relations, all forms of the wave spectra (1-, 2-, and 3-D) can be predicted.**
- **Colin Hines [JAS, 48, 1360, 1991]**
***m*-spectrum is controlled by Doppler spreading of the vertical wavelengths by the random winds of the wave field. This theory has only been applied to *m*-spectrum of horizontal winds.**
- **Jerry Weinstock [JAS, 47, 2211, 1990]**
Wave energy dissipation is caused by strong nonlinear interactions of the wave field, so that dissipation can be modeled as a diffusion process. This theory has only been applied to the *m*-spectrum of horizontal winds.
- **Chet Gardner [JGR, 99,20,601, 1994]**
Wave induced diffusion can be modeled as a filtering process which eliminates from the spectrum waves with vertical phase velocities comparable to or smaller than the effective vertical velocity of momentum diffusion. All forms of the wave spectra (1-, 2-, and 3-D) are predicted by this theory.

- Ed Dewan [*GRL*, 18, 1476, 1991]
Spectra depend on the saturation limit N/m as well as the energy dissipation rate. Dimensional analysis leads to the similitude models for all forms of the 1-D gravity wave spectra.
- Ed Dewan [*GRL*, 21, 817, 1994]
Waves at saturation amplitudes established by shear and convective instabilities simultaneously transfer energy to higher frequency waves via cascade processes. This saturated-cascade model invokes a severe form of non-separability on the joint (m, ω) spectrum but yields all forms of the 1-D gravity wave spectra. The predicted spectra are similar to those of diffusive filtering theory which is also a non-separable model.

Under certain conditions, each of these theories predicts that the vertical wave number spectrum of horizontal winds (u) has the following form in the saturation region.

$$F_u(m) \sim N^2/m^3$$

Linear Instability Theory

[Dewan and Good, 1986]

- Shear and convective instabilities limit horizontal wind velocities to values comparable to the horizontal phase speeds of the wave.
- Gravity wave dispersion relation can be used to express this limit as N/m .

N – buoyancy frequency

m – vertical wave number

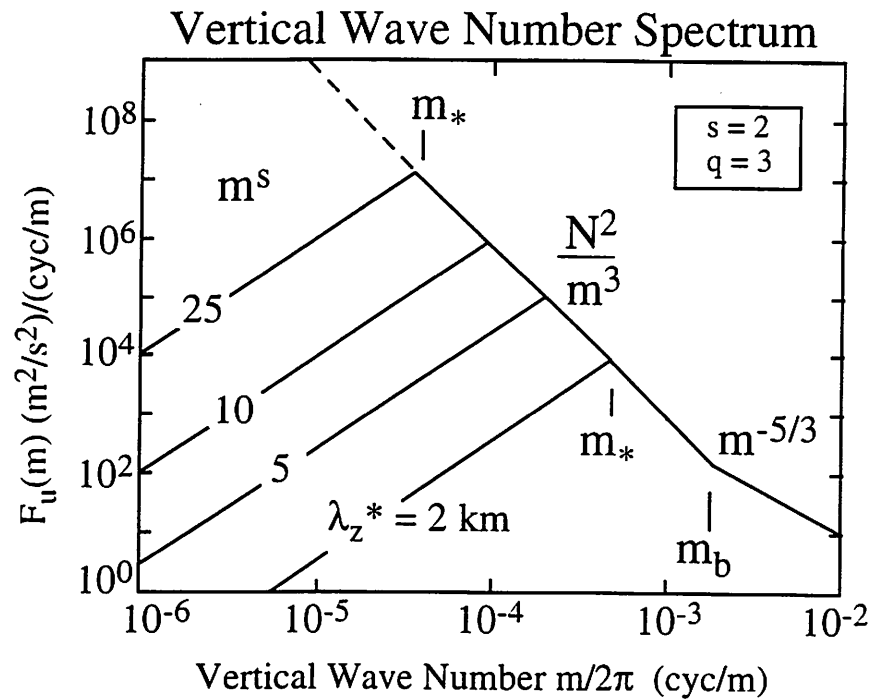
- Vertical wave number spectrum of horizontal winds $F_u(m)$ is the kinetic energy density per unit mass per unit vertical wave number.*
In saturation regime

$$F_u(m) \frac{dm}{2\pi} \simeq \frac{N^2}{m^2}$$

$$F_u(m) \simeq \frac{2\pi N^2}{m^2 dm} = 2\pi \frac{\alpha N^2}{m^3}$$

provided we assume $dm \propto m$.

* $F_u(m)$ has units of $(\text{m}^2/\text{s}^2)/(\text{cyc}/\text{m})$ for this notation.



Amplitude limit $N/m = \lambda_z/T_B$

[Smith, Fritts, and Van Zandt, 1987]

- **As altitude increases, saturation progresses to larger scales (smaller m) as these waves reach their amplitude limit.**

$$F_u(m) = \begin{cases} 2\pi \frac{\alpha N^2}{m_*^2} \left(\frac{m}{m_*}\right)^s & m \leq m_* \\ 2\pi \frac{\alpha N^2}{m^3} & m_* \leq m \leq m_b \end{cases}$$

The parameters α and m_* can be related to $\langle (u')^2 \rangle$ and $\langle (\partial u' / \partial z)^2 \rangle$.

Key Linear Instability Theory Attributes

- $u'_{\text{sat}} \simeq \frac{N}{m} \propto \lambda_z$ in fact for all $\lambda_z < \lambda_z^*$ $u'(\lambda_z) \simeq \lambda_z/T_B$
- $KE_{\text{sat}} \propto u'_{\text{sat}}{}^2 \propto \lambda_z^2$
- $\tilde{R}_i = N^2 / \langle (\partial u' / \partial z)^2 \rangle = 2/P \simeq 1$
- Since saturation does not depend on ω , the joint (m, ω) spectrum must be separable

$$F_u(m, \omega) = (2\pi)^2 \langle (u')^2 \rangle A(m) B(\omega)$$

- The magnitude of $F_u(m)$ in the saturation region $m > m_*$ is constant.

$$F_u(m) \simeq 2\pi \frac{N^2}{6m^3}$$

i.e., we have a universal spectrum in the saturation region.

- Vertical velocity (w') spectrum ($m > m_*$)

$$F_w(m) \propto \frac{1}{m^3} \quad (\text{GW polarization relations and separability})$$

- **The key test of linear instability theory is the measurement of horizontal wind amplitude (u') versus vertical wavelength (λ_z) for saturated waves ($\lambda_z > \lambda_z^*$). The fundamental assumption of the theory is that**

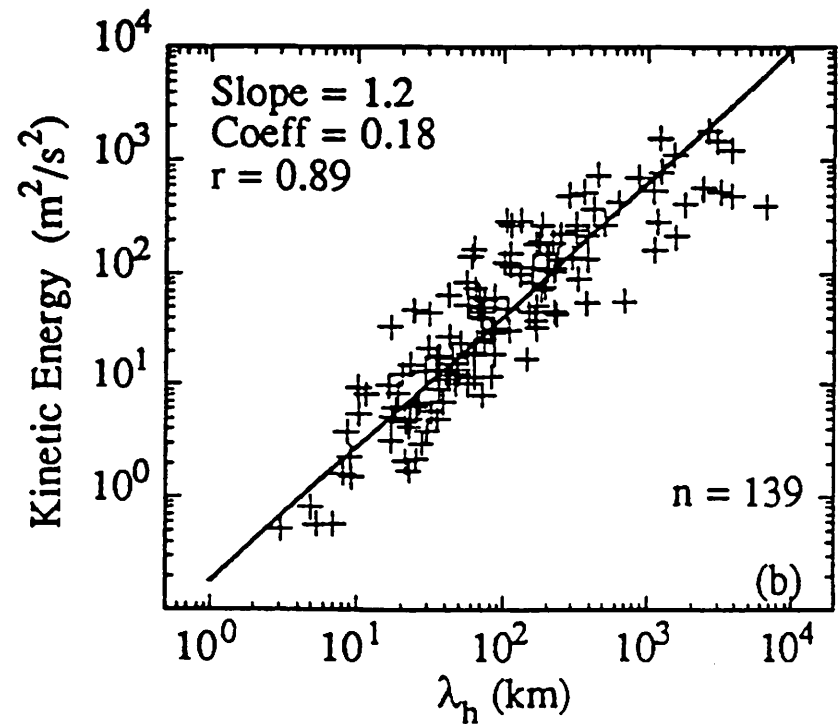
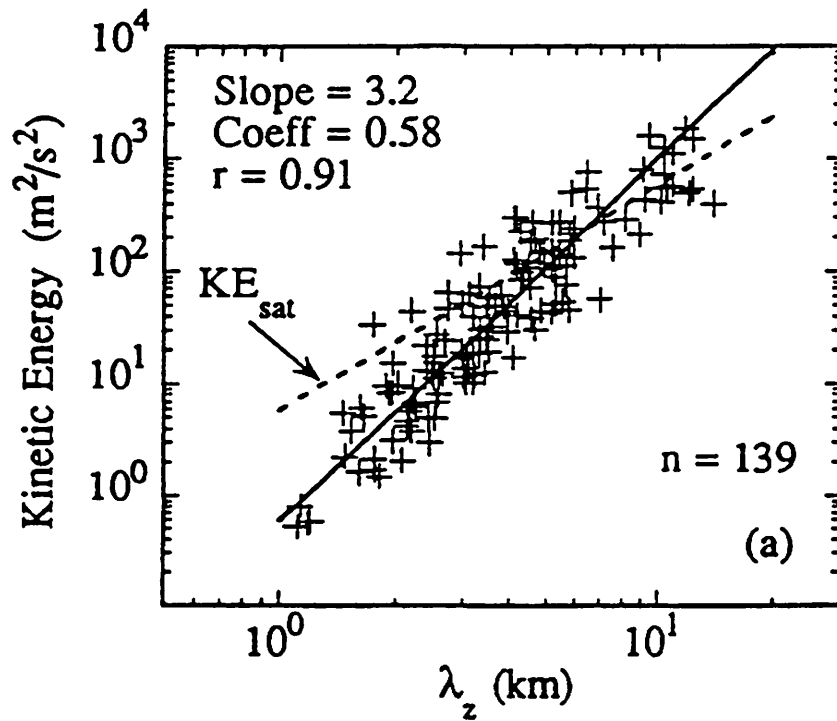
$$u' \sim \frac{N}{m} \sim \lambda_z \Rightarrow KE_{\text{sat}} \propto \lambda_z^2$$

- **The only extensive observations published to date are u' values inferred from Na lidar observations of Na density profiles. These observations show**

$$u' \propto \lambda_z^{1.5} \Rightarrow KE_{\text{sat}} \propto \lambda_z^3$$

However, the results are not definitive since the u' values were inferred from atmospheric density perturbations ($\rho'_a / \bar{\rho}_a$) which were inferred from Na density perturbations ($\rho'_s / \bar{\rho}_s$). Thus the Na density lidar observations provide only indirect experimental evidence that the fundamental assumption of linear instability theory is not valid.

Urbana, IL (40°N) Na Temperature Lidar Data (85-105 km)



$$KE = \frac{1}{2} (u')^2$$

$$KE_{\text{sat}} = \frac{1}{2} \frac{N^2}{m^2} = \frac{\lambda_z^2}{2T_B^2}$$

Urbana (40°N)

Arecibo (18°N)

Syowa (60°S)

South Pole (90°S)

$$\lambda_z^{3.2}$$

$$\lambda_z^3$$

$$\lambda_z^{3.2}$$

$$\lambda_z^{2.9}$$

$n = 139$

$n = 62$

$n = 41$

$n = 53$

$$\lambda_z^3 \quad n = 171$$

$N/m = \lambda_z/T_B$ limit for the amplitude of saturated waves is the fundamental assumption of Linear Instability Theory.

Diffusive Filtering Theory

- Let D be the effective vertical diffusivity of the atmosphere.

$$D = D_{wave} + D_{turb} + D_{mole}$$

- Dissipation is most severe when the vertical diffusive transport, during a time comparable to the wave period, is a significant fraction of the vertical wavelength.
- Alternatively a wave of intrinsic frequency ω and vertical wave number m will be severely damped when the vertical diffusion velocity mD exceeds the vertical phase velocity ω/m .
- Only waves satisfying

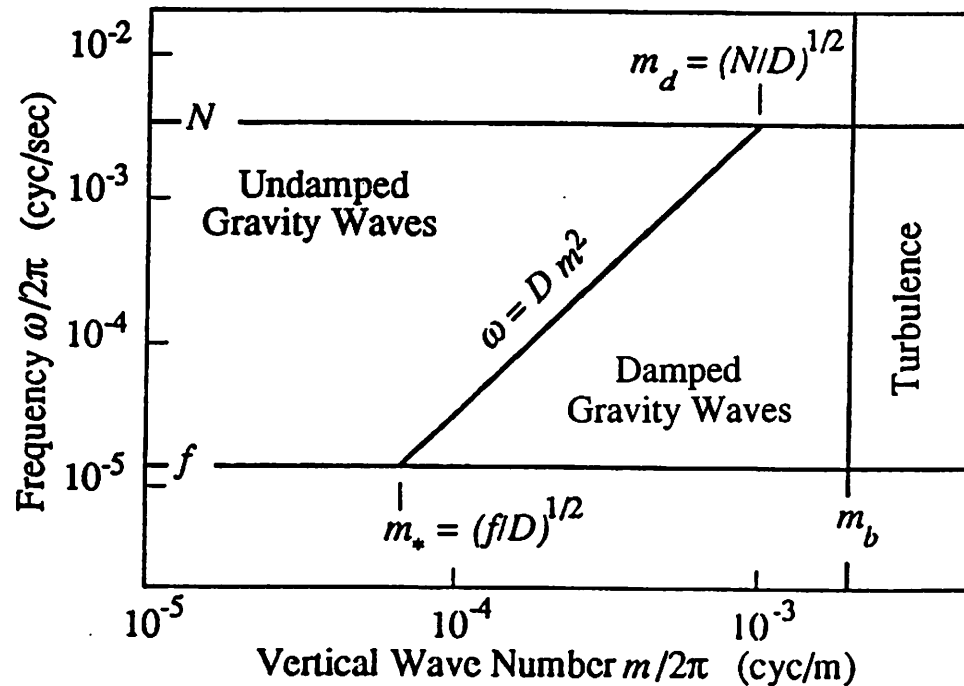
$$mD \leq \omega/m$$

or equivalently

$$Dm^2 \leq \omega$$

$$m \leq (\omega/D)^{1/2}$$

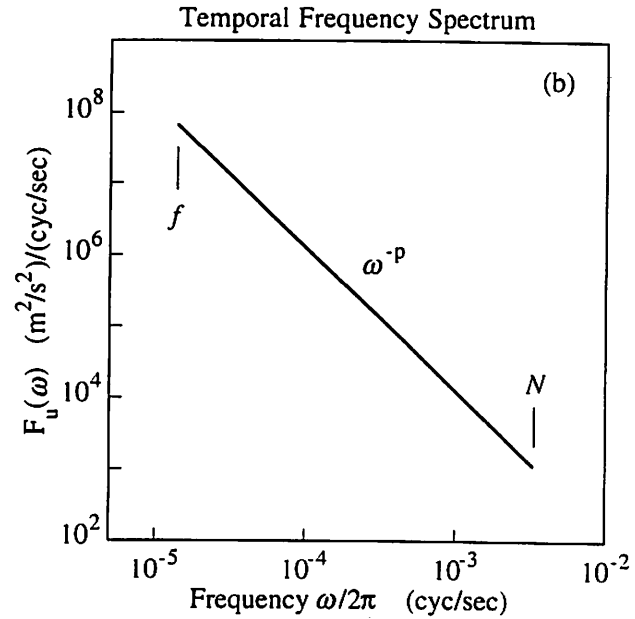
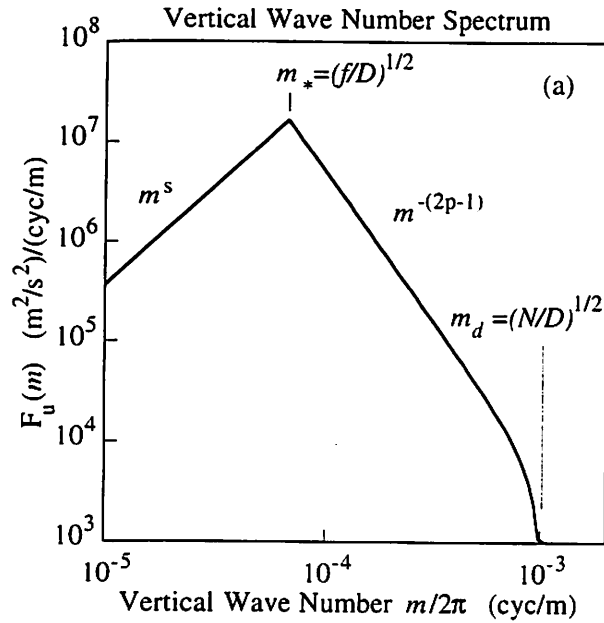
are permitted to grow in amplitude with altitude. The (m, ω) plane can be partitioned into damped and undamped regions.



For $m < m_*$ none of the waves are damped. For $m > m_*$ low frequency waves are damped.

Scale-independent diffusive damping limits for atmospheric gravity waves. The plotted limits are typical of the mid-latitude mesopause region where $f \simeq 2\pi/(20 \text{ h})$, $N \simeq 2\pi/(5 \text{ min})$, $D \simeq 500 \text{ m}^2/\text{s}$, $m_* \simeq 2\pi/(15 \text{ km})$.

Scale-Independent Diffusion Theory



$$F_u(\omega) \propto \frac{1}{\tilde{R}_i f} \left(\frac{f}{\omega} \right)^p \quad f \leq \omega \leq N \quad F_u(m) \propto \frac{N^2}{\tilde{R}_i m_*} \left(\frac{m_*}{m} \right)^{2p-1} \quad m_* \ll m \leq m_d$$

For $p = 2$

$$F_u(m) = \frac{4\pi}{\tilde{R}_i \ln(N/f)} \frac{N^2}{m^3} \approx \frac{2\pi}{3\tilde{R}_i} \frac{N^2}{m^3} \quad m_* \ll m \leq m_d$$

$$D_{wave} = \frac{(s+1)\tilde{R}_i \langle (u')^2 \rangle f}{(s+3)N^2} \xi(p) \quad m_* = (f/D)^{1/2} \sim \frac{N}{u'_{rms}}$$

Key Diffusive Filtering Theory Attributes

- Because of the diffusive filtering cutoff condition $mD = \omega/m$, the joint (m, ω) spectrum is not separable.
- For all undamped waves $\frac{\omega}{m} = C_z \geq mD = 2\pi D/\lambda_z$
- m -spectrum magnitudes and indices are dependent on source spectral characteristics and can be highly variable even for $p = 2$.

$$F_u(m) \propto \frac{N^2}{\tilde{R}_i m^3}$$

- Because waves are not saturated even for $m > m_*$, horizontal wind amplitudes for individual waves can also be highly variable.
- Vertical velocity (w') spectrum ($m > m_*$)

$$F_w(m) \propto m^{5-2p} \sim m \quad (\text{GW polarization relations and non-separability})$$

- **The key test of diffusive filtering theory is the measurement of the intrinsic vertical wave length (λ_z) and intrinsic period (T_i) of the waves.**

For all undamped waves, the theory requires

$$\frac{\omega}{m} = \frac{\lambda_z}{T_i} = C_z \geq mD = 2\pi D/\lambda_z.$$

That is, no waves should lie in the ~~undamped~~ damped region.

- **Compilations of wave parameters measured in the mesopause region by radars, lidars, and airglow imagers show a distinct absence of waves in the ~~undamped~~ damped region characterized by effective diffusivities $\sim 300 \text{ m}^2/\text{s}$ [Reid, *JATP*, 1986; Manson, *JAS*, 1990].**

Unfortunately, for many of these observations it was not possible to measure the intrinsic wave parameters so that the results are contaminated by Doppler shifting effects caused by the background wind field.

16

Horizontal Wavelength vs Observed Period

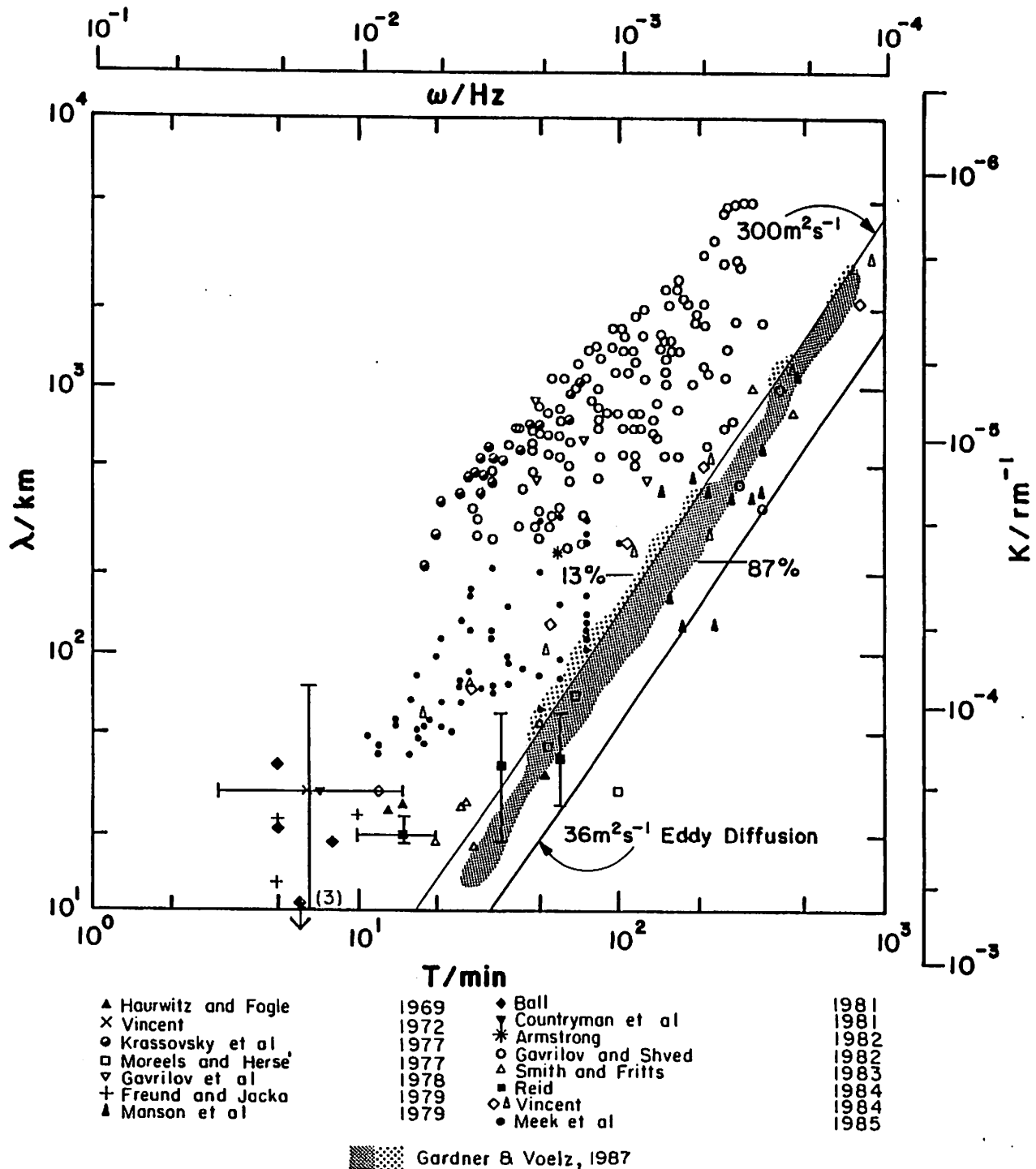
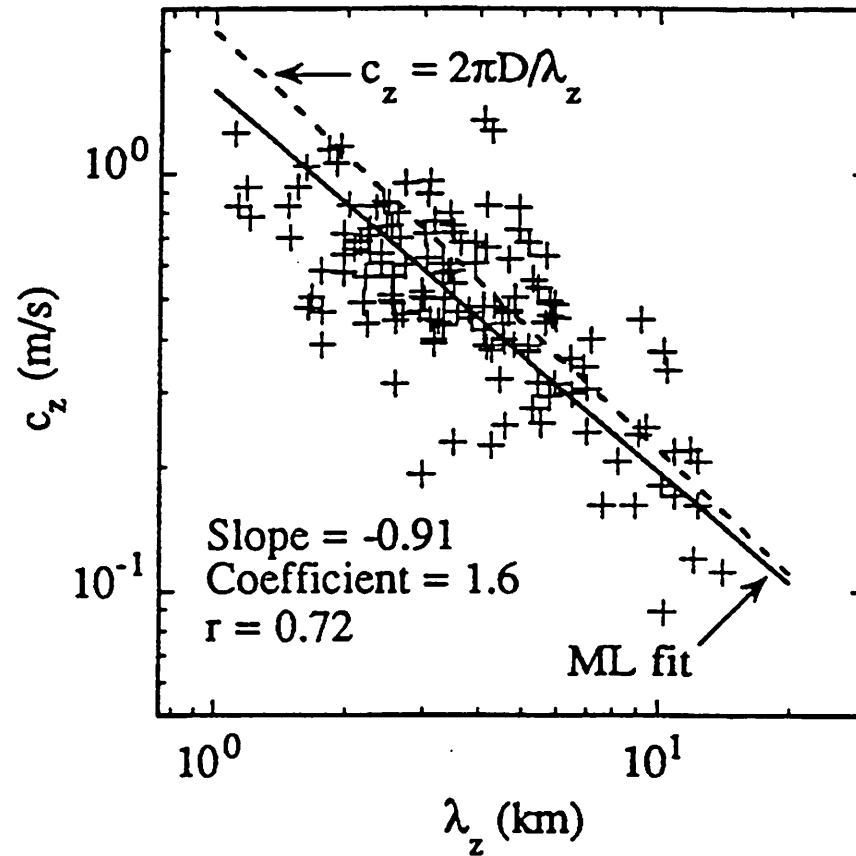


FIG. 4. As in Fig. 1, but the additional data are now from Gardner and Voelz (1987). Lidar sounding (sodium layer) provides vertical wavelengths at observed periods, and use of the dispersion equation provides the plotted horizontal wavelengths: 34 days, 1981-86, 109 waves in summer and winter 80-100 km. (System limits: ~20 minutes-12 hours, λ , 2-17 km).

Urbana, IL (40°N) Na Temperature Lidar (85-105 km)



Urbana (40°N) $\lambda_z^{-0.9}$ $n = 139$ λ_z^{-1} $n = 109$

Arecibo (18°N) $\lambda_z^{-1.2}$ $n = 62$

The fundamental assumption of Diffusive Filtering Theory is that waves are undamped when $C_z > mD = 2\pi D/\lambda_z$.

Saturated-Cascade Model

[Dewan, 1994]

- Shear and convective instabilities limit horizontal wind velocities to values comparable to the horizontal phase speeds of the wave N/m . Thus the wave variance is given by

$$\langle u^2 \rangle \sim N^2/m^2$$

- Simultaneously these waves dissipate energy in a cascade processing by transferring the excess energy to smaller temporal scales, i.e. to higher frequencies. Thus the wave variance is also given by

$$\langle u^2 \rangle \sim \mathcal{E}/\omega$$

Where \mathcal{E} is the energy dissipation rate (units)

$$(\text{m}^2/\text{s}^2/\text{s}).$$

- By equating these two expressions for the wave variance, the wavelength-period relations are obtained

$$\mathcal{E}/\omega \sim N^2/m^2 \Rightarrow \lambda_z \sim T^{1/2}$$

where λ_z is the vertical wave length and T is the intrinsic period of the wave.

Key Saturated-Cascade Theory Attributes

- $U_{\text{sat}} \equiv \frac{N}{m} \propto \lambda_z$
- $U_{\text{sat}} \equiv (\epsilon/\omega)^{1/2} \propto T^{1/2}$
- Since the saturated waves must satisfy the above two conditions simultaneously, all observed saturated-cascade waves must obey

$$\lambda_z \propto T^{1/2} \quad \text{or} \quad m^2 \propto \omega$$

- The above condition requires the joint (m, ω) spectrum to be non-separable and have the following form

$$F_{\mathbf{u}}(\mathbf{m}, \omega) = (2\pi)^2 \langle (\mathbf{u}')^2 \rangle A(\mathbf{m}) \delta(\mathbf{m} - \sqrt{\alpha\omega})$$

where $A(\mathbf{m}) \propto N^2/m^3$.

- **Saturated-Cascade Theory** appears to apply to a subset of the waves, viz., those waves that satisfy $\lambda_z \propto T^{1/2}$. The key test then is to measure the wind amplitude, vertical wavelength, and intrinsic period of these waves to confirm that when

$$\lambda_z \propto T^{1/2}$$

$$u' \propto \lambda_z \quad \text{and} \quad u' \propto T^{1/2} \quad \text{or}$$

$$\text{KE} \propto \lambda_z^2 \propto T$$

- The waves observed by Na lidar systems do obey $\lambda_z \propto T^{1/2}$ but also show that

$$\text{KE} \propto \lambda_z^3 \propto T^{1.5}$$

contradicting the fundamental assumptions of saturated-cascade theory.

However, these lidar observations are not definitive because the results were inferred from Na density perturbations and the measured wave periods were observed periods, not intrinsic periods.

Separability of the Joint (m, ω) Spectra

- Many theoretical studies have employed the assumption that the joint intrinsic (m, ω) spectrum of horizontal winds is separable.

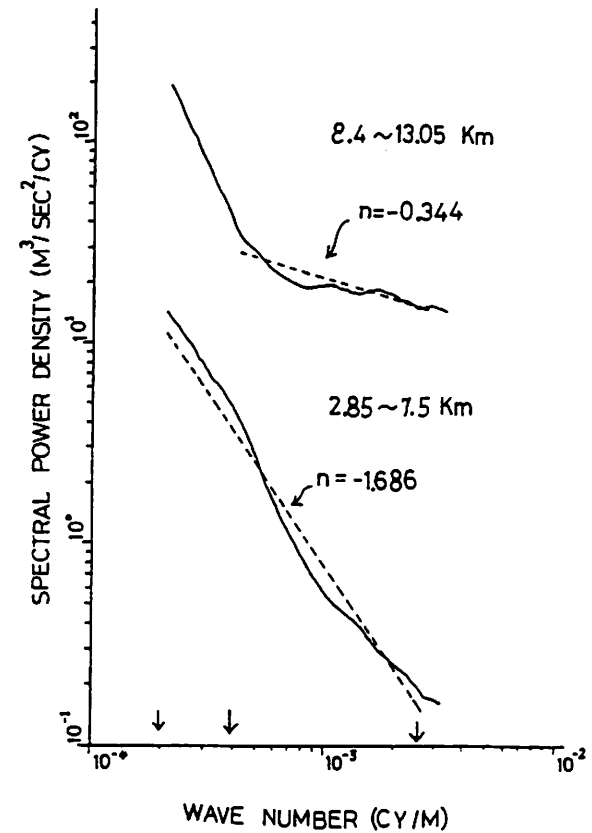
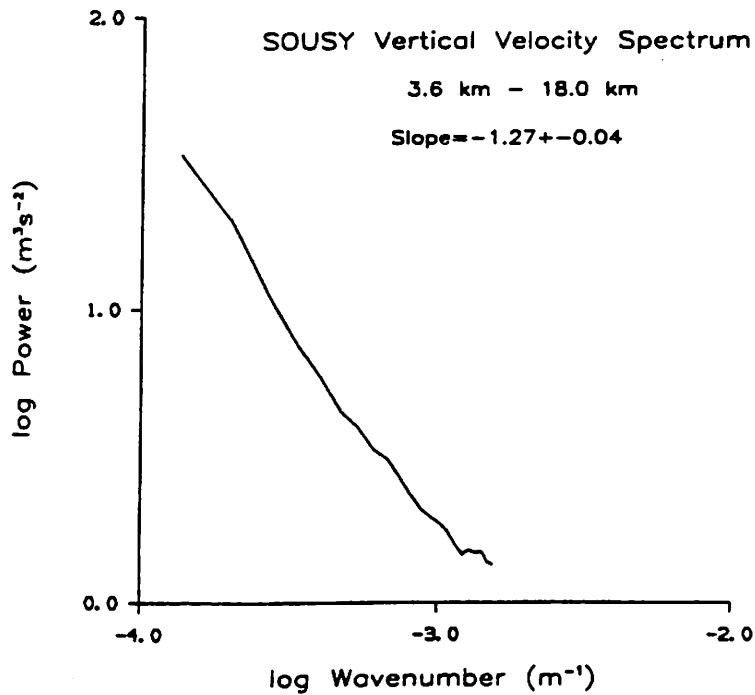
$$F_u (m, \omega) = (2\pi)^2 \langle (u')^2 \rangle A(m) B(\omega)$$

- Because of Doppler shifting effects caused by mean backgrounds the joint observed (m, ω_{obs}) spectrum will not be separable.
- If the intrinsic horizontal wind spectrum is separable, then the polarization relations can be used to show that the m-spectrum of vertical winds is

$$\begin{aligned} F_W (m) &= \frac{1}{2\pi} \int_f^N F_W(m, \omega) d\omega \simeq \frac{1}{2\pi} \int_f^N \left(\frac{\omega}{N}\right)^2 F_u(m, \omega) d\omega \\ &= 2\pi \langle (w')^2 \rangle A(m) = \frac{\langle (w')^2 \rangle}{\langle (u')^2 \rangle} F_u (m) \end{aligned}$$

where we have used $w' \simeq \frac{\omega}{N} u'$

Vertical Velocity Spectra



Larson et al. [*JAS*, 1987] $\frac{1}{m^{1.3}}$

Larson et al. [*JAS*, 1986] $\frac{1}{m^{1.2}}$

Linear Instability Theory

$$F_w(m) \propto \frac{1}{m^3}$$

Kuo et al. [*Radio Sci.*, 1985]

$$\frac{1}{m^{0.3}} \text{ and } \frac{1}{m^{1.7}}$$

Diffusive Filtering Theory

$$F_w(m) \propto m^{5-2p} \sim m$$

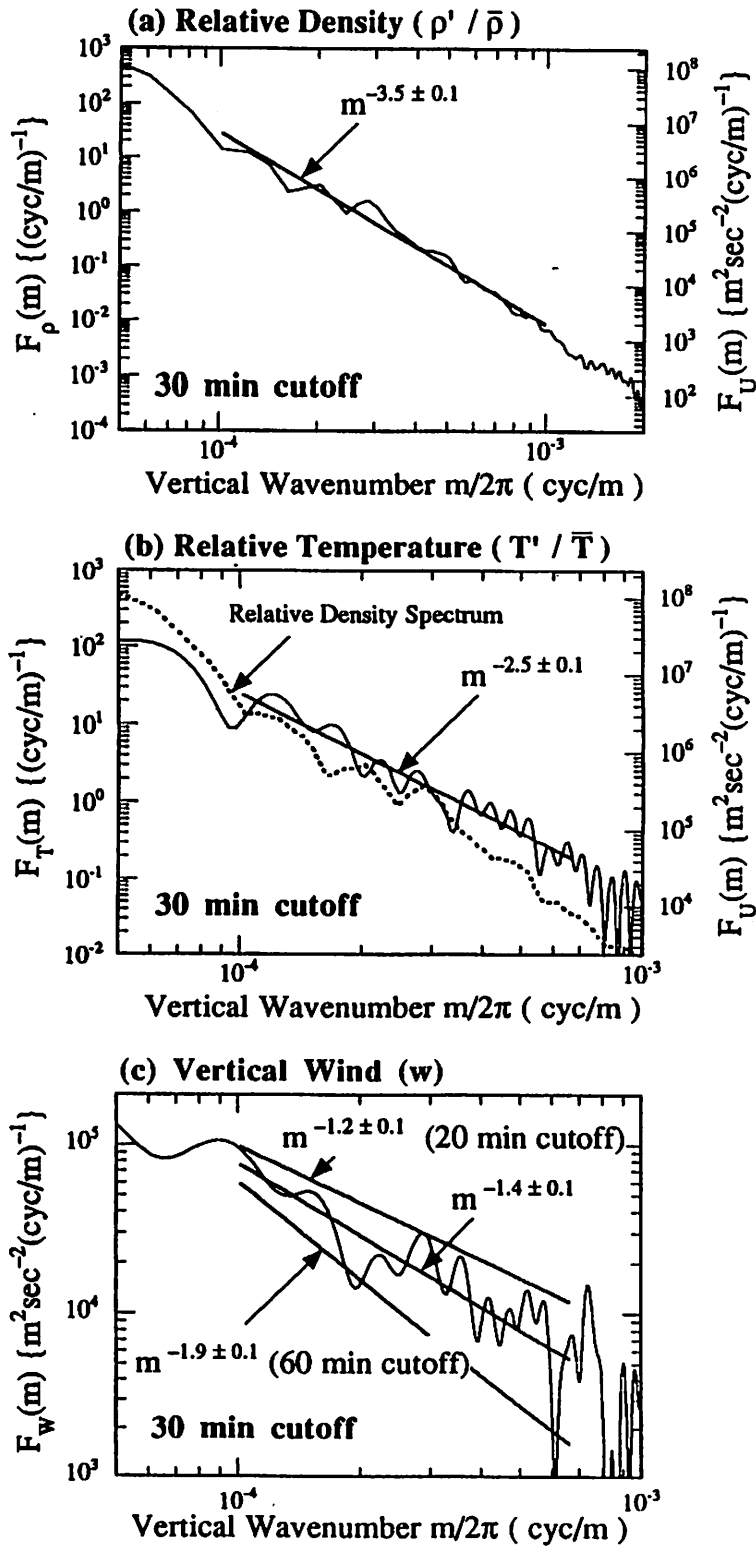


Figure 3

Separability of the 2-D Horizontal Wave Number (h, ϕ) Spectrum

- Several theoretical studies have assumed that (h, ϕ) spectrum is separable

$$F_u(h, \phi) = (2\pi)^2 < (u')^2 > C(h) D(\phi)$$

where

$$2\pi < (u')^2 > C(h) = \frac{1}{2\pi} \int_0^{2\pi} F_u(h, \phi) d\phi$$

$$2\pi < (u')^2 > D(\phi) = \frac{1}{2\pi} \int_0^{\infty} h F_u(h, \phi) dh$$

If we assume that waves propagating along different directions originate from different sources and therefore are statistically independent, then the (h, ϕ) spectrum is given by

$$F_u(h, \phi) = \frac{2 G_u(h, \phi)}{h}$$

where $G_u(h, \phi)$ is the 1-D horizontal wave number spectrum of just the wave propagating along azimuth ϕ .

Gardner, Hostetler, and Franke [*JGR*, 1994] showed how to express $G_u(h, \phi)$ in terms of $F_u(m, \omega, \phi)$ using the gravity wave polarization relations

$$G_u(h, m, \phi) = F_u(m, \omega, \phi) \left| \frac{d\omega}{dh} \right| \quad \text{where } \omega = N \frac{h}{m}$$

$$= \frac{N}{m} F_u(m, Nh/m, \phi)$$

$$G_u(h, \phi) = \frac{1}{2\pi} \int_0^\infty \frac{N}{m} F_u(m, Nh/m, \phi) dm$$

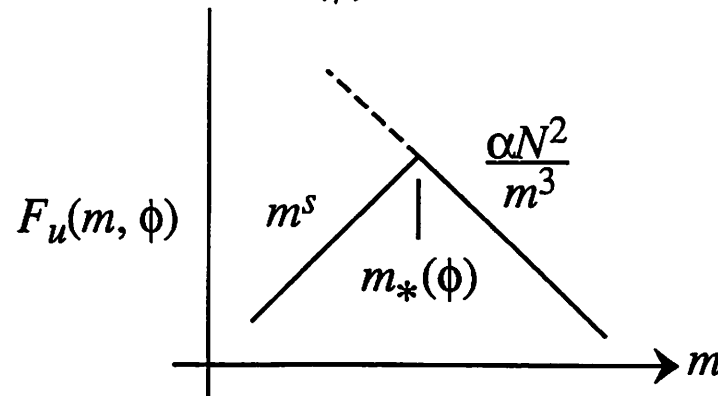
Linear Instability Theory

$$F_u(m, \phi) = \begin{cases} 2\pi \frac{\alpha N^2}{m_*^3} \left(\frac{m}{m_*}\right)^s & m \leq m_* \\ 2\pi \alpha \frac{N^2}{m^3} & m_* \leq m \end{cases}$$

where

$$\langle u'(\phi)^2 \rangle = \frac{1}{2\pi} \int_0^{\infty} F_u(m, \phi) dm = \frac{(s+3)}{2(s+1)} \alpha \frac{N^2}{m_*^2}$$

$$\Rightarrow m_*^2(\phi) = \frac{(s+3)}{2(s+1)} \frac{\alpha N^2}{\langle u'(\phi)^2 \rangle}$$



The shape of $F_u(m, \phi)$ changes with ϕ because m_* changes, hence $G_u(h, \phi)$ cannot be separable.

Diffusive Filtering Theory [Gardner, JGR, 1994]

$$F_u(m, \omega) = (2\pi)^2 \langle (u')^2 \rangle \frac{(s+1)}{m_*} \left(\frac{m}{m_*}\right)^s \frac{(p-1)}{f} \left(\frac{f}{\omega}\right)^{p+(s+1)/2}$$

$$m \leq (\omega/D)^{1/2} = m_* (\omega/f)^{1/2}$$

$$f \leq \omega \leq N$$

$$m_*^2 = f/D = \frac{f \langle (\partial T' / \partial z)^2 \rangle}{\Gamma \langle W' T' \rangle} = \frac{(s+3)}{(s+1) \tilde{R}i \xi(p)} \frac{N^2}{\langle (u')^2 \rangle}$$

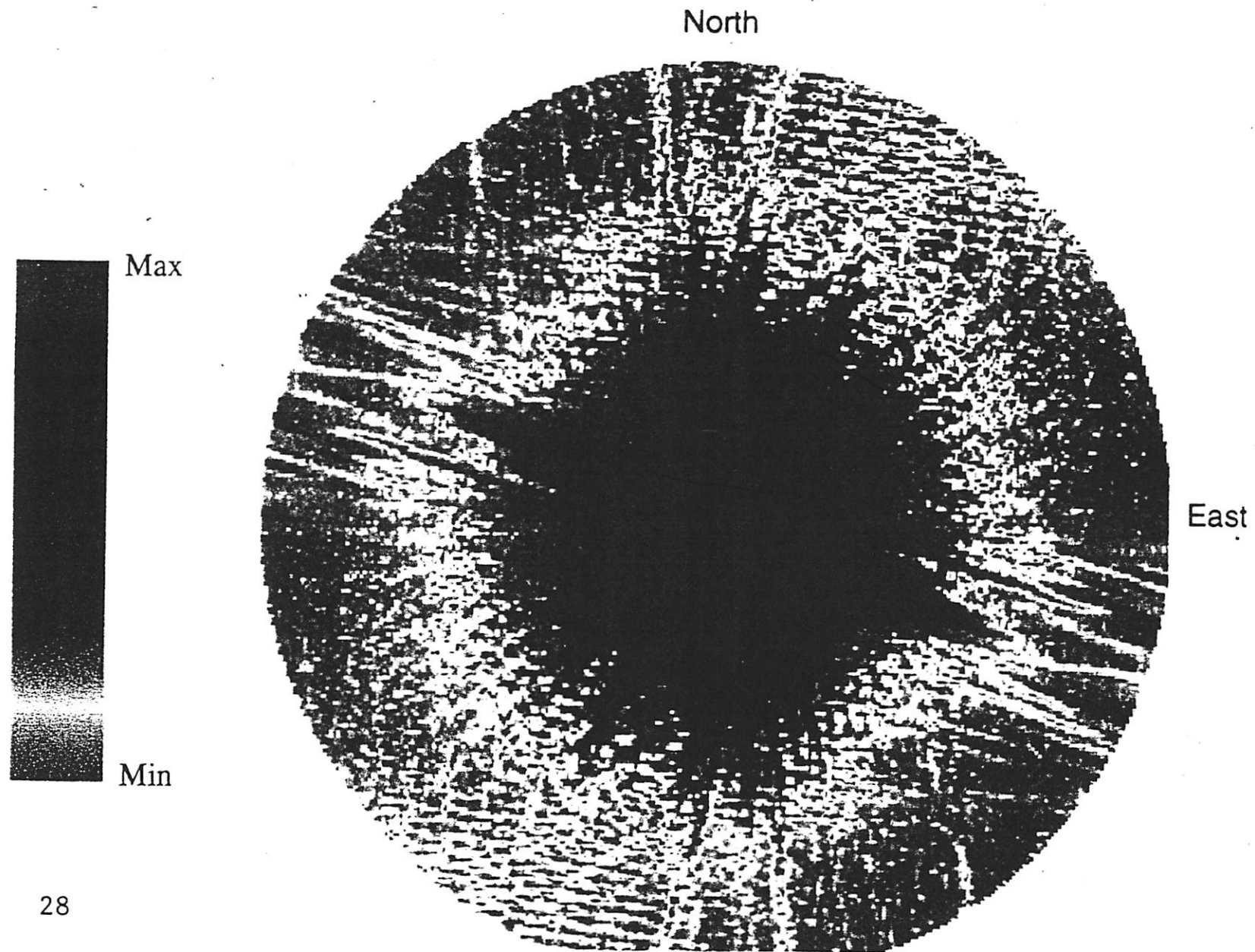
D is total diffusivity contributed by nonlinear interaction of all waves so m_*^2 does not vary with ϕ .

$$F_u(m, \omega, \phi) = (2\pi)^2 \langle u'(\phi)^2 \rangle \frac{(s+1)}{m_*} \left(\frac{m}{m_*}\right)^s \frac{(p-1)}{f} \left(\frac{f}{\omega}\right)^{p+(s+1)/2}$$

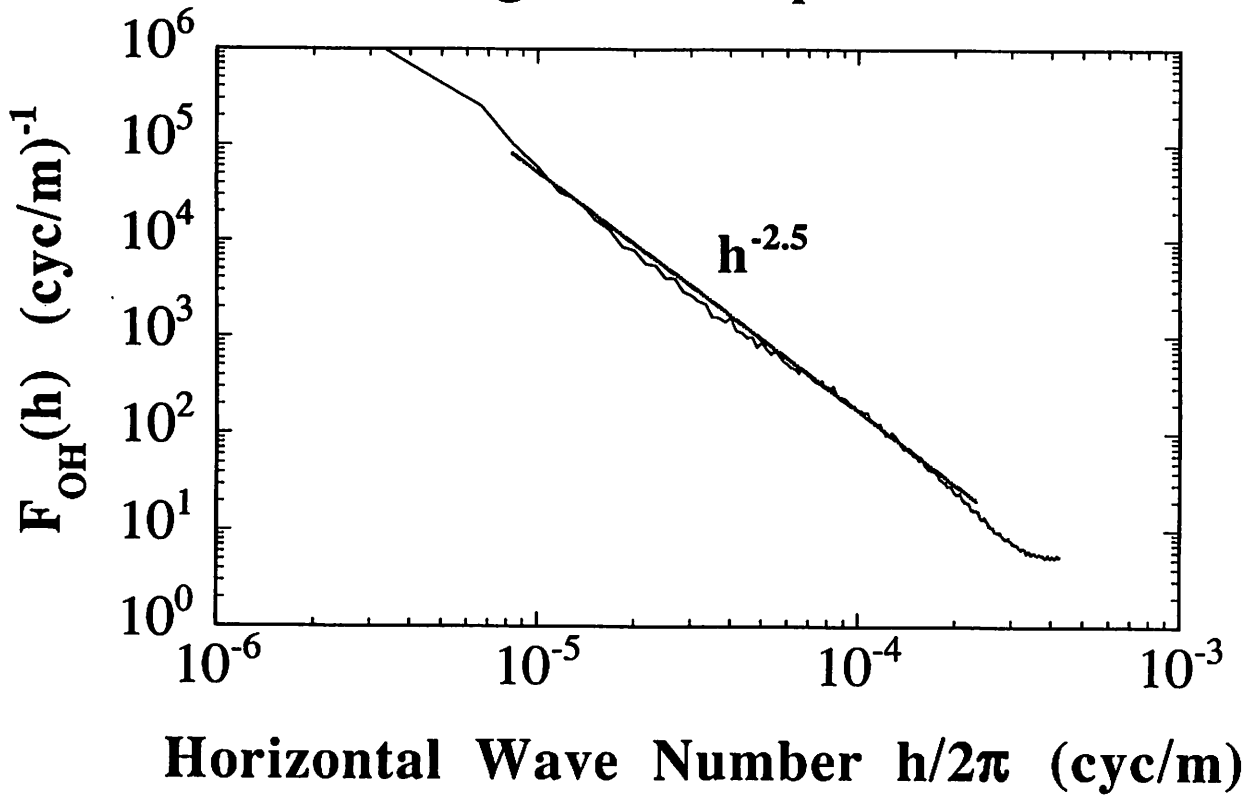
$$m \leq (\omega/D)^{1/2} \quad f \leq \omega \leq N$$

Thus, if we assume s and p , which are source dependent, do not vary appreciably with ϕ , $F_u(m, \omega, \phi) = F_u(m, \omega) D(\phi)$ and so $G_u(h, \phi)$ is separable.

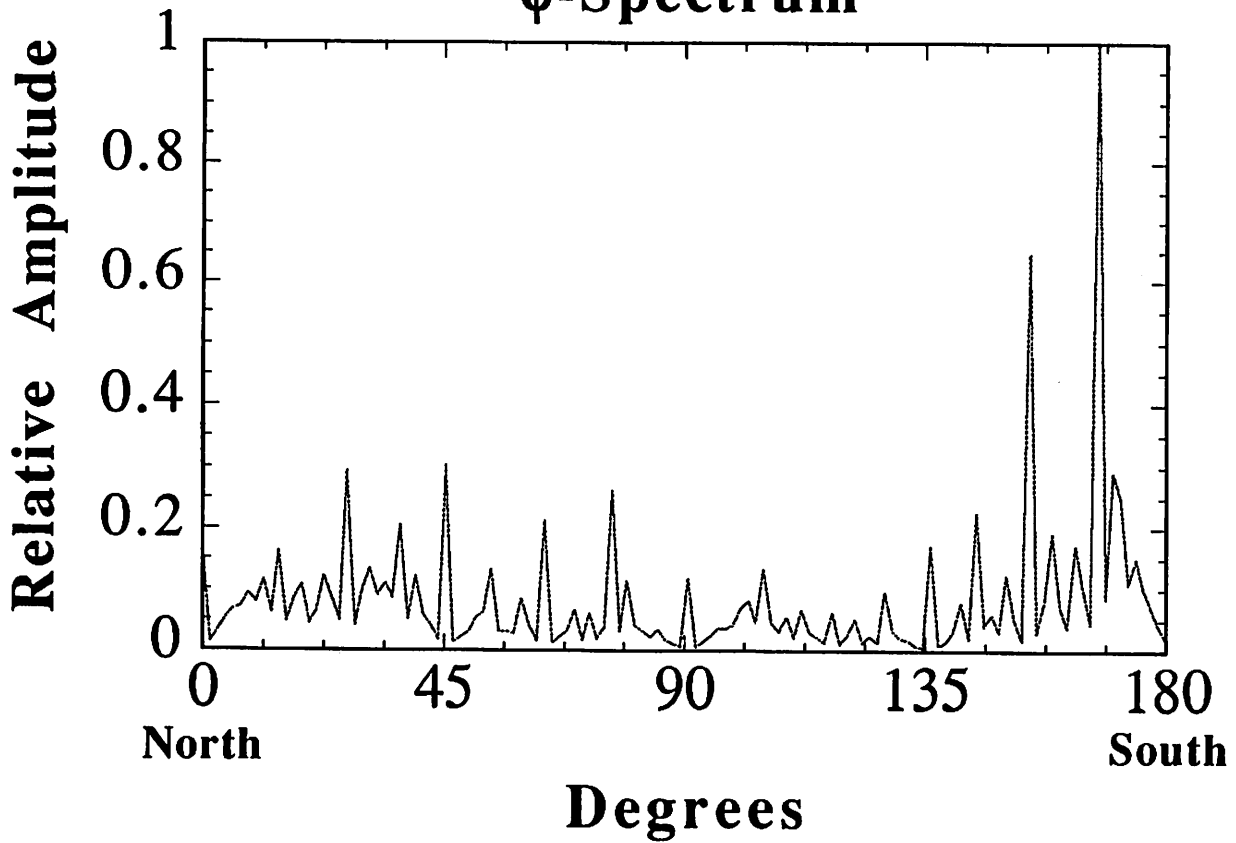
2-D Spectrum from OH Airglow Images
Starfire Optical Range
2 February 1995



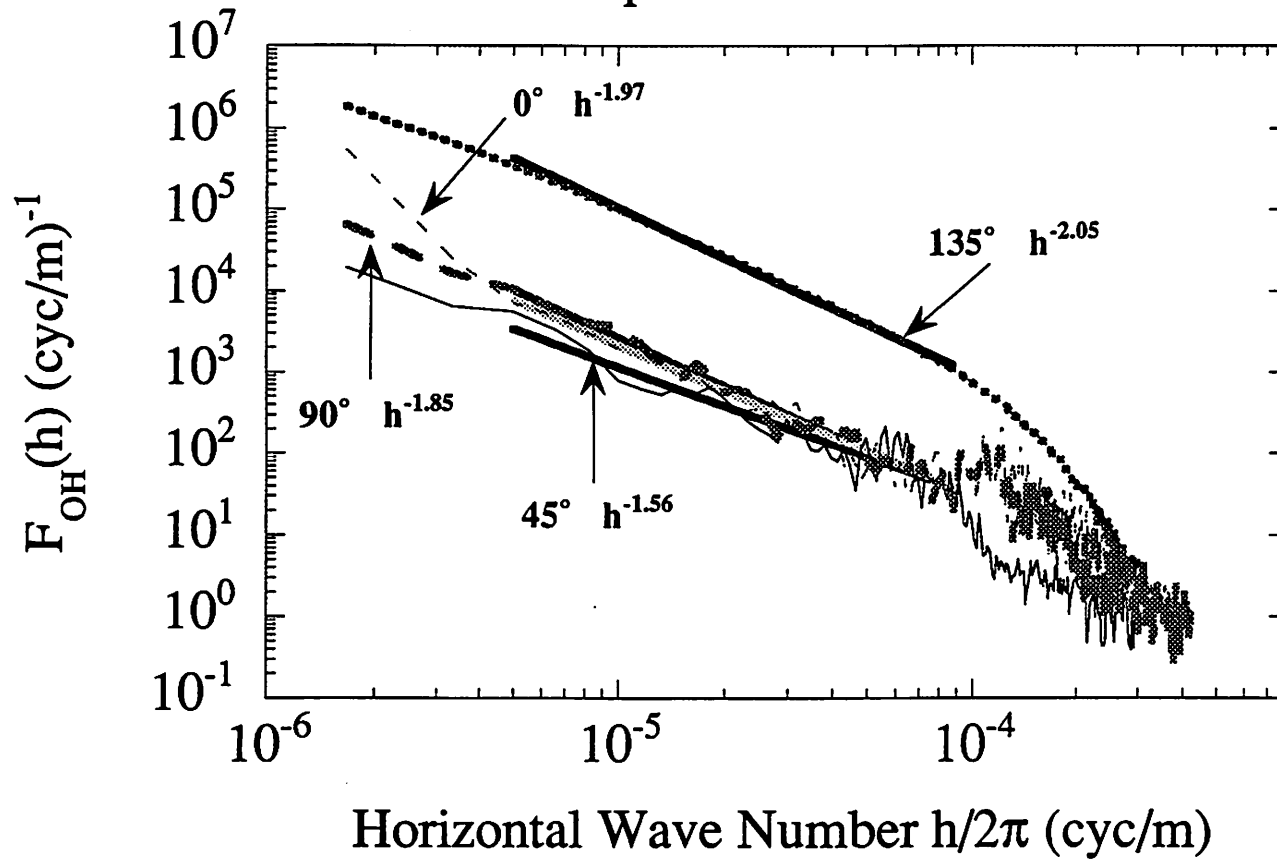
Integrated h-Spectrum



ϕ -Spectrum



Starfire Spectra 10 NOV 1995



Summary

- **Shear and convective instabilities, Doppler spreading, and diffusion (including wave-induced diffusion) all contribute to wave damping and dissipation.**
- **The relative importance of these effects can only be determined by testing the fundamental physics underpinning the wave damping and saturation theories.**
- **The validity of the separability assumptions for the (m, ω) and (h, ϕ) spectra must also be tested.**
- **None of the existing theories incorporate the effects of wave field anisotropies. Observations of the horizontal structure of the wave field with ground-based and space borne imagers can facilitate theoretical progress.**
- **The dominate physics governing wave dissipation must be understood before GW effects can be accurately modeled and incorporated in the global circulation models.**