The Information Content of the Aurora

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SRI International

CEDAR

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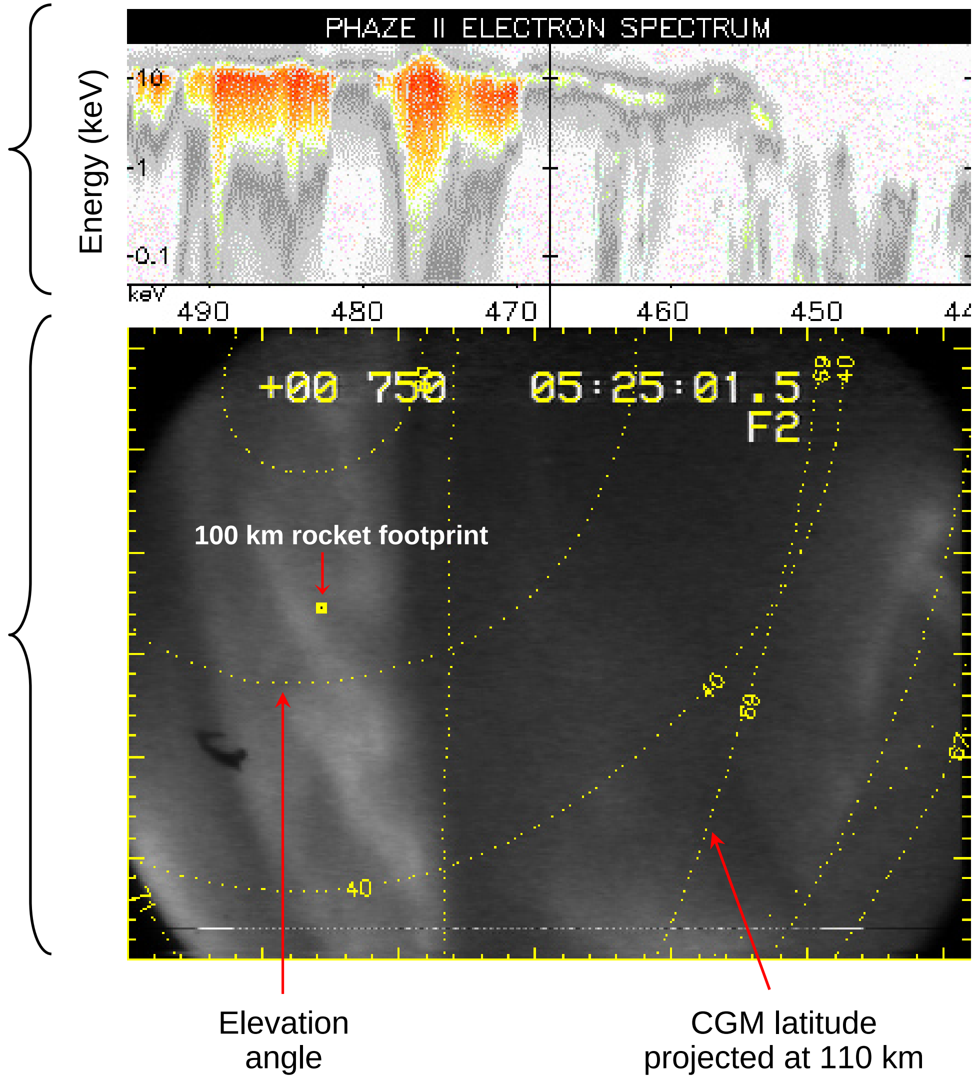
ref.v00-053/v0

PHAZE 2 Movie

Electron differential energy spectrum

$$\left[\frac{\text{eV}}{\text{eV}\,\text{cm}^2\,\text{s}\;\;\text{ster}}\right]$$

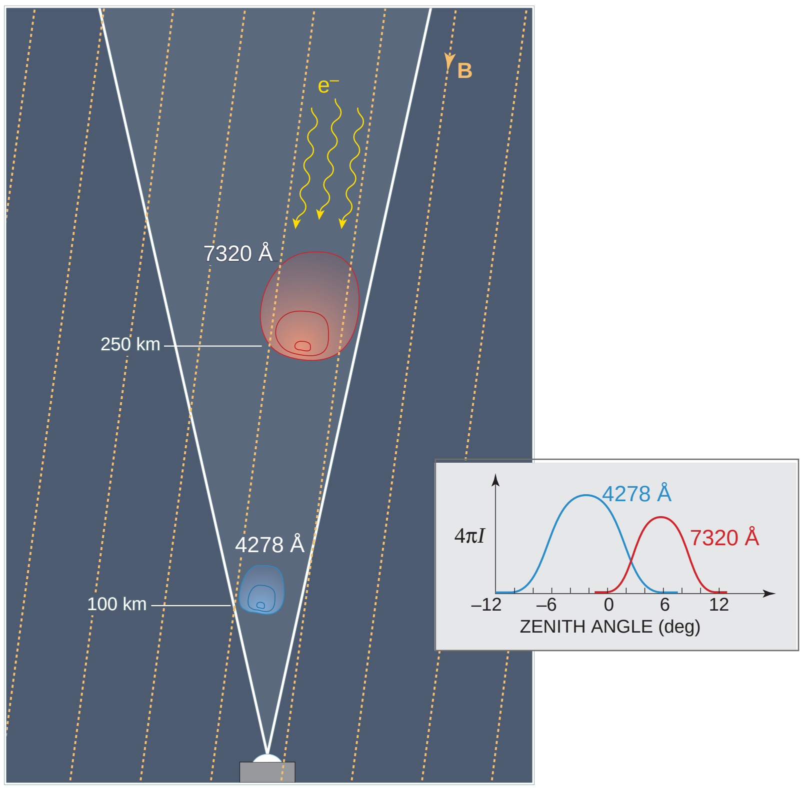
Wide angle 5577Å image

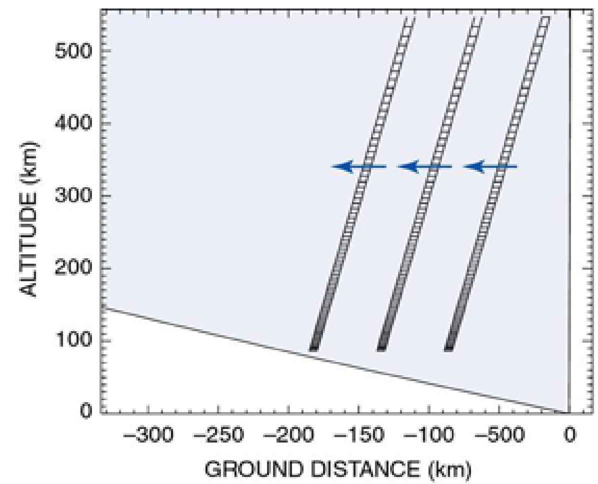


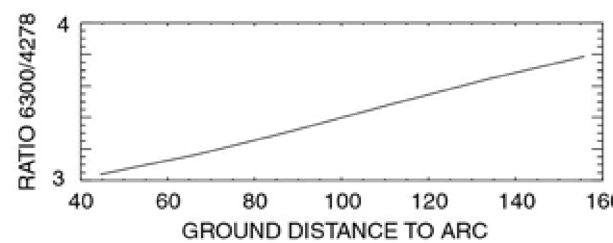
Arnoldy et al., JGRA, 1999; Semeter et al., JGRA, 2001

Color Separation

- **Brightness ratio (e.g. red/blue) is also indeterminate from single station**

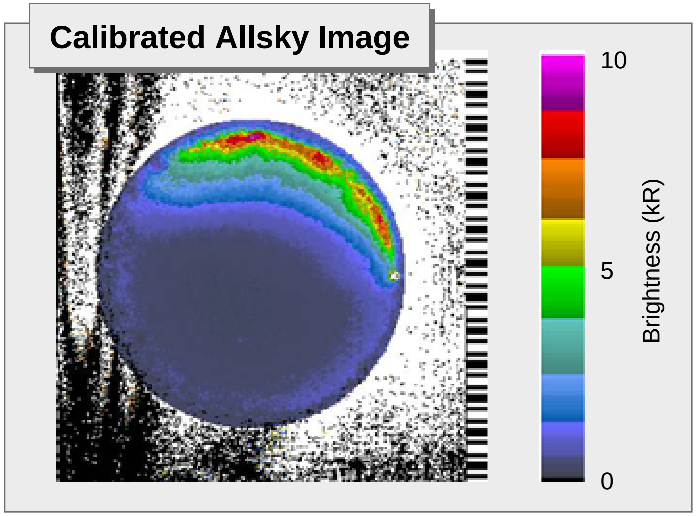


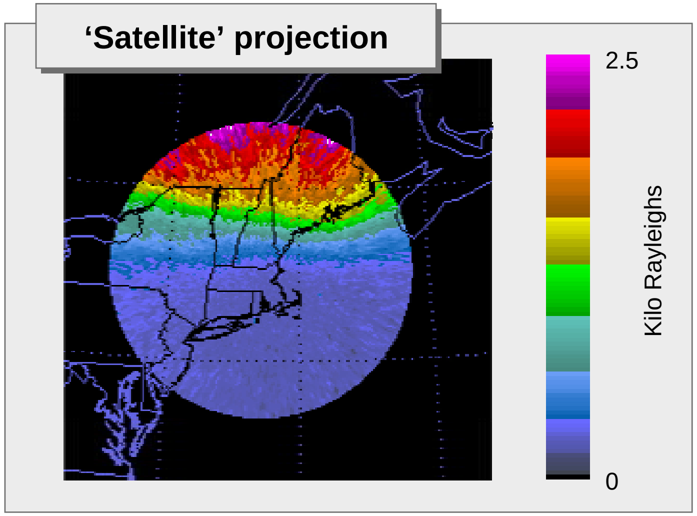




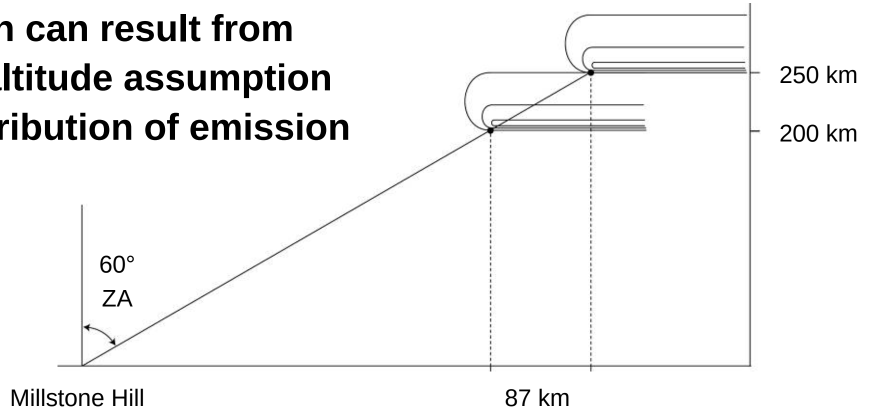
Geographic Projections

Assumes thin layer at constant altitude





- **Misinterpretation can result from**
  - **Erroneous altitude assumption**
  - **Vertical distribution of emission**



Semeter et al., JASTP, 2001

The Rayleigh: A Tomographic Unit

"Because of a lack of general understanding of the units, there was some confusion and misuse of the observational data."

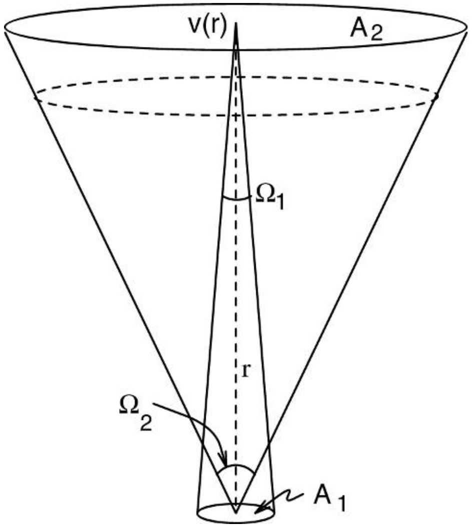
Joseph Chamberlain, 1961

$$I = \text{Surface brightness} = \frac{1}{4\pi}\int_0^\infty v(r)\,dr\left[\frac{\text{photons}}{\text{cm}^2\ \text{s ster}}\right]$$

$$4\pi I = 1\ \text{Rayleigh} = \int_0^\infty v(r)\,dr\left[\frac{\text{photons}}{\text{cm}^2\ \text{s(column) ster}}\right]$$

$4\pi I$ is not a flux

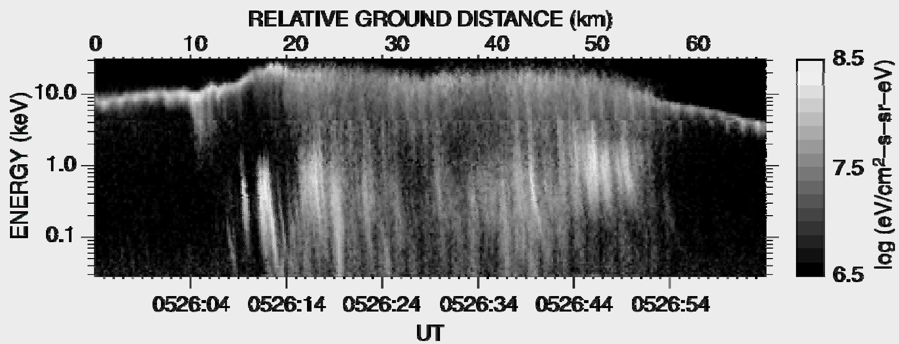
$4\pi I$ is the integrated volume emission rate when the source is isotropic and there is no scattering or extinction in the lower atmosphere. In that case it is also then the integrated rate of atomic or molecular energy transitions related to some excitation process.



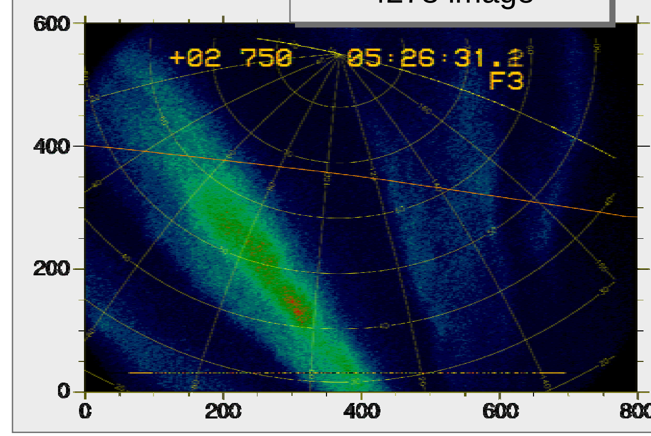
$$A_1\Omega_2 = A_2\Omega_1$$

Forward Model of Auroral Optical Emissions

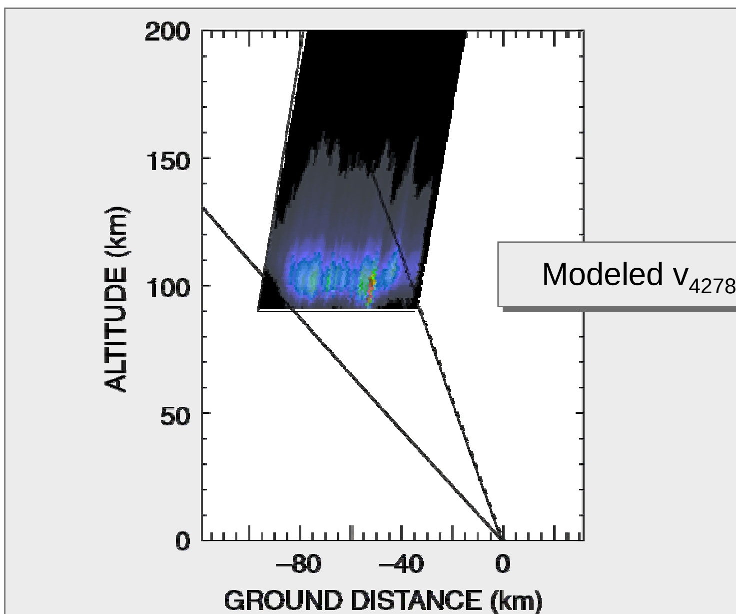
Electron spectrum



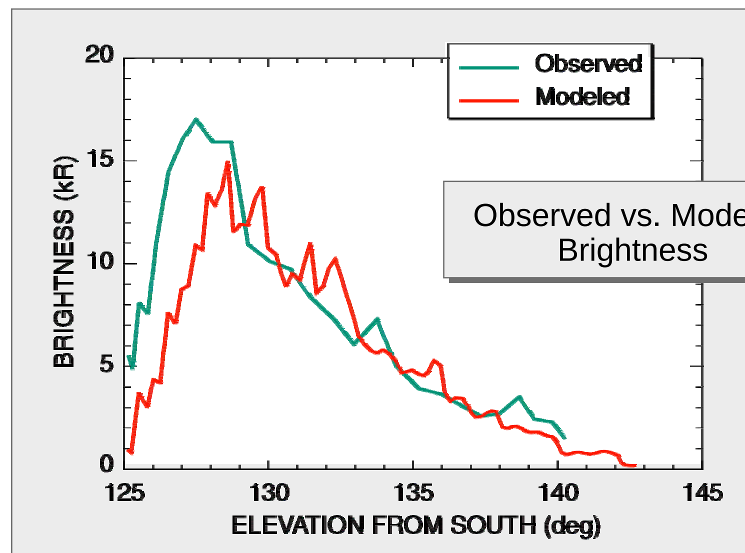
4278 image



Modeled v_{4278}



Observed vs. Modeled Brightness



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Overview

- **Energy transport, energy conversion, and auroral production**
- **Atmospheric response to energetic particles**
- **Inferring the incident energy spectrum from ground-based observations**

  $\Rightarrow$ **The tomographic approach**

  $\Rightarrow$ **The multispectral approach**
- **Simultaneous spatial-spectral-temporal analysis**

Energy Conservation in Auroral Magnetosphere

$$\frac{\partial}{\partial t}(\frac{1}{2}\rho u^2 + U) + \nabla \cdot \left[(\frac{1}{2}\rho u^2 + U)\mathbf{u} + p\mathbf{u} + \mathbf{q}\right] = \mathbf{j} \cdot \mathbf{E} + \rho \mathbf{u} \cdot \mathbf{F}_g / m$$

Consider the steady state simplification:

$$\nabla \cdot \mathbf{Q} = \mathbf{j} \cdot \mathbf{E} \qquad \text{where} \qquad \mathbf{Q} = (\frac{1}{2}\rho u^2)\mathbf{u} = \mathbf{j} \cdot \mathbf{E}$$

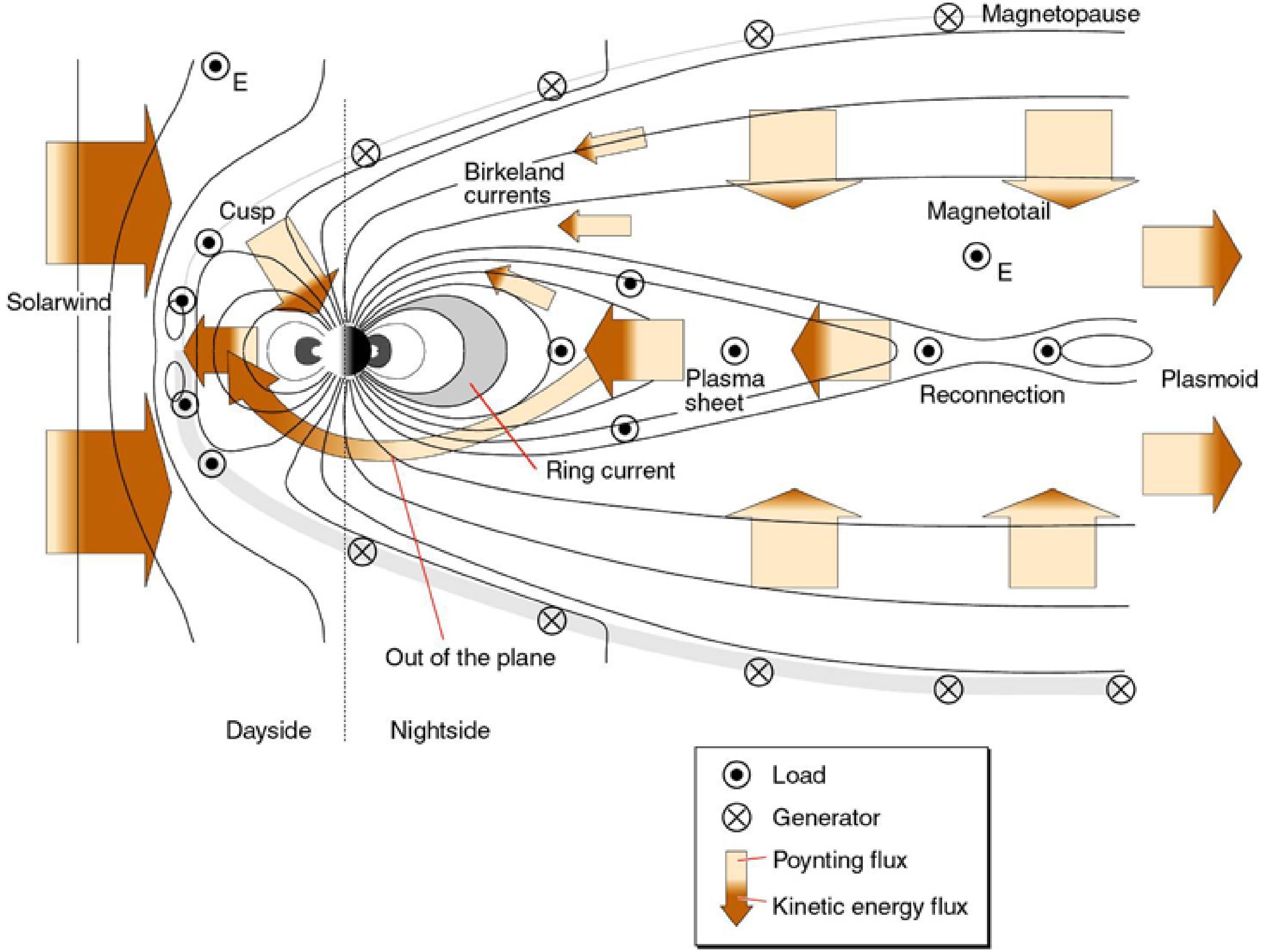
and electromagnetic energy conservation (Poynting's Theorem):

$$\nabla \cdot \mathbf{P} = -\mathbf{j} \cdot \mathbf{E} \qquad \text{where} \qquad \mathbf{P} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_o}$$

Energy is either carried by the electromagnetic field or by the particles.
An electric field appears wherever conversion is taking place such that

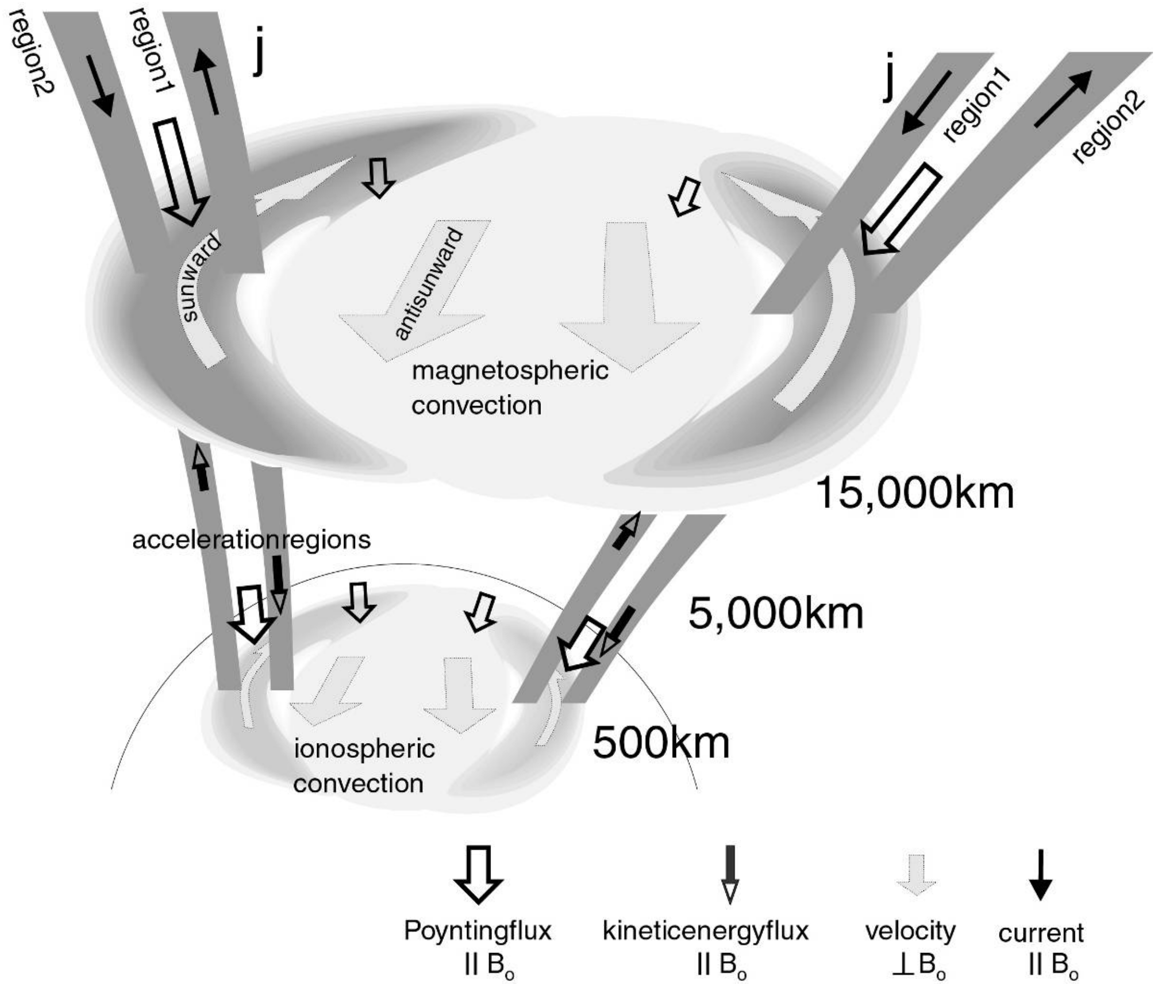
$$\nabla \cdot (\mathbf{P} + \mathbf{Q}) = 0$$

Large Scale Magnetospheric Energy Transport



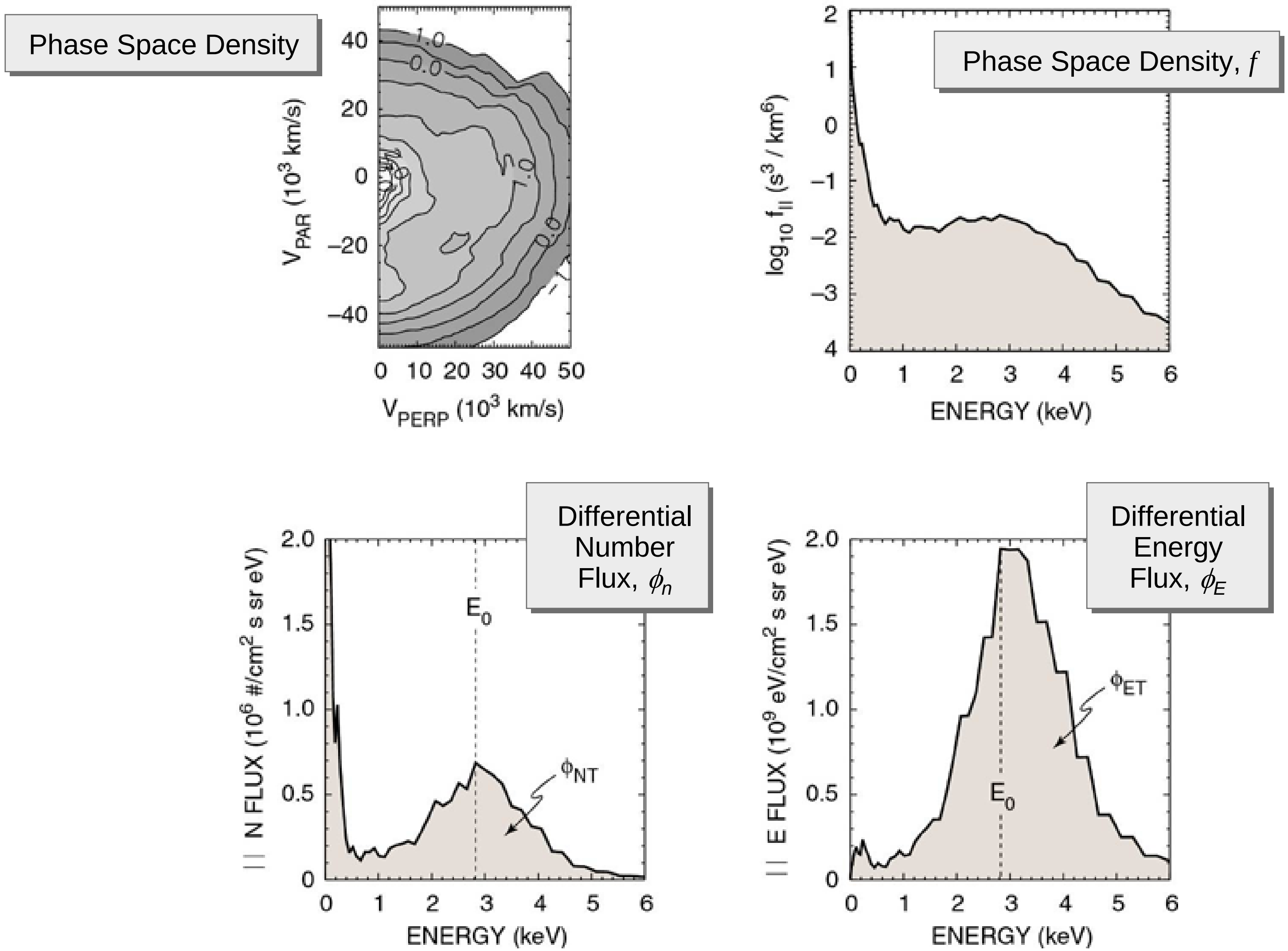
After Vaivads, Ph.D. Dissertation

Energy Transport in the Auroral Zone



Vaivads, Ph.D. Dissertation, 1997

Velocity Distribution Functions for Energetic Aurora



Semeter et al., JASTP, 2001

Current-Voltage Relationship of the Acceleration Region

It is difficult to directly measure the energy conversion rate $j_{||} E_{||}$ in the acceleration region.

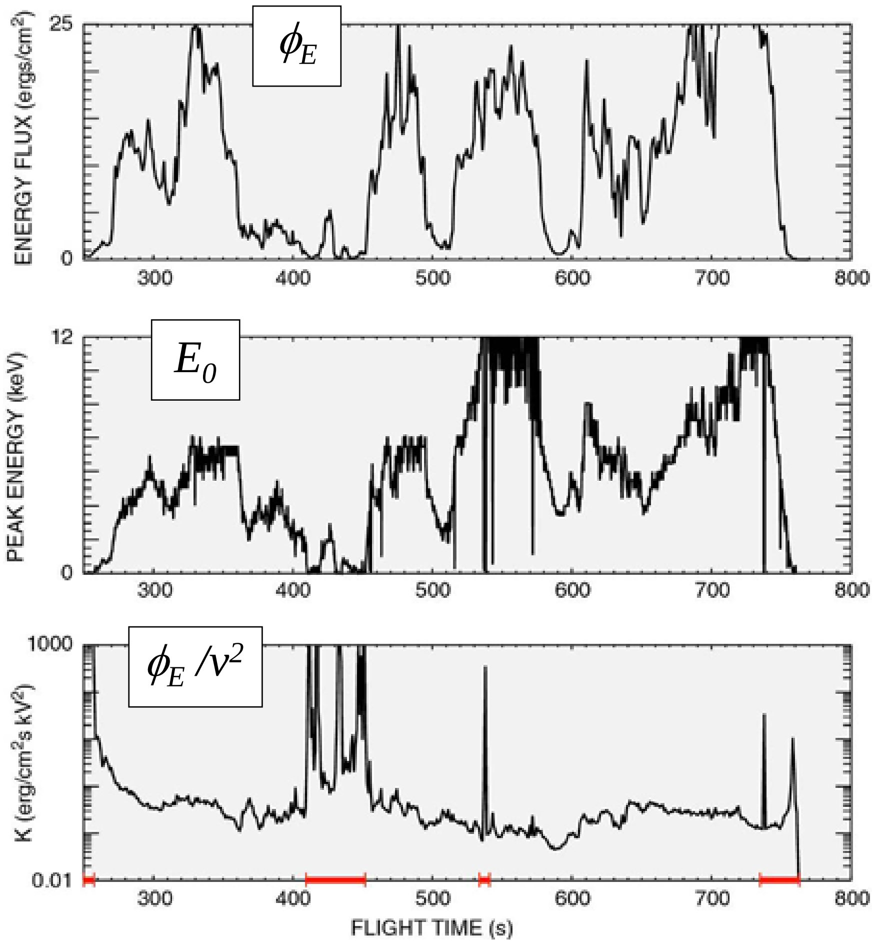
But the related quantities $V = -\int E_{||} \partial l$ and ϕ_E can be estimated from the measured f.

A relationship between ϕ_E and V was derived by Fridman and Lemaire [1981] based on the kinetic model of Knight [1973]. To first order:

$$\phi_E = 0.846 \frac{N_e (E_{0||})^{1/2}}{E_{0\perp}} V^2 \qquad (1)$$

The proportionality was observed experimentally by Lyons et al. [1979]. Does it hold in general?

ϕ_E / v^2 for PHAZE 2

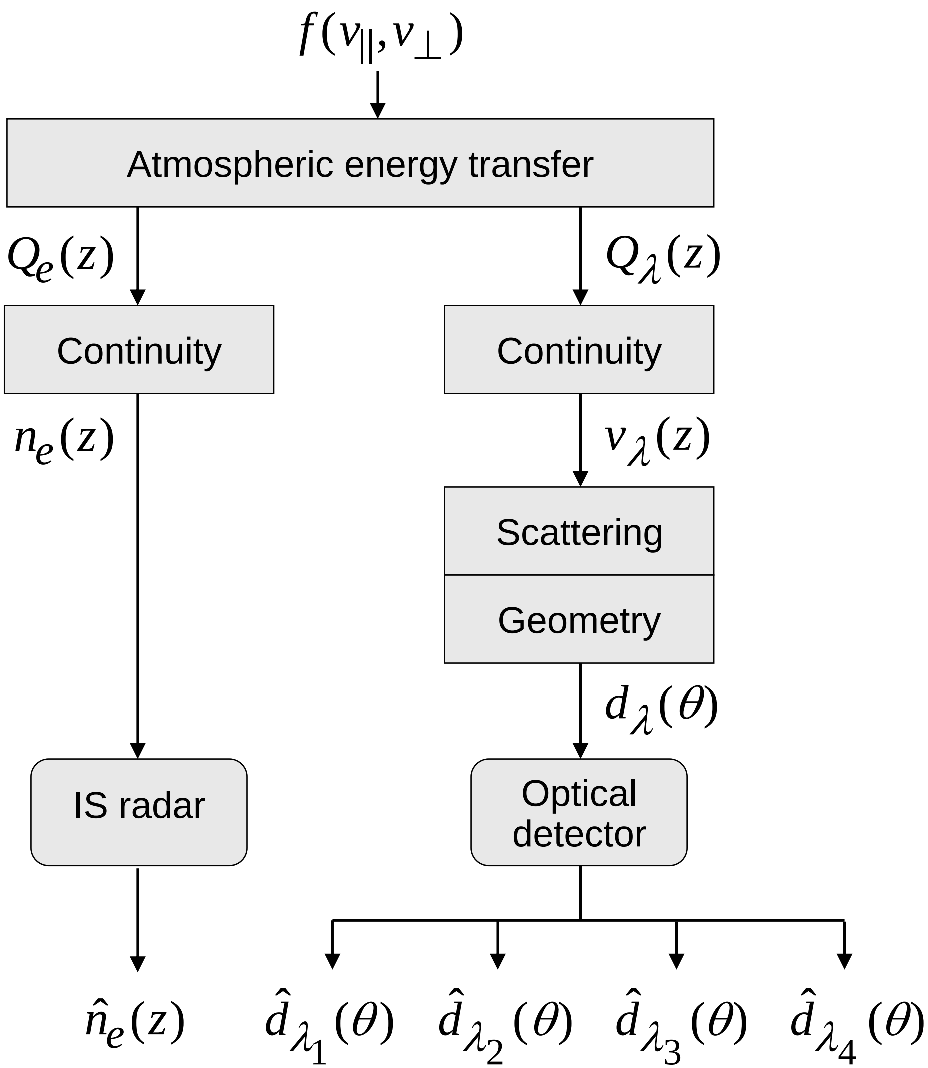


The simple relationships predicted by kinetic theory do not hold in regions where $\phi_E(E)$ is non-Maxwellian, i.e.,

- **When the aurora is very weak**
- **When the aurora is very energetic**
- **When the aurora is very turbulent**

Semeter et al., JGRA, 2001

Atmospheric Response to Energetic Particles



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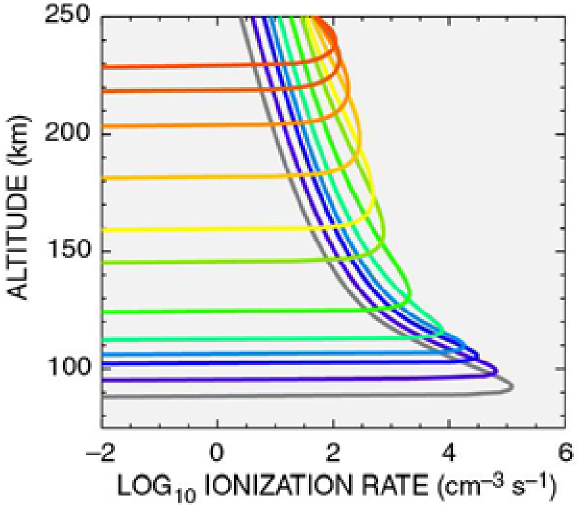
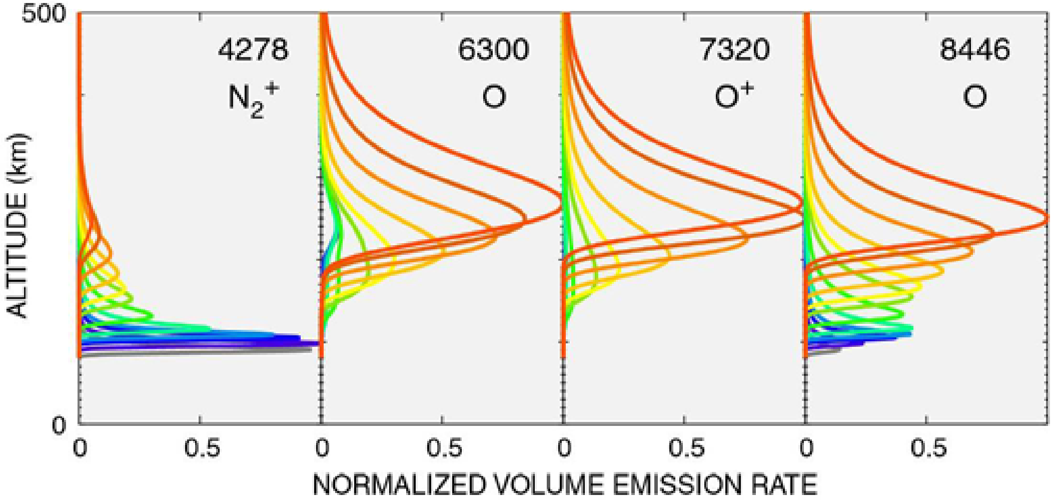
Volume Production Profiles

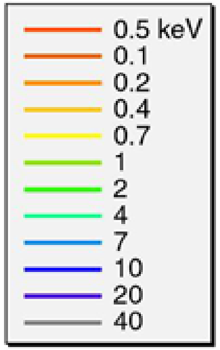
$$q(z) = \int_{allE} Q(E,z)\phi(E)dE$$

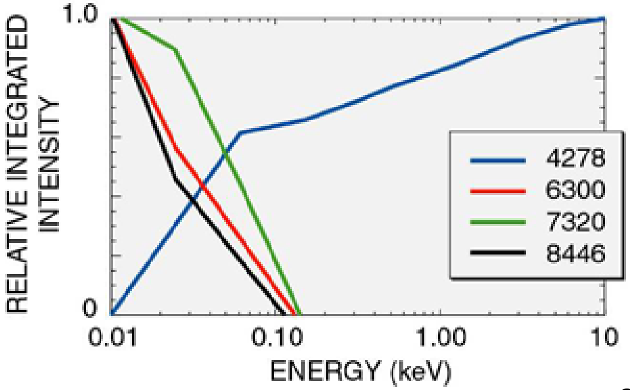
$$n_e(z) = \sqrt{q_e(z)/\alpha(z)}$$

$$v_\lambda(z) = \frac{q_\lambda(z)}{\sum_j k_j n_j(z)}$$



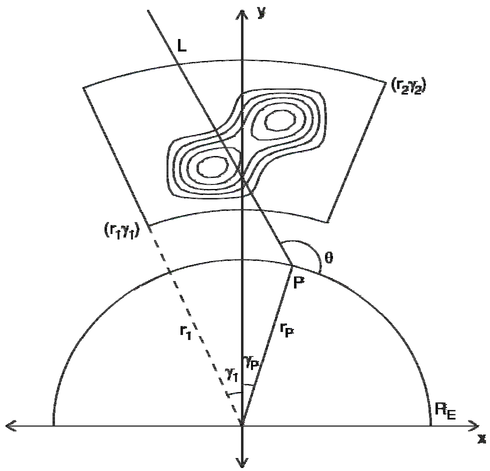






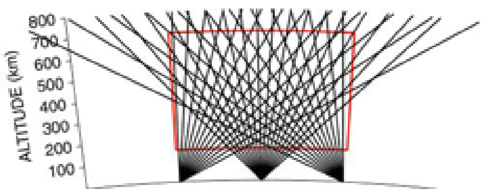
Semeter et al., JASTP, 2001

The Auroral Tomography Problem



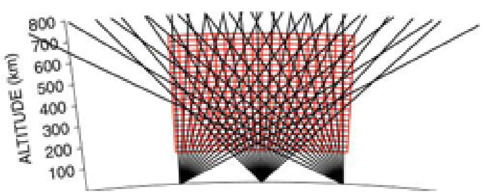
Continuous problem

$$b(\theta) = \int_{\Omega} L(\mathbf{r},\theta)\,v(\mathbf{r})\,d\mathbf{r}$$



Ill-posed problem

$$b(\theta_k) = \int_{\Omega} k(\mathbf{r},\theta_k)\,v(\mathbf{r})\,d\mathbf{r}$$



Ill-conditioned problem

$$b(\theta_k) = \sum_j L(\mathbf{r}_j,\theta_k)\,v(\mathbf{r}_j)\,\Delta\mathbf{r}_j$$

$$b = Lv$$

Semeter et al., IEEE TGARS, 1997.

Forward Auroral Tomographic Equation with Pixel Basis

Canonical forward equation: $d(\theta) = \int_\Omega L(\mathbf{r}, \theta) v(\mathbf{r}) d\mathbf{r} + n(\theta)$

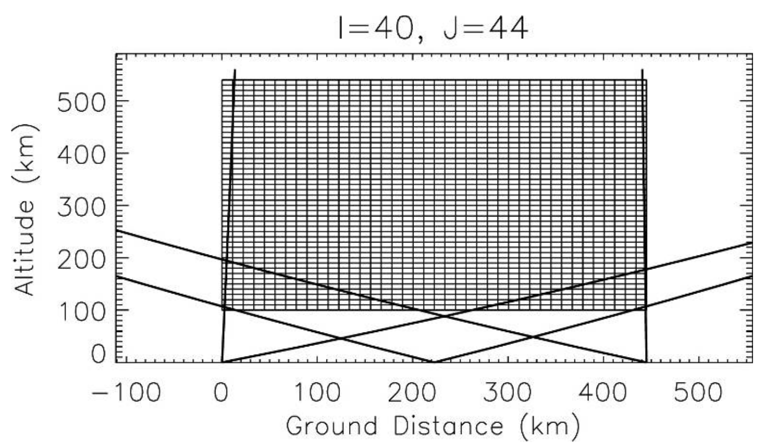
$$\begin{bmatrix} d_1 \\ d_2 \\ \cdot \\ \cdot \\ d_N \end{bmatrix} = \begin{bmatrix} l_{11} & l_{12} & \cdot & \cdot & l_{1M} \\ l_{21} & \cdot & & & \cdot \\ \cdot & & \cdot & & \\ \cdot & & & \cdot & \\ l_{N1} & & & & l_{NM} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ v_M \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \cdot \\ \cdot \\ n_N \end{bmatrix} \iff \boxed{\mathbf{d} = \mathbf{L}\mathbf{v} + \mathbf{n}}$$

$M = I \times J =$ # unknowns

$N = K_1 + K_2 + K_3 =$ # measurements

$N > M \rightarrow$ no solution $\rightarrow$ least squares

$M < N \rightarrow$ infinite solutions $\rightarrow$ "best" one satisfies a side constraint (e.g., least squares minimum norm)



A Unifying Perspective: Tikhonov Regularization

$$\hat{\mathbf{x}}_{\text{Tik}} = \underset{\mathbf{x}}{\text{argmin}} \quad \underbrace{\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2}_{\text{Data Fidelity}} + \underbrace{\gamma^2 \|\mathbf{L}\mathbf{x}\|_2^2}_{\text{Prior Info}}$$

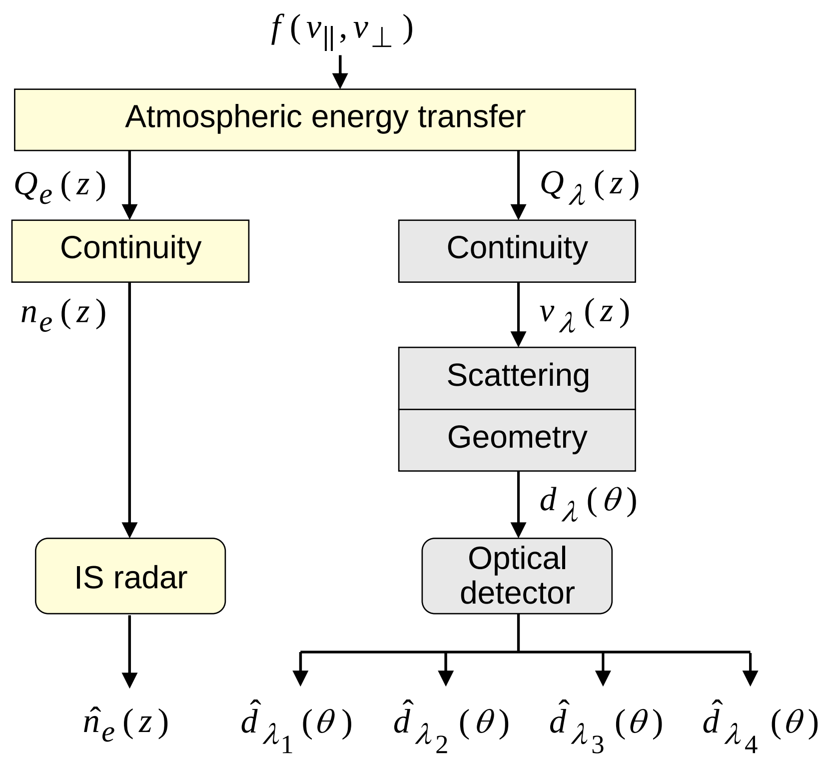
- $\mathbf{L} = \mathbf{I}, \gamma = 1 \Rightarrow$ Least squares minimum norm solution
- $\mathbf{R}_x = r_{\mathbf{x}}\mathbf{I}, \mathbf{R}_n = r_{\mathbf{n}}\mathbf{I}, \gamma = r_{\mathbf{n}}/r_{\mathbf{x}}, \mathbf{L} = \mathbf{I} \Rightarrow$ MAP with prior probability $p_{\mathbf{X}}(\mathbf{x}) \propto e^{-\gamma^2 \mathbf{x}^{\mathbf{T}} \mathbf{L}^{\mathbf{T}} \mathbf{L}\mathbf{x}}$
- $\mathbf{L}\mathbf{x} = \ln \mathbf{x} \Rightarrow$ Maximum Entropy or MART

Estimating $\phi_n(E)$ from $\hat{n}_e(z)$

Consider a classic inverse problem associated with the left-hand side of this chart, namely

Can $\phi_n(E)$ be recovered from a measured E-region auroral ionization profile?

- **UNTANGLE** (Vondrak & Baron, 1975)
- **CARD** (Brekke et al., 1989)



Forward Model of Arc-Related Ionization

$$q_e(z) = \int_{\text{all E}} Q(E, z)\phi_n(E)dE \tag{2}$$

Appropriate discretization in altitude and energy gives us

$$\mathbf{q} = \mathbf{Q}\phi \tag{3}$$

in steady state with $n_{\text{ions}} = n_e$

$$q = L = \alpha n_e^2 \tag{4}$$

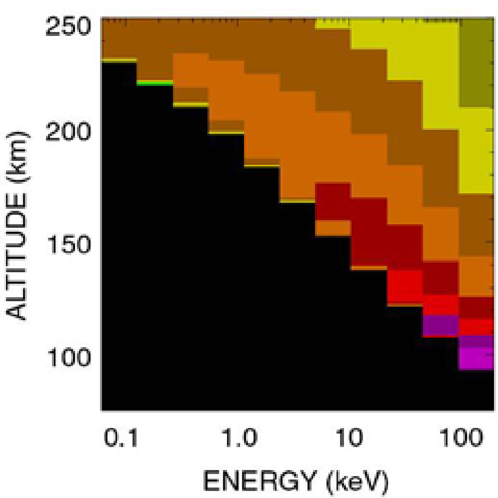
CARD Analysis

Solve linear inverse problem $\mathbf{n_e} = \tilde{\mathbf{Q}}\phi + \mathbf{n}$

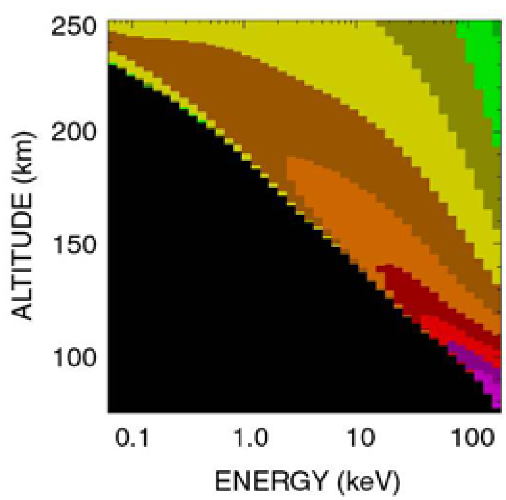
where $\tilde{\mathbf{Q}} = \sqrt{\mathbf{Q}/\alpha}$ and $\mathbf{n}$ is noise.

How many energies can be uniquely resolved?

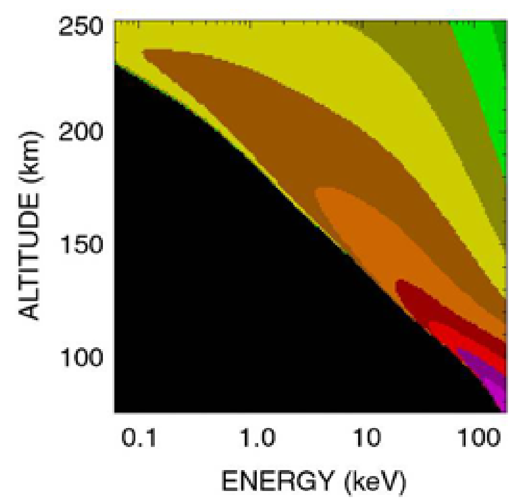
11 energies, min SV = 142



44 energies, min SV = 13



176 energies, min SV = 0



Semeter and Kamalabadi, Radio Science, 2005

Direct Estimation of $\phi_n(E)$ from Ground-Based Photometry

Suppose $v(x,z)$ is related to $\phi_n(E)$ by

$$v(x,z) = \int_{\text{all E}} \phi_n(E,x) Q(E,z) dE \tag{1}$$

Combining this with the tomographic equation gives

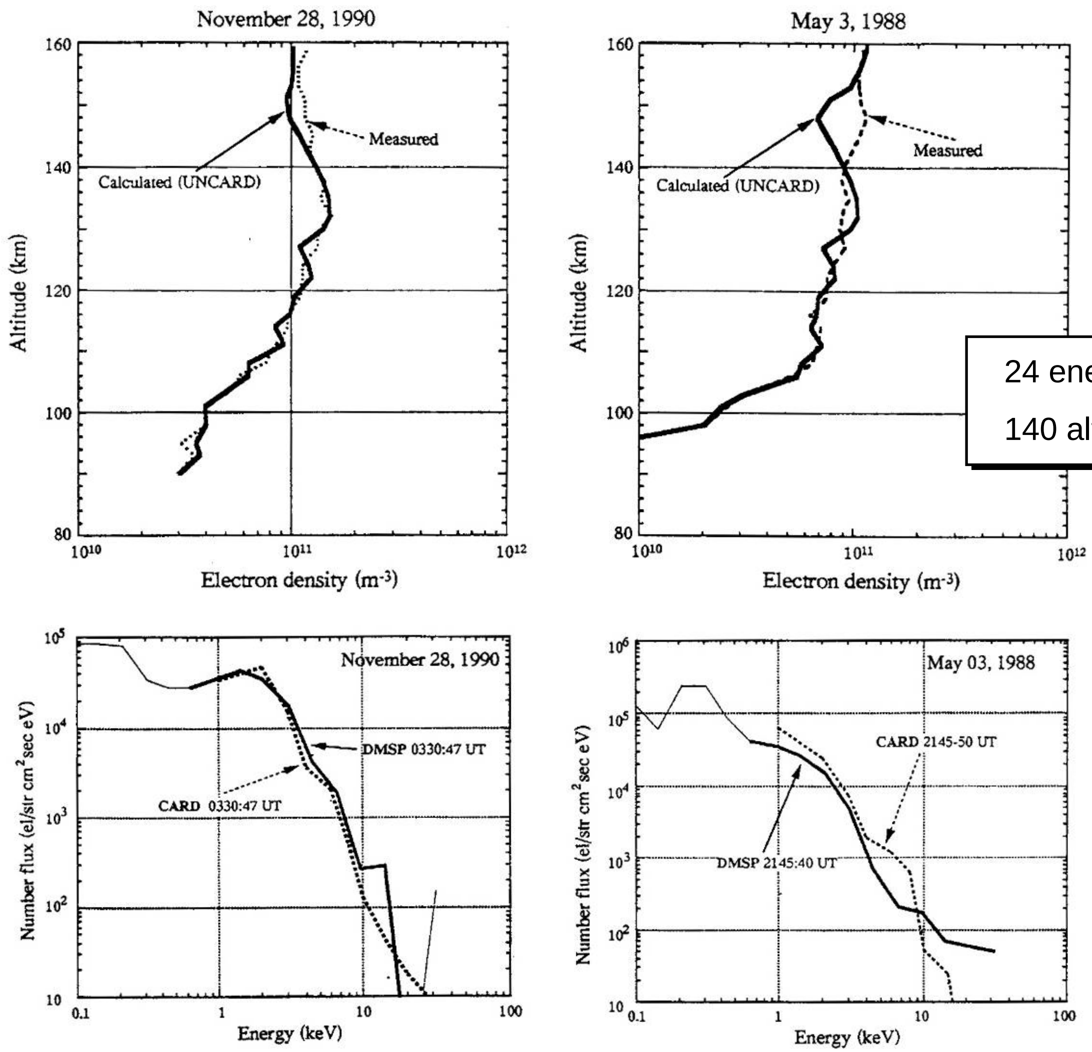
$$d(\theta) = \underbrace{\int_\Omega K(x,z,\theta) \underbrace{\int_{\text{all E}} \phi_n(E,x) Q(E,z) dE dx dz}_{\phi_n(E,x) \Rightarrow v(x,z)} + n(\theta)}_{v(x,z) \Leftrightarrow d_\lambda(\theta)} \tag{2}$$

Or in vector form

$$\mathbf{d} = \mathbf{L}\mathbf{M}\phi + \mathbf{n} \tag{3}$$

Semeter and Kamalabadi, Radio Science, 2005

Experimental test of $\phi_n(E)$ from $n_e(z)$

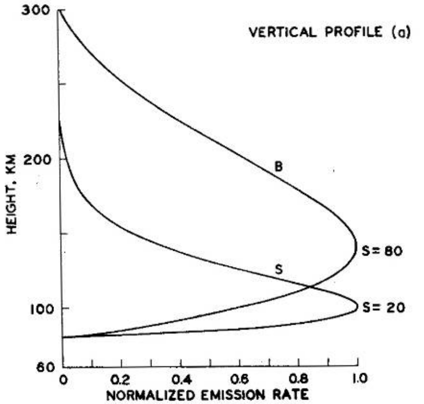


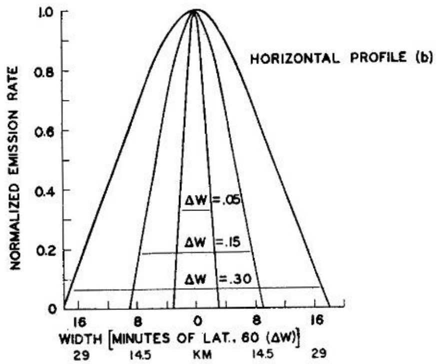
24 energy bins

140 altitude bins

Fujii et al., 1995

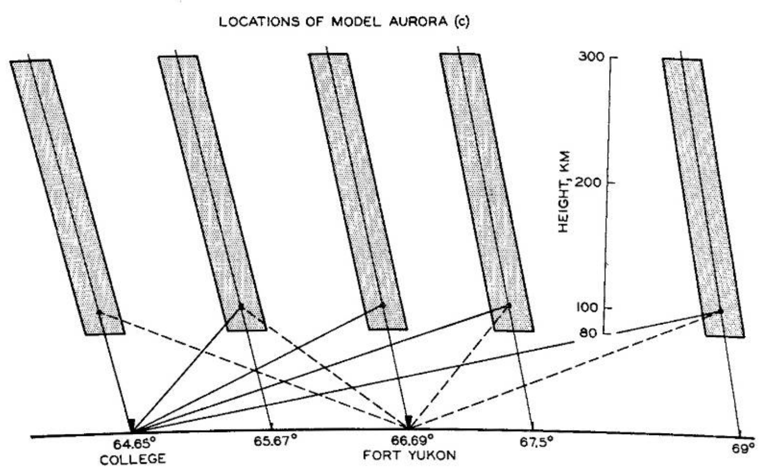
Single Site Measurements: Perspective Effects

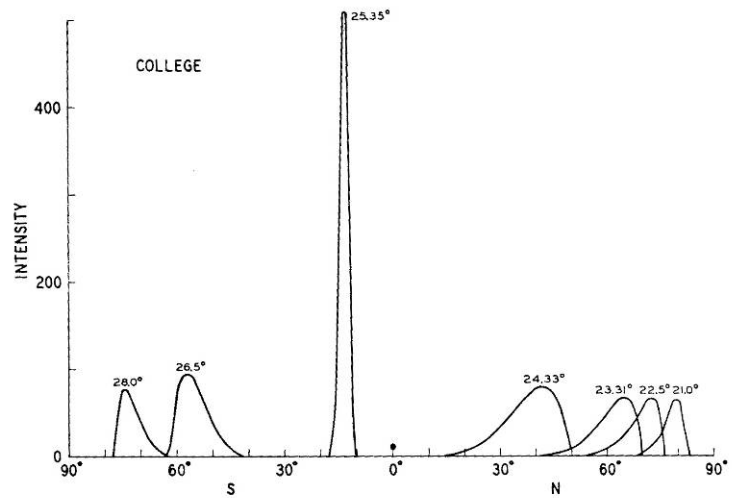




$$v(\text{lat}, \text{alt}) = (H)(V)$$

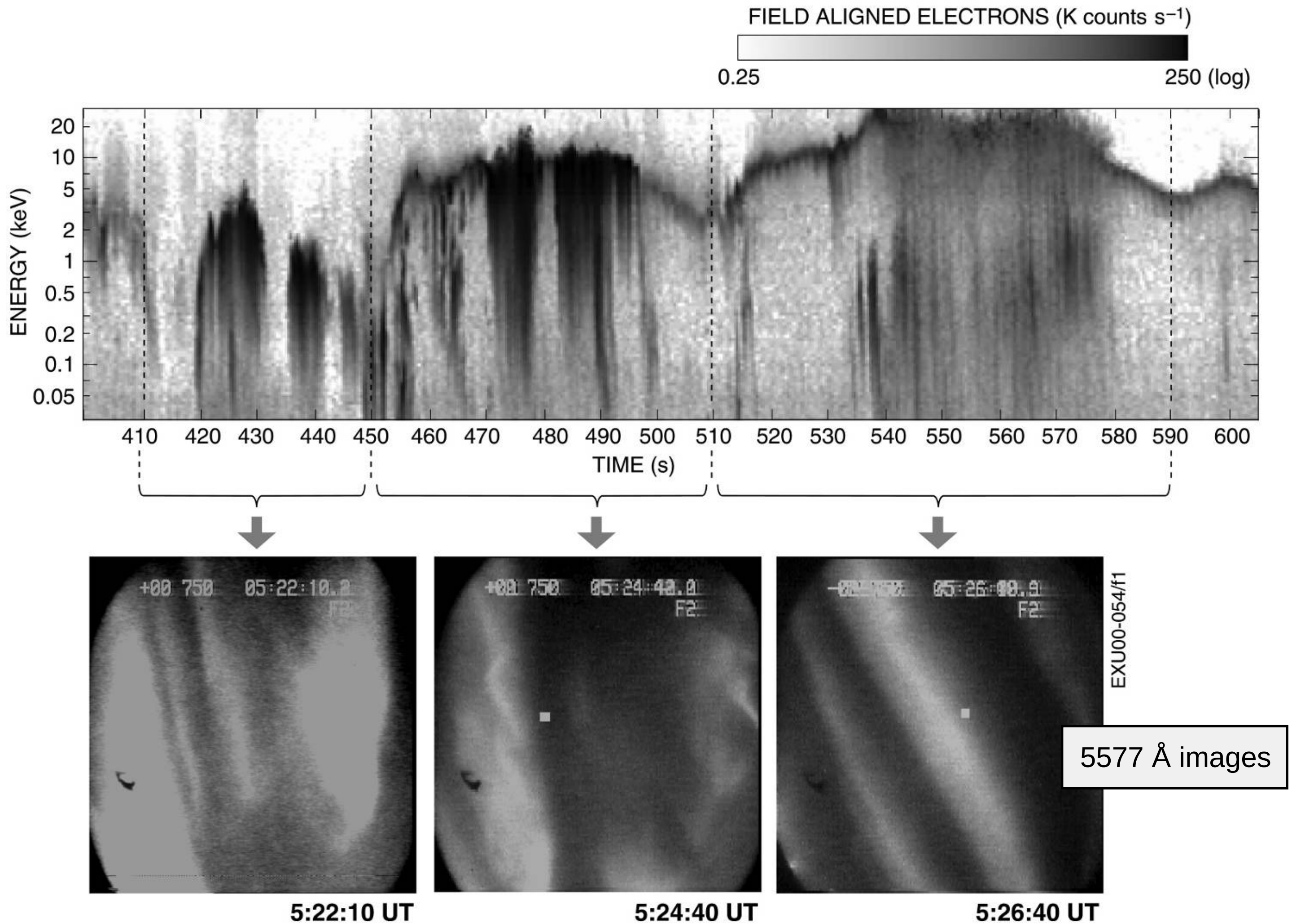
$$b = 10^{-6} \int_{\ell_1}^{\ell_2} HV \sec \chi$$





From Romick and Belon, JATP, 1967

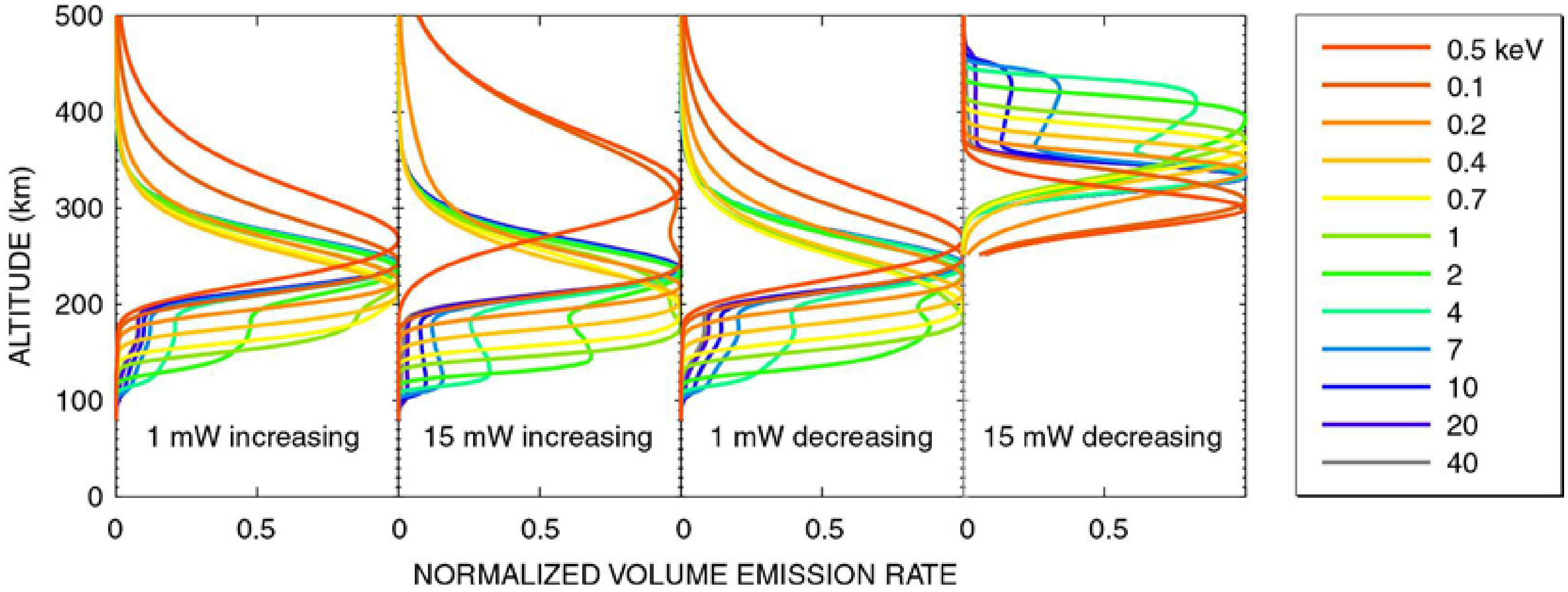
Optical Characteristics of Nonlinear Current/Voltage Relationships



Considerations for Red Line Modeling

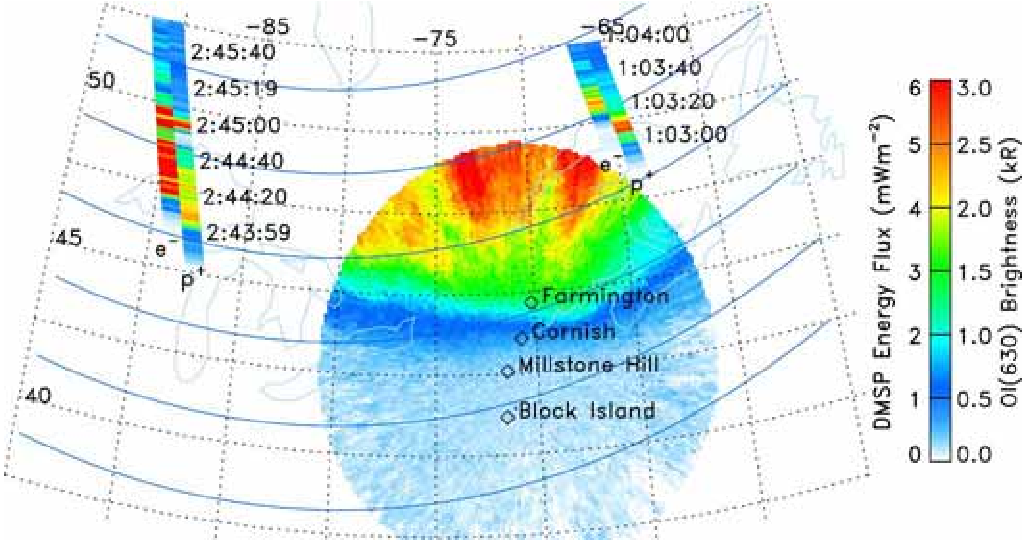
Unlike prompt emissions redline does not vary linearly with energy flux:

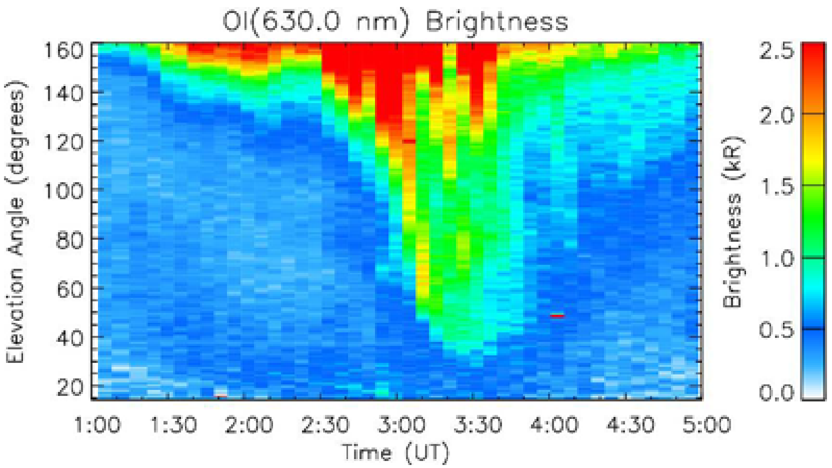
- **The low excitation energy (1.96 eV) of $O(^1D)$ means that heating results in significant production.**
- **The long radiative lifetime (126 s) of $O(^1D)$ causes hysteresis in the emission.**
- **$O(^1D)$ is excited in both proton and electron precipitation.**



Semeter et al., JASTP, 2001

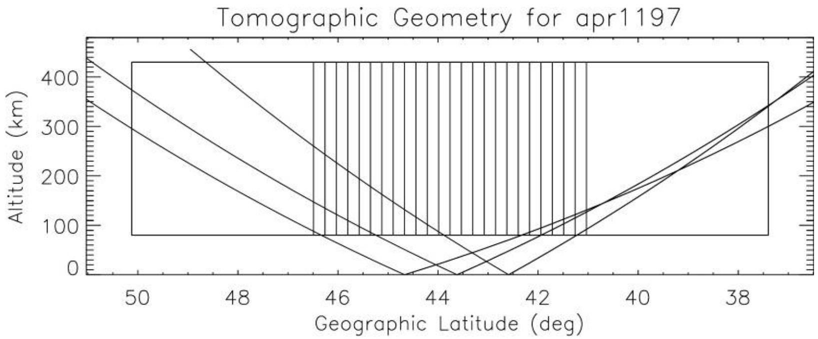
Red Aurora of April 11, 1997

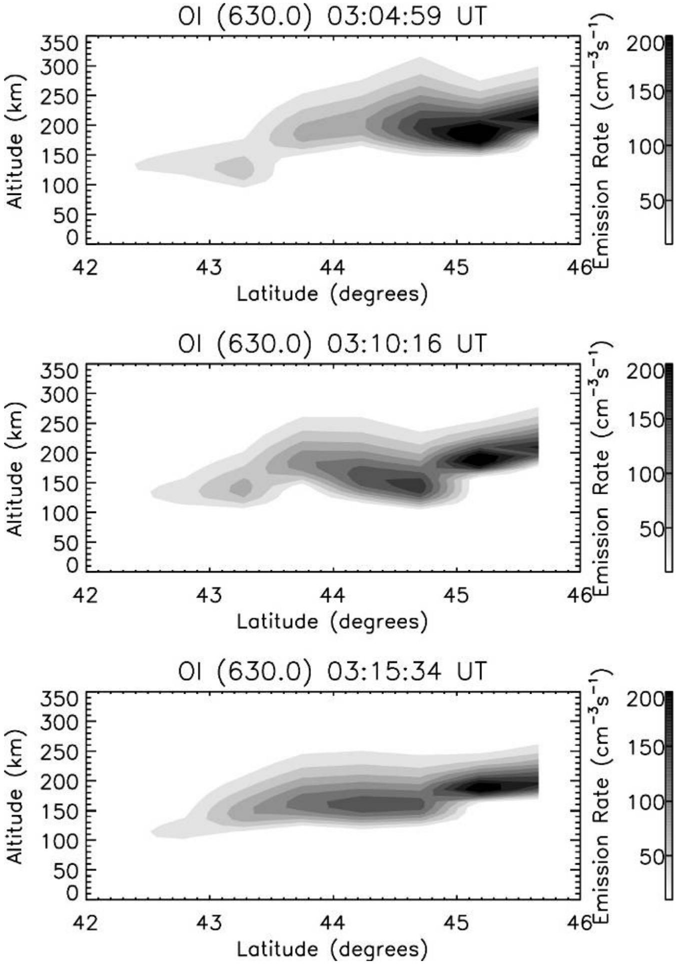




Lummerzheim et al., JGRA, 2001

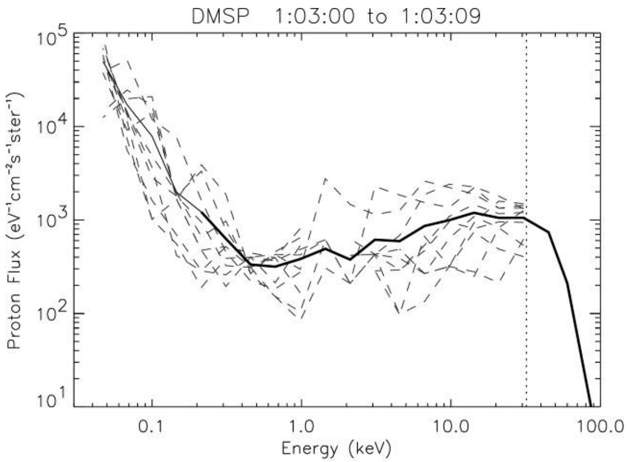
C OTIF Tomographic Analysis of April 11, 1997

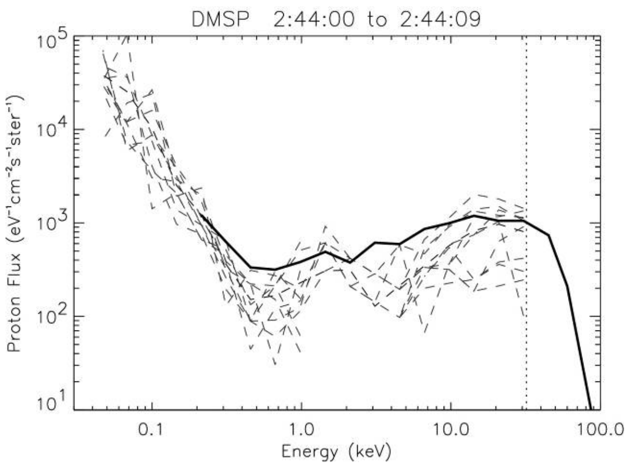


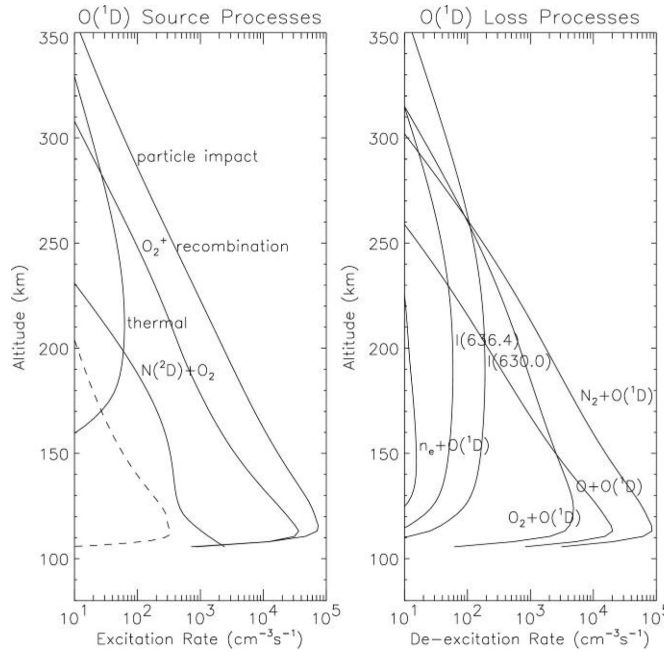


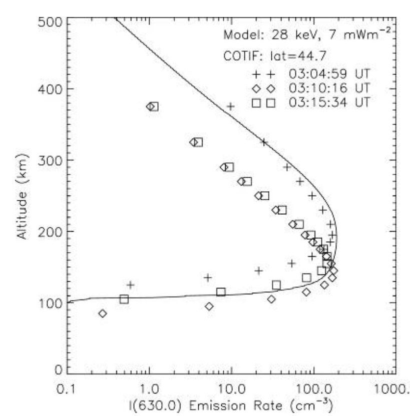
Lummerzheim et al., JGRA, 2001

Explanation for Low Redline Emission Altitude





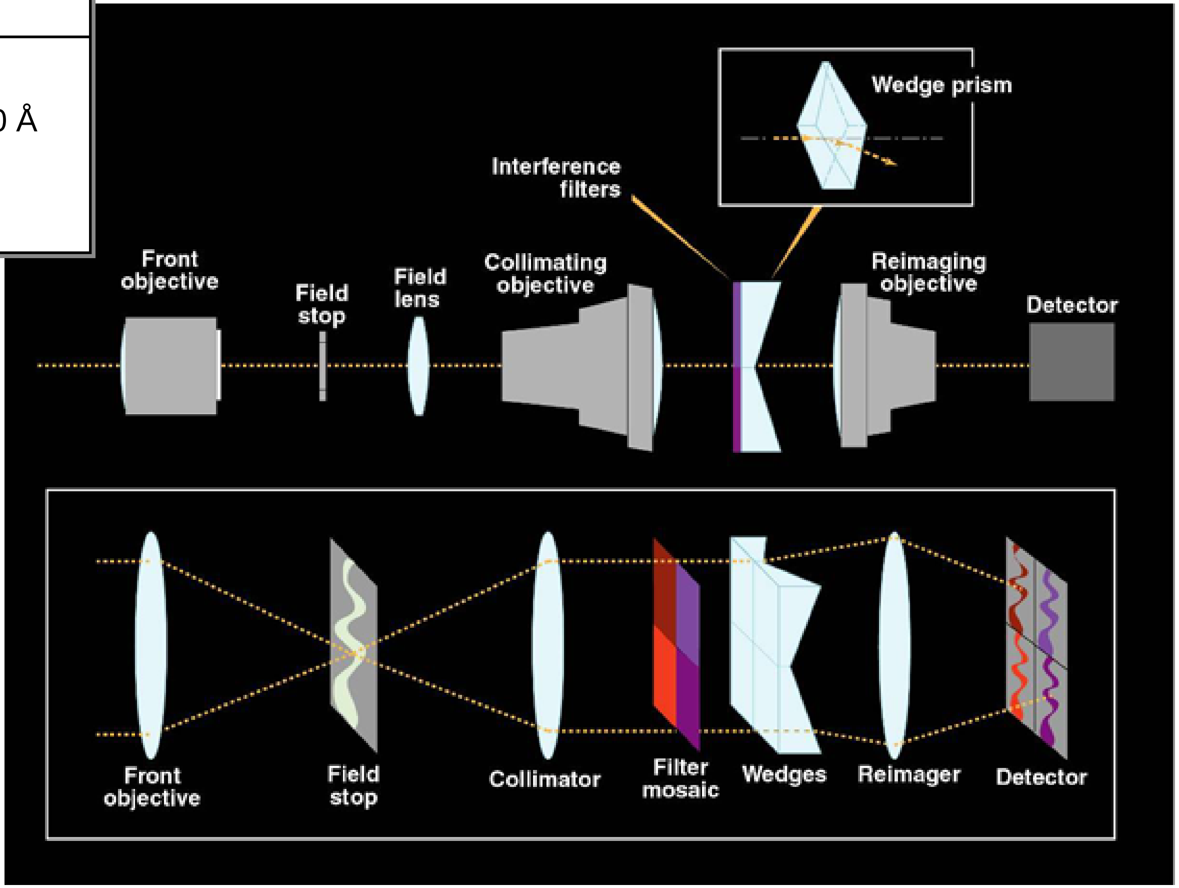




Lummerzheim et al., JGRA, 2001

The Simultaneous Multispectral Imager (SMI)

8446 Å	4278 Å
7320/30 Å	4500 Å



Semeter et al., JASTP, 2001