

# 2001 CEDAR-SCOSTEP

Longmont, Colorado

June 17-22, 2001

CEDAR Prize Lecture

by Hans Mayr

Goddard Space Flight Center, USA

Modeling Wave Driven Non-linear Flow  
Oscillations: The Terrestrial QBO,  
and a Solar Analog

# Modeling Wave Driven Non-linear Flow Oscillations: The Terrestrial QBO, and a Solar Analog

Hans G. Mayr  
in Collaboration with  
John Mengel, Hayden Porter, for QBO  
Richard Hartle, Charles Wolff, for Solar Analog

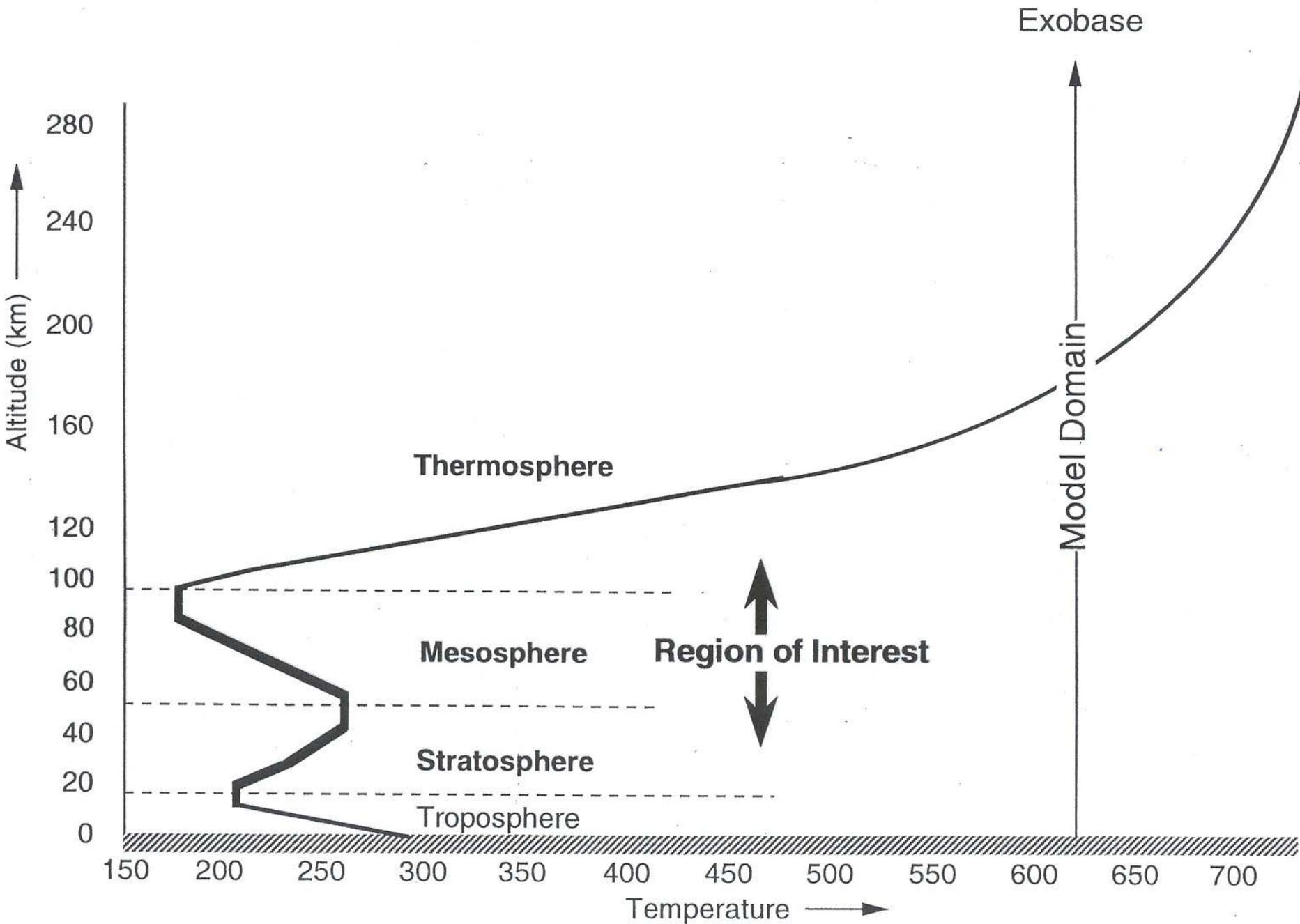
## Terrestrial Quasi-Biennial Oscillation (QBO):

- Lindzen and Holton mechanism, planetary wave forcing
- Gravity wave forcing
- Our numerical model
- QBO & decadal oscillations
- Of non-linear oscillators (QBO, clock)

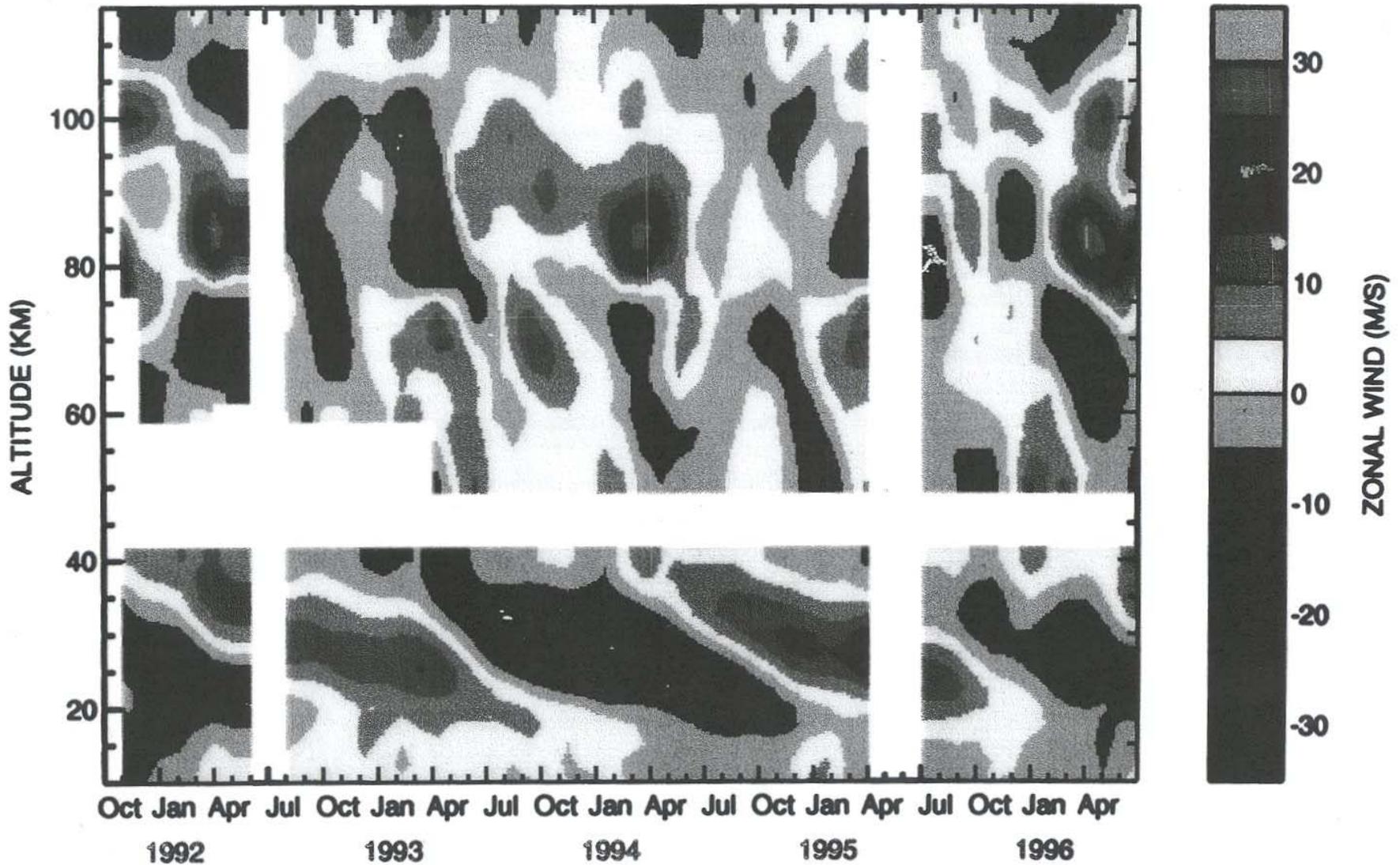
## What we may learn for the Sun from the QBO mechanism:

- Analytic model extracted from terrestrial experience
- Proposed 22-year flow oscillation
- Related dynamo magnetic fields

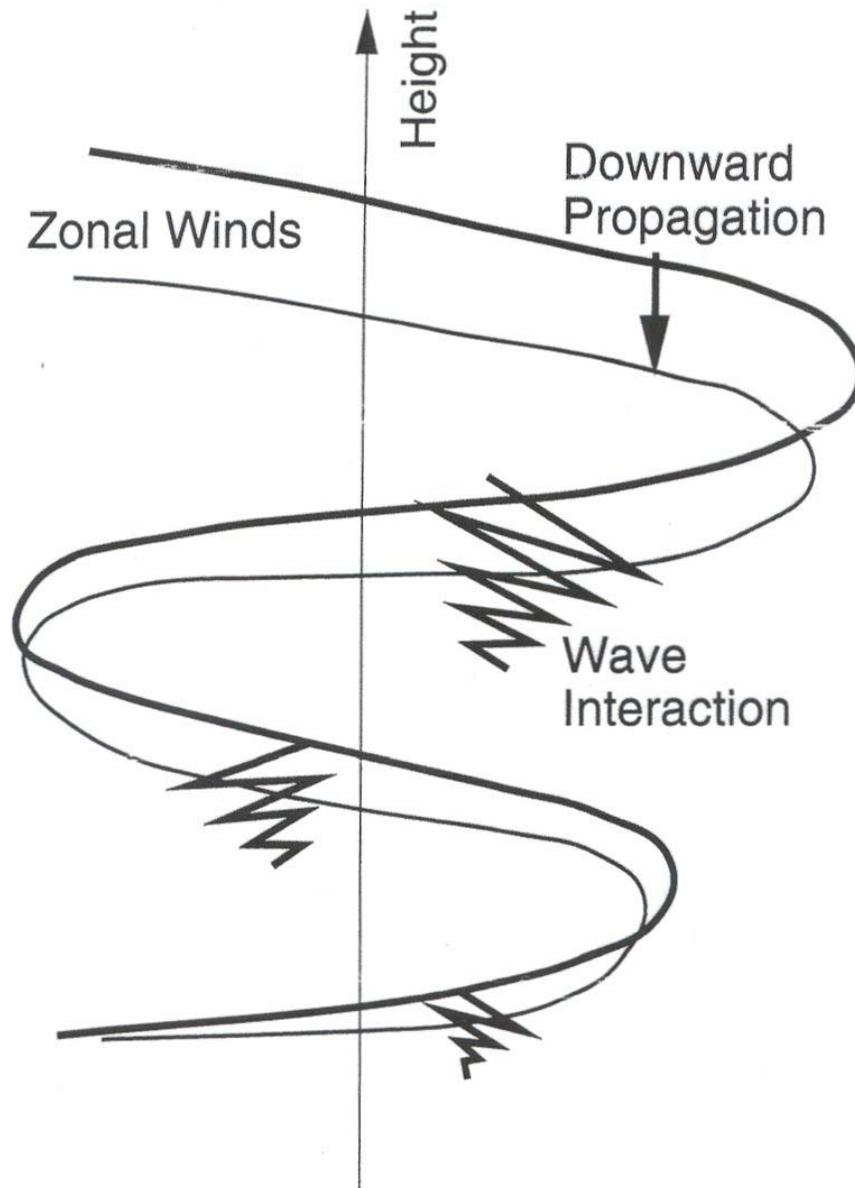
## Conclusions



# HRDI zonal winds at the equator (AO and SAO removed)

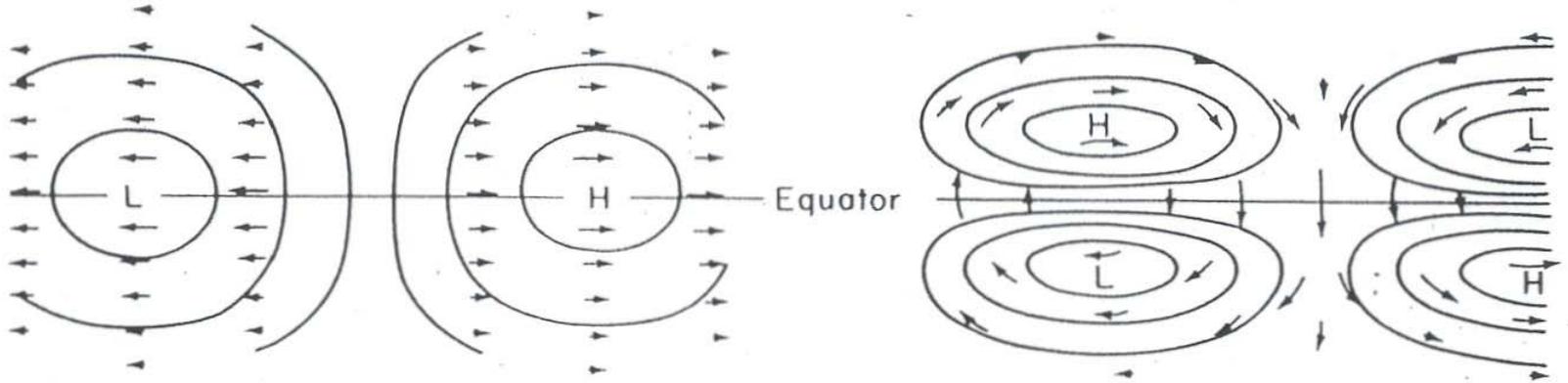


# Quasi-biennial Oscillation (QBO)



Eastward Propagating Kelvin Wave

Westward Propagating Rossby Gravity Wave



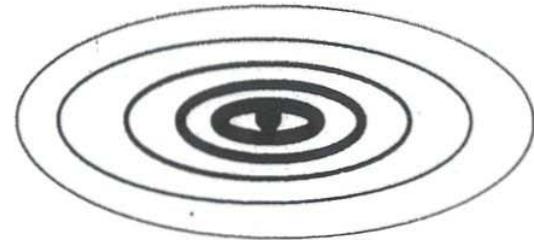
Taken from "An Introduction to Dynamical Meteorology" by James R. Holton

### Gravity Waves

Source

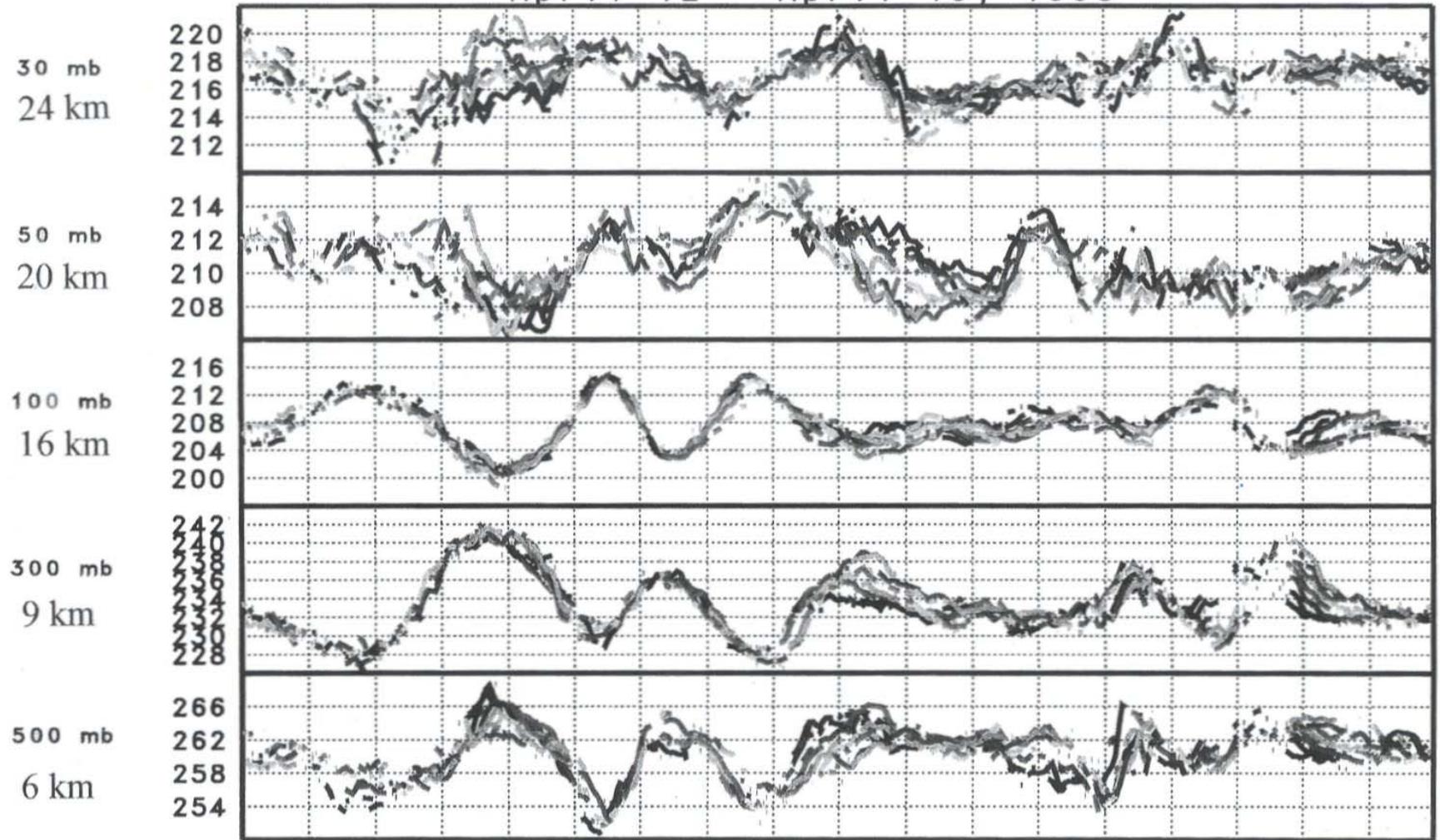


Response



Isotropic Propagation

30 North  
April 12 - April 13, 1998

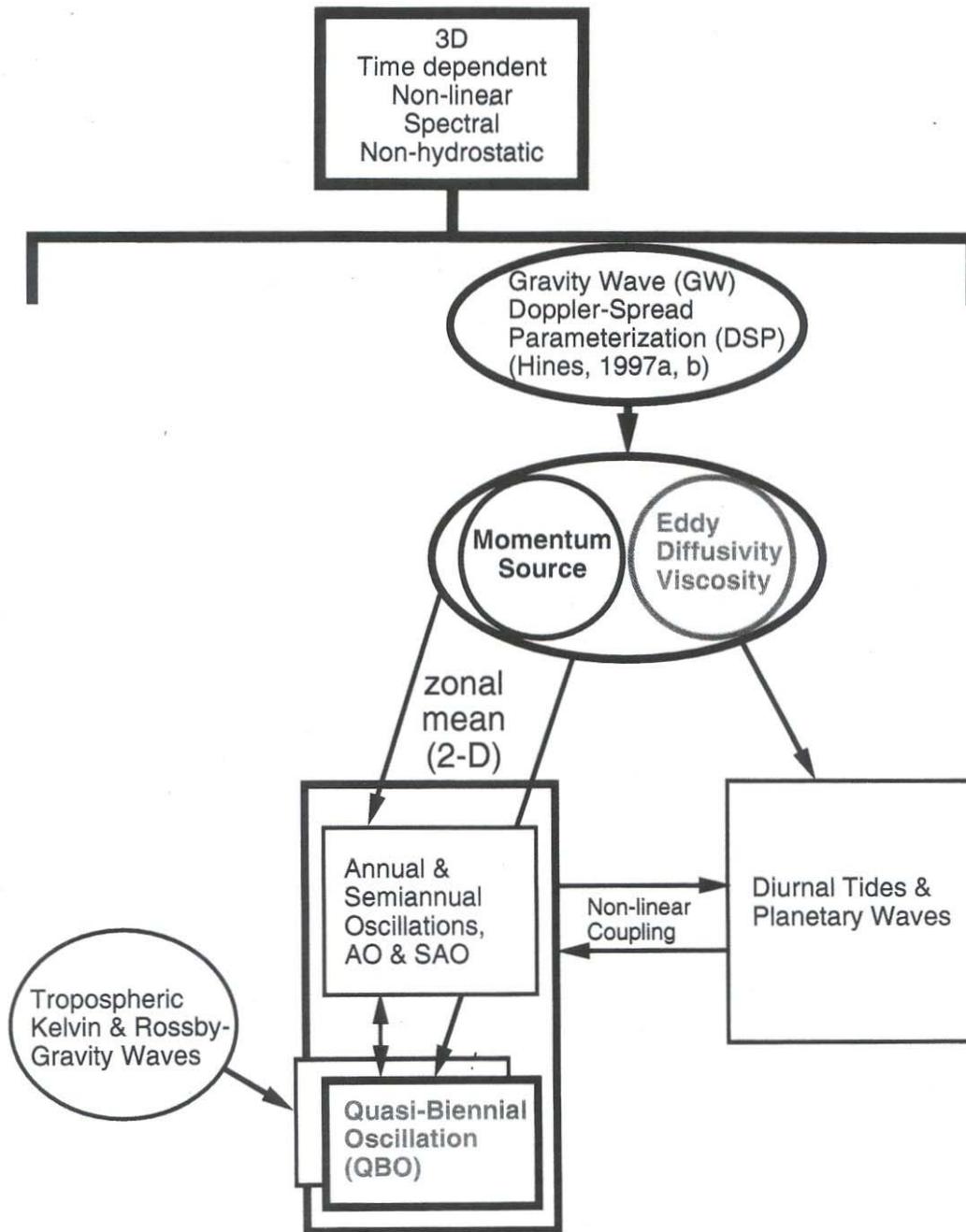


180W 160W 140W 120W 100W 80W 60W 40W 20W 0 20E 40E 60E 80E 100E 120E 140E 160E

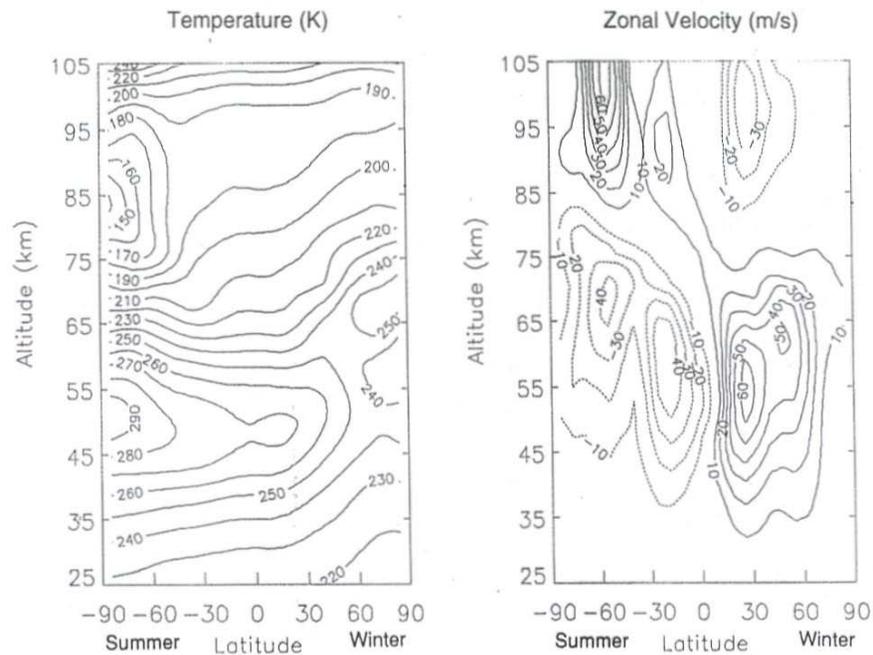
-April 12 NOAA-14 2:45 a.m.	-April 12 NOAA-12 5:59 a.m.	-April 12 NOAA-11 8:57 a.m.
-April 12 NOAA-14 2:45 p.m.	-April 12 NOAA-12 5:59 p.m.	-April 12 NOAA-11 8:57 p.m.
-April 13 NOAA-14 2:45 a.m.	-April 13 NOAA-12 5:59 a.m.	-April 13 NOAA-11 8:57 a.m.
-April 13 NOAA-14 2:45 p.m.	-April 13 NOAA-12 5:59 p.m.	-April 13 NOAA-11 8:57 p.m.

Temperature variations from NOAA satellite measurements, taken from a paper in preparation (Reddy, Susskind, Mayr).

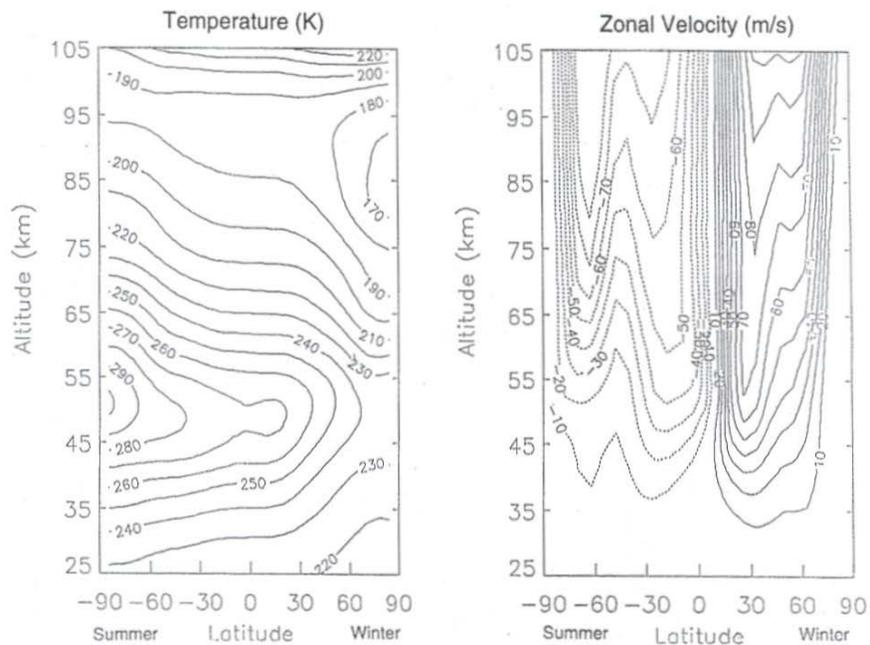
# Numerical Spectral Model (NSM)



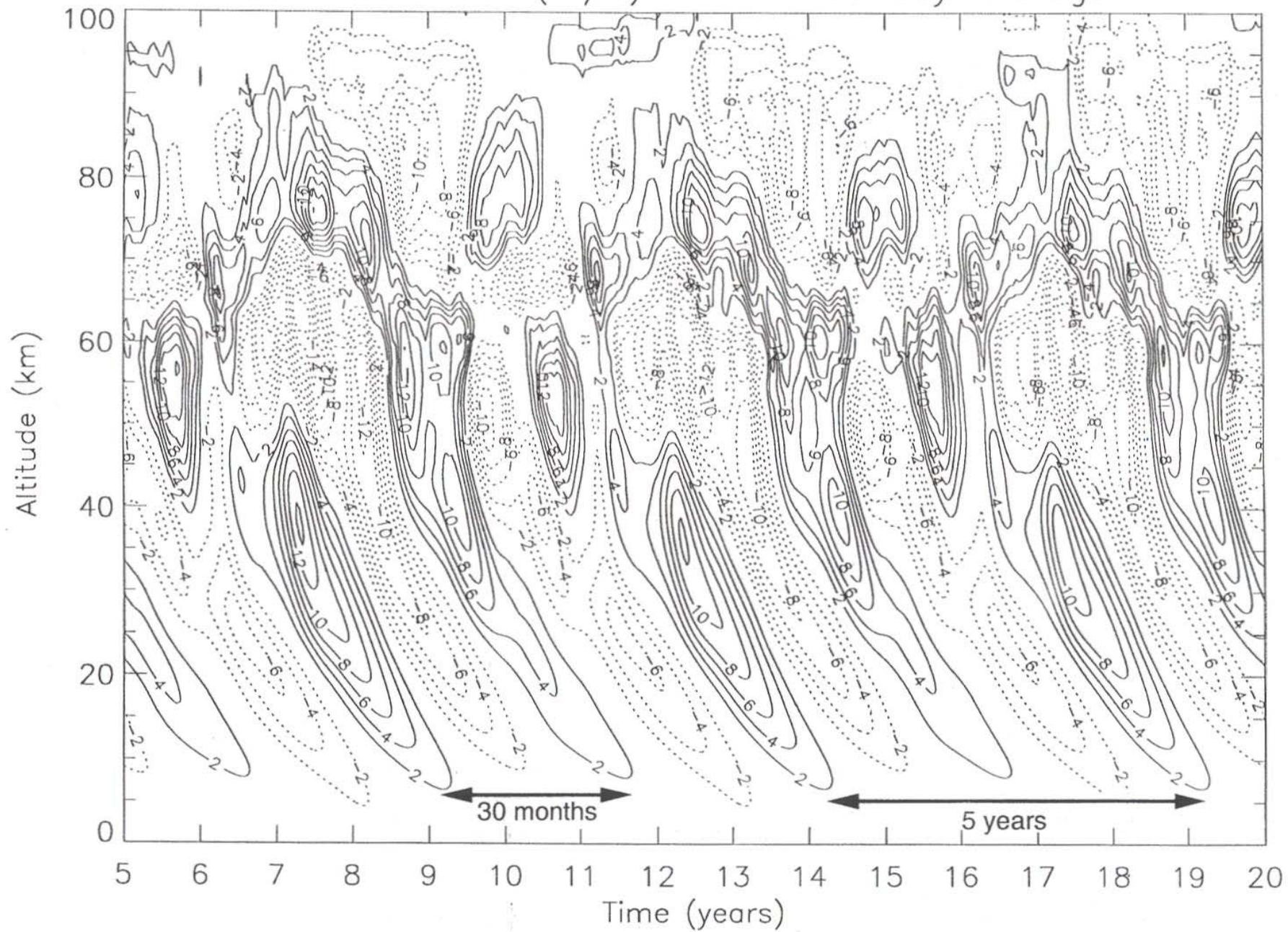
with GW Momentum Source



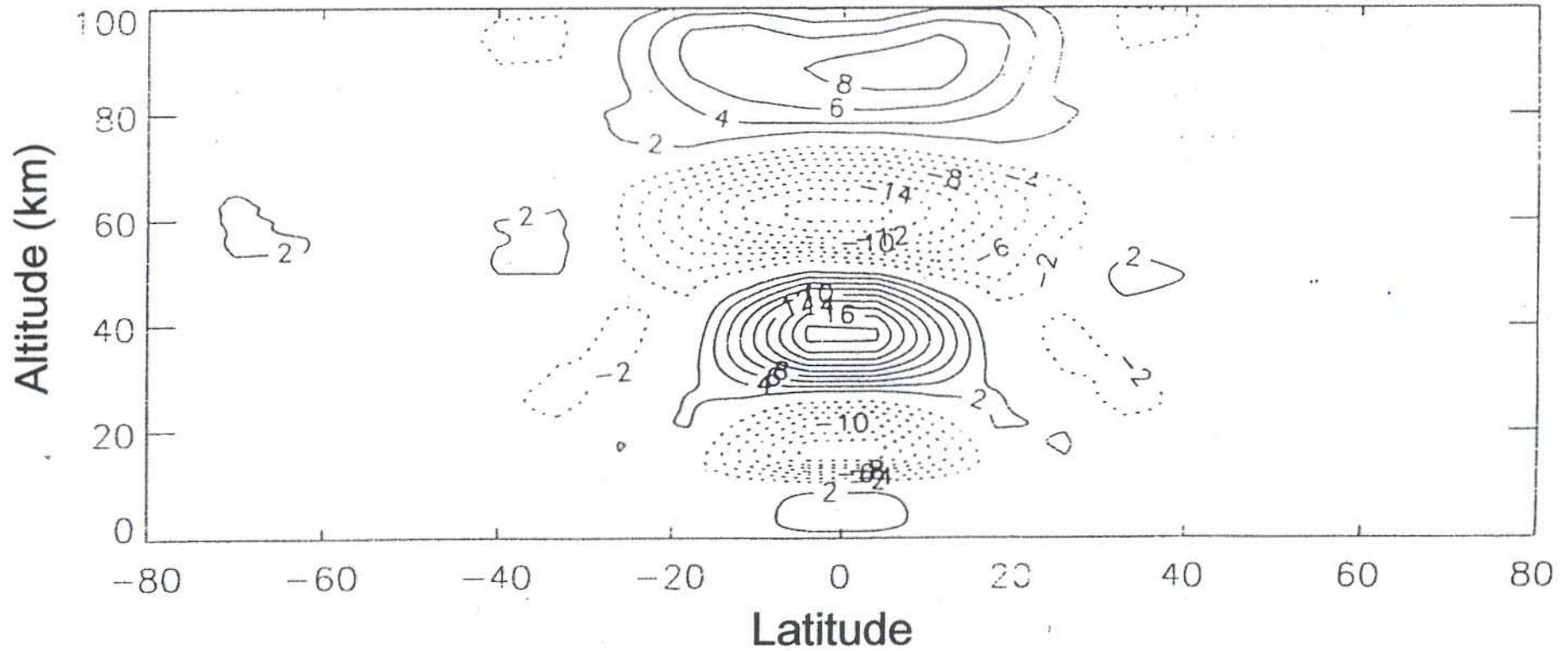
without GW Momentum Source



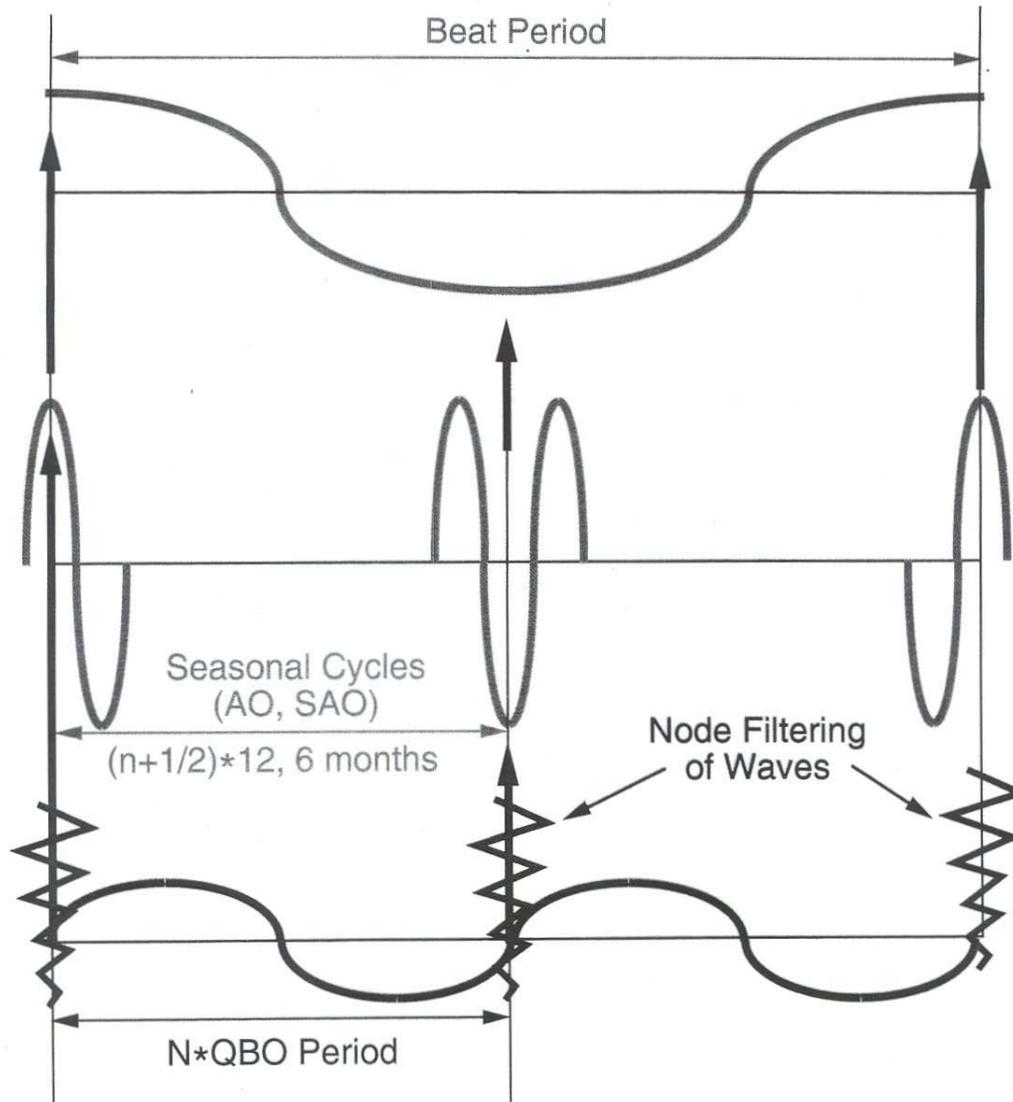
Model Zonal Winds (m/s) 11° N 364day average



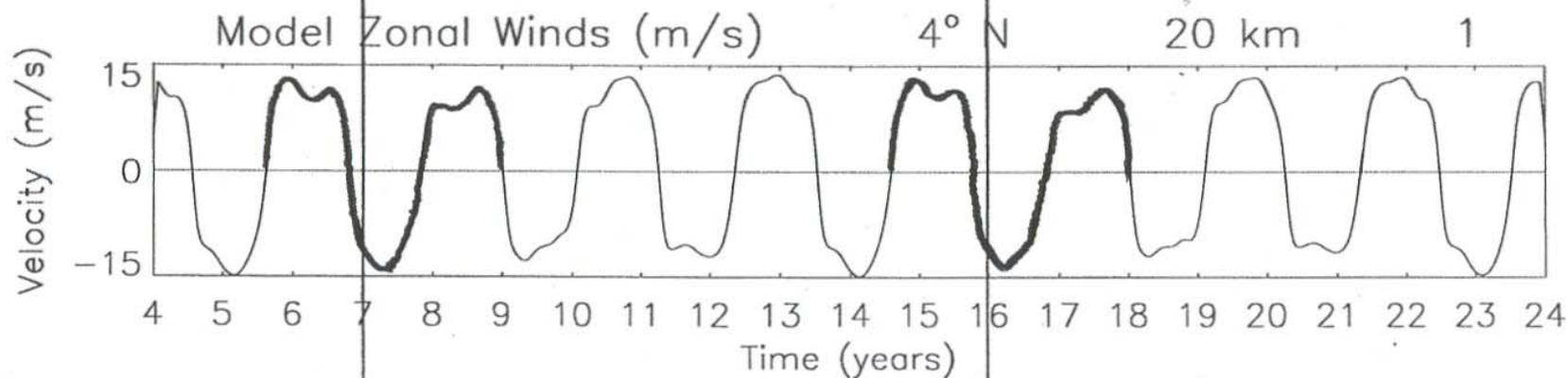
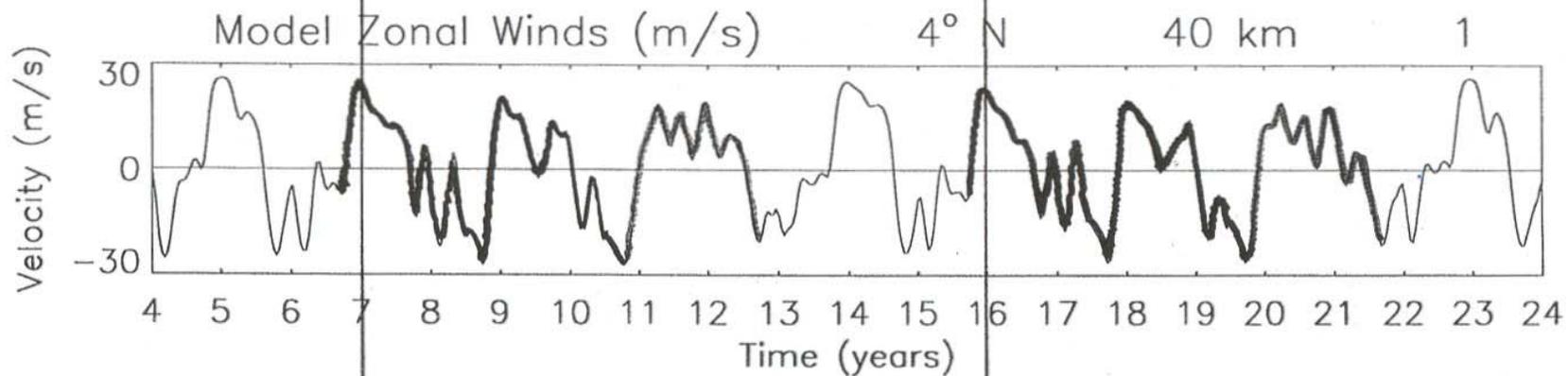
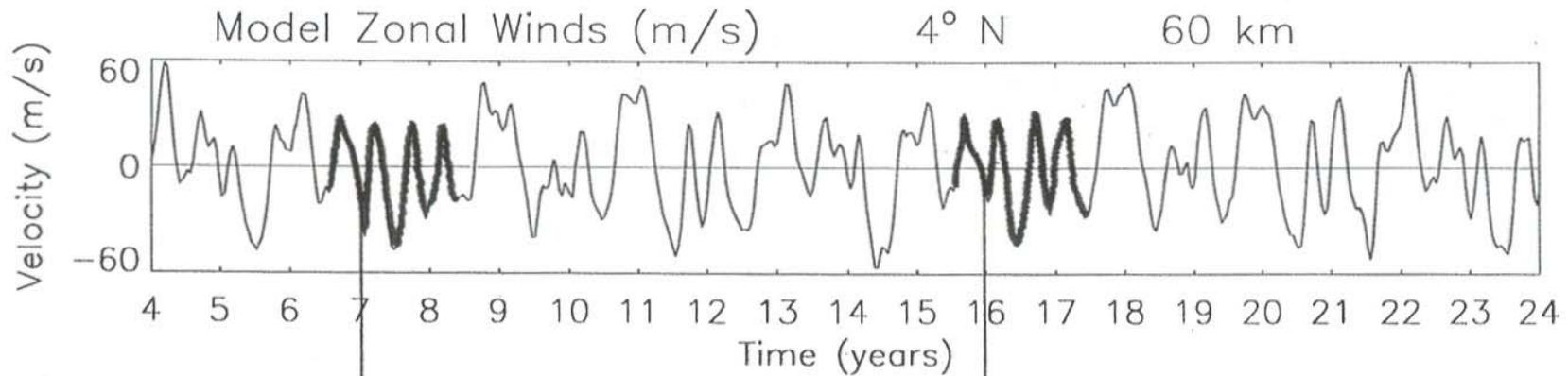
# Zonal Winds (m/s) of 30 Month Period



# Beat Periods Between QBO and Seasonal Cycles Generated by Gravity Wave Node Filtering

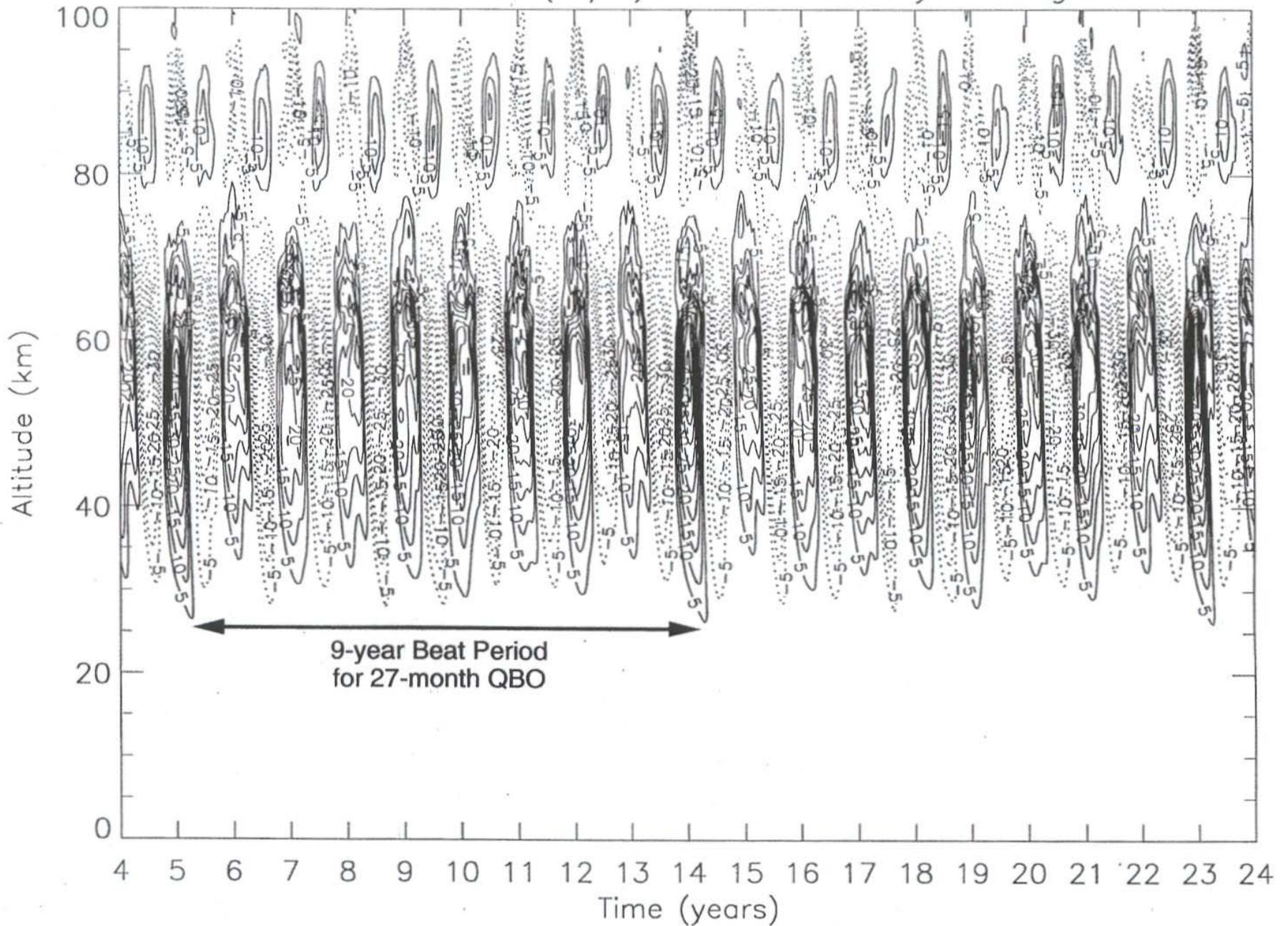


**QBO (30 mo) x AO --> 5-year Beat**  
**QBO (27 mo) x AO --> 9-year Beat**  
**QBO (34.5 mo) x SAO --> 11.5-year Beat**

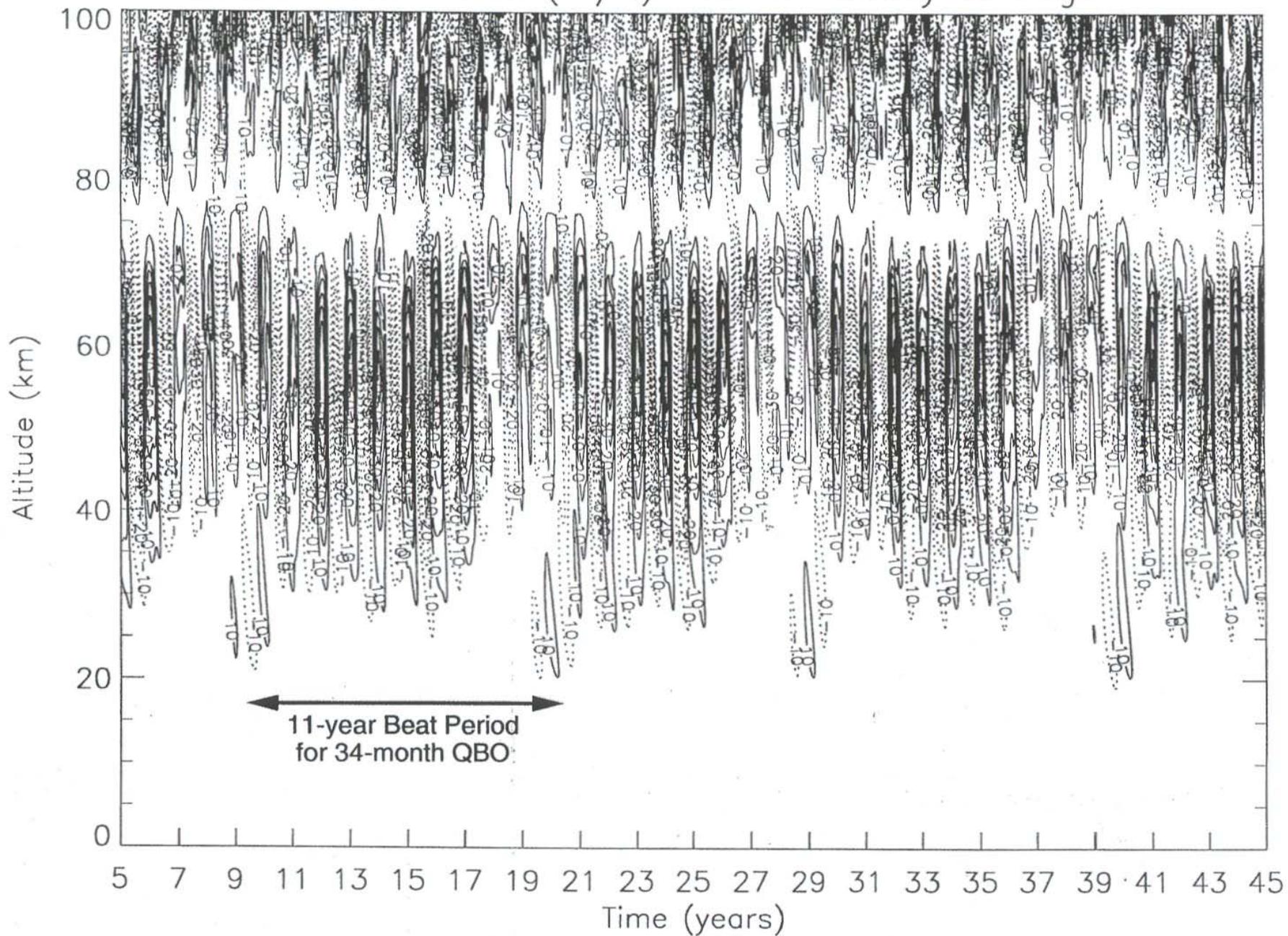


9-year Beat Period  
for 27-month QBO

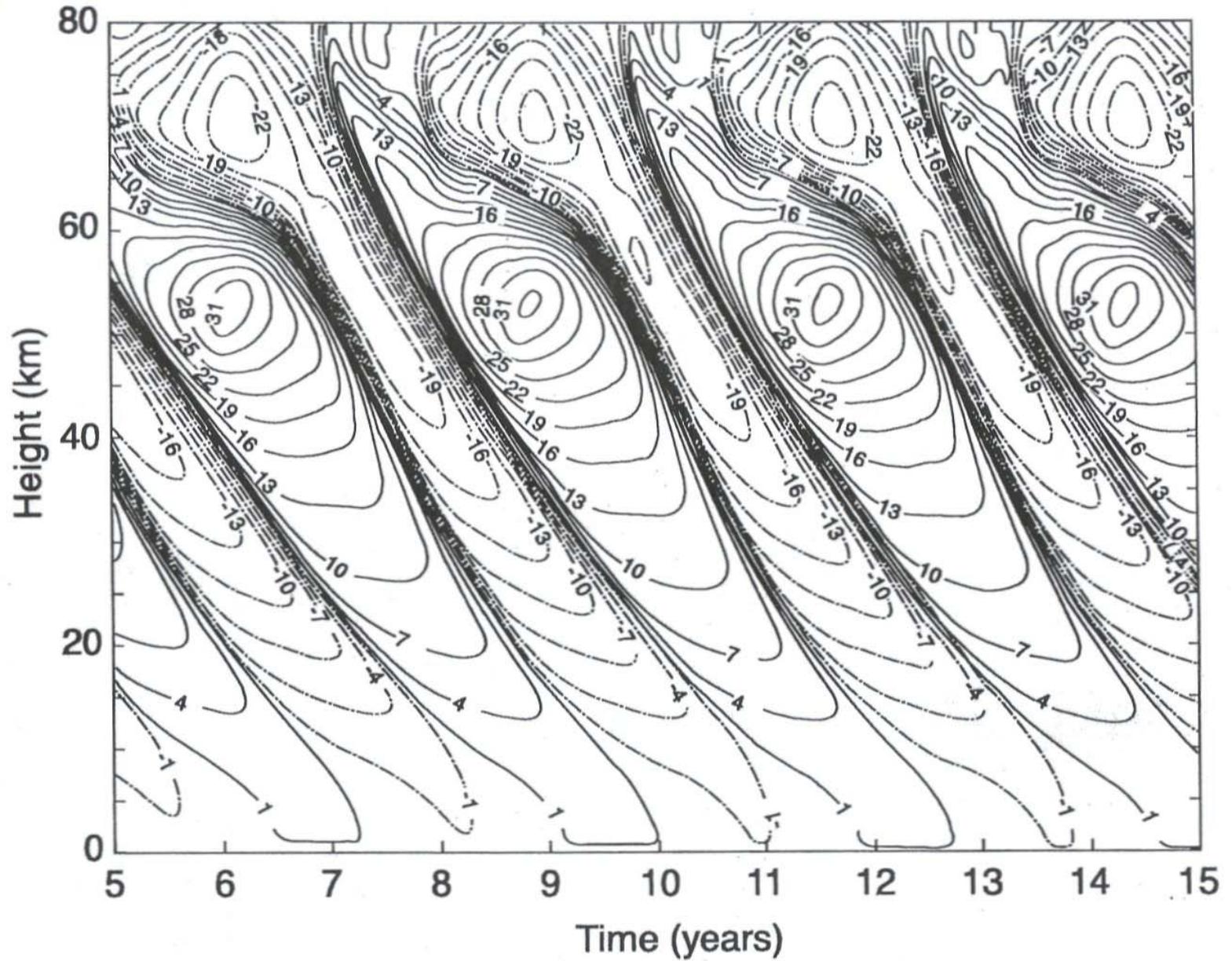
# Model Zonal Winds (m/s) 40° N 28day average



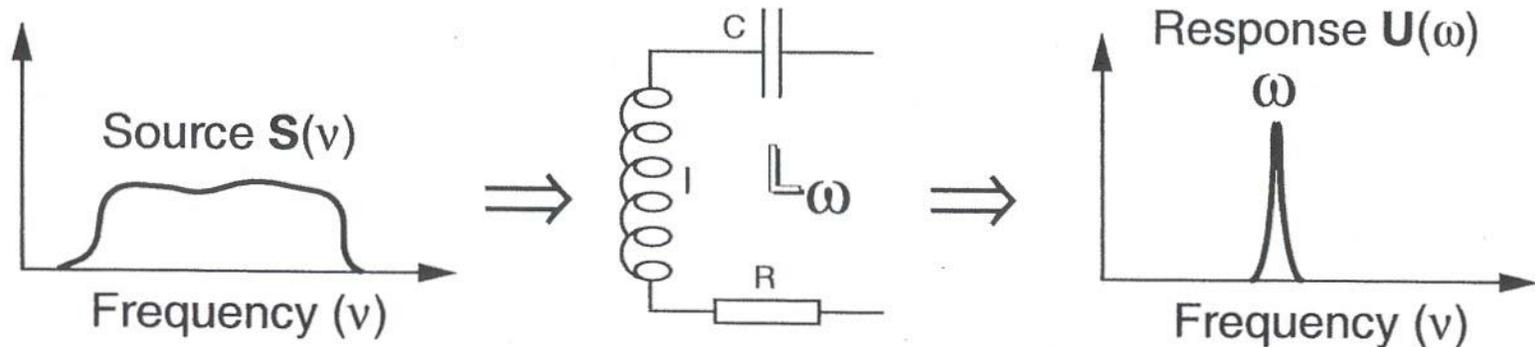
# Model Zonal Winds (m/s) 40° N 28day average



# Perpetual Equinox Zonal Winds



## Linear Oscillators (e.g., circuit, waves):



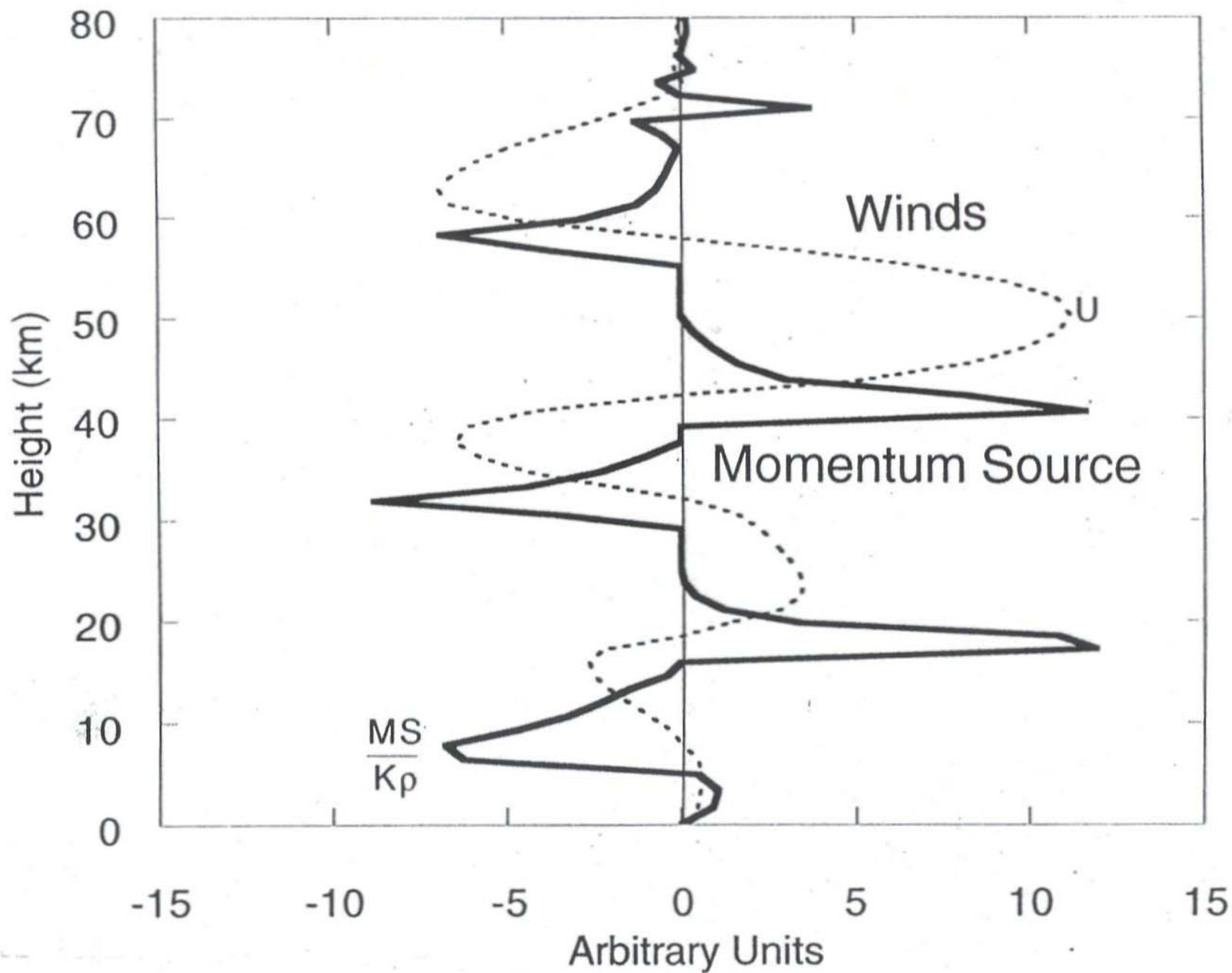
$$L_{\omega} \times \mathbf{U}(\omega) = \mathbf{S}(\nu)$$

## Non-linear Oscillators (e.g., QBO, clock):

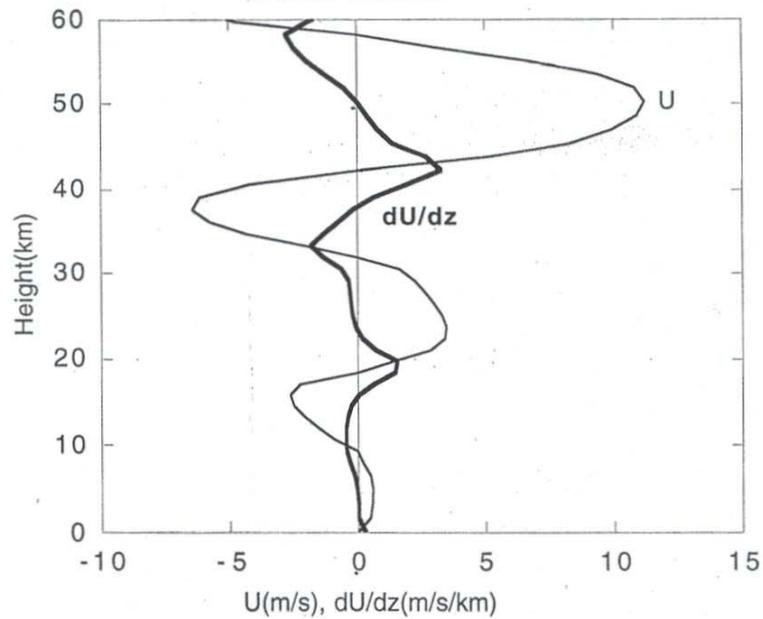
$$N_{\omega} \times \mathbf{U}(\omega) = \mathbf{S}(\mathbf{U}(\omega)) \propto \mathbf{U}(\omega)^3$$

$$\Rightarrow \mathbf{U}(\omega) = \mathbf{U}_1(\omega) + \mathbf{U}_3(3\omega)$$

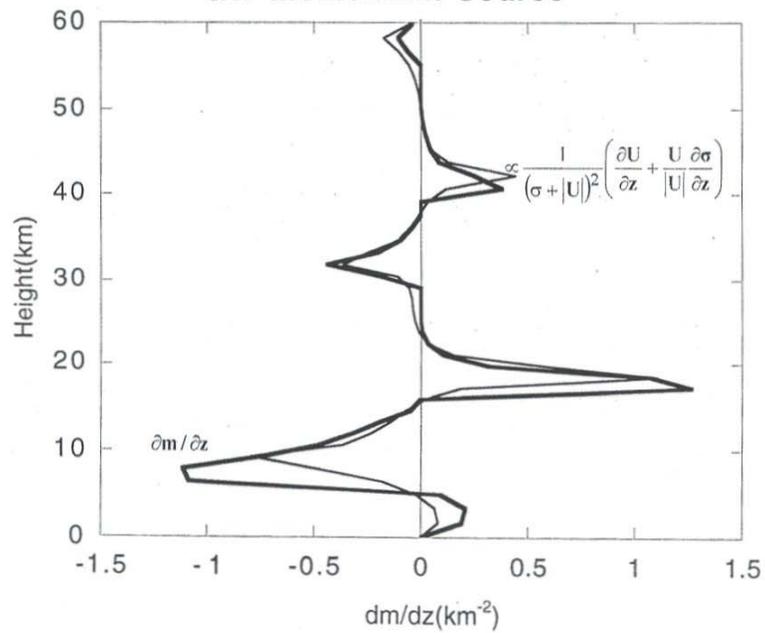
# Effective Gravity Wave Acceleration



### Zonal Winds



### GW Momentum Source

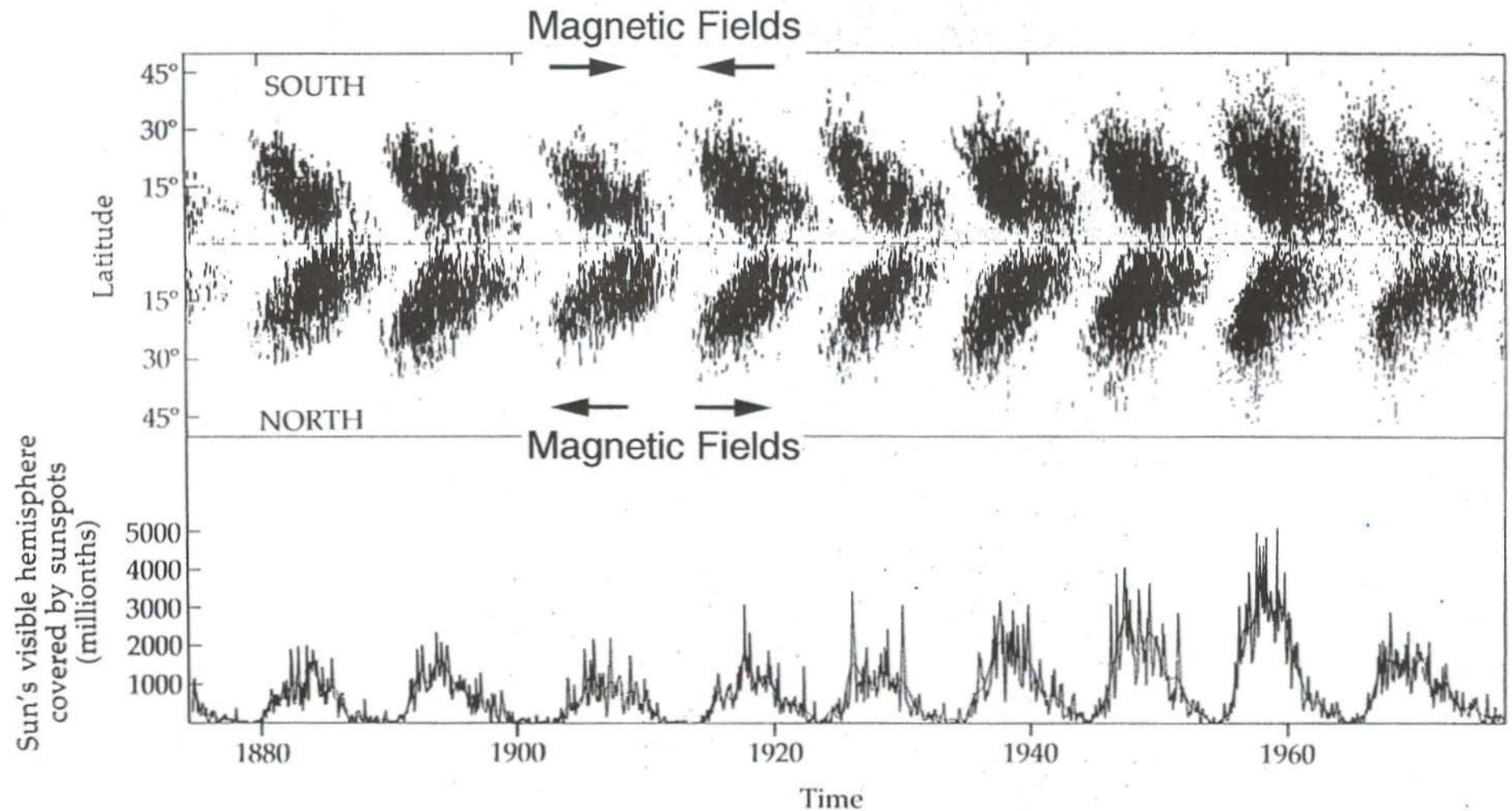


# Conclusions

## Terrestrial Quasi-Biennial Oscillation (QBO):

- 1) At low latitudes, generated by globally uniform wave source.
- 2) Extends into the upper mesosphere.
- 3) Interaction with seasonal cycles generates periods around 10 years.
- 4) Wave driven non-linear flow oscillation -- a fluid chronometer.

A Solar Analog for the QBO?



Solar Activity Cycle = 11 years

**Solar Magnetic Cycle = 22 years**  
**the cause of sunspot activity**

Chiueh, T., in the *Astrophysical Journal*, 2000, asks:

“why the solar dynamo should appear periodic, what makes the polarity of magnetic fields change in a half cycle, and why a new half cycle always starts below the 40° latitude?”

## Is there a chronometer hidden deep in the Sun?

---

**R. H. Dicke** (*Nature*, 1978)

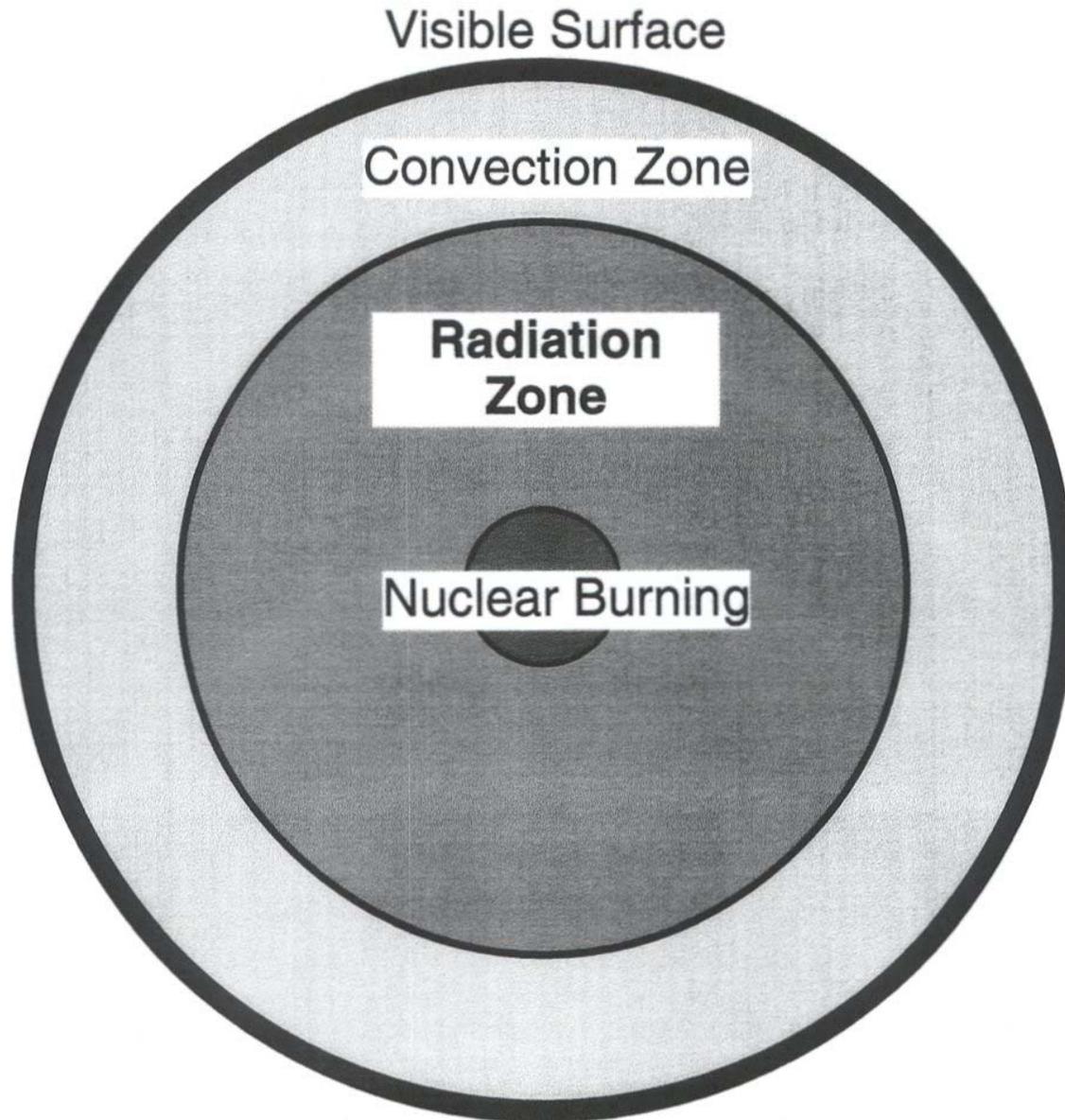
Joseph Henry Laboratories, Physics Department, Princeton University, Princeton, New Jersey 08540

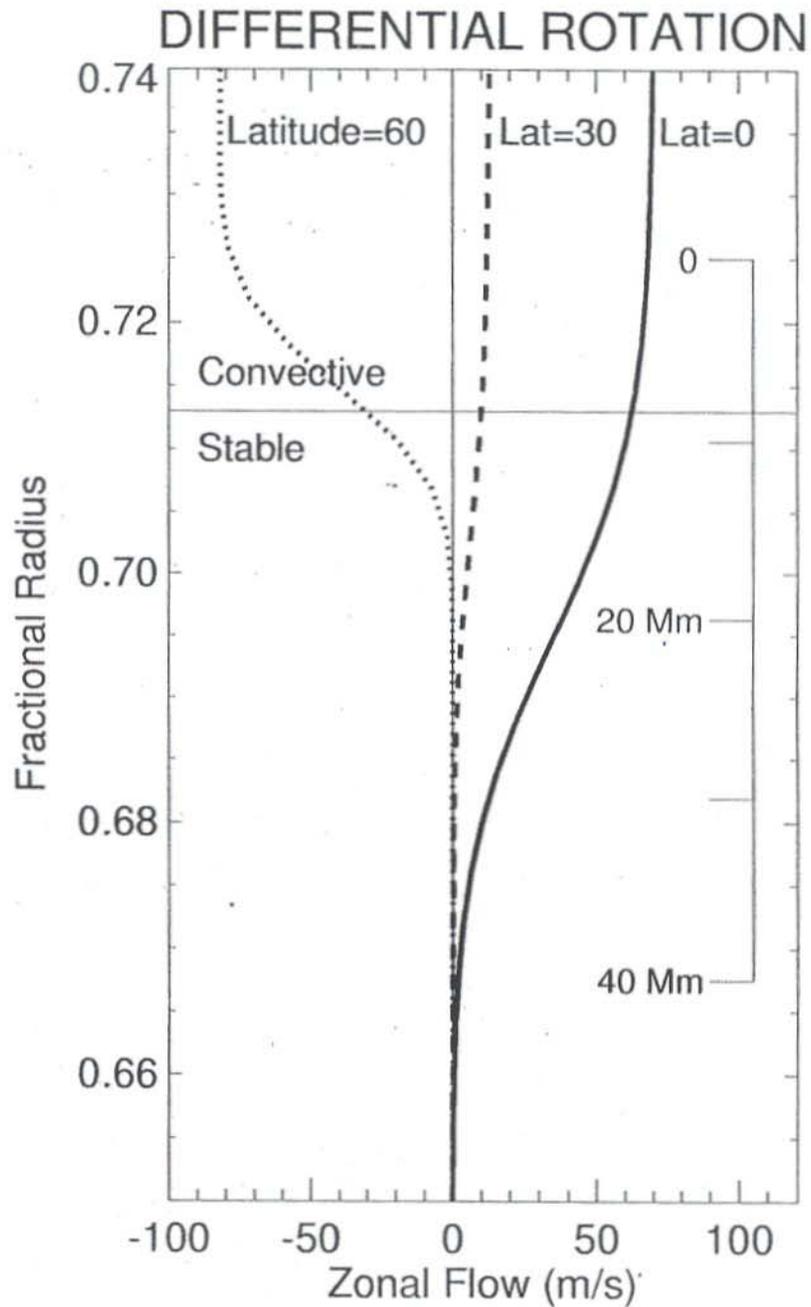
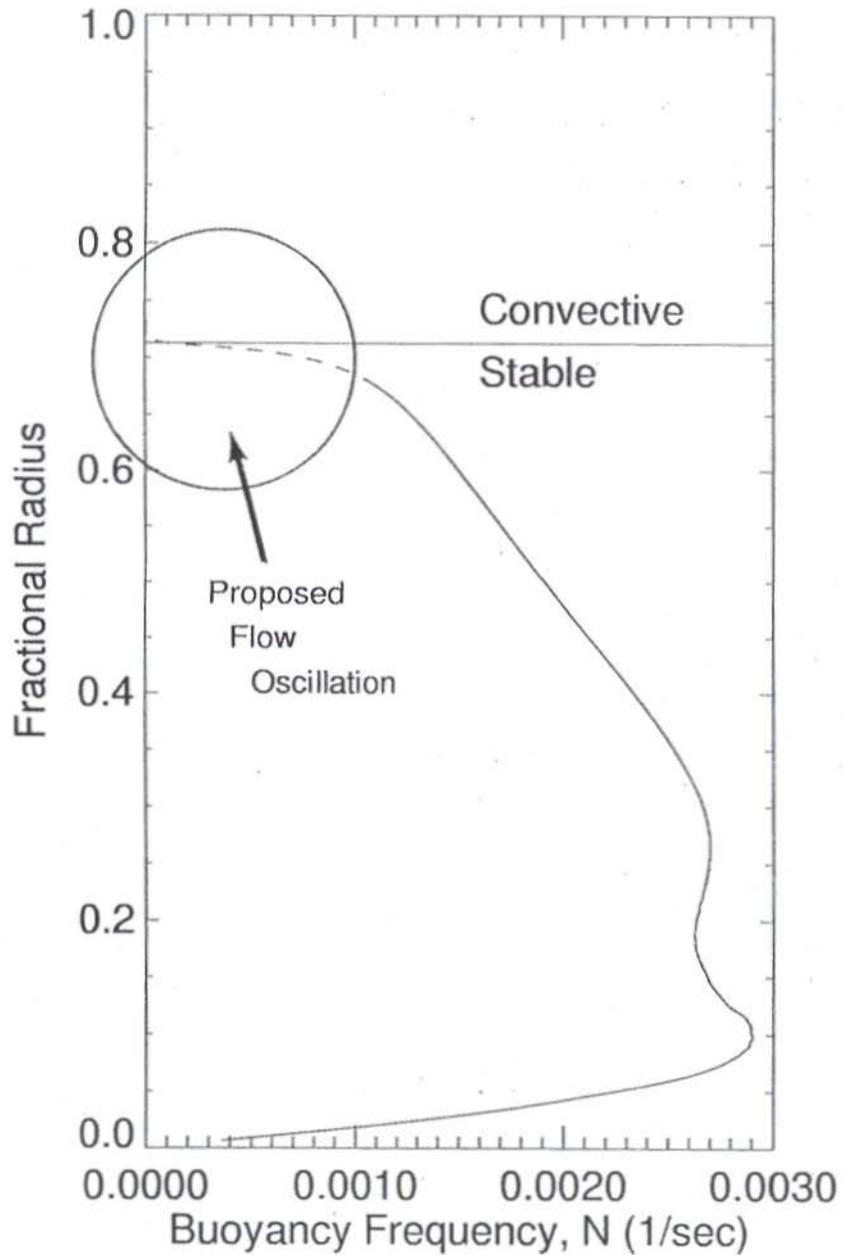
---

*No support is found for the conventional view of the sunspot cycle, that there exists a large random walk in the phase of the cycle. Instead, both sunspots and the [D/H] solar/terrestrial weather indicator seem to be paced by an accurate clock inside the Sun.*

---

# Solar Structure





## Analytic Model

Zonal momentum balance at the equator:

$$\rho \frac{\partial \mathbf{U}}{\partial t} - \frac{\partial}{\partial \mathbf{z}} \rho \mathbf{K} \frac{\partial \mathbf{U}}{\partial \mathbf{z}} = \rho \left[ \frac{\Phi^2 \sigma_h^4}{2 \lambda_h \mathbf{N} (\Phi \sigma_h + |\mathbf{U}|)^2} \right] \left( \frac{\partial \mathbf{U}}{\partial \mathbf{z}} + \frac{\mathbf{U} \Phi}{|\mathbf{U}|} \frac{\partial \sigma_h}{\partial \mathbf{z}} \right)$$

$\mathbf{U}$ , zonal wind velocity;

$\mathbf{N}$ , buoyancy frequency;

$0.95 < \Phi = 1.3 < 1.74$  GW parameter;

$\sigma_h \propto \rho^{-1/3}$ , GW horizontal wind amplitude;  $\rho$ , density;

$\lambda_h$ , characteristic horizontal wave length of GW's;

$\mathbf{K}$ , the vertical eddy diffusivity provided by the DSP.

$$\text{For } \mathbf{U} = \mathbf{U}_0 \exp \left( i 2\pi \left( \frac{\mathbf{t}}{\tau} + \frac{\mathbf{z}}{\lambda_0} \right) \right);$$

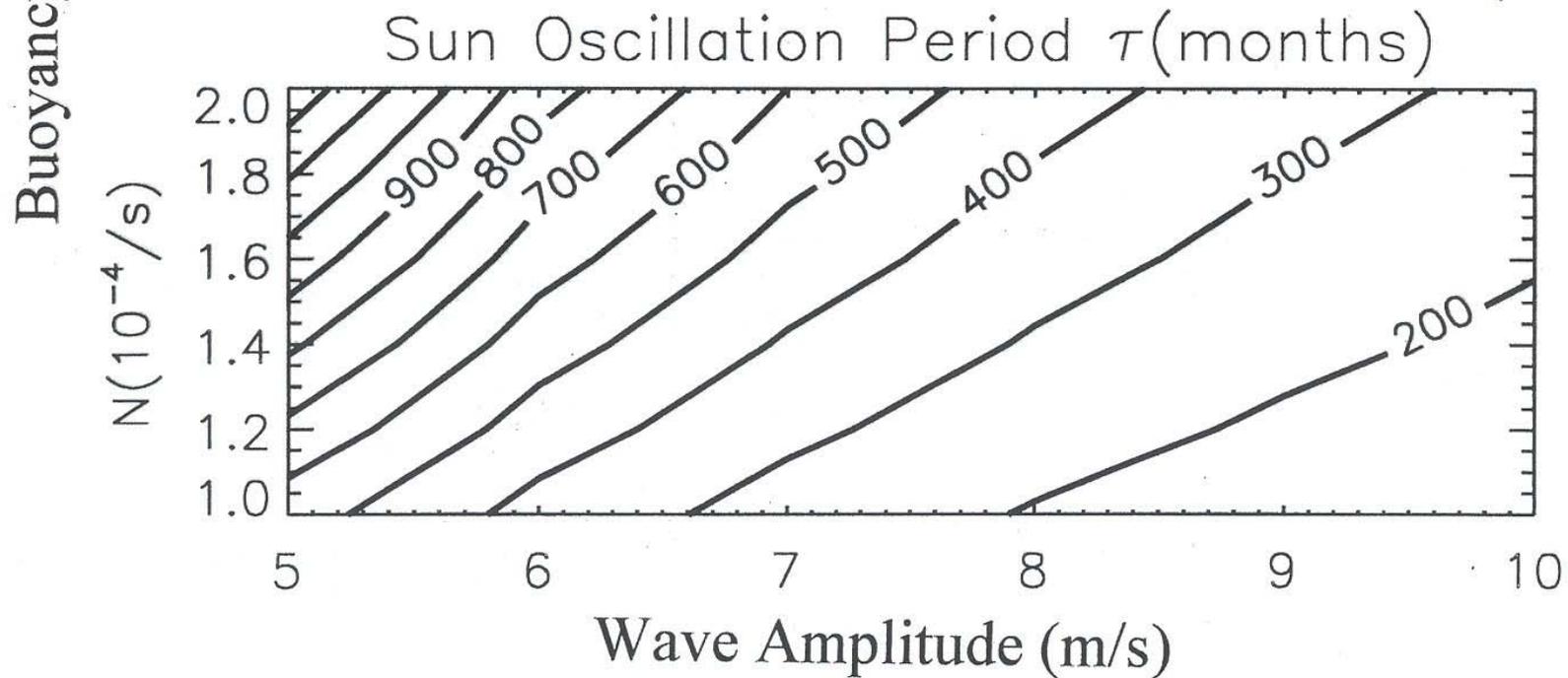
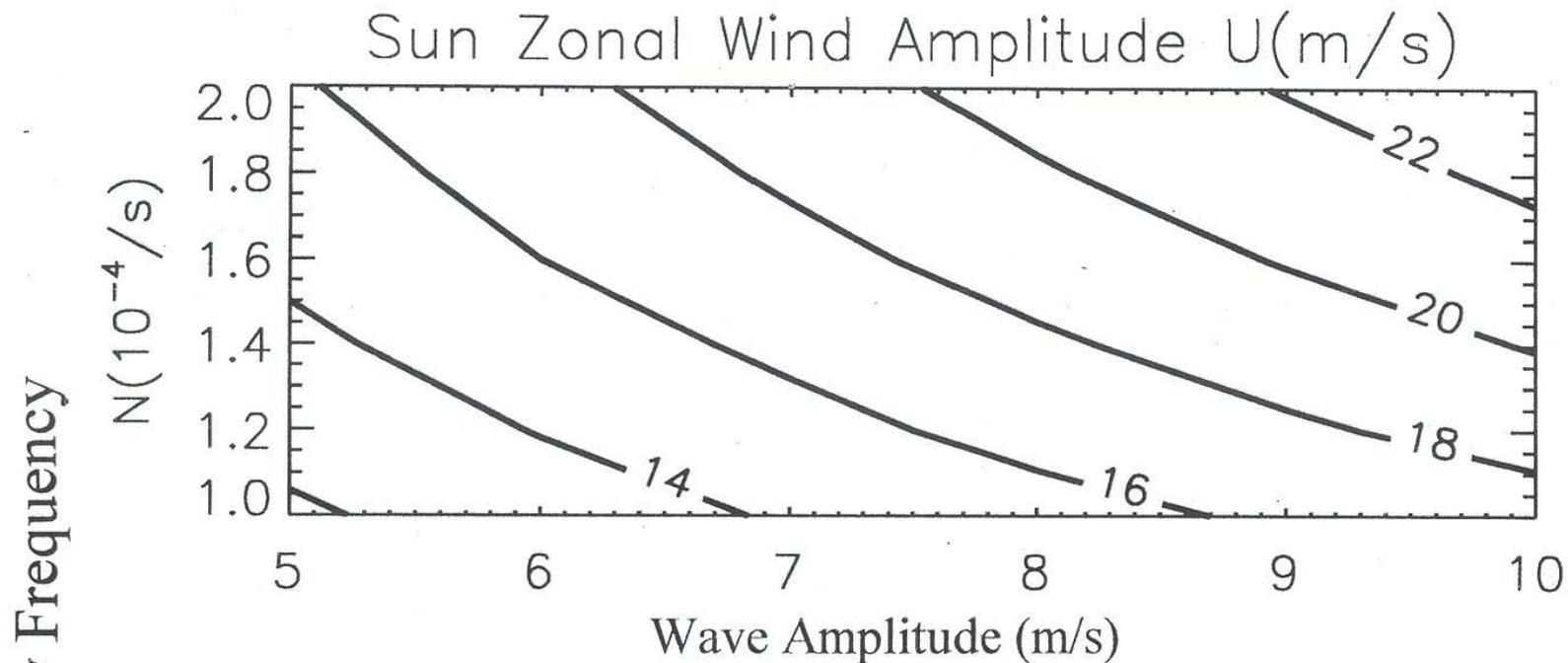
$\lambda_0, \tau$ , vertical wavelength & period of flow oscillation:

$$|\mathbf{U}| \propto \left( \frac{\sigma_h^5 \lambda_0^2}{\lambda_h \mathbf{N} \mathbf{K}} \right)^{1/3}; \quad \tau \propto \frac{\mathbf{N} \lambda_h \lambda_0}{\sigma_h^4} |\mathbf{U}|^2$$

**With larger length scales in the Sun:**

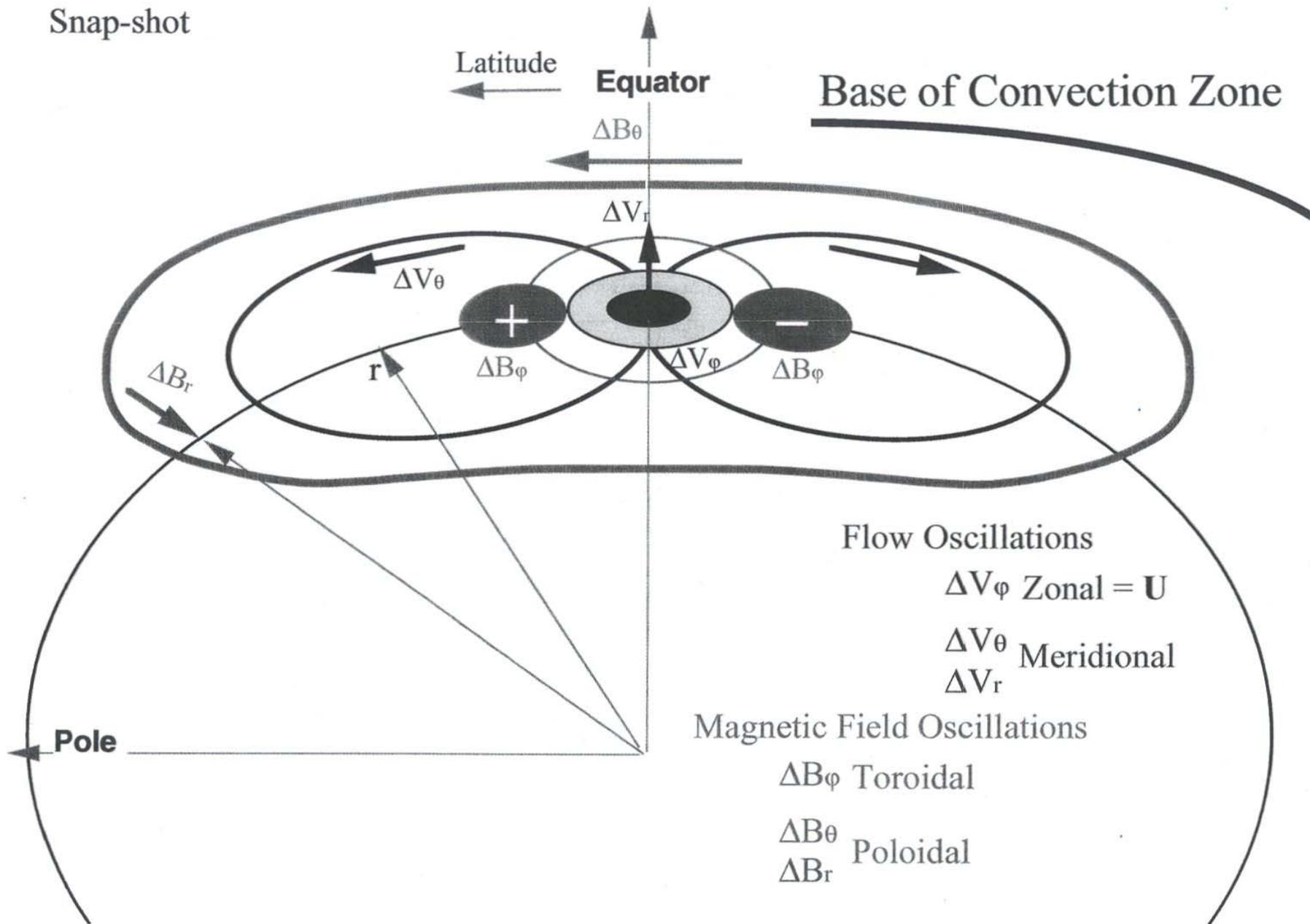
$\lambda_0, \lambda_h$  (sun)  $\gg \lambda_0, \lambda_h$  (earth)  $\rightarrow \tau$  (sun)  $\gg \tau$  (earth).

**Low  $\mathbf{N}$  near base of convection zone  $\rightarrow \tau$  (sun)  $\approx$  22 years.**



# Bi-decadal Oscillations (BDO) of Flows and Magnetic Fields

Snap-shot



## Conclusions

### Solar Analog of QBO:

- 1) Wave-driven flow oscillation generates 22-year magnetic (11-year activity) cycle.
- 2) Below convection region. Low convective stability like upper mesosphere.
- 3) Strong toroidal field, generated by zonal flow, peaks at low latitudes.
- 4) Weaker poloidal field, generated by meridional circulation, peaks when toroidal field changes direction.