

CEDAR Tutorial:  
**Atmospheric Gravity Waves:**  
*Apply Classroom Physics to  
Research*

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High Altitude Observatory



NCAR

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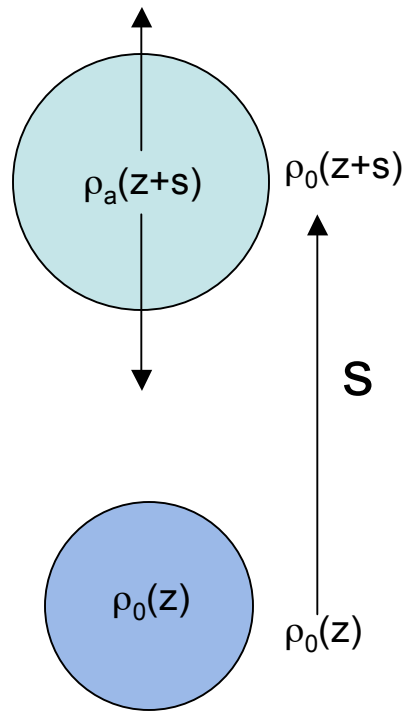


# Outline

- Atmospheric oscillation under buoyancy.
- Gravity wave dispersion/polarization relations.
- Gravity wave energy and momentum fluxes.
- Eliassen-Palm theorems and gravity wave-mean flow interaction.
- Gravity wave sources.
- Gravity wave propagation.
- Gravity wave impacts.

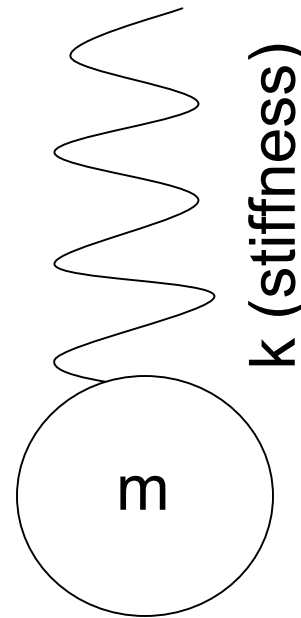
# Atmospheric Oscillation under Buoyancy

Oscillation of an atmospheric parcel



Frequency of atmosphere oscillation?

mechanical oscillator



Oscillation freq:  $\omega = \sqrt{k/m}$

Stiffness: restoring force/displacement

Inertia: Kinetic energy/(velocity<sup>2</sup>/2)

In a stable atmosphere, the restoring force can be found to be:

$$-\left(\frac{g}{c_0^2} + \frac{1}{\rho_0} \frac{d\rho_0}{dz}\right)\rho_0 g \cdot s$$

So the stiffness of the atmos. oscillator is:

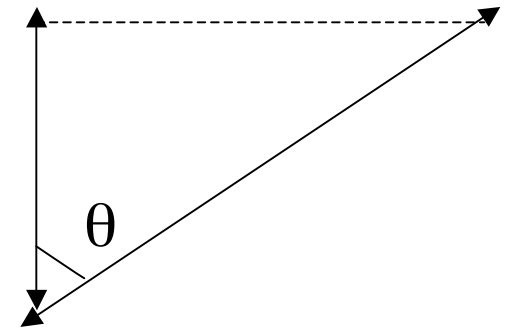
$$-\left(\frac{g}{c_0^2} + \frac{1}{\rho_0} \frac{d\rho_0}{dz}\right)\rho_0 g \quad (> 0 \text{ for stable stratification})$$

The kinetic energy of the fluid parcel is

$$\frac{1}{2} \rho_0 [(ds/dt) / \cos \theta]^2$$

The generalized inertial is thus

$$\rho_0 / \cos^2 \theta$$



The oscillation frequency is then

$$\begin{aligned} \omega^2 &= -\left(\frac{g}{c_0^2} + \frac{1}{\rho_0} \frac{d\rho_0}{dz}\right) \rho_0 g / (\rho_0 / \cos^2 \theta) = -\left(\frac{g}{c_0^2} + \frac{1}{\rho_0} \frac{d\rho_0}{dz}\right) g \cos^2 \theta = \\ &= N^2 \cos^2 \theta \leq N^2 \end{aligned}$$

N is the buoyancy or Brunt-Vaisala frequency, and GW oscillation frequency is always less than N

# GW Excitation

- Any process that can cause vertical displacement of atmosphere parcels: Convection, orography, adjustment of balanced flows and/or instability, aurora heating...
- Challenge: Source distribution of GW
  - Ubiquitous and globally distributed.
  - Vastly different temporal and spatial scales.

## Dispersion relation $\omega(k_x, k_z)$ :

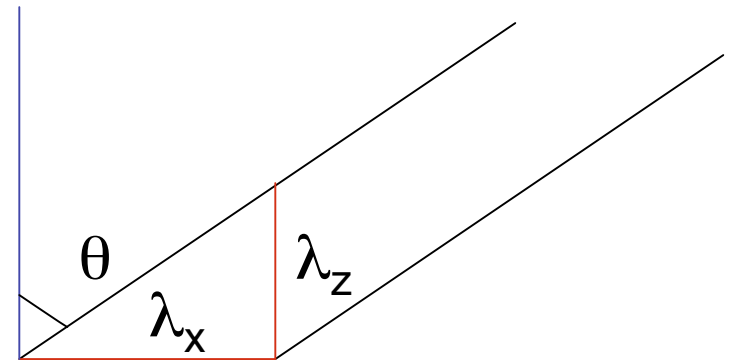
Each wave can be uniquely determined by horizontal and vertical wavenumbers, frequency and amplitude.

From wave geometry:

$$\tan^2 \theta = (k_z / k_x)^2$$

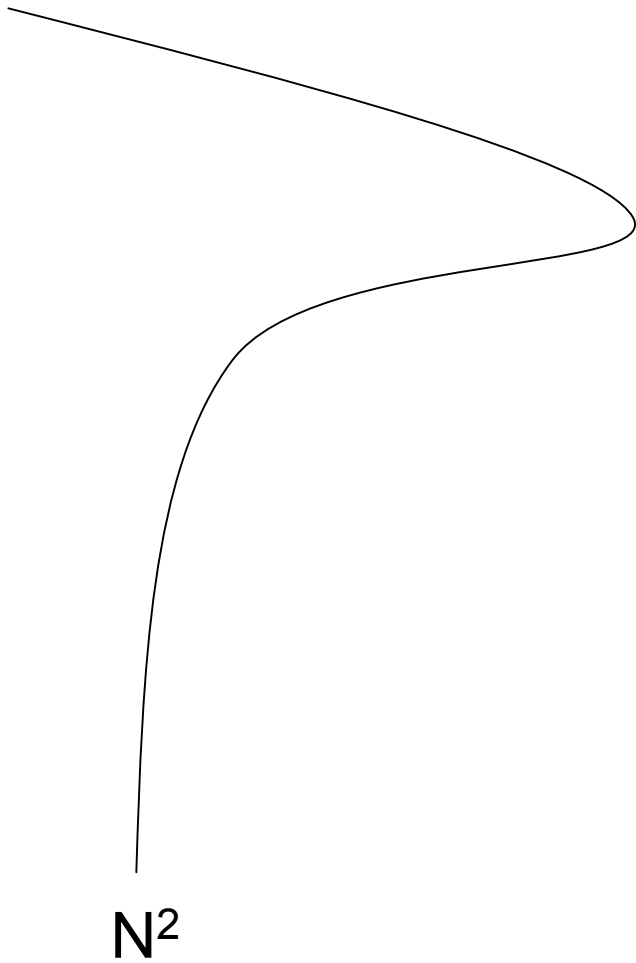
$$\therefore \frac{N^2 - \omega^2}{\omega^2} = (k_z / k_x)^2$$

$$\omega^2 = N^2 \frac{k_x^2}{k_x^2 + k_z^2}$$



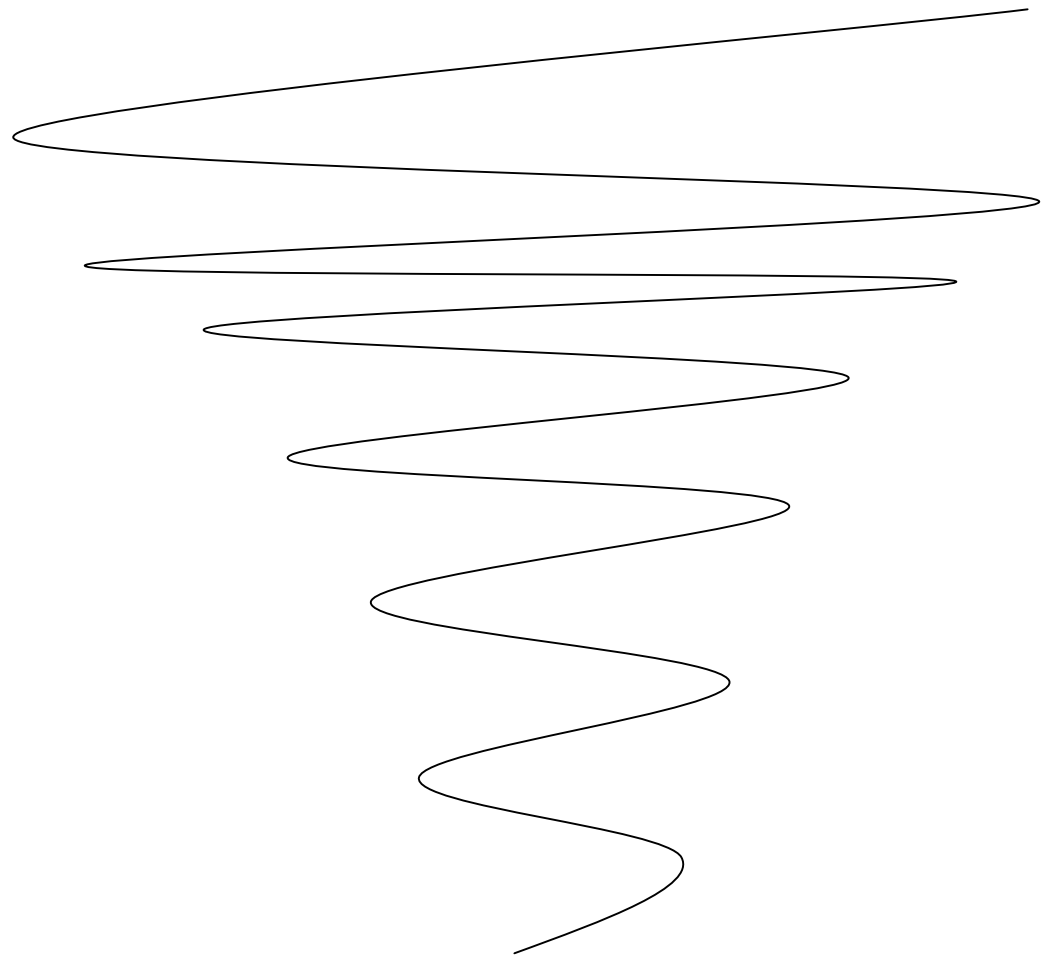
It has many interesting applications...

Example: From  $k_z^2 = \left(\frac{N^2}{\omega^2} - 1\right)k_x^2 \approx \frac{N^2}{\omega^2} k_x^2 = \frac{N^2}{c_{px}^2}$



Perturbative shear:

Perturbative lapse rate:



GW perturbation

$$\frac{du'}{dz} \propto k_z u' \approx \pm \frac{N}{c_{px}} u'$$

$$\frac{dT'}{dz} \propto k_z T' \approx \pm \frac{N}{c_{px}} T'$$



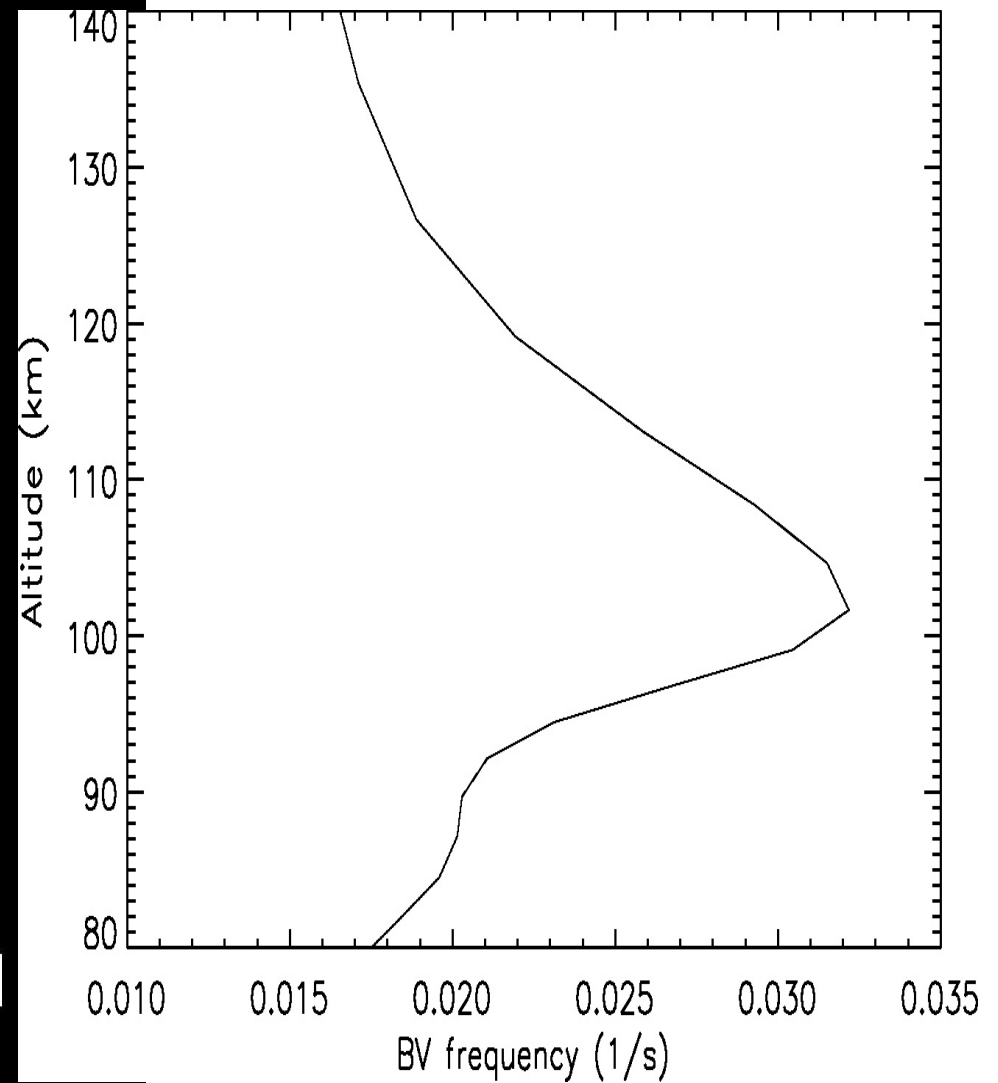
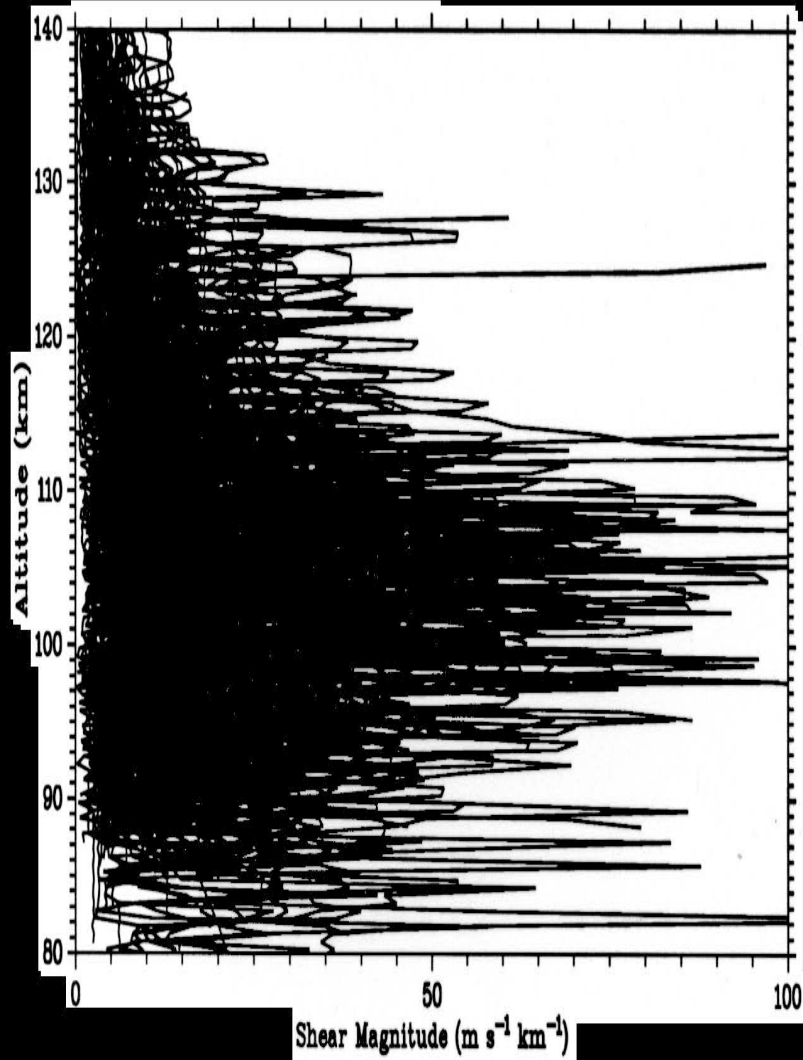


Figure 11. Superposition of the shear magnitude profiles for all of the low-latitude and midlatitude data.

Larsen, 2002

(N also constrain the maximum shear through dynamic instability (Liu, 2007))

Group velocity can be obtained from the dispersion relation:

$$c_{px} = \frac{\omega}{k_x} = \frac{N}{\sqrt{k_x^2 + k_z^2}}, \quad c_{gx} = \frac{\partial \omega}{\partial k_x} = \frac{k_z^2}{(k_x^2 + k_z^2)^{3/2}} N$$

$$c_{pz} = \frac{\omega}{k_z} = \frac{N}{\sqrt{k_x^2 + k_z^2}} \frac{k_x}{k_z}, \quad c_{gz} = \frac{\partial \omega}{\partial k_z} = \frac{-k_x k_z}{(k_x^2 + k_z^2)^{3/2}} N$$

If  $k_z \gg k_x$   $c_{gx} \approx c_{px}$ ,  $c_{gz} \approx -c_{pz}$

GWs propagation velocity are dependent on wave components, thus dispersive. This explains why we often see quasi-monochromatic waves in observations.



Amplitudes of temperature, winds, pressure, and density perturbations: **polarization relations.**

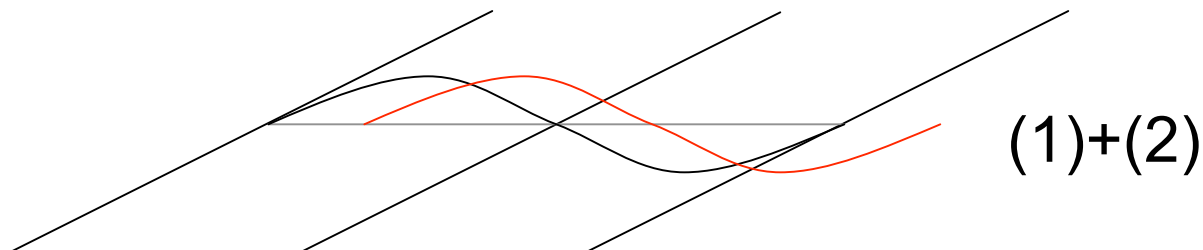
Can be obtained from linear fluid equations.

$$k_x u' = -k_z w'$$

$$\rho_0 (c_{px} - u_0) u' = p'$$

$$T' = i \sqrt{\frac{T}{c_p}} u'$$

Note: The horizontal phase propagation can be achieved by (1) oscillation of air parcels at subsequent locations, (2) translation by horizontal wind. (1) is the intrinsic phase propagation, and wave properties (e.g. dispersion) are tied to intrinsic phase propagation.



# Wave Transport of Energy/Momentum/Heat

Vertical energy flux due to pressure work:  $\overline{p'w'^*}$

Vertical flux of horizontal momentum:  $\rho_0 \overline{u'w'^*}$

Vertical heat flux:  $\overline{w'T'^*}$

From the polarization relation, some important results:

$$\overline{p'w'^*} = -\rho_0 \frac{(c_{px} - u_0)^2}{c_{pz}} |w|^2$$

$$\rho_0 \overline{u'w'^*} = -\rho_0 \frac{k_z}{k_x} |w|^2$$

$$\overline{w'T'^*} = 0$$

- GW produces vertical energy flux and momentum flux.
- Heat flux is zero (in the absence of dissipation).
- Sign of energy flux is uniquely determined by the vertical wave propagation.
- Energy flux and momentum flux are related (Eliassen-Palm's first theorem)

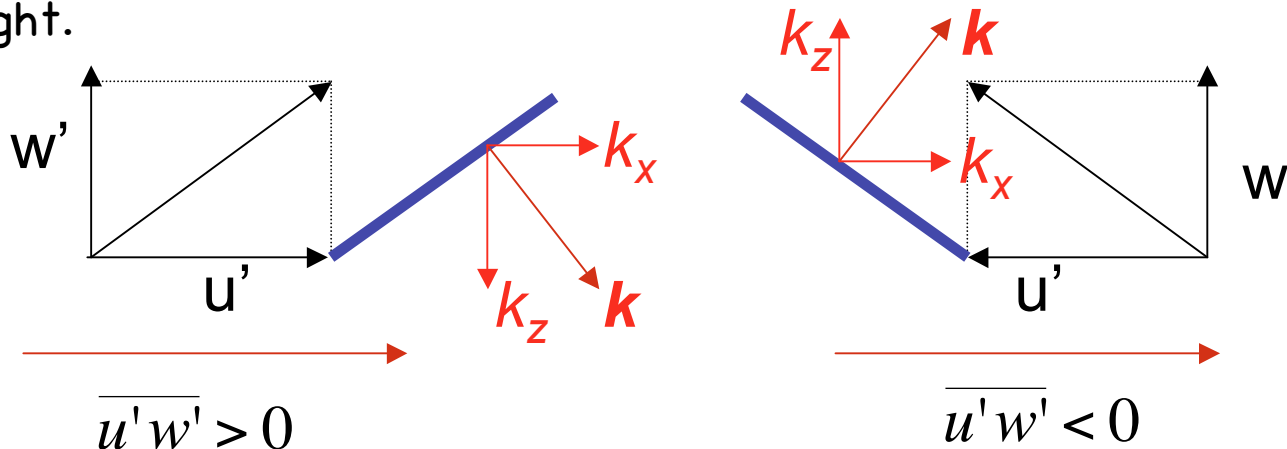
$$\overline{p'w'^*} = \rho_0 (c_{px} - u_0) \overline{u'w'^*}$$

Physical insights gained from dispersion/polarization relations and energy and momentum flux:

a) GW is a transverse wave:  $\vec{k} \cdot \vec{v} = 0$  so particle trajectory // phase line.

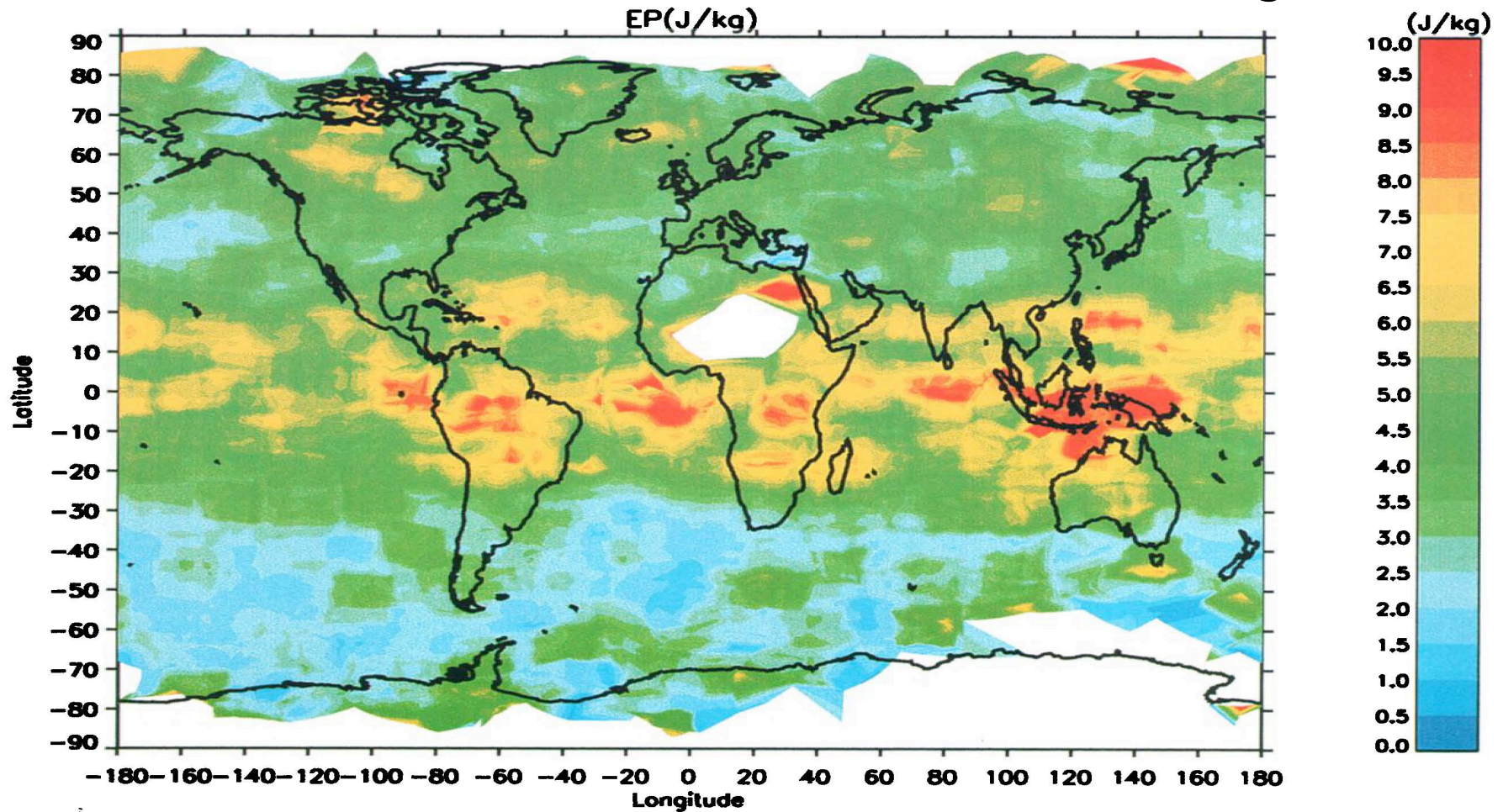
b) Phase line of a vertically propagating wave can't be vertical or horizontal. Otherwise,  $u'$  or  $w'$  will be 0 and momentum flux=0.

c) The phase line of the wave must tilt toward the propagating direction for the vertical flux of momentum in the propagating direction to be positive. In the following schematic plot,  $u'w' > 0$  requires the particle trajectory tilted toward the right. Because GW is a transverse wave, the phase line is parallel to the particle trajectory. Therefore, the phase line moves downward with respect to a ground observer as it moves to the right.



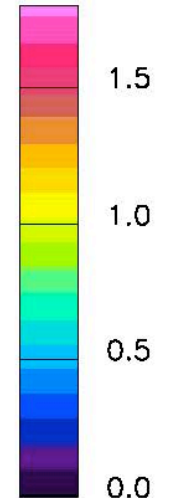
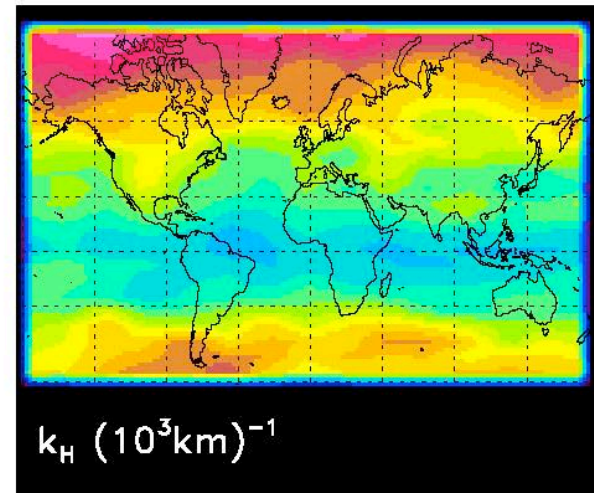
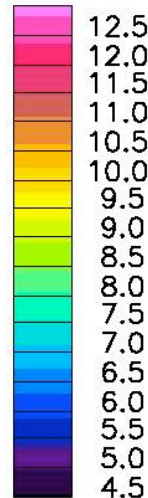
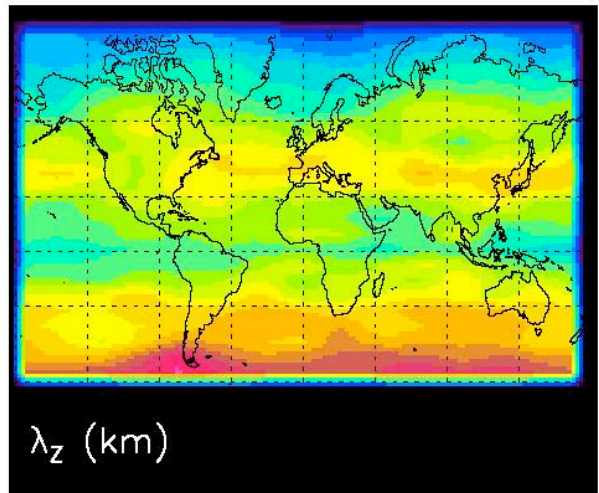
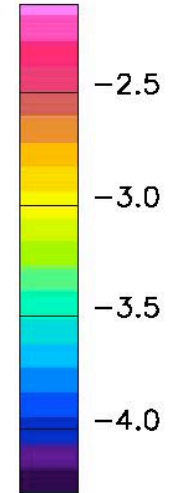
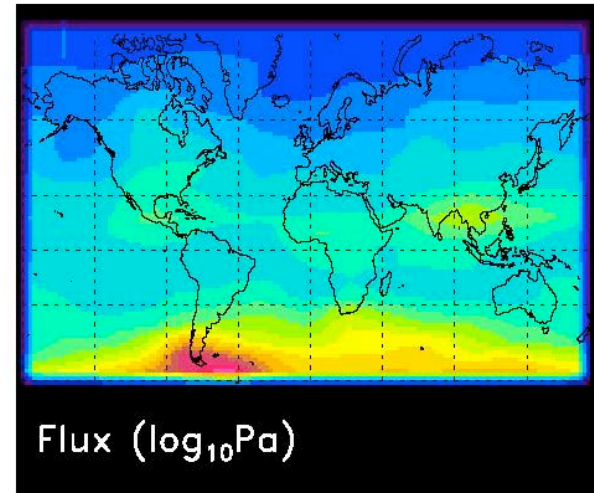
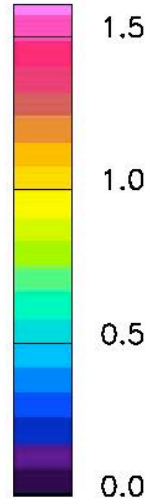
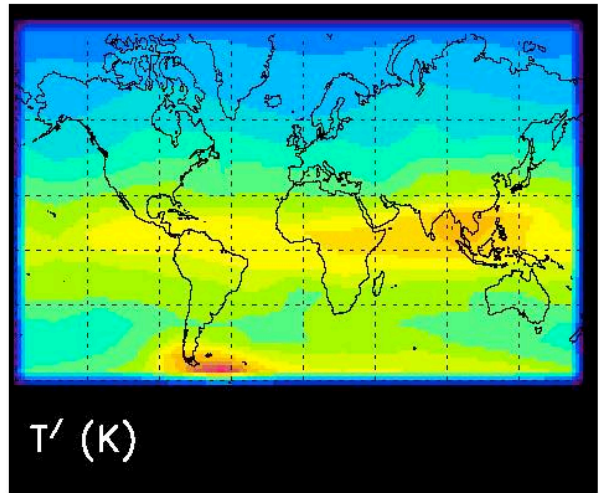


# Satellite Observation of Temperature Variance and GW Potential Energy



**Plate 1.** Global distribution of  $E_p$  from the GPS/MET data at 20–30 km in November–February. The  $E_p$  value is averaged in an area extending  $10^\circ$  and  $20^\circ$  in latitude and longitude, and the center coordinates are shifted every  $1^\circ$  and  $2^\circ$ , respectively.

# Application: Global GW Momentum Flux from Temperature Observations



Alexander et al., 2008

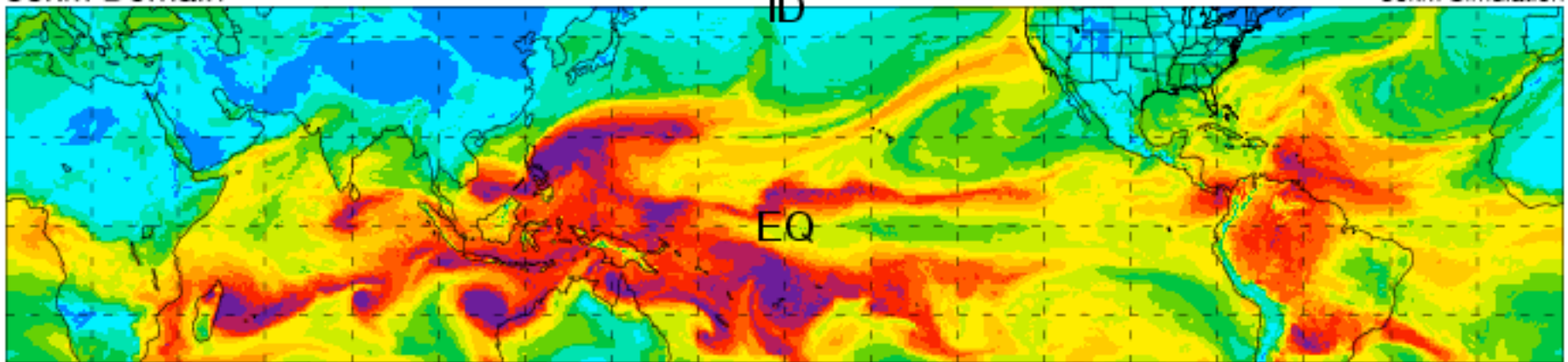


# Another Challenge

PW (mm) - 19970101 00Z

36km Domain

36km Simulation



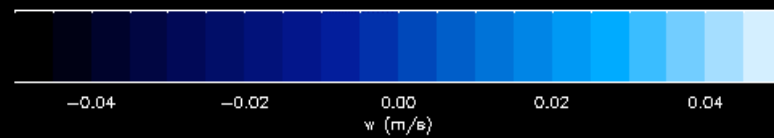
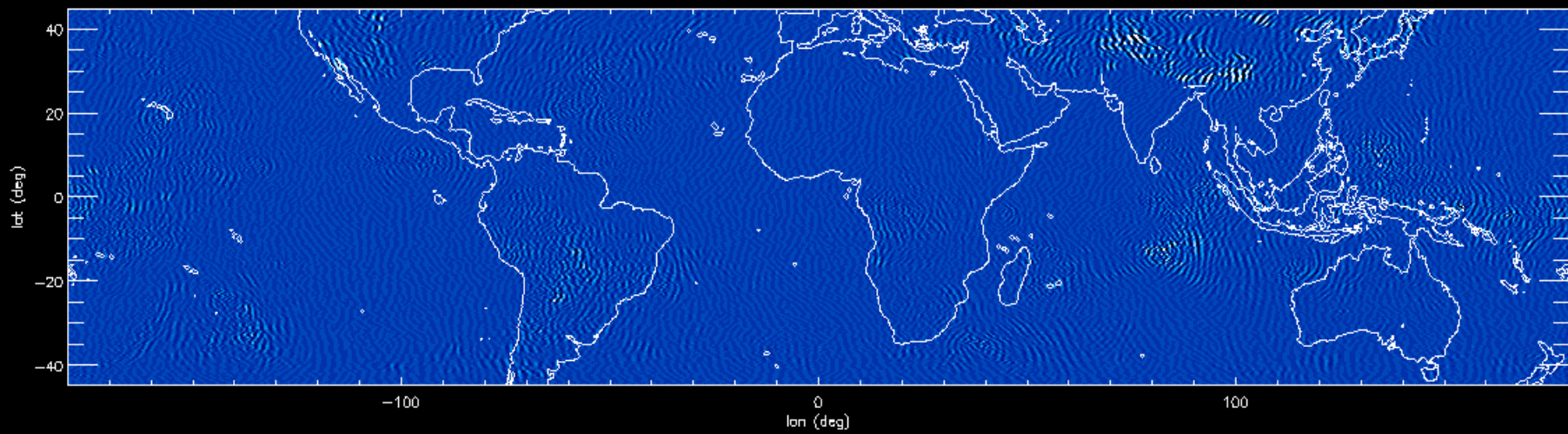


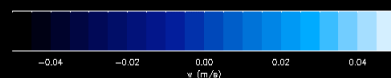
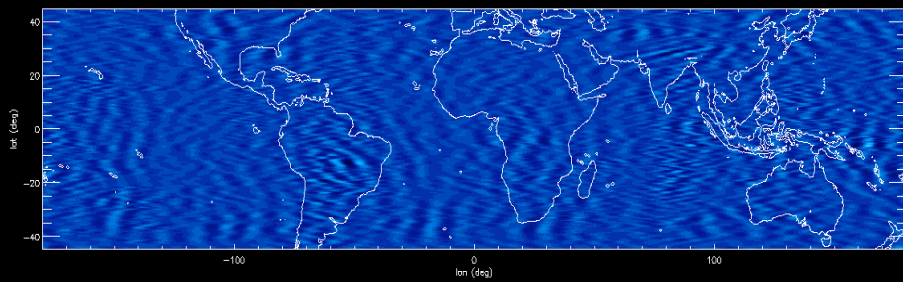
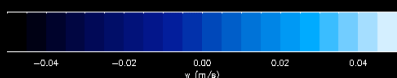
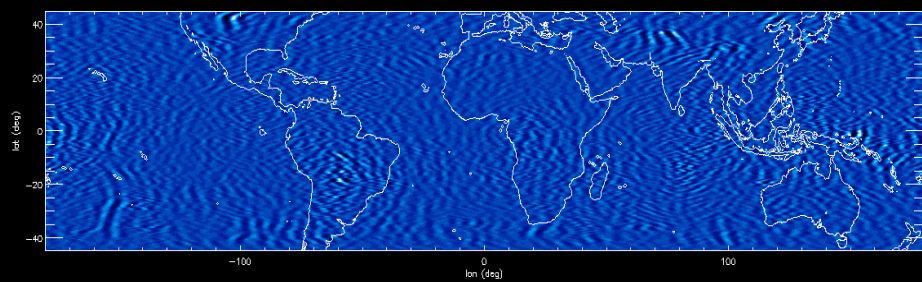
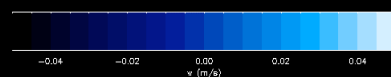
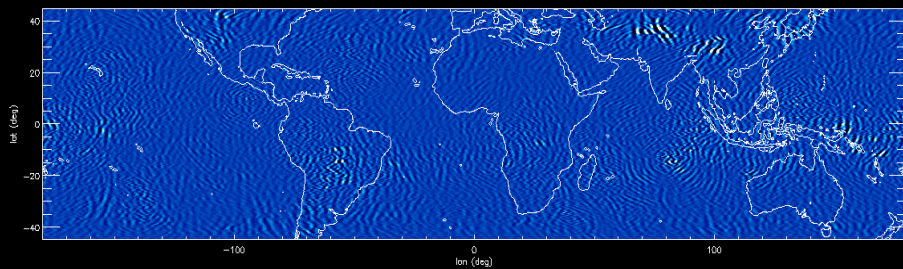
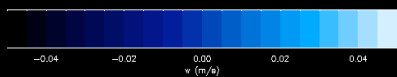
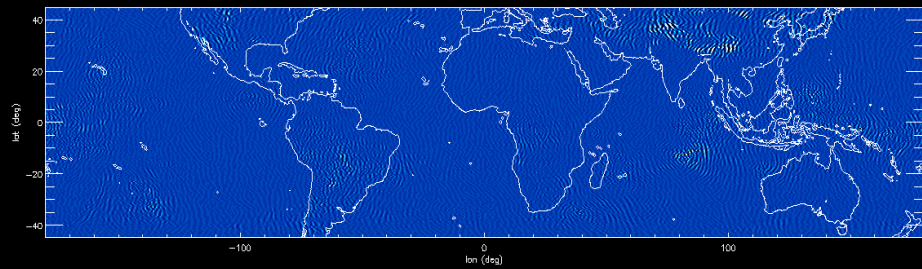
# Diagnostic Analysis (Liu and Dudhia, 2008)

- Reconstruct all field variables in 5 bands in zonal direction: 0-100km, 100-200km, 200-400km, 400-800km, 800-1600km.
- Calculate  $\overline{p'w'^*}$ ,  $\overline{u'w'^*}$ ,  $\overline{v'w'^*}$  over the respective scale ranges.
- Using dispersion and polarization relationship of GW, deduce intrinsic frequency, horizontal wavenumber, vertical wavenumber, propagation direction, and intrinsic phase speed for each zonal scale ranges.

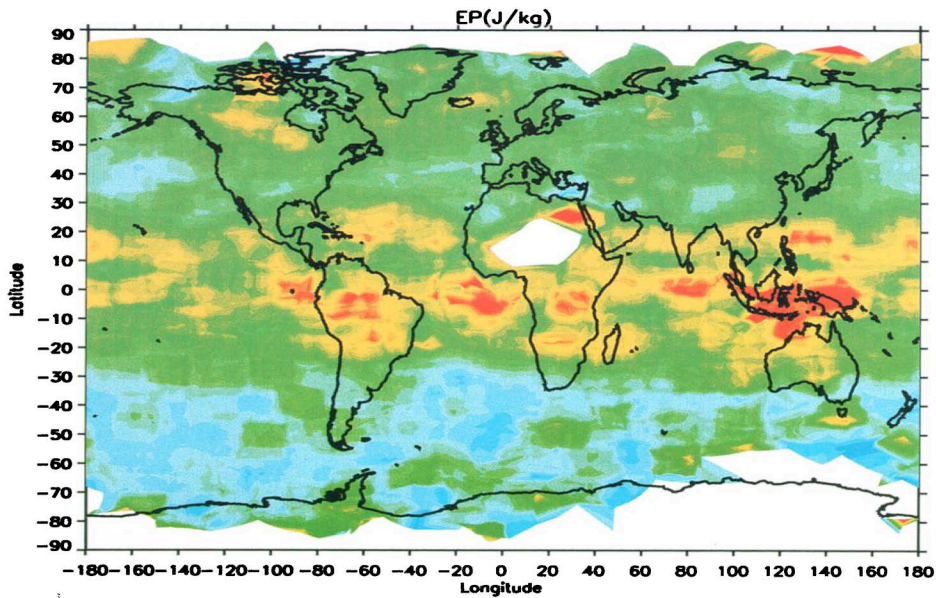
$$\text{Re}(\overline{\rho u'w'^*}) \approx -\left(\frac{\omega}{N}\right)^2 \frac{km}{\omega^2 - f^2} \rho A^2, \quad \text{Re}(\overline{\rho v'w'^*}) \approx -\left(\frac{\omega}{N}\right)^2 \frac{lm}{\omega^2 - f^2} \rho A^2$$

$$\text{Re}(\overline{p'w'^*}) \approx -\frac{\omega m}{N^2} \rho A^2$$



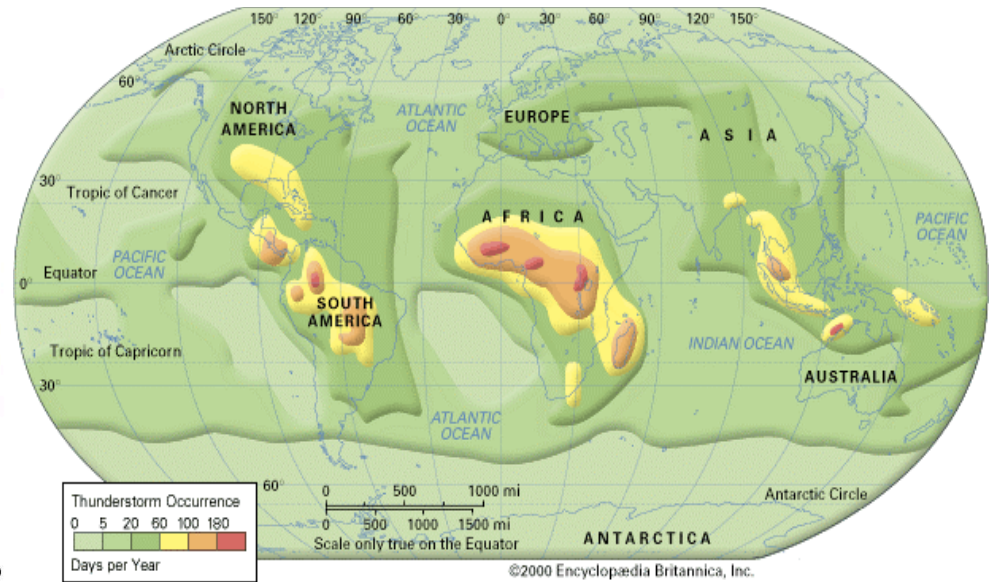




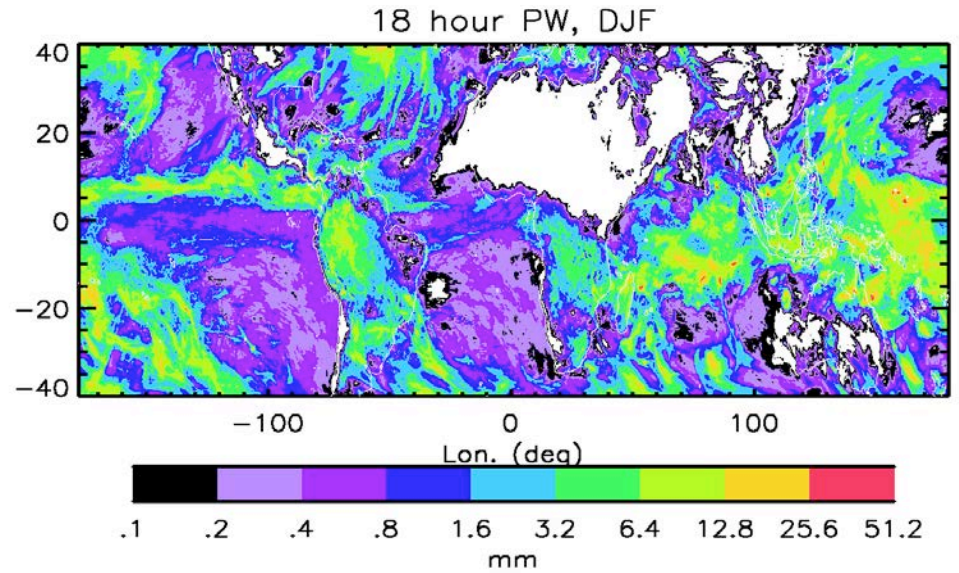
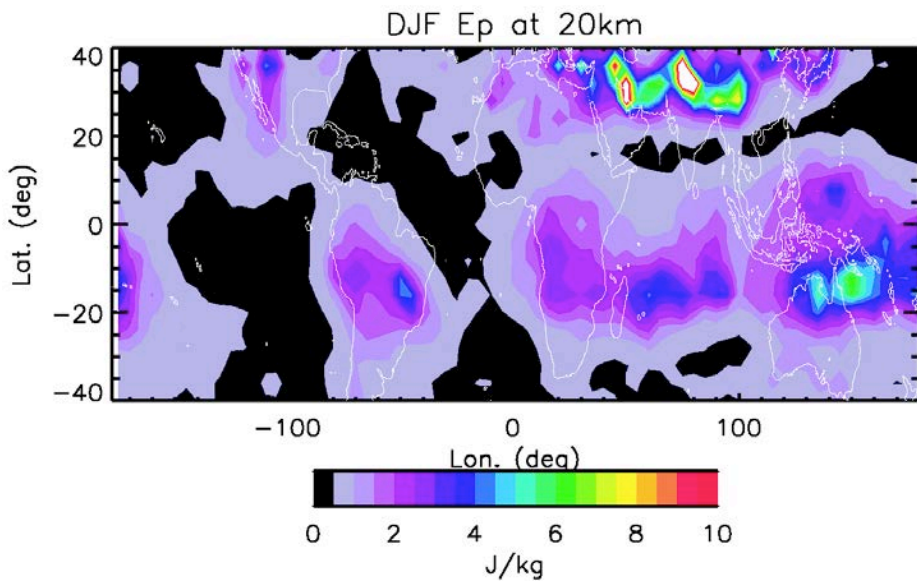


**Plate 1.** Global distribution of  $E_p$  from the GPS/MET data at 20–30 km in November–February. The  $E_p$  value is averaged in an area extending  $10^\circ$  and  $20^\circ$  in latitude and longitude, and the center coordinates are shifted every  $1^\circ$  and  $2^\circ$ , respectively.

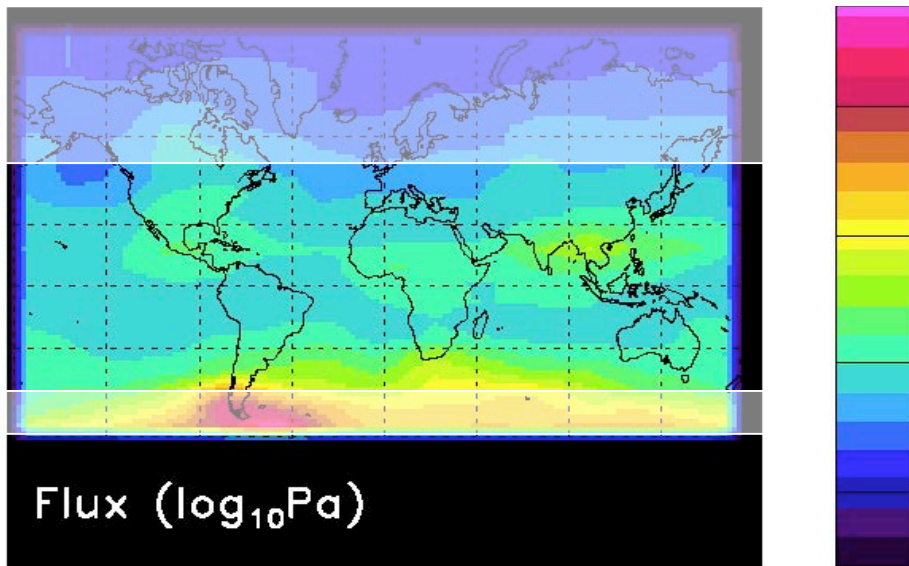
Tsuda et al, 2000



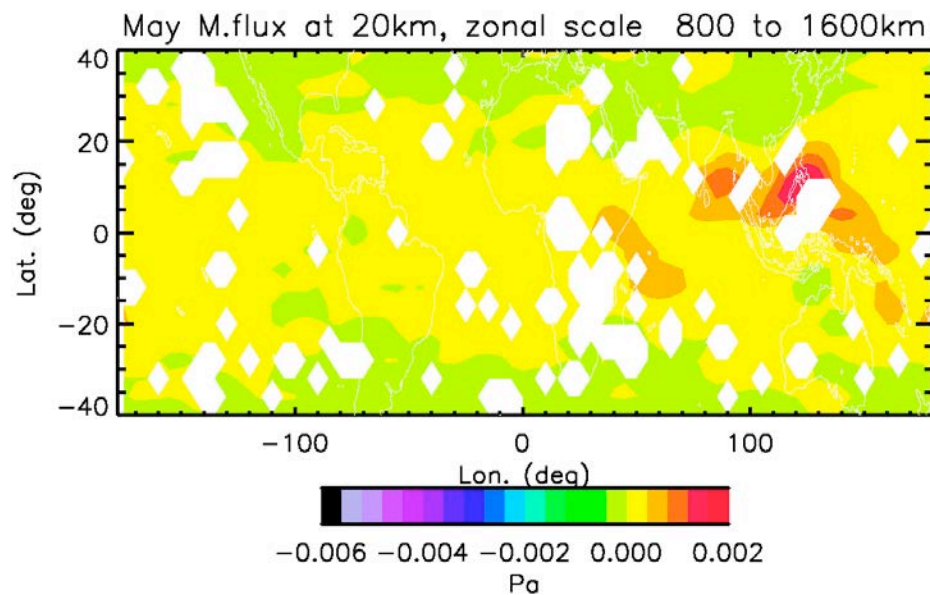
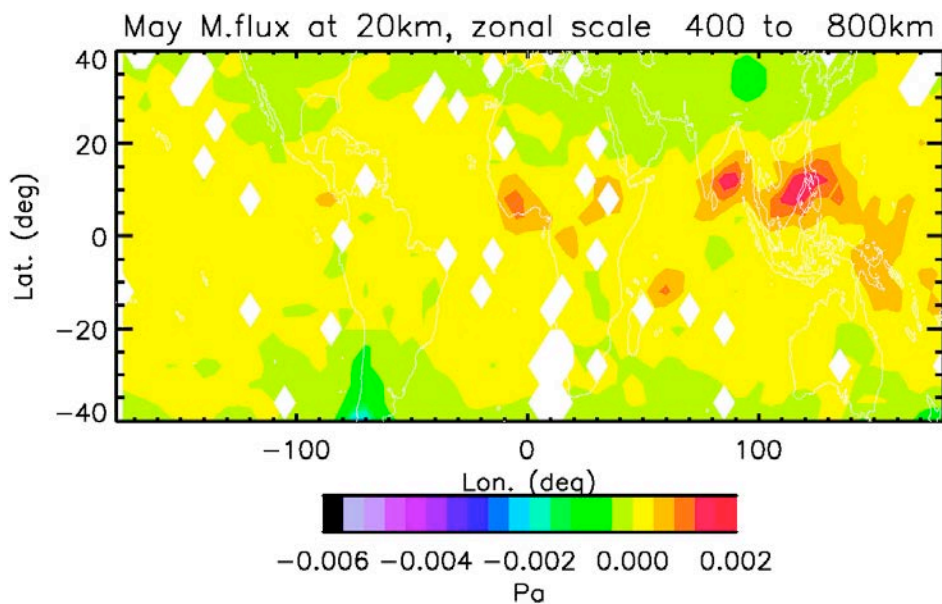
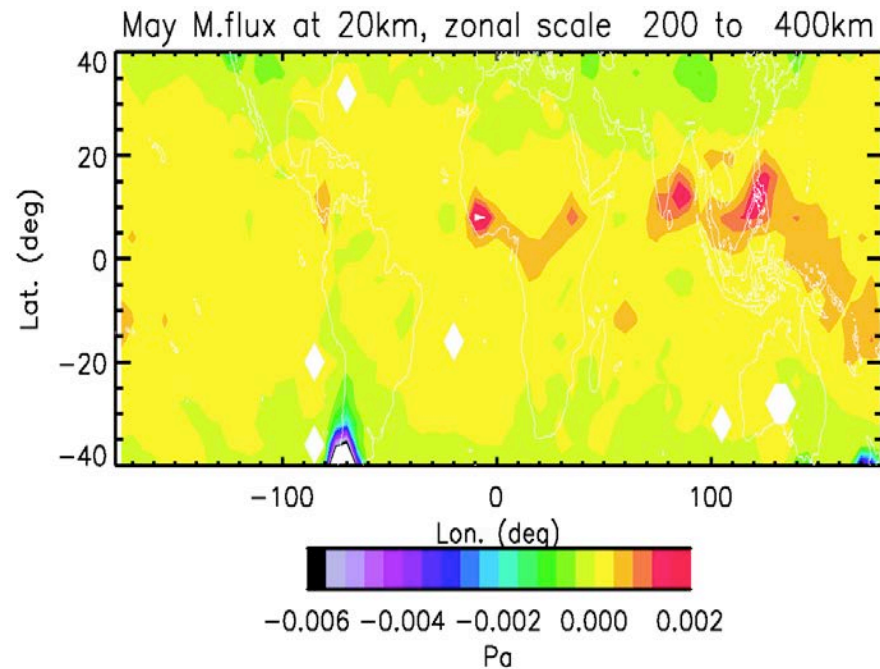
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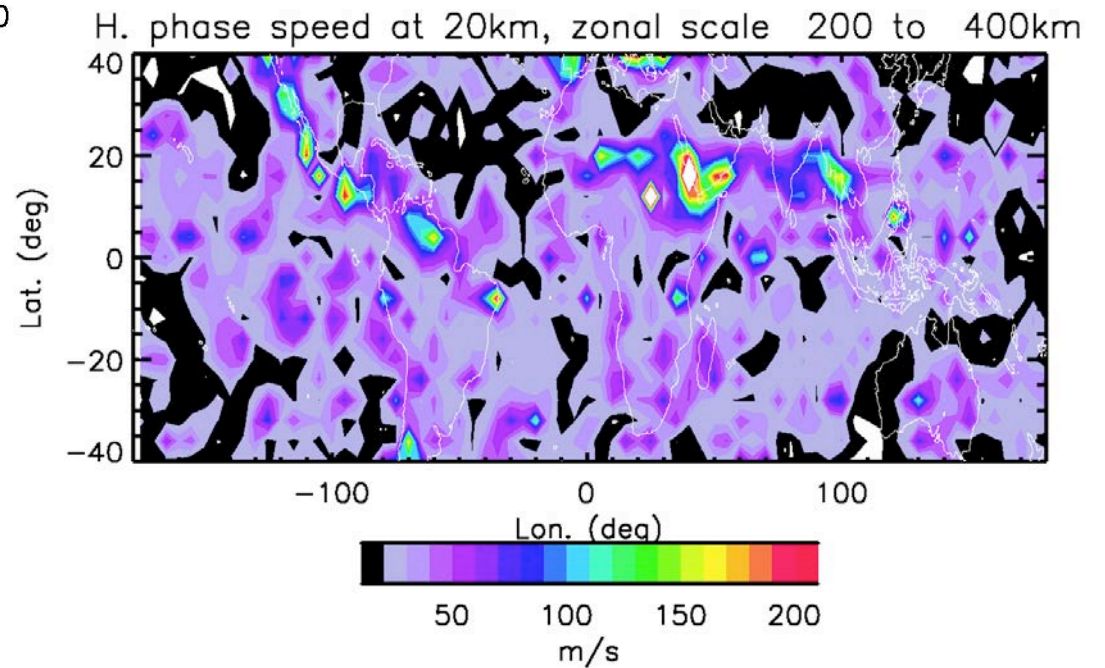
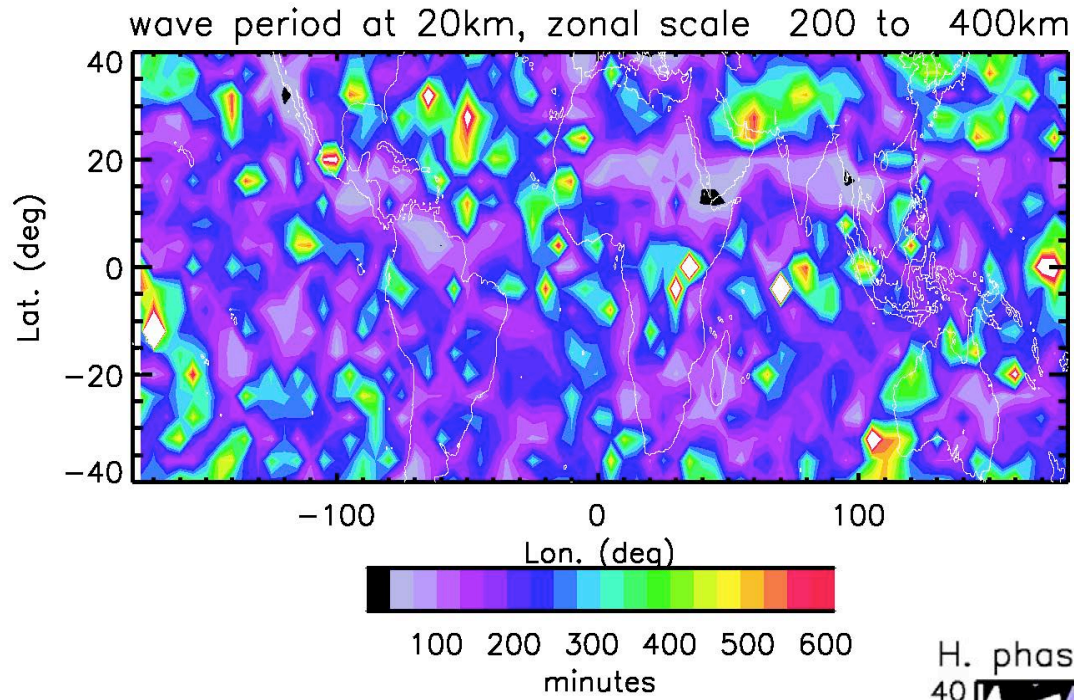




Alexander et al. (2008)

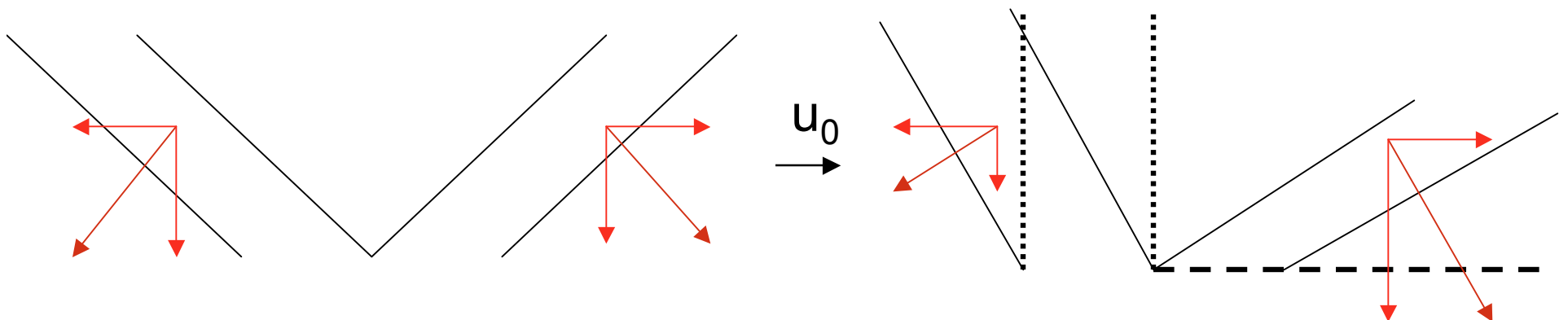


# Deriving Wave Characteristics



# GW Propagation

- GW amplitude grows to compensate for the exponential decrease of density with altitude.
- Atmospheric temperature change leads to change in buoyancy frequency.
- Atmospheric wind leads to changes in wave characteristics.



Change of wave characteristics with background atmosphere:

$$k_z^2 = \left( \frac{N^2}{(\omega - k_x u_0)^2} - 1 \right) k_x^2 = \left( \frac{N^2}{\omega_i^2} - 1 \right) k_x^2$$

$N^2/(\omega - k_x u_0)^2$  increases either as  $N$  increases or intrinsic frequency decreases:  $k_z^2$  increases. Singularity:  $\omega - k_x u_0 = 0$  or  $c_{px} = u_0$ , critical layer.

$N^2/(\omega - k_x u_0)^2$  decreases either as  $N$  decreases or intrinsic frequency increases:  $k_z^2$  decreases. Singularity:  $N^2 = (\omega - k_x u_0)^2$ , evanescence and wave reflection.

Further propagation of GW will stop if there exists instability, critical layer, or evanescence.



# GW Impact on the Mean Flow

The general momentum equation in the horizontal direction:

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2 + p}{\partial x} + \frac{\partial \rho uv}{\partial y} + \frac{\partial \rho uw}{\partial z} = \mu \nabla^2 u$$

Taking an average over the x-direction, and with proper boundary conditions (e.g. periodic, or zero flux)

$$\frac{\partial \overline{\rho u}}{\partial t} + \frac{\partial \overline{\rho uv}}{\partial y} + \frac{\partial \overline{\rho uw}}{\partial z} = \mu \nabla^2 \overline{u}$$

In wave propagation direction,

$$\frac{\partial \overline{\rho u}}{\partial t} + \frac{\partial \overline{\rho uw}}{\partial z} = \mu \nabla^2 \overline{u}$$

The divergence of momentum flux alters the mean horizontal velocity.

# Some Number Crunching

A typical value of GW momentum flux is  $10^{-3}$  Pa. If deposited at various height, the acceleration rates can be estimated (assuming vertical scale of 10km):

Height (density):      Acceleration       $\Delta t$  for  $\Delta u=50\text{m/s}$

20km ( $\sim 10^{-1}\text{kg/m}^3$ ):  $8.6 \times 10^{-2}\text{ms}^{-1}\text{d}^{-1}$       581d

30km ( $\sim 10^{-2}\text{kg/m}^3$ ):  $8.6 \times 10^{-1}\text{ms}^{-1}\text{d}^{-1}$       58d

50km ( $\sim 10^{-3}\text{kg/m}^3$ ):  $8.6\text{ms}^{-1}\text{d}^{-1}$       5.8d      *QBO:  $\Delta u$  in  $\sim 1$  year*

70km ( $\sim 10^{-4}\text{kg/m}^3$ ):  $86\text{ms}^{-1}\text{d}^{-1}$       14 hours

85km ( $\sim 10^{-5}\text{kg/m}^3$ ):  $860\text{ms}^{-1}\text{d}^{-1}$       1.4 hours

*Dominant momentum source*

# Eliassen-Palm's Second Theorem

Goal: Determine divergence of vertical flux of the horizontal momentum, thus the forcing on the mean flow.

From a rather general sets of linear equations, the following is obtained:

$$-\frac{\partial \overline{\rho_0 u' w'}}{\partial z} = \frac{1}{\gamma(u_0 - c_{px})} \overline{D' \left( \frac{s'}{H} + \frac{p'}{p_0} \right)}$$

$D'$ : dissipation/diabatic force.

$s'$ : vertical displacement perturbation

$p'$ : pressure perturbation

GW will not change the mean flow in a "conservative" environment. It is also proven that there is no net heat/mass transport under these conditions.

So under what circumstances is damping important?

# GW Damping and Forcing

- Molecular dissipation (especially in the thermosphere).
- Instability → turbulence.
  - Gravity wave amplitude increases exponentially with altitude.
  - Increase of static stability,  $N$ .
  - Decrease of intrinsic frequency (doppler shift due to wind).
- Critical layer: decreasing vertical wavelength (dissipation inversely proportional to square of vertical wavelength), increasing perturbative shear and lapse rate.

- From E-P's first theorem  $\overline{p'w'^*} = \rho_0(c_{px} - u_0)\overline{u'w'^*}$ 
  - For wave with upward energy propagation the deposition of wave momentum tends to bring the mean flow to the phase velocity of the wave.

$$\overline{p'w'^*} > 0 : c_{px} - u_0 > 0, \rho_0\overline{u'w'} > 0, \text{ if deposited } \frac{\partial u_0}{\partial t} > 0.$$

$$c_{px} - u_0 < 0, \rho_0\overline{u'w'} < 0, \text{ if deposited } \frac{\partial u_0}{\partial t} < 0.$$

$$\text{In both cases : } \frac{\partial |c_{px} - u_0|}{\partial t} < 0$$

- Wave and mean flow will interact around critical layer: intrinsic phase change sign around critical layer, momentum flux will also need to change sign, which implies momentum desposition.

## Constraining GW Forcing in MLT

- Difficult to directly observe acceleration rates caused by GW forcing.
- Direct computation requires detailed knowledge of GW source amplitudes and distributions, atmosphere temperature and winds, good understanding of GW breaking processes, and a good computer.
- Or use classroom scale analysis. Recall:

70km ( $\sim 10^{-4}\text{kg/m}^3$ ):       $86\text{ms}^{-1}\text{d}^{-1}$       14 hours

85km ( $\sim 10^{-5}\text{kg/m}^3$ ):       $860\text{ms}^{-1}\text{d}^{-1}$       1.4 hours

So GW forcing is likely the dominating momentum source in MLT.

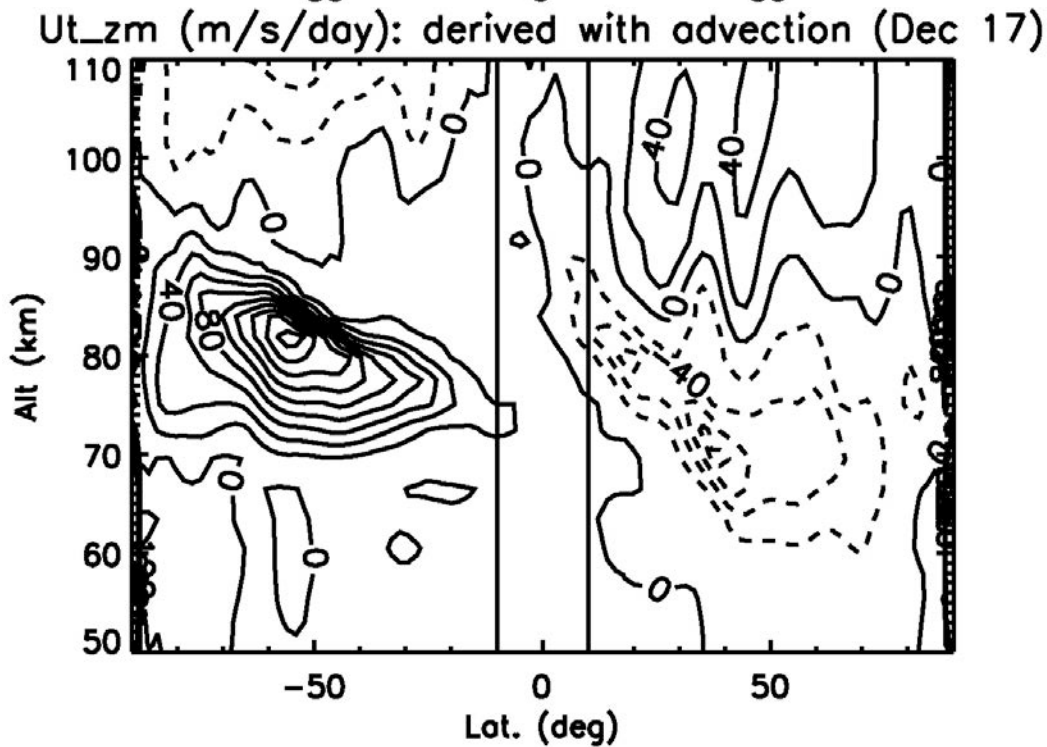
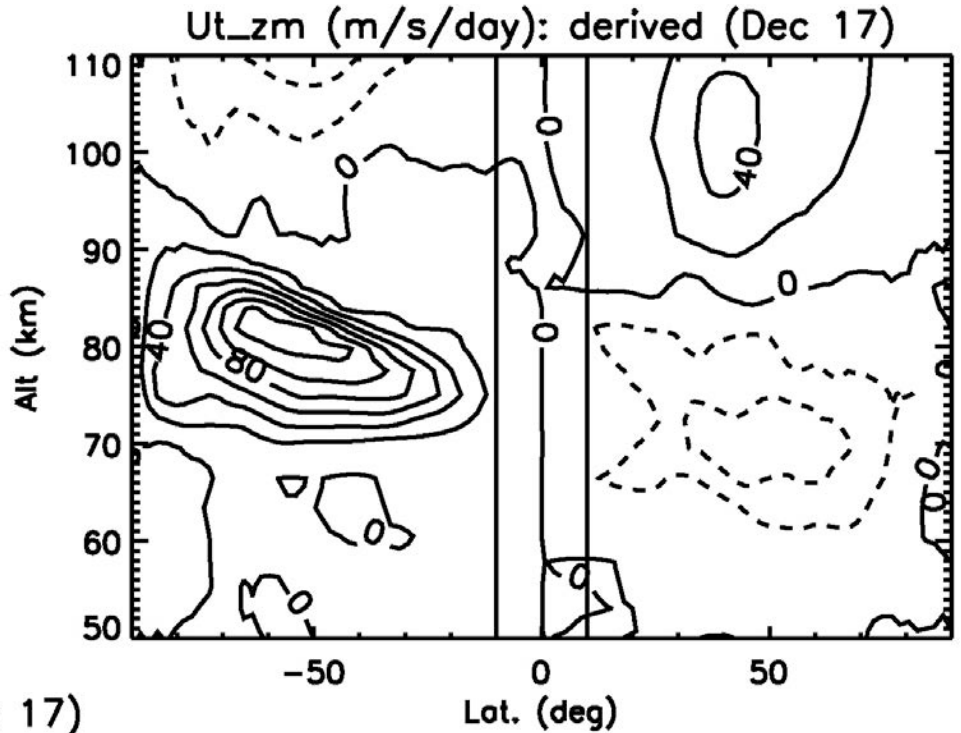
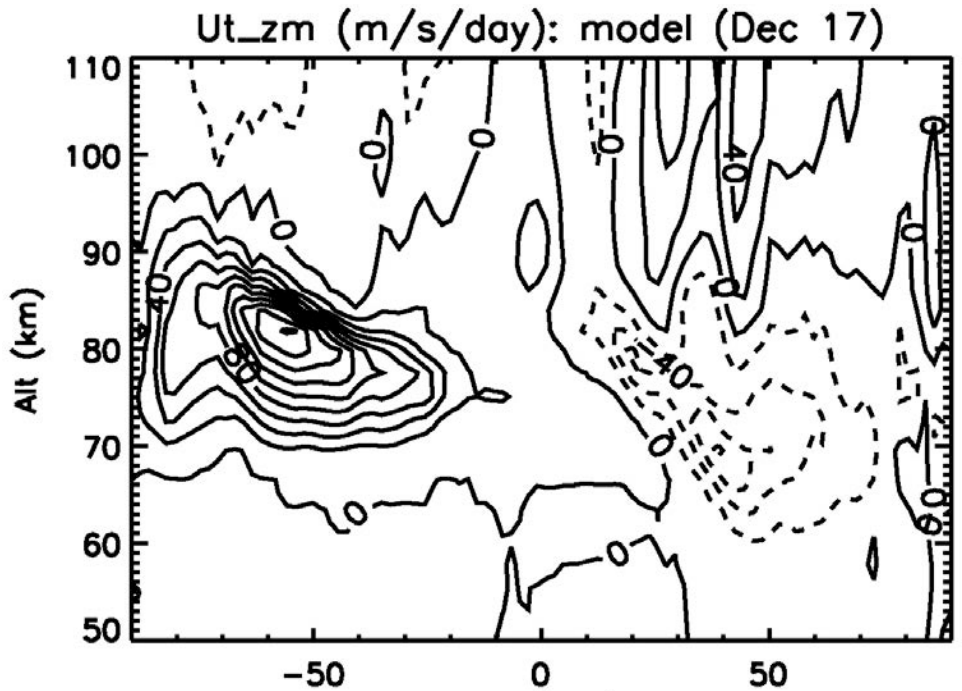
# Scale Analysis: Momentum Balance

$$\left(f + \frac{\bar{u} \tan \phi}{r} + \frac{1}{r} \frac{\partial \bar{u}}{\partial \phi}\right) \bar{v} + \bar{F}_x = \frac{\partial \bar{u}}{\partial t} + \frac{1}{a \cos^2 \phi} \frac{\partial \overline{u'v'} \cos^2 \phi}{\partial \phi} + \frac{1}{\rho} \frac{\partial \overline{\rho u'w'}}{\partial z}$$

Planetary waves, tides

- For climatology,  $\partial u / \partial t = 0$ .
- If both mean winds and PW and tidal winds could be accurately determined, the climatological zonal mean GW forcing can be derived.
- If PW or tidal winds cannot be determined, but if they are secondary compared to GW forcing, then GW forcing can be determined to the first order.
- For local measurements, we will further assume  $|f| \gg 1/r \approx |\partial u / \partial \phi|$ .

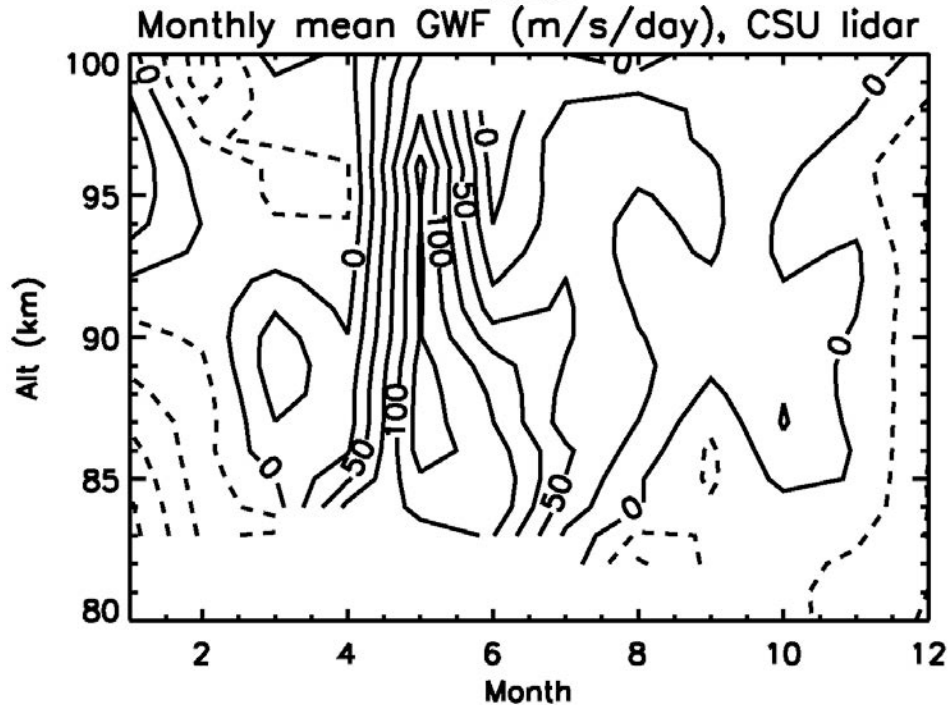
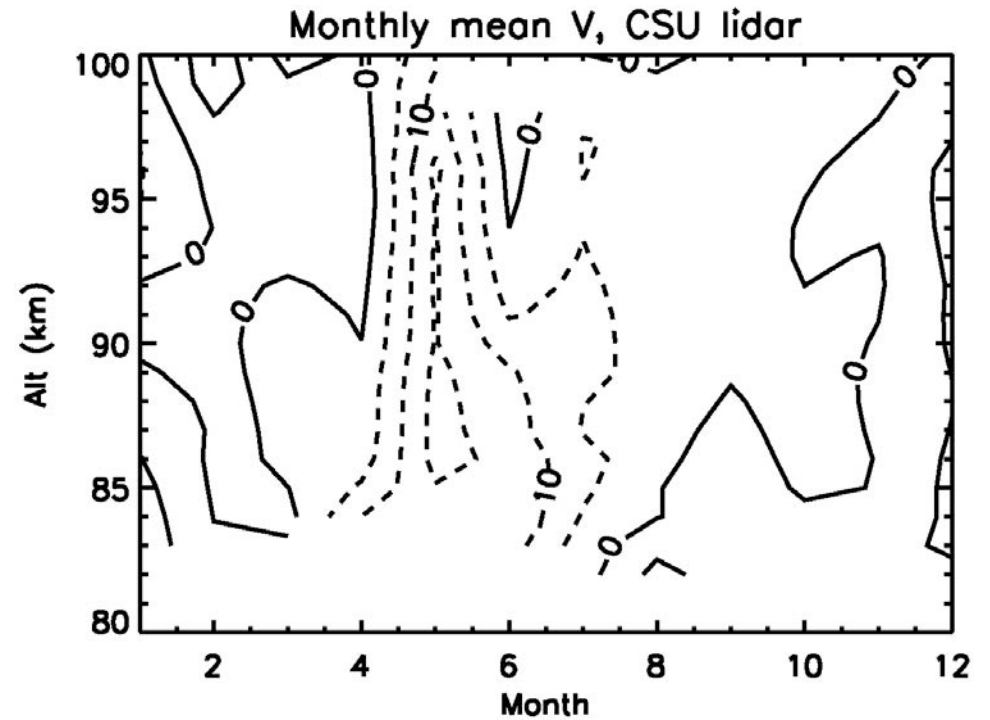
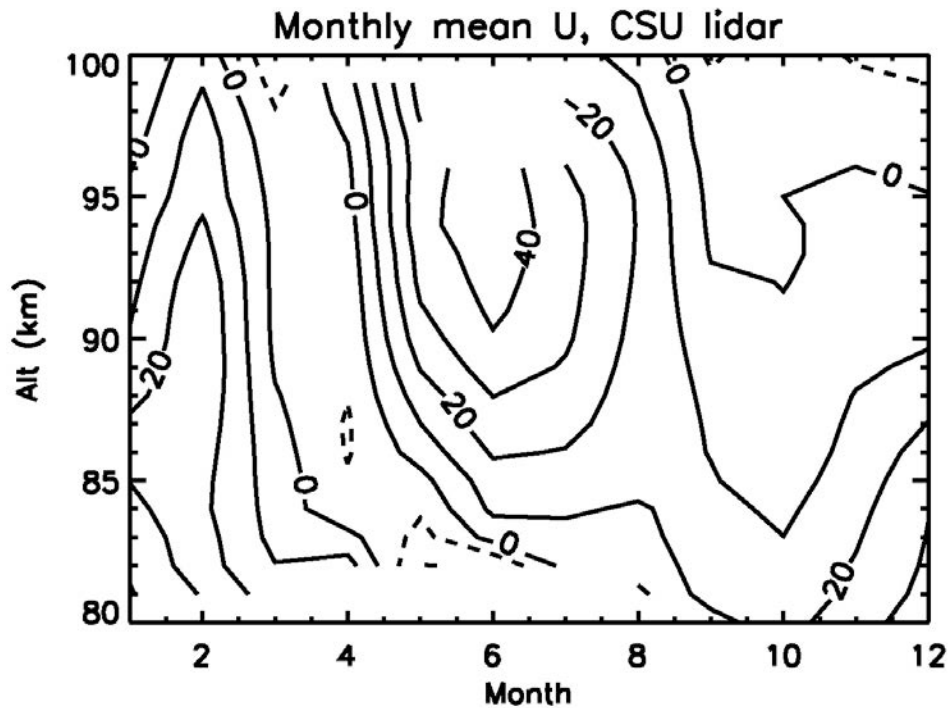
$$\bar{F}_x \approx -\left(f + \frac{\bar{u} \tan \phi}{r}\right) \bar{v}$$



**GWF<sub>zm</sub>: Actual vs  
Derived (WACCM)**

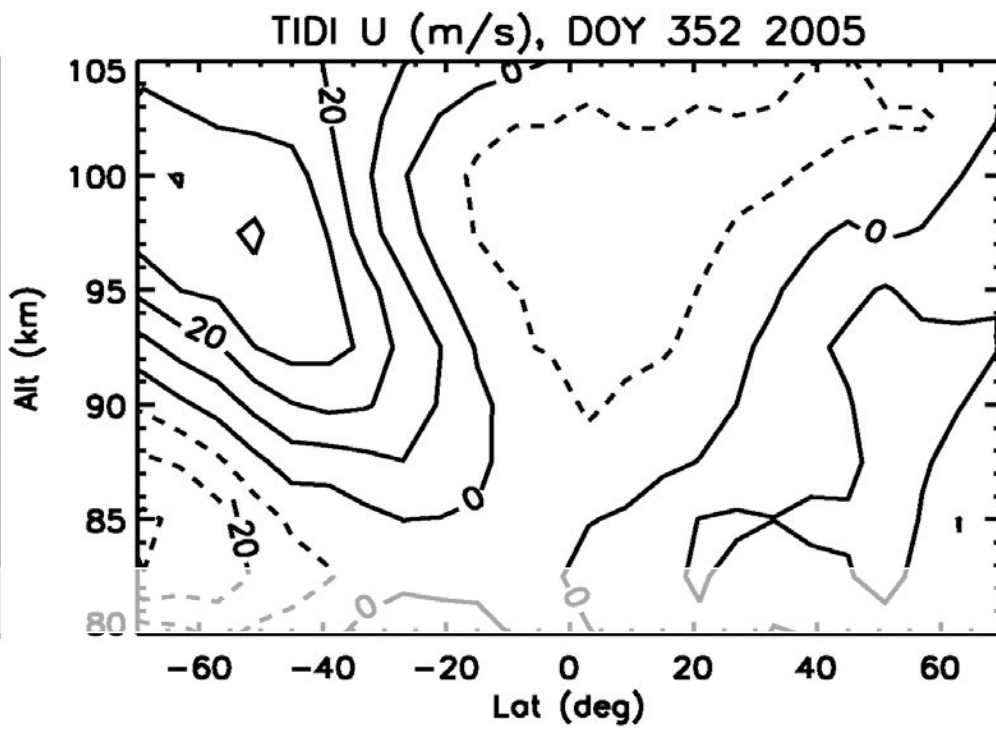
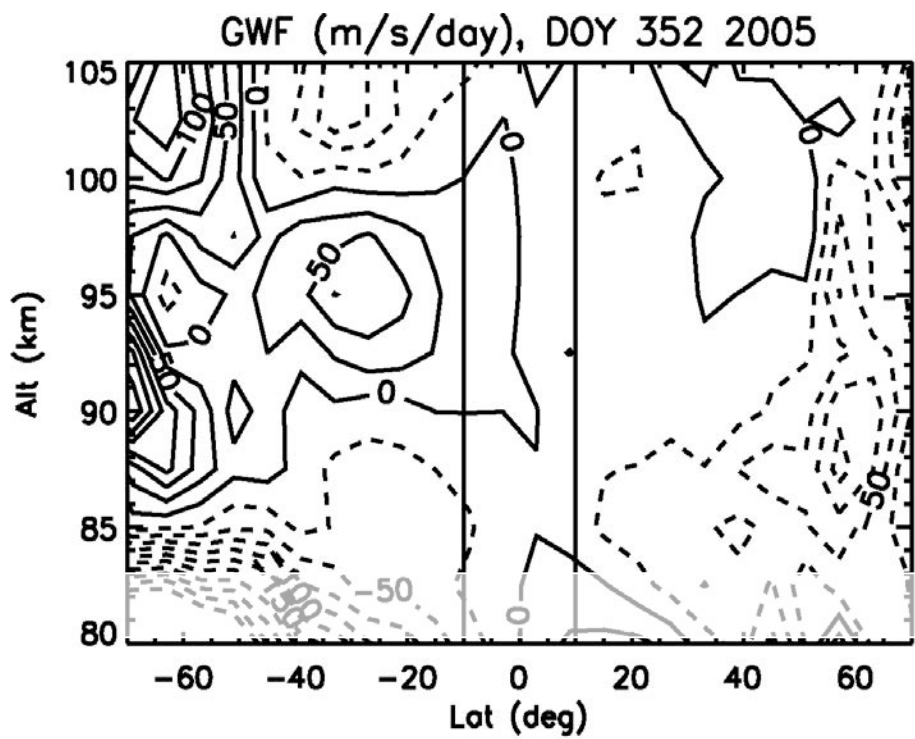
$$\bar{F}_x = -\left(f + \frac{\bar{u}_G \tan \phi}{r}\right)\bar{v}$$





# U, V and Derived GW Force: CSU Lidar Climatology (41N)

Liu et al, 2008.



# Challenges in Gravity Waves Research

- An important agent for atmosphere coupling by transporting momentum and energy from lower atmosphere to upper atmosphere.
- Sources: Global distribution and challenging to characterize.
- Propagation: In a complex wind system.
- The impacts: Global scale to microscale, and involve interactions of the multiple scales.
- Detailed GW breaking process and interaction with turbulence.
- Role of GW in regulating/modulating precipitation.
- Role of GW in ionosphere and thermosphere.

# Role of CEDAR in GW Study

- C for Coupling: GW from lower to upper atmosphere.
- Observational networks plus portable systems: Characterize GW at “hot spots”.
- Ground based observations with high temporal/spatial resolution: Potential for detailed GW quantification.
- Extended temporal coverage: Potential for constraining GW forcing and studying longer term variability.

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