Geospace Electrodynamics

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June 20, 2016

Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho_c}{\epsilon_0} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$

$$\frac{\partial \rho_c}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

- Solutions where $\rho_c = 0$ and $\mathbf{J} = 0$ are a completely solved problem
- Solutions where ρ_c and $\mathbf{J}$ are known a priori are a completely solved problem
- In media (like geospace plasmas) $\mathbf{J}$ depends on the fields $\mathbf{E}$ and $\mathbf{B}$
- A generalized Ohm's law (GOL) relating $\mathbf{J}$ to $\mathbf{E}$ and $\mathbf{B}$ is needed to close the system of equations

Vlasov - Maxwell Equations

$$\frac{\partial f_e}{\partial t} + \mathbf{v} \cdot \nabla f_e + \left[-\frac{e}{m_e} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) + \mathbf{g} \right] \cdot \nabla_{\mathbf{v}} f_e = \frac{\delta f_e}{\delta t}$$

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla f_i + \left[\frac{q_i}{m_i} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) + \mathbf{g} \right] \cdot \nabla_{\mathbf{v}} f_i = \frac{\delta f_i}{\delta t}$$

$$\mathbf{J} = \sum_i q_i \int \mathbf{v} f_i \, d\mathbf{v} - e \int \mathbf{v} f_e \, d\mathbf{v}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$

- $f \left(\mathbf{x}, \mathbf{v}, t \right)$ are 7-dimensional particle distribution functions
- $\frac{\delta}{\delta t}$ denotes collisional terms
- Completely impractical to use in most situations

Constructing Approximate Theories of Electrodynamics

Theories of geospace electrodynamics differ depending on:

- Inclusion of displacement current $\frac{\partial \mathbf{E}}{\partial t}$
    - Only important for radio-waves and high-frequency phenomena
- Inclusion of inductive fields $\frac{\partial \mathbf{B}}{\partial t}$
    - Electrostatic approximation common in ionosphere
- Approximations of particle motion (simplifications of the GOL)
    - Fluid vs kinetic
    - Guiding center approximation
    - Adiabatic assumptions

Areas of Geospace Electrodynamics

1. Ionospheric Electrostatics
2. Inner Magnetospheric Kinetic Electrodynamics
3. Magnetohydrodynamics
4. Solar Wind-Magnetosphere-Ionosphere Coupling
5. The Polar Wind and Auroral Acceleration Region

Electrostatic Approximation

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \cancelto{0}{\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}} \longrightarrow \nabla \cdot \mathbf{J} = 0 \qquad (\text{Recall: } \nabla \cdot \nabla \times \mathbf{B} = 0)$$

$$\nabla \times \mathbf{E} = -\cancelto{0}{\frac{\partial \mathbf{B}}{\partial t}} \longrightarrow \mathbf{E} = -\nabla \Phi \qquad (\text{Recall: } \nabla \times \nabla \Phi = 0)$$

Ohm's Law for the ionosphere:

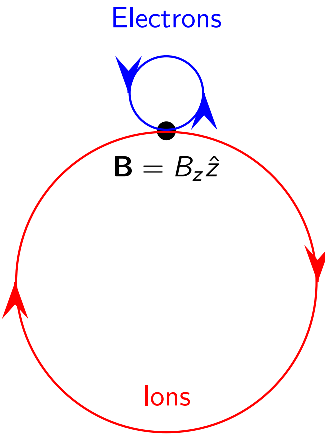
$$\mathbf{J} = \sigma \cdot \mathbf{E} + \mathbf{J}_0$$

Putting everything together yields a boundary value problem:

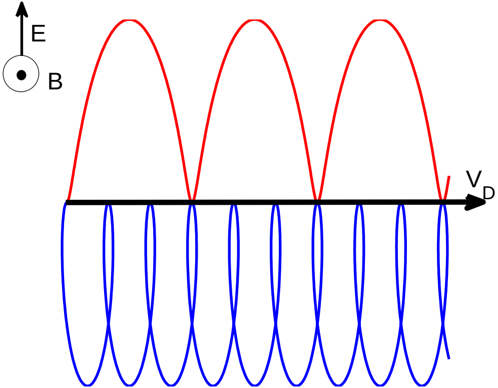
$$\nabla \cdot \sigma \cdot \nabla \Phi = \nabla \cdot \mathbf{J}_0$$

Motion of Particles in Uniform Fields: $\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$

Uniform **B** Field



Crossed Uniform **E** and **B**



$v_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$

Note $\mathbf{E} + \frac{\mathbf{E} \times \mathbf{B}}{B^2} \times \mathbf{B} = 0$ as long as $\mathbf{E} \cdot \mathbf{B} = 0$

Effects of Collisions: Ohm's Law for the Ionosphere

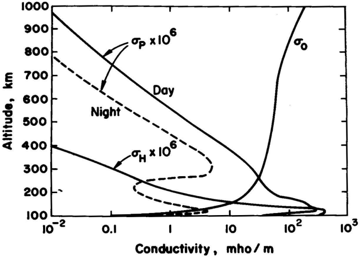
Steady-state momentum equation for each species (zero neutral wind case):

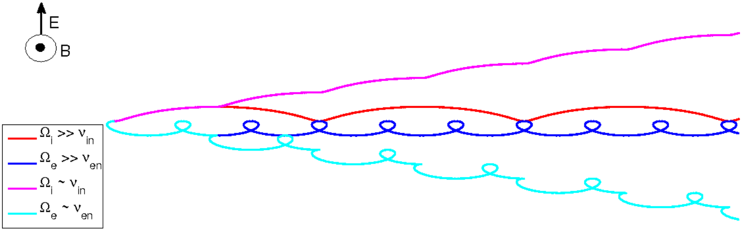
$$0 = n_\alpha q_\alpha \left(\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}\right) - \nu_{\alpha n} m_\alpha n_\alpha \mathbf{u}_\alpha$$

Resulting Ohm's Law:

$$\mathbf{J} = \sum_\alpha n_\alpha q_\alpha \mathbf{u}_\alpha \longrightarrow \mathbf{J} = \begin{pmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix} \cdot \mathbf{E}$$

Conductivity Profiles

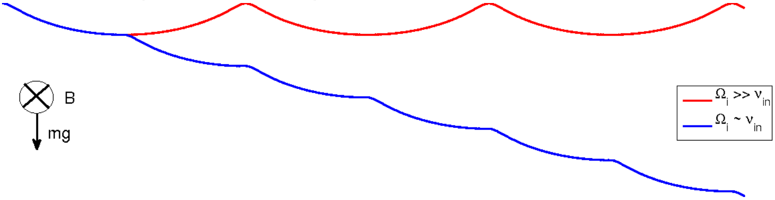




Other Kinds of Current: Complete Dynamo Equation

Substitute $\mathbf{F}$ for $q_\alpha \mathbf{E}$ in steady state momentum equation.

- Wind drag: $\mathbf{F} = \nu_{\alpha n} m_\alpha \mathbf{u}_n \longrightarrow \mathbf{J} = \sigma \cdot (\mathbf{u}_n \times \mathbf{B})$
- Gravity: $\mathbf{F} = m_\alpha \mathbf{g} \longrightarrow \mathbf{J} = \mathbf{\Gamma} \cdot \mathbf{g}$



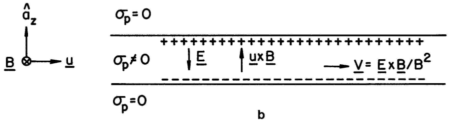
- Pressure Gradients (Diamagnetic Currents):
  $\mathbf{F} = -\frac{1}{n_\alpha} \nabla p_\alpha \longrightarrow \mathbf{J} = \mathbf{D} \cdot \nabla \sum_\alpha p_\alpha$

Complete Dynamo Equation:

$$\nabla \cdot \sigma \cdot \nabla \Phi = \nabla \cdot \left(\sigma \cdot (\mathbf{u}_n \times \mathbf{B}) + \mathbf{\Gamma} \cdot \mathbf{g} + \mathbf{D} \cdot \nabla \sum_\alpha p_\alpha \right)$$

Slab Models of F- and E-region Dynamos

F-region

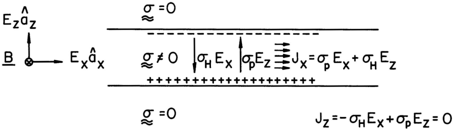


$$\mathbf{J} = \sigma_P \left(\mathbf{E} + \mathbf{u}_n \times \mathbf{B}\right)$$

$$\mathbf{J} = 0 \longrightarrow \mathbf{E} = -\mathbf{u}_n \times \mathbf{B}$$

$$\mathbf{v}_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2} = \frac{-\mathbf{u}_n \times \mathbf{B} \times \mathbf{B}}{B^2} = \mathbf{u}_n$$

E-region



A vertical electric field forms to oppose the vertical Hall current.

$$\sigma_H E_x = \sigma_P E_z \Longrightarrow E_z = \frac{\sigma_H}{\sigma_P} E_x$$

The Hall current from this new E_z adds to the existing Pedersen current from E_x

$$\mathbf{J}_x = \sigma_H E_z + \sigma_P E_x = \left[(\sigma_H/\sigma_P)^2 + 1\right] \sigma_P E_x \equiv \sigma_C E_x$$

Closure of Field Aligned Currents in a Slab Ionosphere

3D potential equation with magnetospheric currents:

$$\nabla \cdot \sigma \cdot \nabla \Phi = \nabla \cdot \mathbf{J}^{\mathrm{iono}} + \nabla \cdot \mathbf{J}^{\mathrm{mag}}$$

Integrate over altitude, assume equipotential field lines:

$$\nabla_{\perp} \cdot \Sigma \cdot \nabla_{\perp} \Phi = \int \nabla \cdot \mathbf{J}^{\mathrm{iono}}\, dz + \int \nabla \cdot \mathbf{J}^{\mathrm{mag}}\, dz \qquad \mathbf{K}^{\mathrm{iono}} \equiv \int \mathbf{J}^{\mathrm{iono}}\, dz$$

Expand the divergence:

$$\nabla \cdot \mathbf{J}^{\mathrm{mag}} = \nabla_{\perp} \cdot \mathbf{J}_{\perp}^{\mathrm{mag}} + \frac{\partial J_{\parallel}^{\mathrm{mag}}}{\partial z}$$

Above ionosphere, $\mathbf{J}_{\perp}^{\mathrm{mag}} = 0$

$$\int \nabla \cdot \mathbf{J}_{\mathrm{mag}}\, dz = J_{\parallel}^{\mathrm{mag}}$$

2D slab ionosphere potential equation:

$$\nabla_{\perp} \cdot \Sigma \cdot \nabla_{\perp} \Phi = \nabla_{\perp} \cdot \mathbf{K}^{\mathrm{iono}} + J_{\parallel}^{\mathrm{mag}}$$

Conjugacy and Mapping

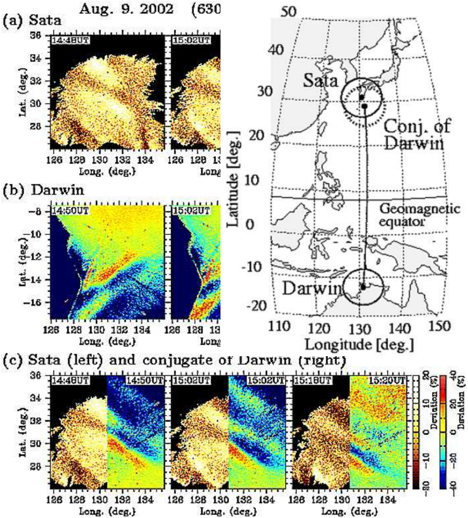
In low latitudes current out of northern hemisphere (N) equals current into southern hemisphere (S)

$$J_{\parallel}^{\mathrm{N}} = -J_{\parallel}^{\mathrm{S}}$$

Assuming equipotential field lines:

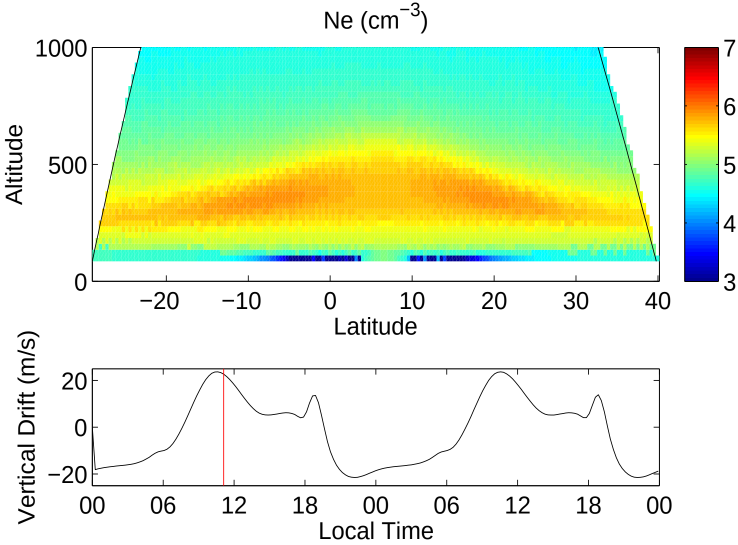
$$\nabla_{\perp} \cdot \Sigma^{\mathrm{N}} \cdot \nabla_{\perp}\Phi - \nabla \cdot \mathbf{K}^{\mathrm{Niono}} = -\nabla_{\perp} \cdot \Sigma^{\mathrm{S}} \cdot \nabla_{\perp}\Phi + \nabla \cdot \mathbf{K}^{\mathrm{Siono}}$$

$$\nabla_{\perp} \cdot \left(\Sigma^{\mathrm{N}} + \Sigma^{\mathrm{S}}\right) \cdot \nabla_{\perp}\Phi = \nabla \cdot \left(\mathbf{K}^{\mathrm{Niono}} + \mathbf{K}^{\mathrm{Siono}}\right)$$

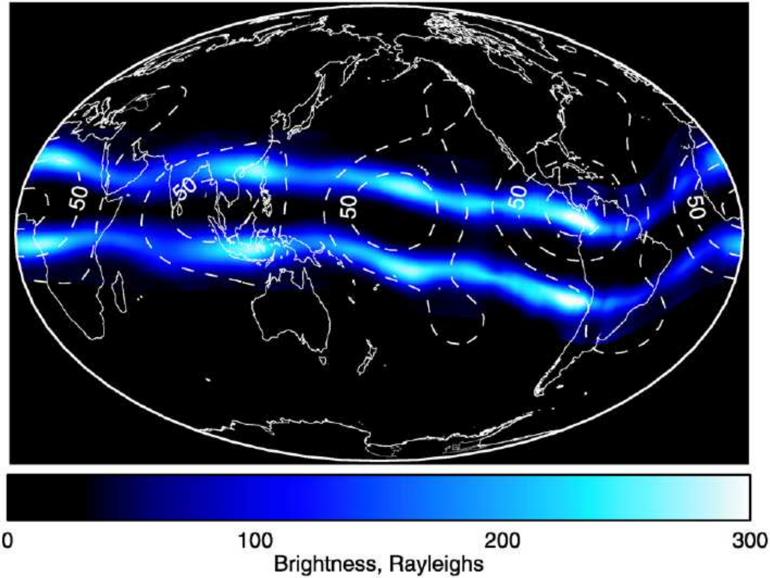


Otsuka et al. (2004)

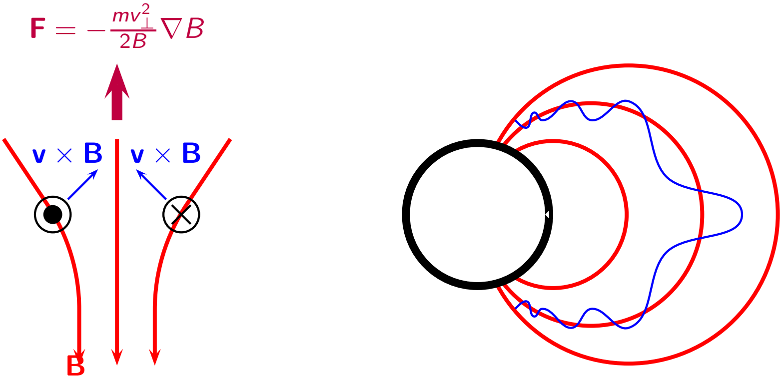
Equatorial Fountain Effect



Influences of Atmospheric Tides (Immel et al. 2006)

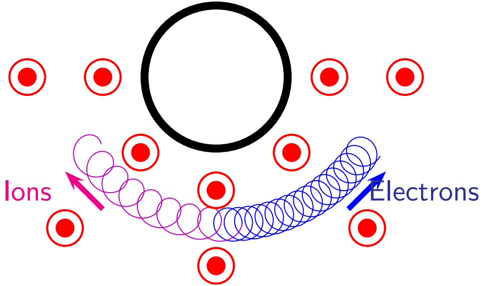


Magnetic Mirror Force and Bounce Motion



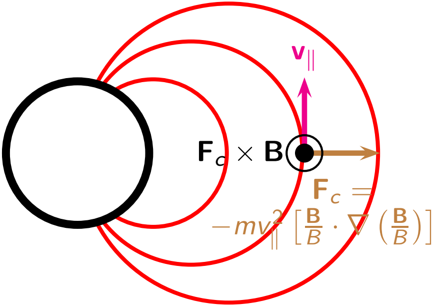
Gradient-Curvature Drift

Gradient Drift



$$\mathbf{v}_{\nabla B} = -\frac{mv_\perp^2}{2qB^3}\nabla B \times \mathbf{B}$$

Curvature Drift



$$\mathbf{v}_c = -\frac{mv_\parallel^2}{qB^2}\left[\frac{\mathbf{B}}{B}\cdot\nabla\left(\frac{\mathbf{B}}{B}\right)\right]\times\mathbf{B}$$

- Both drifts are energy dependent
- Both drifts move ions CW and electrons CCW

Adiabatic Invariants

Type of Periodic Motion	Adiabatic Invariant
Gyromotion	$\mu = \frac{mv_\perp^2}{2B}$
Bounce Motion	$\mathcal{J} = \oint_{\text{Bounce}} mv_\parallel \, ds$
Drift Motion	$\Phi = \oint_{\text{Drift}} q\mathbf{A} \cdot \, ds$

Average over periodic motion to reduce the dimensionality of the problem

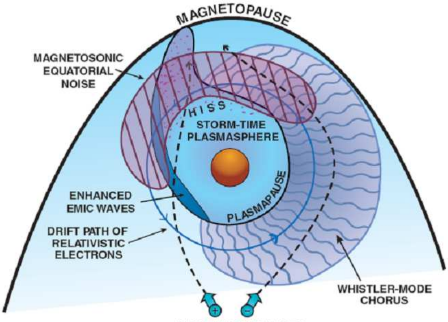
Velocity-like coordinates:

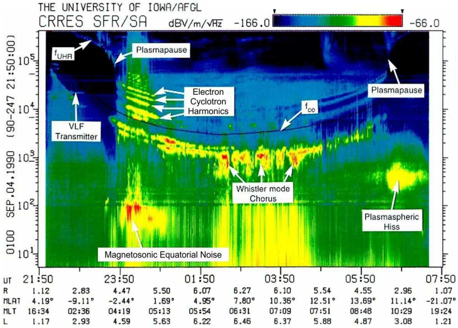
Energy	μ	gyrophase
		Avg. over gyromotion

Position coordinates:

L-shell	pos. along field line	MLT
	Avg. over bounce motion	Avg. over drift motion

Breaking the Adiabatic Invariants: Wave Environment

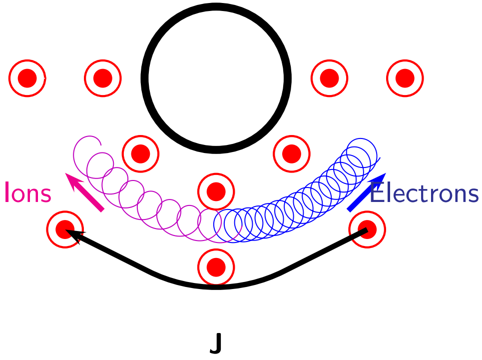




Cumulative effect of wave particle interactions modeled as phase-space diffusion coefficients

Images courtesy the U. of Iowa EMFISIS Team

Ring Current

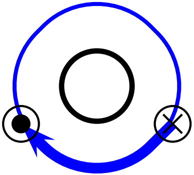


Collective behavior in the inner magnetosphere

- Gradient-curvature drifts result in currents
- Currents affect fields via $\mathbf{J} = \frac{1}{\mu_0}\nabla \times \mathbf{B}$
- Fields affect particle gradient-curvature drifts

Region 2 Field-Aligned Currents, Coupling to Ionosphere

$\nabla \cdot \mathbf{J} = 0$ still applies



$$J_{\|} = \nabla_{\perp} \cdot \int \mathbf{J}_{\perp}^{\mathrm{Ring}}\, ds$$

This current closes in both ionospheres

$$\zeta J_{\|} = \nabla_{\perp} \cdot \Sigma^{\mathrm{N}} \cdot \nabla_{\perp} \Phi - \nabla_{\perp} \cdot \mathbf{K}^{\mathrm{Niono}}$$

$$(1-\zeta) J_{\|} = \nabla_{\perp} \cdot \Sigma^{\mathrm{S}} \cdot \nabla_{\perp} \Phi - \nabla_{\perp} \cdot \mathbf{K}^{\mathrm{Siono}}$$

$$J_{\|} = \nabla_{\perp} \cdot \left(\Sigma^{\mathrm{N}} + \Sigma^{\mathrm{S}}\right) \cdot \nabla_{\perp} \Phi - \nabla_{\perp} \cdot \left(\mathbf{K}^{\mathrm{Niono}} + \mathbf{K}^{\mathrm{Siono}}\right)$$

Solve boundary-value problem for Φ to get **E**-fields in ionosphere and inner-magnetosphere.

2-Fluid Equations

Describe ions and electrons as separate fluids:

$$
\begin{aligned}
\frac{\partial}{\partial t} n_e \quad & + \nabla \cdot [n_e u_e] && = 0 \\
\frac{\partial}{\partial t} (m_e n_e \mathbf{u}_e) \quad & + \nabla \cdot [m_e n_e \mathbf{u}_e + p_e \mathbf{I}] && = -e n_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) + \mathbf{R}_e^{\mathrm{coll}} \\
\frac{\partial}{\partial t} \left(\frac{1}{2} m_e n_e u_e^2 + \frac{3}{2} p_e \right) & + \nabla \cdot \left[\frac{1}{2} m_e n_e u_e^2 \mathbf{u_e} + \frac{5}{2} p_e \mathbf{u}_e \right] && = -m_e n_e e \mathbf{u}_e \cdot \left[\mathbf{E} - \frac{\mathbf{R}_e^{\mathrm{coll}}}{e n_e} \right] \\
\frac{\partial}{\partial t} n_i \quad & + \nabla \cdot [n_i u_i] && = 0 \\
\frac{\partial}{\partial t} (m_i n_i \mathbf{u}_i) \quad & + \nabla \cdot [m_i n_i \mathbf{u}_i + p_i \mathbf{I}] && = e n_i (\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) + \mathbf{R}_i^{\mathrm{coll}} \\
\frac{\partial}{\partial t} \left(\frac{1}{2} m_i n_i u_i^2 + \frac{3}{2} p_i \right) & + \nabla \cdot \left[\frac{1}{2} m_i n_i u_i^2 \mathbf{u_i} + \frac{5}{2} p_i \mathbf{u}_i \right] && = m_i n_i e \mathbf{u}_i \cdot \left[\mathbf{E} + \frac{\mathbf{R}_i^{\mathrm{coll}}}{e n_i} \right]
\end{aligned}
$$

Plasma as A Single Fluid

Define single fluid quantities:

$$\rho = m_i n_i + m_e n_e$$

$$\mathbf{u} = \frac{m_i n_i \mathbf{u}_i + m_e n_e \mathbf{u}_e}{m_i n_i + m_e n_e}$$

$$p = p_i + p_e$$

$$\mathbf{J} = e n_i \mathbf{u}_i - e n_e \mathbf{u}_e$$

Make a few approximations

Quasineutrality: $n_e = n_i$

Mass Ratio: $\frac{m_e}{m_i} \ll 1$

$$\rightarrow \rho \approx m_i n_i$$

$$\rightarrow \mathbf{u} \approx \mathbf{u}_i$$

$$\rightarrow \mathbf{J} \approx en\left(\mathbf{u} - \mathbf{u}_e\right)$$

With these definitions and approximations the 2-fluid equations can be rearranged into the Extended MHD equations

Extended MHD

$$
\begin{aligned}
\frac{\partial}{\partial t}\rho &+ \nabla\cdot[\rho\mathbf{u}] &= 0 \\
\frac{\partial}{\partial t}\rho\mathbf{u} &+ \nabla\cdot[\rho\mathbf{u}\mathbf{u} + p\mathbf{I}] &= \mathbf{J}\times\mathbf{B} \\
\frac{\partial}{\partial t}\left(\frac{p}{\rho^{2/3}}\right) &+ \nabla\cdot\left[\mathbf{u}\frac{p}{\rho^{2/3}}\right] &= \frac{2}{3}\rho^{-2/3} \\
\frac{\partial}{\partial t}\mathbf{B} &+ \nabla\times\mathbf{E} &= 0 \\
\frac{\partial}{\partial t}\mathbf{E} &- c^2\nabla\times\mathbf{B} &= -\frac{1}{\epsilon_0}\mathbf{J}
\end{aligned}
$$

- All $\frac{\partial}{\partial t}$ terms retained in derivation
- This set of equations can be used for initial value problems

$$
\frac{\partial}{\partial t}\mathbf{J} + \nabla\cdot\left[\mathbf{J}\mathbf{u} + \mathbf{u}\mathbf{J} - \frac{1}{en}\mathbf{J}\mathbf{J} - \frac{e}{m_e}p_e\mathbf{I}\right]
= \frac{e^2 n}{m_e}\left[\mathbf{E} + \mathbf{u}\times\mathbf{B} - \frac{1}{en}\mathbf{J}\times\mathbf{B} - \nu_{ei}\mathbf{J}\right]
$$

Limiting Cases of the GOL

$$\frac{m_e}{e^2 n}\left\{\frac{\partial}{\partial t}\mathbf{J} + \nabla\cdot\left[\mathbf{Ju} + \mathbf{uJ} - \frac{1}{en}\mathbf{JJ}\right]\right\}$$
$$= \mathbf{E} + \mathbf{u}\times\mathbf{B} + \frac{1}{en}\nabla p_e - \frac{1}{en}\mathbf{J}\times\mathbf{B} - \frac{m_e \nu_{ei}}{e^2 n}\mathbf{J}$$

- Electron Inertia: negligible on length scales $> \lambda_e = \sqrt{\frac{m_e}{e^2 n \mu_0}}$
- Ambipolar Field: negligible in cold plasma
- Hall Term: negligible on length scales $> \lambda_i = \sqrt{\frac{m_i}{e^2 n \mu_0}}$ in collisionless plasma
- Resistive Term: negligible in collisionless plasma

Ideal MHD

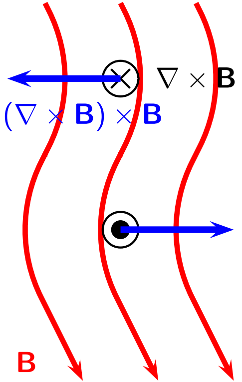
Assumptions:

- $\mathbf{E} + \mathbf{u} \times \mathbf{B} = 0$
- $\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B}$

Equations:

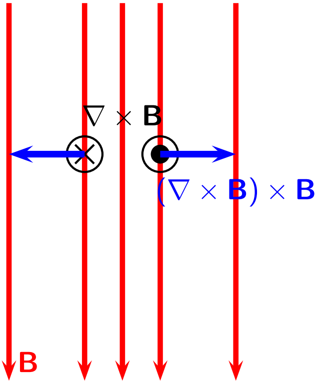
$$\begin{aligned}
\frac{\partial}{\partial t}\rho &+ \nabla \cdot [\rho \mathbf{u}] &= 0 \\
\frac{\partial}{\partial t}\rho \mathbf{u} &+ \nabla \cdot [\rho \mathbf{u}\mathbf{u} + p\mathbf{I}] &= \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} \\
\frac{\partial}{\partial t}\left(\frac{p}{\rho^{2/3}}\right) &+ \nabla \cdot \left[\mathbf{u}\frac{p}{\rho^{2/3}}\right] &= \frac{2}{3}\rho^{-2/3} \\
\frac{\partial}{\partial t}\mathbf{B} &- \nabla \times [\mathbf{u} \times \mathbf{B}] &= 0
\end{aligned}$$

Magnetic Tension, Magnetic Pressure, and Alfvén Waves



Shear Alfvén Waves

$v_A = \frac{B}{\sqrt{\mu_0 \rho}}$

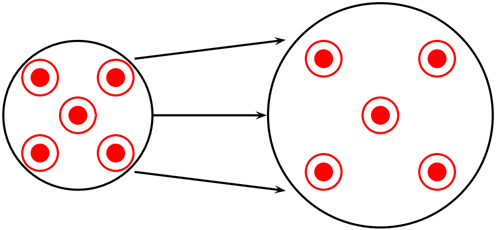


Compressional Alfvén Waves
(Magnetosonic Waves)

$v_M = \sqrt{v_s^2 + v_A^2}$

Flux Tubes and the Frozen-in Condition

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) \Longrightarrow \frac{D\phi_m}{Dt} = 0$$



In electrostatic fields

$$\mathbf{u} = \frac{1}{B^2}(-\nabla\Phi \times \mathbf{B})$$

$$\nabla \times (\mathbf{u} \times \mathbf{B}) = 0$$

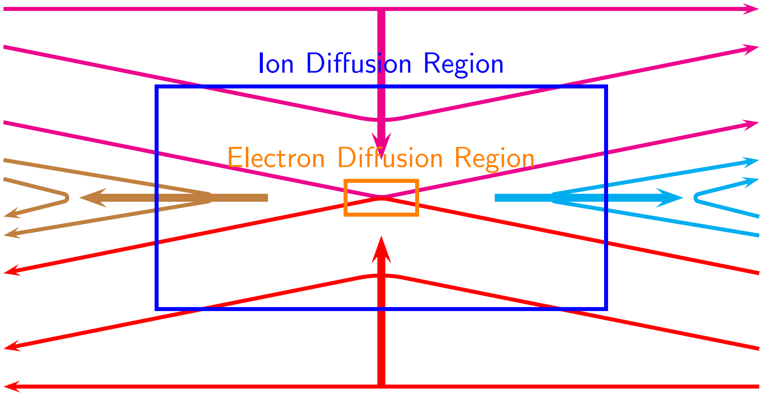
The flux tubes expand and contract to always enclose the same flux

In inductive fields

$$\nabla \times (\mathbf{u} \times \mathbf{B}) \neq 0$$

The magnetic field changes to preserve the enclosed flux

Magnetic Reconnection

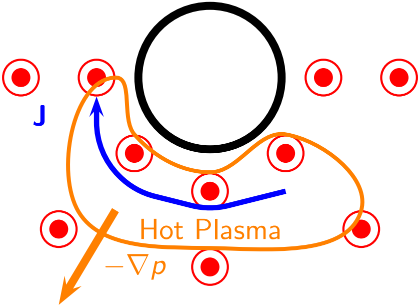


$$\frac{m_e}{e^2 n}\left\{\frac{\partial}{\partial t}\mathbf{J} + \nabla\cdot\left[\mathbf{J}\mathbf{u} + \mathbf{u}\mathbf{J} - \frac{1}{en}\mathbf{J}\mathbf{J} - \frac{e}{m_e}\mathbf{P}_e\right]\right\} = \mathbf{E} + \mathbf{u}\times\mathbf{B} - \frac{1}{en}\mathbf{J}\times\mathbf{B}$$

Force/Stress Balance and the Ring Current

$$\cancelto{0}{\frac{\partial}{\partial t}\rho \mathbf{u}} + \cancelto{0}{\nabla \cdot [\rho \mathbf{u}\mathbf{u}]} = -\nabla p + \rho \mathbf{g} + \mathbf{J} \times \mathbf{B}$$

$$\mathbf{J}_\perp = -\frac{1}{B^2}\nabla p \times \mathbf{B} + \frac{1}{B^2}\rho \mathbf{g} \times \mathbf{B}$$



- MHD diamagnetic currents are a poor approximation of the ring current because using a single MHD pressure misses the energy and pitch-angle dependences of the gradient-curvature drift.

Recovering the Ionospheric Limit

GOL with neutral collisions and neutral winds:

$$0 = \mathbf{E} + \mathbf{u} \times \mathbf{B} - \frac{1}{en}\mathbf{J} \times \mathbf{B} - \frac{m_e}{e^2 n}\left(\nu_{ei} + \nu_{en} + \frac{m_e}{m_i}\nu_{in}\right)\mathbf{J} + en\left(\nu_{en} - \nu_{in}\right)\left(\mathbf{u} - \mathbf{u}_n\right)$$

Steady state momentum equation:

$$0 = \mathbf{J} \times \mathbf{B} + \rho\mathbf{g} - \nabla p - \nu_{in}\left(\mathbf{u} - \mathbf{u}_n\right)$$

$$\mathbf{u} = \mathbf{u}_n + \frac{1}{\nu_{in}}[\mathbf{J} \times \mathbf{B} + \rho\mathbf{g} - \nabla p]$$

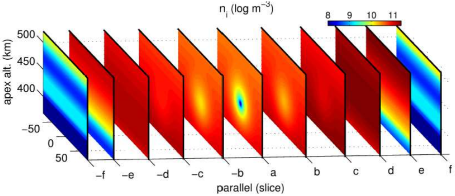
Substitute for **u** in GOL

$$\begin{aligned} 0 = \mathbf{E} + \mathbf{u}_n \times \mathbf{B} + \frac{1}{\nu_{in}}[\mathbf{J} \times \mathbf{B} + \rho\mathbf{g} - \nabla p] \times \mathbf{B} \\ - \frac{1}{en}\mathbf{J} \times \mathbf{B} - \frac{m_e}{e^2 n}\left(\nu_{ei} + \nu_{en} + \frac{m_e}{m_i}\nu_{in}\right)\mathbf{J} \\ + en\frac{(\nu_{en} - \nu_{in})}{\nu_{in}}[\mathbf{J} \times \mathbf{B} + \rho\mathbf{g} - \nabla p] \end{aligned}$$

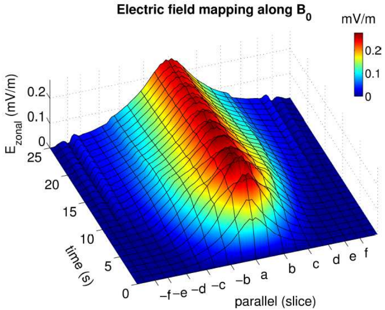
$$\mathbf{J} = \sigma \cdot [\mathbf{E} + \mathbf{u}_n \times \mathbf{B}] + \mathbf{D} \cdot \nabla p + \mathbf{\Gamma} \cdot \mathbf{g}$$

Transient MHD Behavior of the Ionosphere

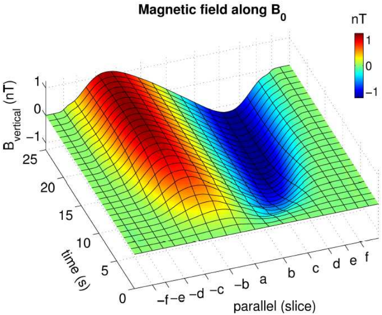
Eugene Dao
Ph.D. Dissertation
Cornell, 2013



(a) Simulation setup for a bubble at the equator.

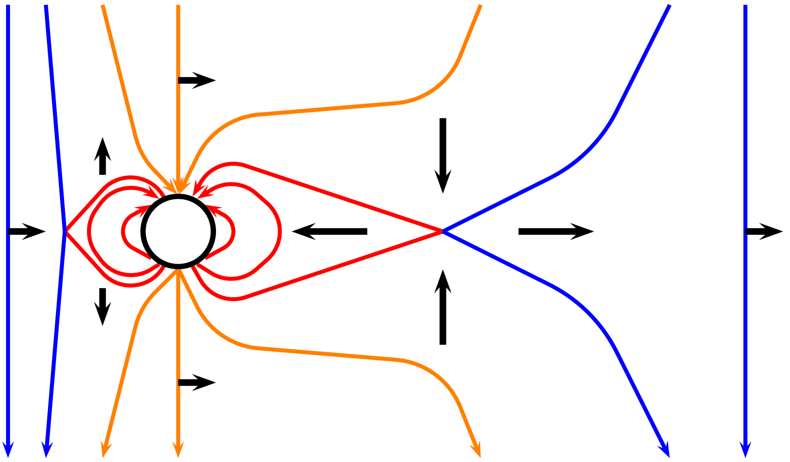


(b) E_{x1} for a bubble at the equator.

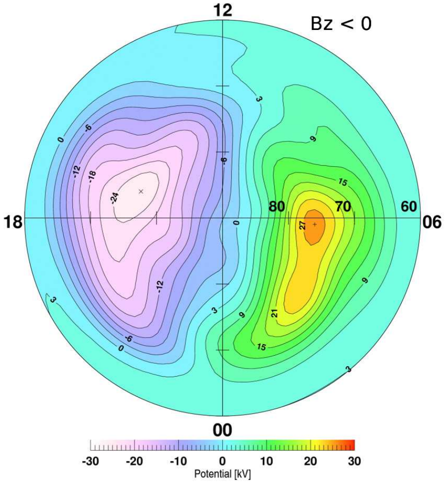


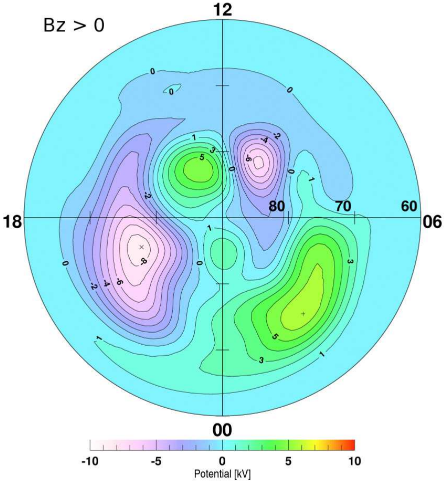
(c) B_{z1} for a bubble at the equator.

Convection with IMF B_z South (Dungey Cycle)

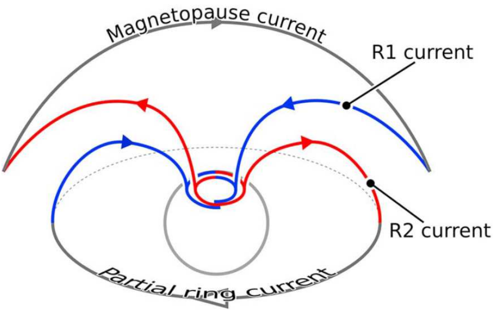


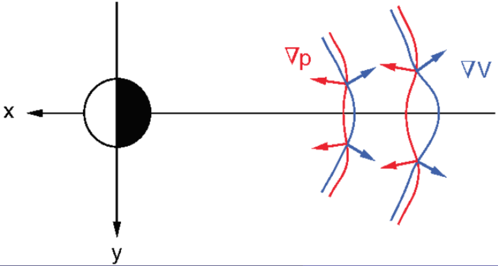
High-Latitude Ionospheric Convection





Current Systems





Force Balance:

$$\mathbf{J}_\perp = -\frac{1}{B^2}\nabla p \times \mathbf{B}$$

$\nabla \cdot \mathbf{J} = 0$:

$$J_\| = \int \nabla \cdot \mathbf{J}_\perp \, ds$$

$$J_\| = -\frac{\mathbf{B}_{\mathrm{eq}}}{B_{\mathrm{eq}}^2} \cdot \nabla p_{\mathrm{eq}} \times \nabla V$$

$$V = \int_{\mathrm{eq}}^{\mathrm{iono}} \frac{ds}{B}$$

Energy Transport and Poynting's Theorem

Poynting's Theorem:

$$\frac{\partial}{\partial t}\underbrace{\left[\frac{\epsilon_0 |\mathbf{E}|^2}{2} + \frac{|\mathbf{B}|^2}{2\mu_0}\right]}_{\text{Energy Density}} + \nabla \cdot \underbrace{\left[\frac{\mathbf{E}\times\mathbf{B}}{\mu_0}\right]}_{\text{Energy Flux}} = \underbrace{-\mathbf{J}\cdot\mathbf{E}}_{\text{Joule Heating}}$$

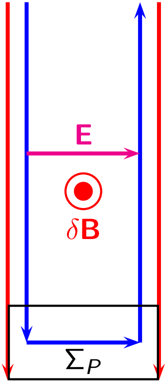
Ionospheric Joule Heating: Use **E** field in the neutral wind frame

$$\begin{aligned}\mathbf{J}\cdot\mathbf{E}' &= \left(\sigma\cdot\mathbf{E}'\right)\cdot\mathbf{E}' \\ &= \sigma_P \left|\mathbf{E} + \mathbf{u}_n \times \mathbf{B}\right|^2 \\ &= n_i m_i \nu_{in} \left|\mathbf{u}_i - \mathbf{u}_n\right|^2\end{aligned}$$

See Appendix A of Thayer and Semeter, 2004, JASTP.

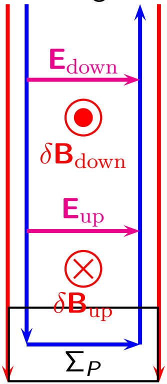
Alfvén Wave Transmission Lines

Electrostatic:



$$\mu_0 \frac{E}{\delta B} = \frac{1}{\Sigma_P}$$

Electromagnetic:



$$\mu_0 \frac{E}{\delta B} = \mu_0 v_A \equiv \frac{1}{\Sigma_A}$$

Reflected Alfvén wave forms such that

$$\frac{E_{\mathrm{down}} + E_{\mathrm{up}}}{\delta B_{\mathrm{down}} - \delta B_{\mathrm{up}}} = \frac{1}{\Sigma_P}$$

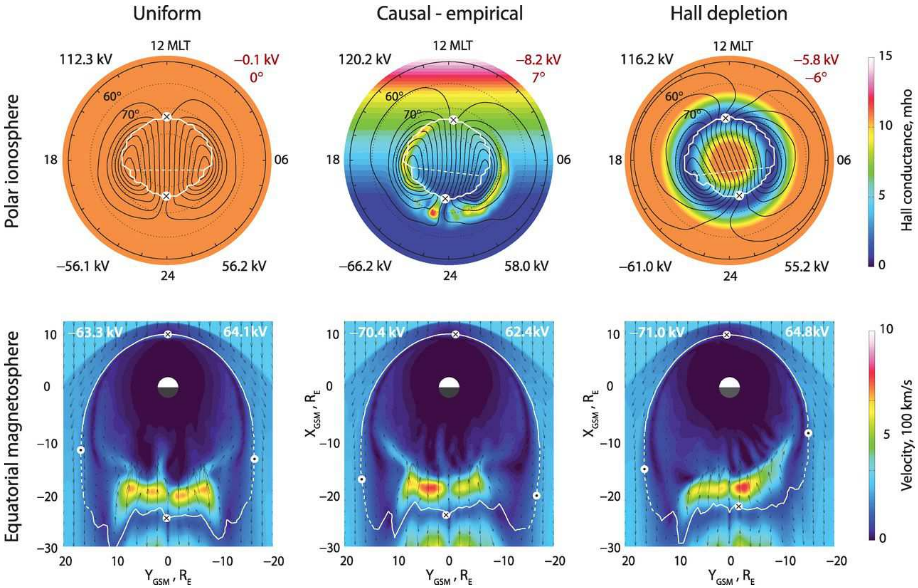
Reflection coefficient:

$$\frac{E_{\mathrm{up}}}{E_{\mathrm{down}}} = \frac{\Sigma_A - \Sigma_P}{\Sigma_A + \Sigma_P}$$

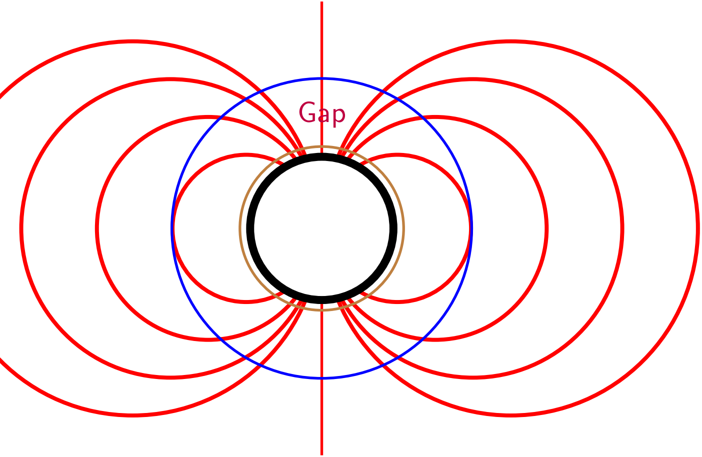
This simple transmission line model assumes

- Ionosphere is thin slab
- Alfvén speed above ionosphere is constant

Effects of Conductance Distributions (Lotko et al. 2014)

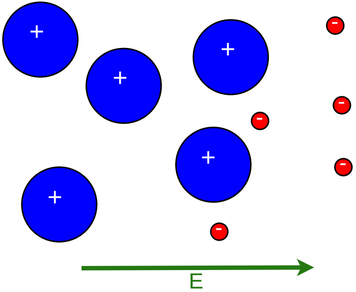


The Magnetosphere-Ionosphere "Gap" Region



- Magnetosphere models operate outside of $2-3\ \mathrm{R_E}$
- Ionosphere-thermosphere models operate up to $\sim 600\ \mathrm{km}$ altitude $(1.1\ \mathrm{R_E})$
- Electrostatic fields assumed to map along field lines in between

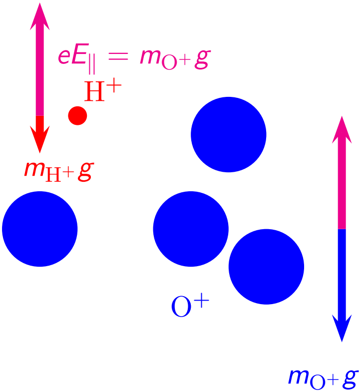
Ambipolar Electric Fields

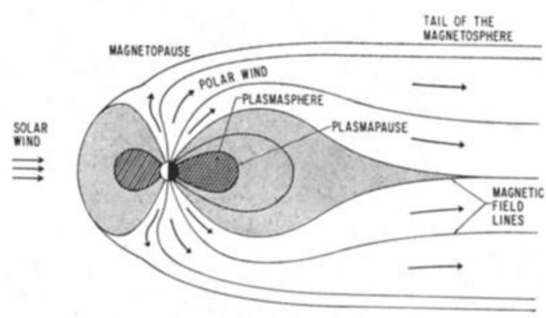


$$\mathbf{E} + \mathbf{u} \times \mathbf{B} - \frac{1}{en}\mathbf{J} \times \mathbf{B} = -\frac{1}{en}\nabla p_e$$

$$E_{\parallel} = -\frac{1}{en}\nabla_{\parallel} p_e$$

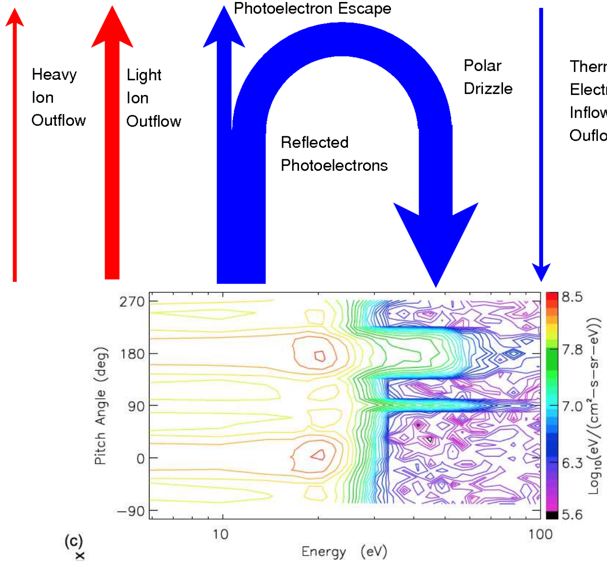
Classical Polar Wind





- In steady state ambipolar field balances gravity for major ion species (O^+)
- Light minor ions (H^+ and He^+) feel same field

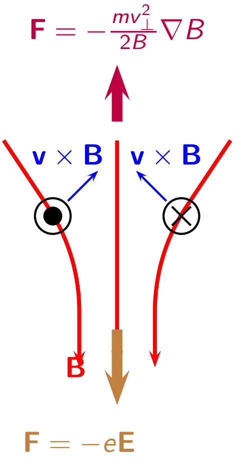
Photoelectron Escape and Zero Current



See Wilson et al. (1997), Kitamura et al. (2012, 2013), and Varney et al. (2014).

The Knight Relation and Mono-energetic Aurora

How can field lines carry upwards FAC?



"Ambipolar Term" with anisotropy

$$
\begin{aligned}
-en\mathbf{E} &= \nabla \cdot \mathbf{P}_e \\
&= \nabla \cdot \left[p_\parallel \hat{b}\hat{b} + p_\perp \left(\mathbf{I} - \hat{b}\hat{b} \right) \right] \\
&= \nabla p_\parallel + \left(\mathbf{I} - \hat{b}\hat{b} \right) \cdot \nabla \left(p_\perp - p_\parallel \right) \\
&\quad - \left(p_\perp - p_\parallel \right) \left(\mathbf{I} - 2\hat{b}\hat{b} \right) \cdot \frac{1}{B} \nabla B
\end{aligned}
$$

$$
-en\hat{b} \cdot \mathbf{E} = \hat{b} \cdot \nabla p_\parallel + \left(p_\perp - p_\parallel \right) \hat{b} \cdot \frac{1}{B} \nabla B
$$

Fields required to overcome the mirror-force term can produce $> 1\ \mathrm{kV}$ potential drops!

Energetic Ion Outflow

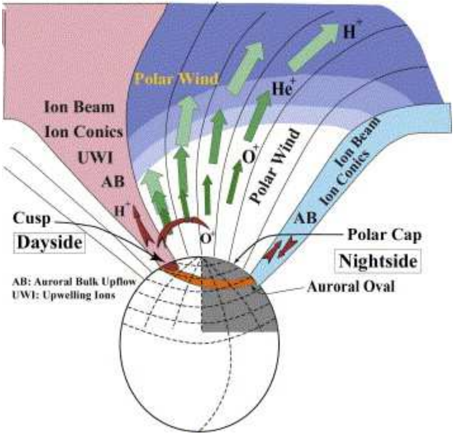
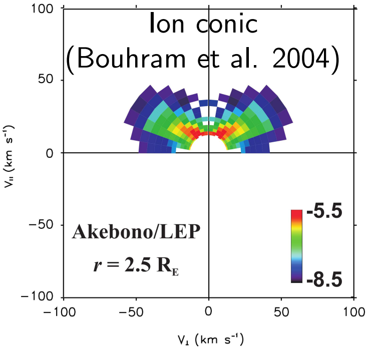


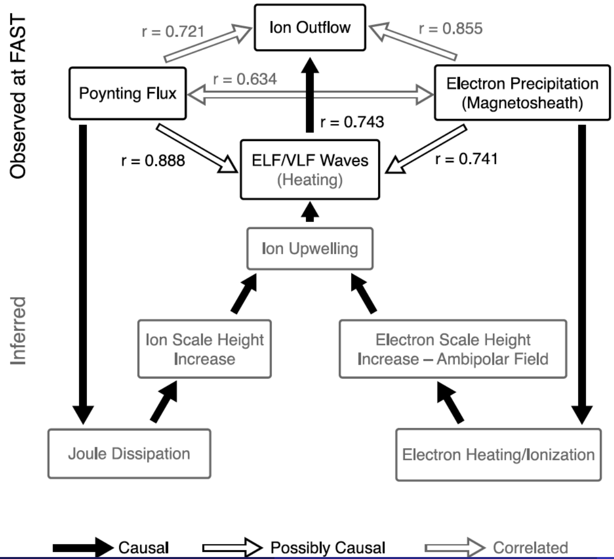
Image courtesy of the ePOP team

How do heavy ions escape gravity?

- Parallel electric fields
- Transverse acceleration combined with mirror force lifting



Ion Outflow as a Multistep Process



Strangeway et al. (2005)

Some Open Research Areas